

CSCE 420 Homework #2 (2023 Fall)

Game Playing ; Logic and Theorem Proving

Total 100 points
See Canvas for submission details.

1 Game Playing

1.1 Minmax Search

Question 1 (5 pts): Using the following figure 1, use minmax search to assign utility values for each internal node (i.e., non-leaf node) and indicate which path is the optimal solution for the MAX node at the root of the tree. Assume you explore the successors from left to right.

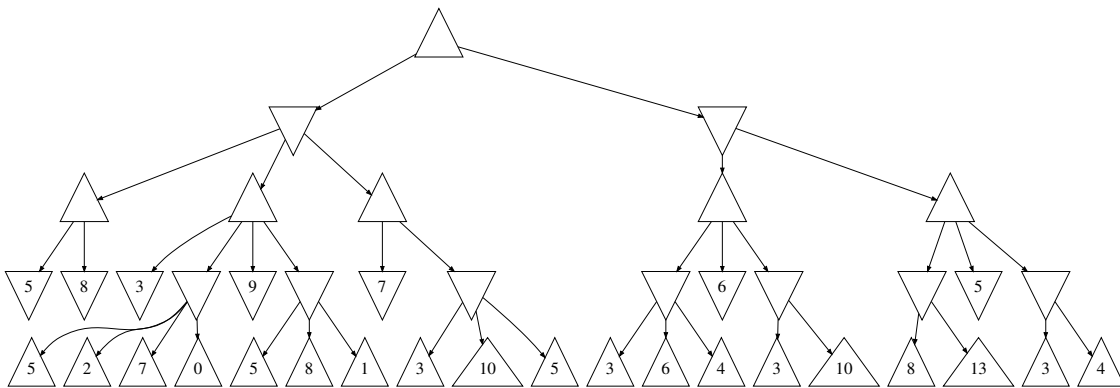


Figure 1: **Game Tree.** Solve using minmax search. Note: \triangle is the MAX node, ∇ is the MIN node.

1.2 $\alpha - \beta$ pruning

Question 2 (15 pts): Using the following figure 2, use $\alpha - \beta$ pruning to (1) assign utility values for each internal node (i.e., non-leaf node) and indicate which path is the optimal solution for the MAX node at the root of the tree. (2) For each node, indicate the final α and β values. (Note that initial values at the root are $\alpha = -\infty, \beta = \infty$.) (3) For each cut that happens, draw a line to cross out that subtree.

Hint: There are 6 places that need to be cut (see how to count cuts: figure 3).

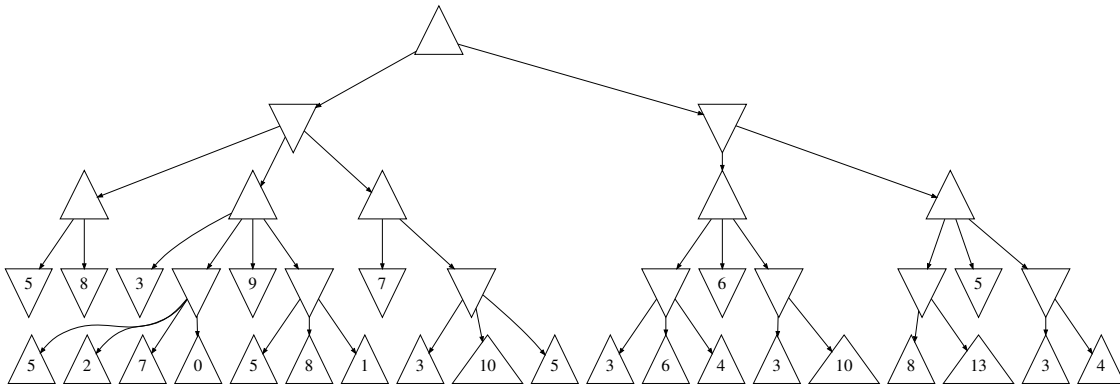
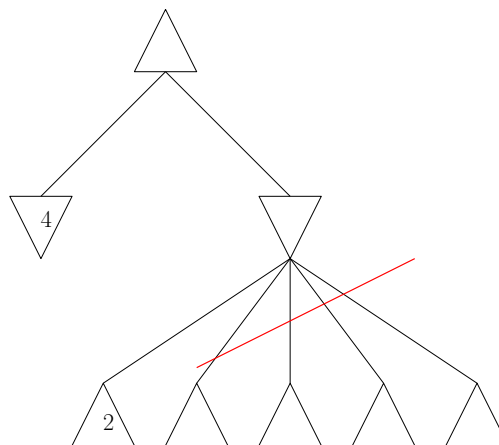


Figure 2: **Game Tree.** Solve using $\alpha - \beta$ pruning. This tree is the same as figure 1. Note: \triangle is the MAX node, ∇ is the MIN node.



This cut counts as "one" cut, not four.

Figure 3: **Game Tree.** Counting cuts

Question 3 (5 pts): In Minmax search, we used a depth-first exploration through the use of recursion. We know that Minmax gives an optimal solution, however, we also know that depth-first search is suboptimal. Explain why Minmax gives an optimal solution even when it is using a depth-first exploration.

Programming Exercise (0 pts): As an exercise, you can implement and test the alpha-beta pruning algorithm. This is an exercise, so it is not required, and it will not be graded. The `minmax.ipynb` file in the hw2 folder implements the minmax algorithm. You can browse this to get an idea on how to represent the game tree and do a recursive search. Test it on the problem in Figure 1.

2 Propositional Logic

In this section, assume P, Q, R, S, T, U, V, W are atoms (propositions).

2.1 Inference rule

Question 4 (10 pts): Using a truth table, show that the resolution inference rule is valid (if the premises are true, the conclusion is also true, or, $((P \vee Q) \wedge (\neg Q \vee R)) \rightarrow (P \vee R)$ is valid). Note: valid means “true under all interpretations”.

$$\frac{P \vee Q, \neg Q \vee R}{P \vee R}$$

P	Q	R	$(P \vee Q)$	$(\neg Q \vee R)$	$(P \vee Q) \wedge (\neg Q \vee R)$	$(P \vee R)$	$((P \vee Q) \wedge (\neg Q \vee R)) \rightarrow (P \vee R)$
T	T	T					
T	T	F					
T	F	T					
T	F	F					
F	T	T					
F	T	F					
F	F	T					
F	F	F					

2.2 Normal forms

In all of the problems in this section, show each step of the derivation and indicate which law (or other rules) you used: For example, *distributive law*, *by definition*, *etc.*

Question 5 (10 pts):

- (1) Convert $(Q \wedge \neg P) \vee (Q \wedge R) \vee S$ into conjunctive normal form.
- (2) Convert $\neg(\neg P \vee Q) \vee R$ into conjunctive normal form.
- (3) Convert $\neg((\neg P \rightarrow Q) \wedge (R \vee S))$ into disjunctive normal form.

2.3 Theorem proving

Using resolution, show that $Q \vee W$ is a logical consequence of the following premises:

1. $R \rightarrow W$
2. $R \vee (\neg P \wedge S)$
3. $S \rightarrow (P \vee Q)$
4. $S \vee R$

Question 6 (10 pts): Transform the above problem into a set of clauses (premises and the conclusion), suitable for resolution-based theorem proving.

- Turn each axiom in the list of premises above into conjunctive normal form.
 - One premise may result in multiple clauses.
 - For example, a premise $\neg((P \wedge \neg R) \vee S)$ will convert to CNF as $(\neg P \vee R) \wedge \neg S$, which results in two clauses:
 - * Clause 1: $\neg P \vee R$
 - * Clause 2: $\neg S$
- Don't forget to negate the conclusion $(Q \vee W)$, before adding to the clause list. Multiple clauses may (or may not) result from the negated conclusion.

Write your resulting clauses in the following format:

C1:

C2:

C3:

C4:

C5:

C6:

...

Question 7 (10 pts): Use resolution to derive **False**. Show every step. DO NOT USE any other inference rule.

3 First Order Logic (Basics)

Important: In this section, assume that w, x, y, z are variables; A, B, C, D are constants; and $f(\cdot), g(\cdot), h(\cdot)$ are functions; and $P(\cdot), Q(\cdot), R(\cdot)$ are predicates.

Question 8 (6 pts): Convert to prenex normal form (2 points each):

1. $\neg \forall x ((\exists y Q(x, y)) \rightarrow P(x))$
2. $\forall x \neg (\exists y Q(x, y) \wedge \neg R(x))$
3. $\neg \exists x (\neg (\forall y Q(x, y)) \vee \neg P(x))$

Question 9 (5 pts): Skolemize the expressions (1 point each):

1. $\exists x P(x)$
2. $\forall x \exists y P(x, y)$
3. $\exists x \exists y \forall z P(x, y) \wedge Q(y, z)$
4. $\forall x \exists y \exists z P(x, y, z) \wedge Q(y, z)$

5. $\forall x \forall y \exists z P(x, y) \wedge Q(x, y, z)$

Question 10 (9 pts): Convert the following into a standard form (prenex, CNF, skolemization: 3pt each):

$$\forall x [\neg(\exists z(P(z) \wedge Q(x, z))) \rightarrow \exists y R(x, y)]$$

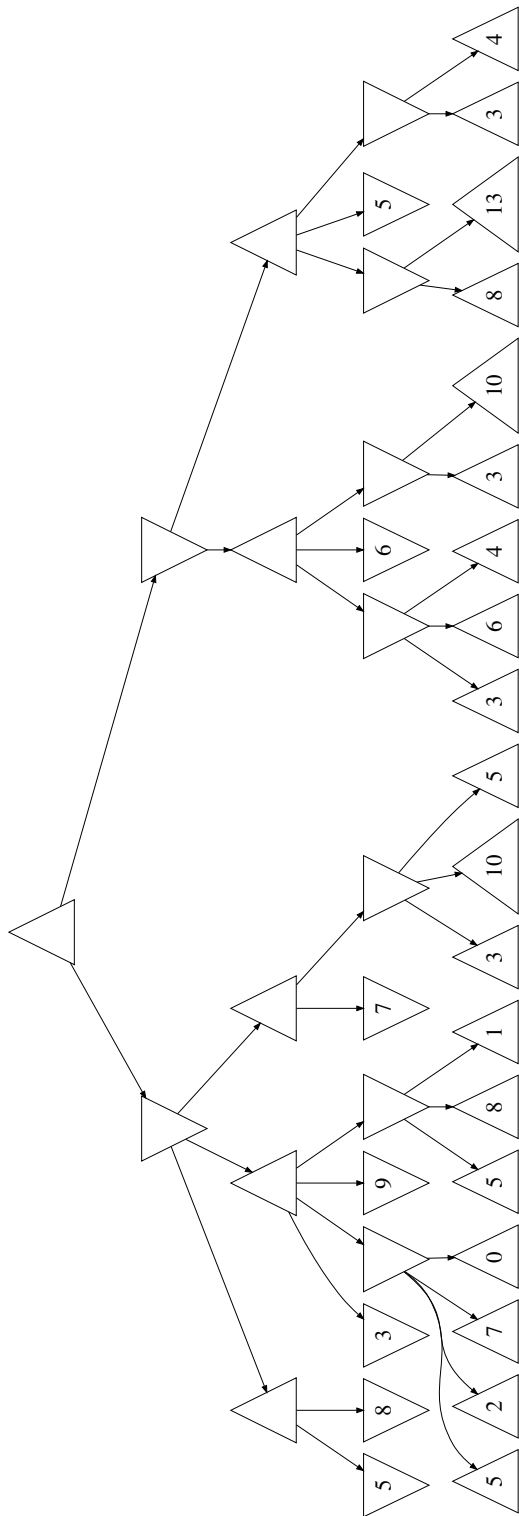
Question 11 (3 pts): Apply the following substitutions to the expressions (1 point each);

1. Apply $\{x/f(A)\}$ to $P(x, y) \vee Q(x)$.
2. Apply $\{x/A, y/f(z)\}$ to $P(x, y) \vee Q(x)$.
3. Apply $\{y/x\}$ to $P(x, y) \vee Q(x)$.

Question 12 (12 pts): For each of the following, (1) find the unifier, and (2) show the unified expression. For example, given $P(A)$ and $P(x)$, the unifier would be $\{x/A\}$, and the unified expression $P(A)$. If the pair of expressions is not unifiable, indicate so. (3 points each):

1. $P(x, f(x), y)$ and $P(A, f(g(w)), z)$
2. $P(x, A)$ and $P(y, y)$
3. $P(x, f(g(x)), g(A), w)$ and $P(A, f(y), y, y)$
4. $P(x, f(x))$ and $P(A, f(B))$

Full-size print of the game tree, for practice, etc. (copy 1)



Full-size print of the game tree, for practice, etc. (copy 2)

