Problem Set X

Huy Quang Lai
132000359

Texas A&M University

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Problem 1

Let Φ := number of elements in the HashTable, n.

Insertion (without resize):
$\Delta\Phi = \Phi(T_i) - \Phi(T_{i-1}) = n - (n-1) = 1$
Amortized cost: $a_i = C_i + \Delta\Phi = 1 + 1 = 2$
In this case, amortized cost is $O(1)$

Insertion (with resize):
$\Delta\Phi = \Phi(T_i) - \Phi(T_{i-1}) = 2n - 2(n-1) = 2$
Amortized cost: $a_i = C_i + \Delta\Phi = 1 + 2 = 3$
In this case, amortized cost is $O(1)$

When hashtable is empty: $\Phi(T_0) = 0$
$\Phi(T_i) = n$
$\therefore \Phi(T_i) \geq \Phi(T_0)$

Problem 2

A. Since a deque can be access starting from the front or back, because all the stated operations access elements at differing places in the deque, either the element in the front or the back.
So to make these operations run in constant amortized time, we can utilize the fact that a deque can be accessed from the front and back in order to access the desired element based on the operation we perform in constant time rather than iterating from one end to the other to reach the desired element.

B. `push(x)`:
Φ := number of elements in deque, n
Since you must maintain heap order, inserting a new element must percolate this element down the heap
$\Phi(T_i) - \Phi(T_{i-1}) = \log(n+1) - \log n = \log\left(\frac{n+1}{n}\right)$ $a_i = C_i + \Phi(T_i) - \Phi(T_{i-1}) = \log n + \log\left(\frac{n+1}{n}\right) = \log\left(n\left(\frac{n+1}{n}\right)\right) = \log(n+1)$
Amortized cost: $O(\log n)$

`pop()`:
Φ := number of elements in deque, n
$\Phi(T_i) - \Phi(T_{i-1}) = 1 - 1 = 0$
$a_i = C_i + \Phi(T_i) - \Phi(T_{i-1}) = 1 + 0 = 1$
Amortized cost: $O(1)$

`inject(x)`:
Φ := number of elements in deque, n
Since you must maintain heap order, inserting a new element must percolate this element up the heap. $\Phi(T_i) - \Phi(T_{i-1}) = (\log(n+1)+1) - (\log(n)+1) = \log\left(\frac{n+1}{n}\right)$
$a_i = C_i + \Phi(T_i) - \Phi(T_{i-1}) = \log(n) + 1 + \log\left(\frac{n+1}{n}\right) = \log\left(n\left(\frac{n+1}{n}\right)\right) = \log(n+1) + 1$
Amortized cost: $O(\log n)$