CSCE 222 (Carlisle), Discrete Structures for Computing
Spring 2022
Homework 12

Type your name below the pledge to sign
On my honor, as an Aggie, I have neither given nor received unauthorized aid on this academic work.
HUY QUANG LAI

Instructions:

- The exercises are from the textbook. You are encouraged to work extra problems to aid in your learning; remember, the solutions to the odd-numbered problems are in the back of the book.
- Grading will be based on correctness, clarity, and whether your solution is of the appropriate length.
- Always justify your answers.
- Don't forget to acknowledge all sources of assistance in the section below, and write up your solutions on your own.
- *Turn in .pdf file to Gradescope by the start of class on Monday, April 25, 2022.* It is simpler to put each problem on its own page using the LaTeX clearpage command.

Help Received:

- Rosen, Kenneth H. *Discrete Mathematics and Its Applications*. McGraw-Hill, 2019.

Exercises for Section 9.1:

4(a-d): (2 points).

Determine whether the relation R on the set of all people is reflexive, symmetric, antisymmetric, and/or transitive, where $(a, b) \in \mathbb{R}$ if and only if

a) a is taller than b
Not reflexive: a cannot be taller than a.
Antisymmetric: If a is taller than b, b cannot be taller than a
Not symmetric: a is taller than b, b cannot be taller than a
Transitive: If a is taller than b and b is taller than c, then a must be taller than c.

b) a and b were born on the same day.
Reflexive: a is born on a's birthday.
Symmetric: If a was born on b's birthday, then b will be born on a's birthday.
Not antisymmetric: If a was born on b's birthday, then b will be bor on a's birthday.
Transitive: If a and b share a birthday, and b and c share a birthday, then a and c must share a birthday.

c) a has the same first name as b.
Reflexive: a has the same first name as a
Symmetric: If a has the same first name as b, then b has the same first name as a.
Not antisymmetric: If a has the same first name as b, then b has the same first name as a.
Transitive: If a has the same first name as b and b has the same first name as c, then a has the same first name as c

d) a and b have a common grandparent.
Reflexive: a has a common grandparent with a.
Symmetric: If a has a common grandparent with b, then b has a common grandparent with a.
Not antisymmetric: If a has a common grandparent with b, then b has a common grandparent with a.
Not Transitive: a and b can share a common grandparent g while b and c can share a common grandparent h. However, a and c are not guaranteed to have a common grandparent.

6(a,c,e,g): (2 points).

Determine whether the relation R on the set of all real numbers is reflexive, symmetric, antisymmetric, and/or transitive, where $(x, y) \in \mathbb{R}$ if and only if

a) $x + y = 0$
Not reflexive: $(1, 1) \notin R$
Symmetric: $x + y = 0 = y + x$
Not antisymmetric: $(-1, 1) \wedge (1, -1) \in R$
Not transitive: $(-1, 1) \wedge (1, -1) \in R$ but $(1, 1) \notin R$

c) $x - y \in \mathbb{Q}$
Reflexive: $x - x = 0 \in \mathbb{Q}$
Symmetric: If $x - y \in \mathbb{Q}$, then $y - x = -(x - y) \in \mathbb{Q}$
Not antisymmetric: If $x = 1, y = 2$ then $x - y \in \mathbb{Q} \wedge y - x \in \mathbb{Q}$ but $x \neq y$
Transitive: If $x - y \in \mathbb{Q} \wedge y - z \in \mathbb{Q}$, then $x - z = (x - y) + (y - z) \in \mathbb{Q}$

e) $xy \geq 0$
Reflexive: $x^2 \geq 0$
Symmetric: $xy = yx \geq 0$
Not antisymmetric: If $x = 1, y = 2$, then $xy \geq 0 \wedge yx \geq 0$ but $x \neq y$
Not transitive: If $x = 1, y = 0, z = -1$, then $xy \geq 0 \wedge yz \geq 0$ but $xz < 0$

g) $x = 1$
Not reflexive: $(2, 2) \notin R$
Not symmetric: $(1, 2) \in R$ but $(2, 1) \notin R$
Antisymmetric: If (x, y) and (y, x) are in R, then $x = 1 = y$
Transitive: If (x, y) and (y, z) are in R, then $x = 1$ and $(x, z) = (1, z)$ which is in R.

34(a,c,e,g): (2 points).

$R_1 = \{(a,b) \in \mathbb{R}^2 | a > b\}$
$R_2 = \{(a,b) \in \mathbb{R}^2 | a \geq b\}$
$R_3 = \{(a,b) \in \mathbb{R}^2 | a < b\}$
$R_4 = \{(a,b) \in \mathbb{R}^2 | a \leq b\}$
$R_5 = \{(a,b) \in \mathbb{R}^2 | a = b\}$
$R_6 = \{(a,b) \in \mathbb{R}^2 | a \neq b\}$

Find

a) $R_1 \bigcup R_3$
   R_6

c) $R_2 \bigcap R_4$
   R_5

e) $R_1 - R_2$
   $\emptyset$

g) $R_1 \oplus R_3$
   R_6

Exercises for Section 9.5:

2(a,c,e): (2 points).

Which of these relations on the set of all people are equivalence relations? Determine the properties of an equivalence relation that the others lack.

a) $\{(a, b)|a$ and b are the same age$\}$
This is a equivalence relation.

c) $\{(a, b)|a$ and b share a common parent$\}$
This is not an equivalence relation, since it is not transitive.
A is the child of W and X, B is the child of X and Y, and C is the child of Y and Z.
Then A is related to B, and B is related to C, but A is not related to C.

e) $\{(a, b)|a$ and b speak a common language$\}$
This is not an equivalence relation, since it is not transitive.
A and B both can speak English, B and C both can speak Chinese.
However A and C cannot communicate.

12: (2 points).

Show that the relation R consisting of all pairs (x, y) such that x and y are bit strings of length three or more that agree except perhaps in their first three bits is an equivalence relation on the set of all bit strings of length three or more.

Suppose A is the set of all bit strings of length three or more.
Define a relation R on A as $R = \{(x, y)|x$ and y agree in their first three bits$\}$

Reflexive since x shares the first three bits with x.

Symmetric
Suppose $x, y \in A, (x, y) \in R$
If x and y agree in their first three bits, then y and x agree in their first three bits.

Transitive
Suppose $x, y, z \in A, (x, y), (y, z) \in R$
If x and y agree in their first three bits and y and z agree in their first three bits, then x and z agree in their first three bits.

Hence, R is reflexive symmetric and transitive on A and therefore, R is an equivalence relation on A.

16: (2 points).

Let R be the relation on the set of ordered pairs of positive integers such that $((a, b), (c, d)) \in \mathbb{R}$ if and only if $ad = bc$. Show that R is an equivalence relation.

Reflexive
Since $a \cdot b = b \cdot a$.
Then $((a, b), (a, b)) \in R$

Symmetric
Since $ad = bc \equiv bc = ab$
Then $((c, d), (a, b)) \in R$

Transitive
If $((a, b), (c, d) \in R \wedge ((c, d), (e, f)) \in R$
Then $ab = bc \wedge cf = de$

44(b,c,d): (2 points).

Which of these collections of subsets are partitions of the set of integers?

b) the set of positive integers and the set of negative integers
This is not a partition, since 0 is in neither set.

c) the set of integers divisible by 3, the set of integers leaving a remainder of 1 when divided by 3, and the set of integers leaving a remainder of 2 when divided by 3.
This is a partition by the division algorithm

d) the set of integers less than -100, the set of integers with absolute value not exceeding 100, and the set of integers greater than 100
This is a partition, since the second set mentioned is the set of all number between -100 and 100, inclusive.

Exercises for Section 9.6:

8(a-c): (2 points).

Determine whether the relations represented by these zero–one matrices are partial orders.

a) $\begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Reflexive because the matrix's diagonal is only 1
Antisymmetric because $m_{ij} = 1 \wedge mji = 1$ is only true when $i = j$.
Not transitive $m_{21} = 1 \wedge m_{13} = 1 \wedge m_{23} = 0$.
Not a partial ordering since R is not transitive.

b) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$

Reflexive because the matrix's diagonal is only 1
Antisymmetric because $m_{ij} = 1 \wedge mji = 1$ is only true when $i = j$.
Transitive because $m31 = 1$ does not cause transitivity problems.
R is a partial ordering.

c) $\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \end{bmatrix}$

Reflexive because the matrix's diagonal is only 1
Antisymmetric because $m_{ij} = 1 \wedge mji = 1$ is only true when $i = j$.
Not transitive $m_{13} = 1 \wedge m_{34} = 1 \wedge m_{14} = 0$.
Not a partial ordering since R is not transitive.

34(a,c,f,h): (2 points).

Answer these questions for the poset $(\{2, 4, 6, 9, 12, 18, 27, 36, 48, 60, 72\}, |)$.

a) Find the maximal elements.
   27, 48, 60, and 72 are maximal.
   Each divides no number in the list other than itself.
   All of the other numbers divide 72 so they are not maximal

c) Is there a greatest element? There is no greatest element.
   There is no number in the set that both 60 and 72 divide

f) Find the least upper bound of $\{2, 9\}$, if it exists.
   18
   This is the smallest number that 2 and 9 divide.

h) Find the greatest lower bound of $\{60, 72\}$, if it exists. 12
   This is the largest number that divide both 60 and 72.

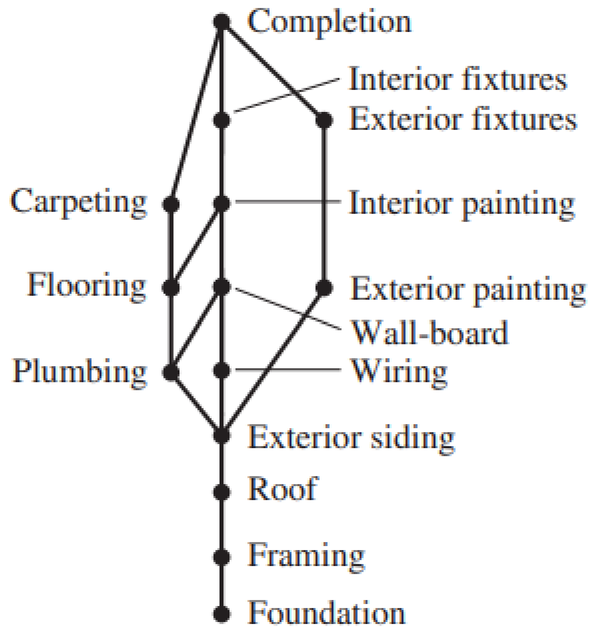
52b: (1 point).

Give an example of an infinite lattice with a least but not a greatest element.

$(\mathbb{N}, \leq)$
Since there is no upper limit to the natural numbers, there is no greatest element.
However, the lower limit is 1 as it is the smallest natural number.

66: (1 point).

Schedule the tasks needed to build a house, by specifying their order, if the Hasse diagram representing these tasks is as shown in the figure.



Foundation $\prec$ Framing $\prec$ Roof $\prec$ Exterior siding $\prec$ Wiring $\prec$ Plumbing $\prec$ Flooring $\prec$ Wall-board $\prec$ Exterior painting $\prec$ Interior painting $\prec$ Carpeting $\prec$ Interior fixtures $\prec$ Exterior fixtures $\prec$ Completion.