CSCE 222 (Carlisle), Discrete Structures for Computing
Spring 2022
Homework 3

Type your name below the pledge to sign
On my honor, as an Aggie, I have neither given nor received unauthorized aid on this academic work.
YOUR NAME HERE

Instructions:

- The exercises are from the textbook. You are encouraged to work extra problems to aid in your learning; remember, the solutions to the odd-numbered problems are in the back of the book.
- Grading will be based on correctness, clarity, and whether your solution is of the appropriate length.
- Always justify your answers.
- Don't forget to acknowledge all sources of assistance in the section below, and write up your solutions on your own.
- *Turn in .pdf file to Gradescope by the start of class on Monday February 7, 2022.* It is simpler to put each problem on its own page using the LaTeX clearpage command.

Help Received:

- List any help received here, or "NONE".

Exercises for Section 1.8:

8: (2 points)
Prove using the notion of without loss of generality that $5x + 5y$ is an odd integer when x and y are integers of opposite parity.

Without loss of generality,
Let $x = 2a, a \in \mathbb{Z}$
Let $y = 2b + 1, b \in \mathbb{Z}$

$5x + 5y = 5(2a) + 5(2b + 1)$
$= 10a + 10b + 5 = 2(5a + 5b + 2) + 1$
$2(5a + 5b + 2) + 1$ is odd.
$\blacksquare$

20: (2 points)
Show that if r is an irrational number, there is a unique integer n such that the distance between r and n is less than $\frac{1}{2}$.

Let $n \in \mathbb{Z}, m \in \mathbb{Z}, |n - m| \geq 1$
With this, suppose that $(|r - n| < \frac{1}{2}) \wedge (|r - m| < \frac{1}{2})$.
From this, $|n - m| \leq |r - n| + |r - m| < \frac{1}{2} + \frac{1}{2} = 1$.
This forces $|n - m| < 1$. However this contradicts with the fact that $|n - m| \geq 1$.
Because of this only one integer n can exist such that the distance between r and n is less than $\frac{1}{2}$.
$\blacksquare$

32: (2 points)
Prove that there are no solutions in integers x and y to the equation $2x^2 + 5y^2 = 14$.
Let $x \in \mathbb{Z}, y \in \mathbb{Z}$
With this, $x^2 \geq 0 \wedge y^2 \geq 0$.
If $y \leq -2 \vee y \geq 2$, then $y^2 \geq 4$ and $5y^2 \geq 20$. However $5y^2 \geq 20$ is a contradiction. This forces that $y = -1, y = 0, y = 1$. We can look at each of these cases separately.

When $y = -1 \vee y = 1$; then $2x^2 = 9$; the left-hand side is even, when the right-hand side is odd. This equation has no integer solution.

When $y = 0$ then $2x^2 = 14$ or $x^2 = 7$. We can check all possible values of x.

- $x = 0$; then $x^2 = 0 \neq 7$
- $x = \pm 1$; then $x^2 = 1 \neq 7$
- $x = \pm 2$; then $x^2 = 4 \neq 7$
- $|x| \geq 3$; then $x^2 \geq 9 > 7$

With these cases, there are no integers x and y that satisfy the equation $2x^2 + 5y^2 = 14$
■

Exercises for Section 2.1:

24: (2 points). (Prove your answer is correct.)
Can you conclude that $A = B$ if A and B are two sets with the same power set?
Assume $A \neq B$.
$x \in A, x \notin B$
Because of this $\{x\} \in \mathcal{P}(A), x \notin \mathcal{P}(B)$
However, this contradicts with the fact that $\mathcal{P}(A) = \mathcal{P}(B)$ which must contain the same elements.
■

34(a-d): (2 points)
Let $A = \{a, b, c\}$, $B = \{x, y\}$, and $C = \{0, 1\}$. Find

1. $A \times B \times C$
   $\{(a, x, 0), (a, x, 1), (a, y, 0), (a, y, 1),$
   $(b, x, 0), (b, x, 1), (b, y, 0), (b, y, 1),$
   $(c, x, 0), (c, x, 1), (c, y, 0), (c, y, 1)\}$

2. $C \times B \times A$
   $\{(0, x, a), (0, x, b), (0, x, c), (0, y, a), (0, y, b), (0, y, c),$
   $(1, x, a), (1, x, b), (1, x, c), (1, y, a), (1, y, b), (1, y, c)\}$

3. $C \times A \times B$
   $\{(0, a, x), (0, a, y), (0, b, x), (0, b, y), (0, c, x), (0, c, y),$
   $(1, a, x), (1, a, y), (1, b, x), (1, b, y), (1, c, x), (1, c, y)\}$

4. $B \times B \times B$
   $\{(x, x, x), (x, x, y), (x, y, x), (x, y, y), (y, x, x), (y, x, y), (y, y, x), (y, y, y)\}$

46(a-d): (2 points)

Translate each of these quantifications into English and determine its truth value.

1. $\exists x \in \mathbb{R}(x^3 = -1)$
   There exists a real number x whose cube is -1.
   True $x = -1$

2. $\exists x \in \mathbb{Z}(x + 1) > x$
   There exists an integer x such that $x + 1$ is greater than x.
   True $x = 0$

3. $\forall x \in \mathbb{Z}(x - 1 \in \mathbb{Z})$
   For all integers x, $x - 1$ is also an integer.
   True
   Let $f(x) = x - 1$.
   $f : \mathbb{Z} \to \mathbb{Z}$

4. $\forall x \in \mathbb{Z}(x^2 \in \mathbb{Z})$
   For all integers x, x^2 is also an integer.
   Let $f(x) = x^2$
   $f : \mathbb{Z} \to \mathbb{Z}$

Exercises for Section 2.2:

14: (2 points)

Find the sets A and B if $A - B = \{1, 5, 7, 8\}$, $B - A = \{2, 10\}$, and $A \cap B = \{3, 6, 9\}$.
If $A - B = \{1, 5, 7, 8\}$, then A contains $1, 5, 7, 8$.
If $B - A = \{2, 10\}$, then B contains $2, 10$.
If $A \cap B = \{3, 6, 9\}$, then both A and B contain $3, 6, 9$.
Therefore:
$A = \{1, 5, 7, 8, 3, 6, 9\}$
$B = \{2, 10, 3, 6, 9\}$

20d: (2 points)

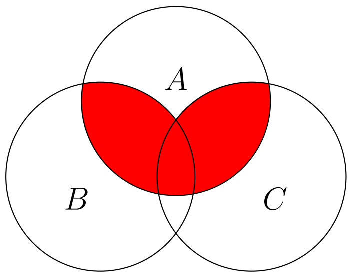
Let A, B, and C be sets. Show that $(A - C) \cap (C - B) = \emptyset$

Assume $(A - C) \cap (C - B) \neq \emptyset$
$\exists x, x \in (A - C) \cap (C - B)$
By the definition of intersection, this means that $x \in A - C$ and $x \in C - B$.
By the definition of set difference, it follows that $x \in A$ and $x \notin C$. However, from the statement $x \in C - B$, $x \in C$ and $x \notin B$. This is a contradiction.
Therefore, $(A - C) \cap (C - B) = \emptyset$
■

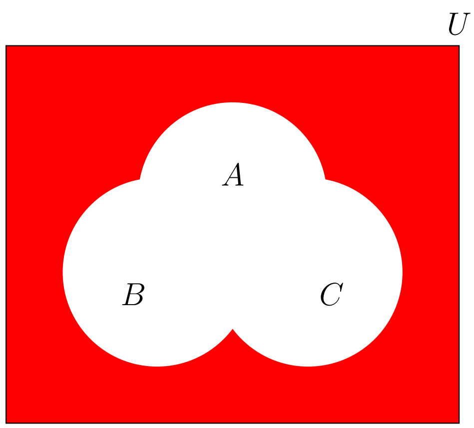
28(a-c): (2 points)

Draw the Venn diagrams for each of these combinations of the sets A, B, and C.

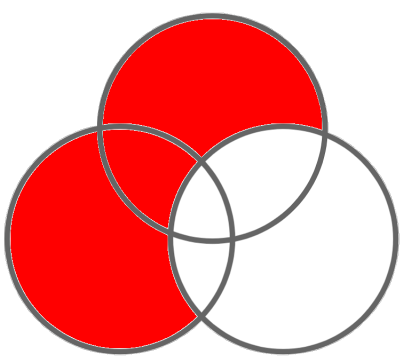
1. $A \cap (B \cup C)$



2. $\overline{A} \cap \overline{B} \cap \overline{C}$



3. $(A - B) \cup (A - C) \cup (B - C)$
   Top Circle is A, Bottom Left Circle is B, Bottom Right Circle is C



50: (2 point)

Show that if A and B are finite sets, then $A \cup B$ is a finite set.

If one of the two sets is empty, then $A \cup B$ is only the non-empty set, and thus, finite. We can move one element of the non-empty set into the empty set. This process does not change $A \cup B$ as this set contains all the elements originally in the non-empty set. We can keep moving one element from the non-empty set into the empty set until the non-empty set becomes empty. Since the sets are finite, this process will eventually swap the contents of the two sets.