CSCE 222 (Carlisle), Discrete Structures for Computing
Spring 2022
Homework 13

Type your name below the pledge to sign
On my honor, as an Aggie, I have neither given nor received unauthorized aid on this academic work.
Huy QUANG LAI

Instructions:

- The exercises are from the textbook. You are encouraged to work extra problems to aid in your learning; remember, the solutions to the odd-numbered problems are in the back of the book.
- Grading will be based on correctness, clarity, and whether your solution is of the appropriate length.
- Always justify your answers.
- Don't forget to acknowledge all sources of assistance in the section below, and write up your solutions on your own.
- *Turn in .pdf file to Gradescope by the start of class on Monday, May 2, 2022.* It is simpler to put each problem on its own page using the LaTeX clearpage command.

Help Received:

- Rosen, Kenneth H. *Discrete Mathematics and Its Applications*. McGraw-Hill, 2019.
- Wallace, Evan. *Finite State Machine Designer*, 2010, madebyevan.com/fsm/.

Exercises for Section 13.1:

4(a-c): (3 points).

Let $G = (V, T, S, P)$ be the phrase-structure grammar with $V = \{0, 1, A, S\}, T = \{0, 1\}$, and set of productions P consisting of $S \to 1S, S \to 00A, A \to 0A$, and $A \to 0$.

a) Show that 111000 belongs to the language generated by G.
$S \Rightarrow 1S \Rightarrow 11S \Rightarrow 111S \Rightarrow 11100A \Rightarrow 111000$

b) Show that 11001 does not belong to the language generated by G.
Every production results in a string ending in S, A or 0. Therefore, any string ending with 1 is not possible.

c) What is the language generated by G?
The string can start any number of 1's, including zero, by iterating the production $S \to 1S$.
Eventually the S must turn into $00A$, so at least two 0's must come next.
The string can then contain additional 0's using $A \to 0A$ repeatedly.
Because $A \to 0$ is called at the end, this adds at least one more 0 (and therefore a total of at least three 0's).
So the language generated by G is the set of all strings consisting of zero or more 1's followed by three or more 0's. We can write this as $\{0^n 1^m | n \geq 0$ and $m \geq 3\}$.

18(a,c): (2 points).

Construct phrase-structure grammars to generate each of these sets.

a) $\{01^{2n} | n \geq 0\}$
Exactly one 0. Followed by an even number of 1.
$S \to 0A$
$A \to 11A$
$A \to \lambda$

c) $\{0^n 1^m 0^n | m \geq 0$ and $n \geq 0\}$
Grow 0's from the center. Convert the center into a 1-making machine.
$S \to 0S0$
$S \to A$
$A \to 1A$
$A \to \lambda$

24b: (1 point).

Let G be the grammar with $V = \{a, b, c, S\}; T = \{a, b, c\}$; starting symbol S; and productions $S \rightarrow abS, S \rightarrow bcS, S \rightarrow bbS, S \rightarrow a$, and $S \rightarrow cb$. Construct derivation trees for $bbbcbba$.

1. $S \rightarrow bbS$
2. $S \rightarrow bcS$
3. $S \rightarrow bbS$
4. $S \rightarrow a$

Exercises for Section 13.2:

4(a-c): (3 points).

Find the output generated from the input string 10001 for the finite-state machine with the state diagram in

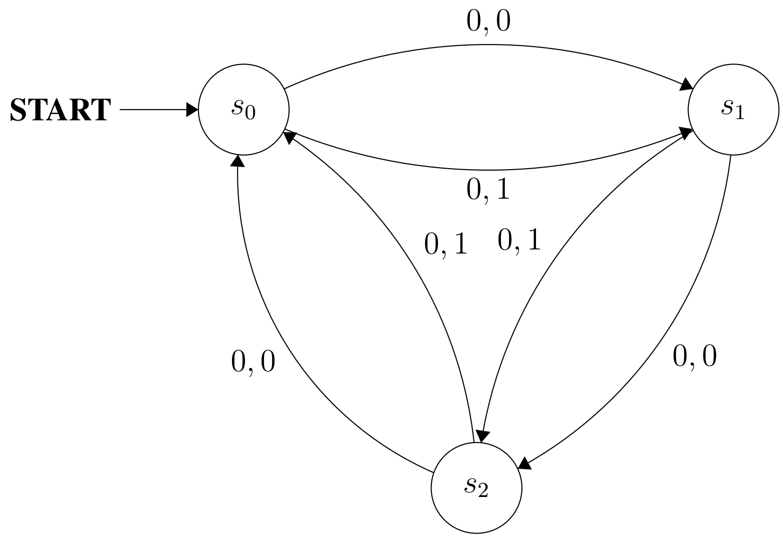
a) Exercise 2(a).
The machine starts in state s_0.
On input 1, it moves to state s_2 and outputs 0.
The next three inputs of 0's drive it to state s_3, then to s_1, then back to s_0. This will output 011.
The final 1 drives it back to s_2 and outputs 0.
The output is 00110.

b) Exercise 2(b).
The machine starts in state s_0.
On input 1, it moves to state s_2 and outputs 1.
The next three inputs of 0's keep it at s_2, outputting 1 each time. The final 1 drives it back to s_0 and outputs 0.
The output is 11110.

c) Exercise 2(c).
The machine starts in state s_0.
On input 1, it moves to state s_1 and gives 1 as output.
The next input symbol is 0, so the machine moves back to state s_0 and gives 0 as output.
The third input is 0, so the machine moves to state s_3 and gives 0 as output.
The fourth input is 0, so the machine moves to state s_1 and gives 0 as output.
The fifth input is 1, so the machine stays in state s_1 and gives 1 as output.
The output is 10001.

16: (2 points).

Construct a finite-state machine that gives an output of 1 if the number of input symbols read so far is divisible by 3 and an output of 0 otherwise.



Exercises for Section 13.3:

10(a,c,e): (3 points).

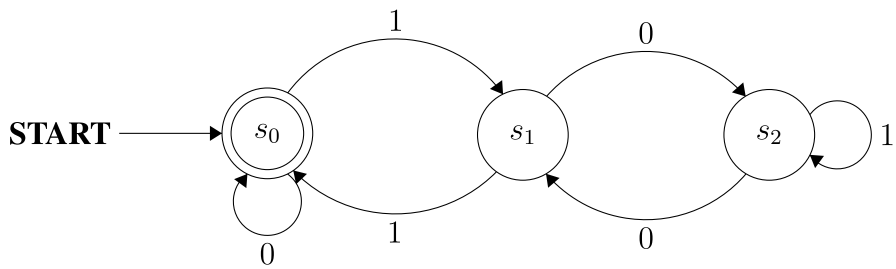
Determine whether the string 01001 is in each of these sets

a) {0,1}*
   This is the set of all bit strings. Therefore, the string is within the set.

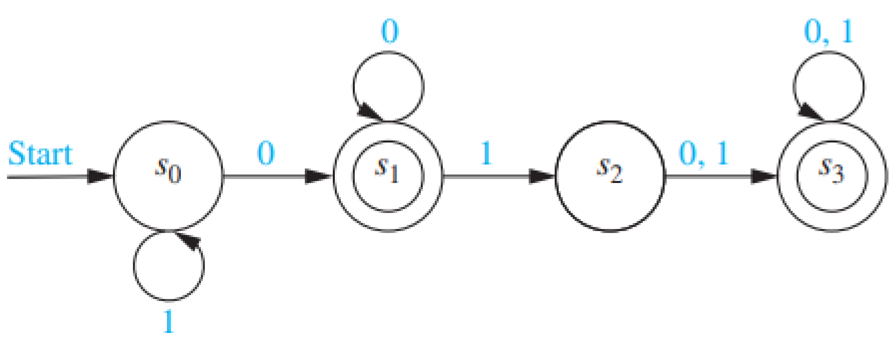
c) {010}*{0}*{1}
   The string is $(010)^{1}0^{1}1$ so it is in the set.

e) {00}{0}*{01}
   Any string in this set must start with 00. Since the string does not, the string is not in the set.

CUSTOM: Create a DFA that recognizes binary numbers that are a multiple of 3: (2 points).



20: (2 points).



$\{1\}^*\{0\}\{0\}^* \bigcup \{1\}^*\{0\}\{0\}^*\{10, 11\}\{0, 1\}^*$

26: (2 points).

Construct a deterministic finite-state automaton that recognizes the set of all bit strings that do not contain three consecutive 0s.

Have four states, with only s_3 non-final. For $i = 0, 1, 2$, transition from s_i to s_{i+1} on input 0 but back to s_0 on input 1. Both transitions from s_3 are to itself.