CSCE 420: Artificial Intelligence

Homework 4

Due date: 3 December 2023 *Name: Huy Lai*

Problem 1. Given the belief network as shown below, calculate the joint probability
$P(JohnCalls, \neg MaryCalls, Alarm, \neg Earthquake, Burglary)$

Solution:
The joint probability here is computed using the given set of conditional and marginal probabilities from the given tree as:

$$\begin{aligned} P(J, \neg M, A, \neg E, B) &= P(B)P(\neg E)P(A \mid B, \neg E)P(J \mid A)P(\neg M \mid A) \\ &= (0.001)(1 - 0.002)(0.94)(0.9)(0.01) \\ &= 0.00000844308 \end{aligned}$$

Problem 2. Consider the belief network construction algorithm. (1) If the node order is "JohnCalls", "Marycalls", …, and "JohnCalls" is already added to the belief network, should "JohnCalls" point to "MaryCalls" when "MaryCalls" is newly added? (2) Explain by referring to certain probabilities that need to be compared in this case, and whether these probabilities are equal or not.

Solution:
If there's some conditional dependence between John and Mary calling, say John calling increases the likelihood of Mary calling, then there might be an edge between "JohnCalls" and "MaryCalls".

However, since these two events are conditionally independent, there would not exist

Problem 3. Consider random variables Earthquake,Burglary, and Alarm. Using these three nodes, give an example of "intercausal inference". Explain why. Hint: Consider the cause and effect.

Solution:
Suppose in this scenario:

- Earthquake can cause Alarm to go off.
- Burglary can also cause Alarm to go off.
- But Earthquake and Burglary are independent events.

Intercausal inference occurs when the knowledge of a common effect (Alarm) influences reasoning about the relationship between two causes (Earthquake and Burglary).

For example, if we observe the Alarm going off:

- We might initially think it's more likely that there was either an Earthquake or a Burglary.
- However, knowing that Earthquake and Burglary are independent, observing the Alarm doesn't give us direct information about whether there was an Earthquake or a Burglary.
- This situation represents an intercausal inference, as the common effect (Alarm) doesn't provide information about the specific cause (Earthquake or Burglary) when the causes are independent of each other.

Problem 4. Decision Tree

Solution:

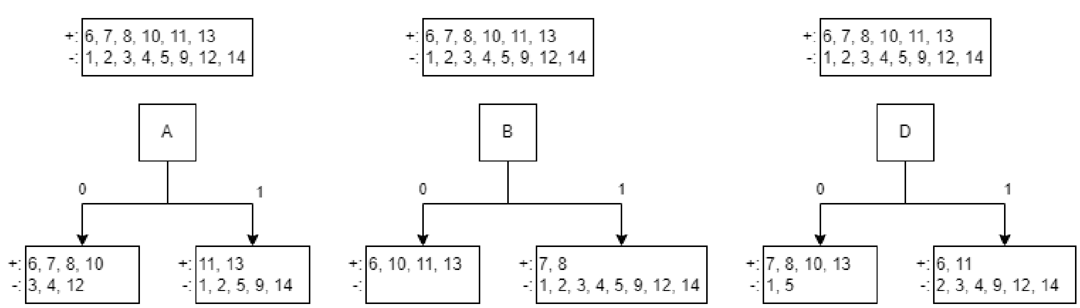


Figure 1: Question 4 Decision Tree

Problem 5. Information Gain

Solution:
The calculations for the infomation gained from A, B, D are as follows

$$
\begin{aligned}
Entropy(E) &= -\frac{6}{14}\log_2\left(\frac{6}{14}\right) - \frac{8}{14}\log_2\left(\frac{8}{14}\right) \\
&= 0.985 \\
Entropy(A_{E_0}) &= -\frac{4}{7}\log_2\left(\frac{4}{7}\right) - \frac{3}{7}\log_2\left(\frac{3}{7}\right) \\
&= 0.985 \\
Entropy(A_{E_1}) &= -\frac{2}{7}\log_2\left(\frac{2}{7}\right) - \frac{5}{7}\log_2\left(\frac{5}{7}\right) \\
&= 0.863 \\
Gain(E, A) &= Entropy(E) - \frac{7}{14}Entropy(A_{E_0}) - \frac{7}{14}Entropy(A_{E_1}) \\
&= 0.985 - \frac{7}{14}(0.985) - \frac{7}{14}(0.863) \\
&= 0.061
\end{aligned}
$$

$$
\begin{aligned}
Entropy(B_{E_0}) &= -\frac{4}{4}\log_2\left(\frac{4}{4}\right) - \frac{0}{4}\log_2\left(\frac{0}{4}\right) \\
&= 0 \\
Entropy(B_{E_1}) &= -\frac{2}{10}\log_2\left(\frac{2}{10}\right) - \frac{8}{10}\log_2\left(\frac{8}{10}\right) \\
&= 0.722 \\
Gain(E, B) &= Entropy(E) - \frac{4}{14}Entropy(B_{E_0}) - \frac{10}{14}Entropy(B_{E_1}) \\
&= 0.985 - \frac{4}{14}(0) - \frac{10}{14}(0.722) \\
&= 1.469
\end{aligned}
$$

$$
\begin{aligned}
Entropy(D_{E_0}) &= -\frac{4}{6}\log_2\left(\frac{4}{6}\right) - \frac{2}{6}\log_2\left(\frac{2}{6}\right) \\
&= 0.918 \\
Entropy(D_{E_1}) &= -\frac{2}{8}\log_2\left(\frac{2}{8}\right) - \frac{6}{8}\log_2\left(\frac{6}{8}\right) \\
&= 0.811 \\
Gain(E, D) &= Entropy(E) - \frac{6}{14}Entropy(D_{E_0}) - \frac{8}{14}Entropy(D_{E_1}) \\
&= 0.985 - \frac{6}{14}(0.918) - \frac{8}{14}(0.811) \\
&= 0.128
\end{aligned}
$$

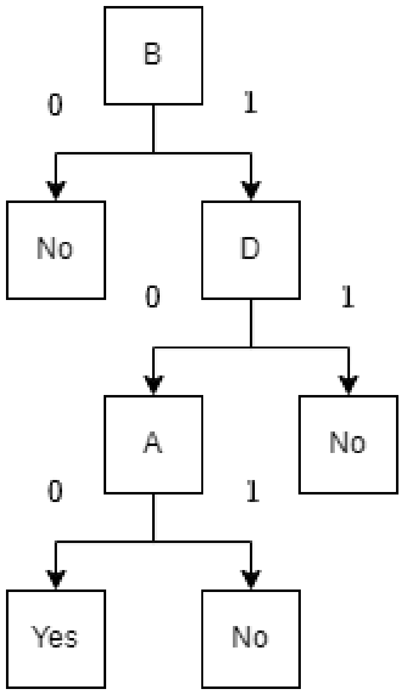
Based on the calculated values for the information gains, we should test Attribute B first since it has the highest information gain.

Problem 6. Programming Information Gain

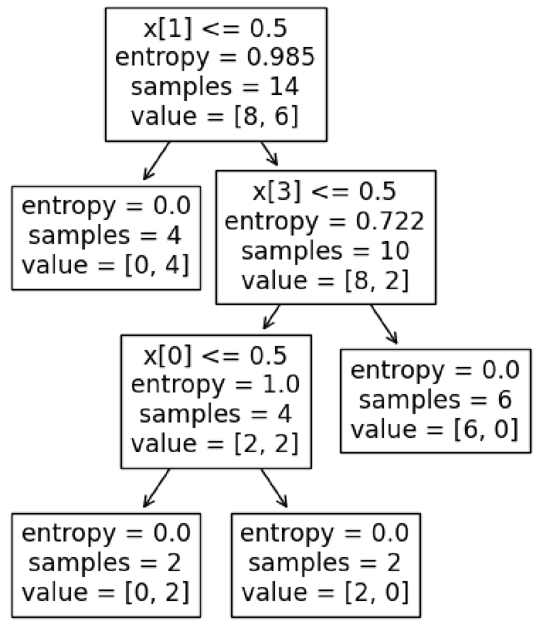
Solution:
The code is included in the submission folder as requested.

Problem 7. Decision Tree

Solution:

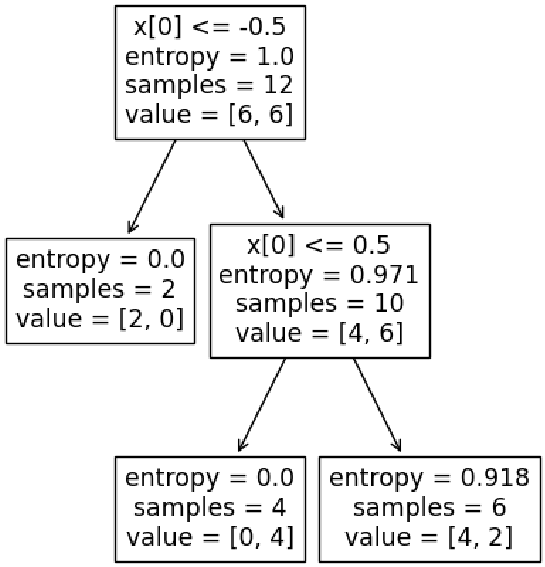


(a) Simplified Decision Tree

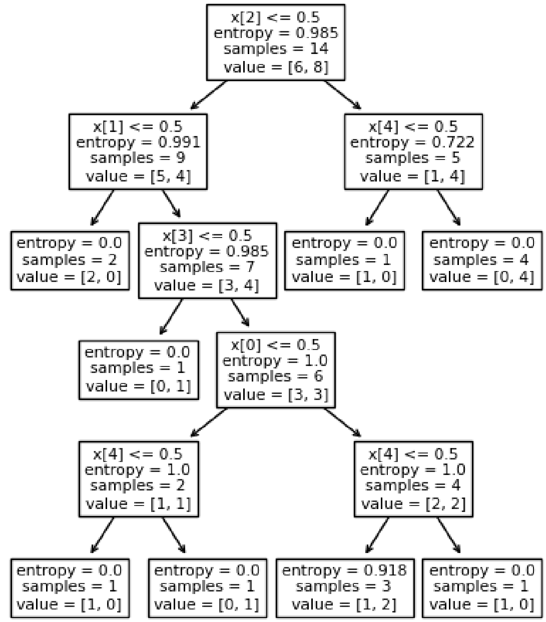


(b) Original Decision Tree

Figure 2: Comparison of Decision Trees



(a) Novel Tree 1



(b) Novel Tree 2

Figure 3: Novel Trees

Problem 8. Perceptron

Solution:
The code is included in the submission folder as requested.

Problem 9. Perceptron Drawing

Solution:

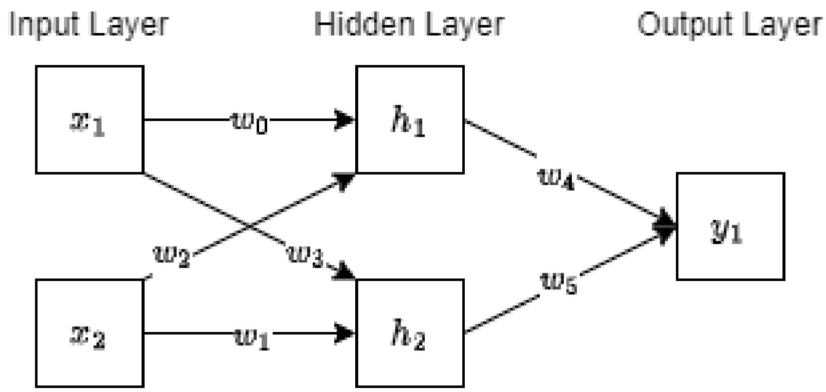


Figure 4: XOR Perceptron

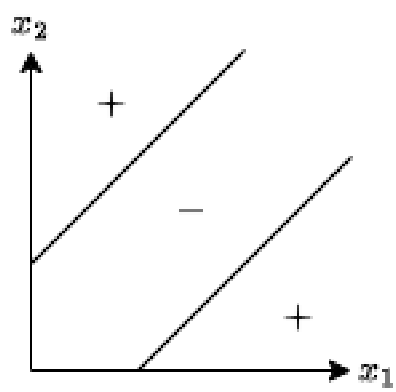


Figure 5: XOR Decision Boundary