CSCE 222 (Carlisle), Discrete Structures for Computing
Spring 2022
Homework 9

Type your name below the pledge to sign
On my honor, as an Aggie, I have neither given nor received unauthorized aid on this academic work.
HUY QUANG LAI

Instructions:

- The exercises are from the textbook. You are encouraged to work extra problems to aid in your learning; remember, the solutions to the odd-numbered problems are in the back of the book.
- Grading will be based on correctness, clarity, and whether your solution is of the appropriate length.
- Always justify your answers.
- Don't forget to acknowledge all sources of assistance in the section below, and write up your solutions on your own.
- *Turn in .pdf file to Gradescope by the start of class on Monday, March 28, 2022.* It is simpler to put each problem on its own page using the LaTeX clearpage command.

Help Received:

- Rosen, Kenneth H. *Discrete Mathematics and Its Applications*. McGraw-Hill, 2019.

Exercises for Section 6.1:

8: (1 point).
How many different three-letter initials with none of the letters repeated can people have?

$26 \cdot 25 \cdot 24 = 15600$

16: (2 points).
How many strings are there of four lowercase letters that have the letter x in them?
There are a total of 26^4 four-letter strings. There are 25^4 strings that do not contain an x.
Therefore, the total number of strings that contain the letter x is $26^4 - 25^4$ or 66351.

28: (1 point).
How many license plates can be made using either three digits followed by three uppercase English letters or three uppercase English letters followed by three digits?
There are 10 digits and 26 uppercase English letters.
Therefore, the total number of license plates that can be made are $2 \cdot 10^3 \cdot 26^3$ or 35152000.

48c: (1 point).
In how many ways can a photographer at a wedding arrange 6 people in a row from a group of 10 people, where the bride and the groom are among these 10 people, if exactly one of the bride and the groom is in the picture?

Since either the bride or the groom be in the picture, that leaves 5 more people to be chosen from the group of 10. Therefore, the total number of wedding arrangements is $2\binom{10}{5}$ or 504.

Exercises for Section 6.2:

18: (2 points).
How many numbers must be selected from the set $\{1, 3, 5, 7, 9, 11, 13, 15\}$ to guarantee that at least one pair of these numbers add up to 16?

We can split the list into the following four groups: $\{1, 15\}, \{3, 13\}, \{5, 11\}, \{7, 9\}$. Using the pigeon hole principle, we can one number from each pair that will not add up to 16. Therefore, at least 5 numbers must be chosen from the list to guarantee that at least one pair of the numbers adds up to 16.

20b: (2 points).
Suppose that there are nine students in a discrete mathematics class at a small college. Show that the class must have at least three male students or at least seven female students.

Assume that the class has less than 3 male and less than seven female students. Then the number of students in the class is: $S = M + F = 2 + 6 = 8$.
To add an additional student into the class to get a total of nine students, the student has to be either male or female and would cause there to be at least three male students or at least seven female students.

40: (2 points).
Find the least number of cables required to connect eight computers to four printers to guarantee that for every choice of four of the eight computers, these four computers can directly access four different printers. Justify your answer.

Connect C_1 to P_1, C_2 to P_2, C_3 to P_3, and C_4 to P_4. This will create four connections.

Next you connect C_5 through C_8 to the four printers. This will create an additional sixteen connections.

The total number of connections is 20. With these connections, it is guaranteed through the pigeon hole principle that any four computers are connected to four different printers.

Exercises for Section 6.3:

30: (1 point).
A professor writes 40 discrete mathematics true/false questions. Of the statements in these questions, 17 are true. If the questions can be positioned in any order, how many different answer keys are possible?

$\binom{40}{17}$

32b: (1 point).
Seven women and nine men are on the faculty in the mathematics department at a school. How many ways are there to select a committee of five members of the department if at least one woman and at least one man must be on the committee?

38: (2 points).
How many bit strings contain exactly five 0s and 14 1s if every 0 must be immediately followed by two 1s?

Since every 0 must be immediately followed by two 1s, there exists five blocks of "011" and four 1s. This gives us nine objects to order.

Since every block of "011" can be swapped without changing the string, we can divide out the number of these arrangements. Similarly, we can divide out the number of arrangements of the "1" blocks.

Therefore, the number of strings that fit the criteria is $\frac{9!}{4! \cdot 5!} = \binom{9}{4} = \binom{9}{5}$.

Exercises for Section 6.4:

8: (1 point).

What is the coefficient of x^8y^9 in the expansion of $(3x+2y)^{17}$?

$$(x+y)^n = \sum_{k=0}^{n} \binom{n}{k} x^{n-k}y^k$$

$$\binom{17}{8}(3x)^8(2y)^9$$

The coefficient would be $\binom{17}{8} \cdot 3^8 \cdot 2^9$

26b: (2 points).

Prove the identity $\binom{n}{r}\binom{r}{k} = \binom{n}{k}\binom{n-k}{r-k}$, whenever n, r, and k are non-negative integers with $r \leq n$ and $k \leq r$, using an argument based on the formula for the number of r-combinations of a set with n elements.

32b: (2 points).

Show that if n is a positive integer, then $\binom{2n}{n} = 2\binom{n}{2} + n^2$ by algebraic manipulation.

$$\binom{2n}{2} = \frac{(2n)!}{2!(2n-2)!}$$
$$= \frac{(2n)(2n-1)}{2} = 2n^2 - n$$

$$2\binom{n}{2} + n^2 = 2\frac{n!}{2!(n-2)!} + n^2$$
$$= n(n-1) + n^2 = 2n^2 - n$$

$$\because \binom{2n}{n} = 2n^2 - n = \binom{n}{2}$$
$$\therefore \binom{2n}{n} = \binom{n}{2}$$