CSCE 420: Artificial Intelligence

Homework 2

Due date: 17 September 2023 *Name: Huy Lai*

Problem 1. Use minmax search to assign utility values for each internal node (i.e., non-leaf node) and indicate which path is the optimal solution for the MAX node at the root of the tree. Assume you explore the successors from left to right.

Solution:
The optimal path is highlighted in Red.

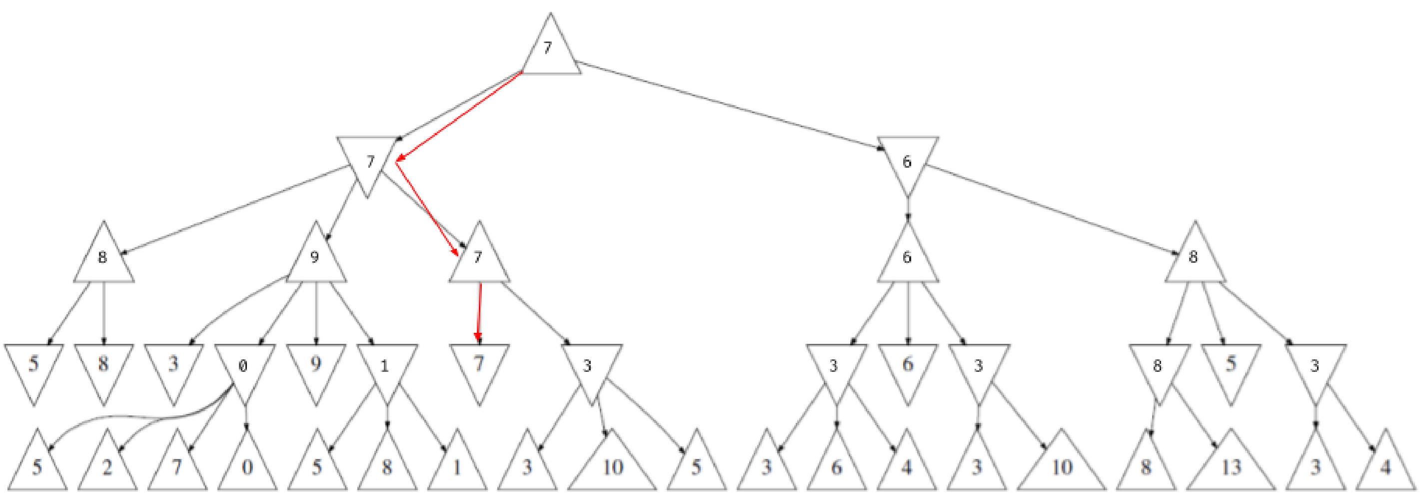

Figure 1: MinMax Search

Problem 2. Using the following figure 2, use $\alpha - \beta$ pruning to (1) assign utility values for each internal node (i.e., non-leaf node) and indicate which path is the optimal solution for the MAX node at the root of the tree. (2) For each node, indicate the final α and β values. (3) For each cut that happens, draw a line to cross out that subtree.

Solution:
The optimal path is highlighted in Red. Cuts are made with red lines.

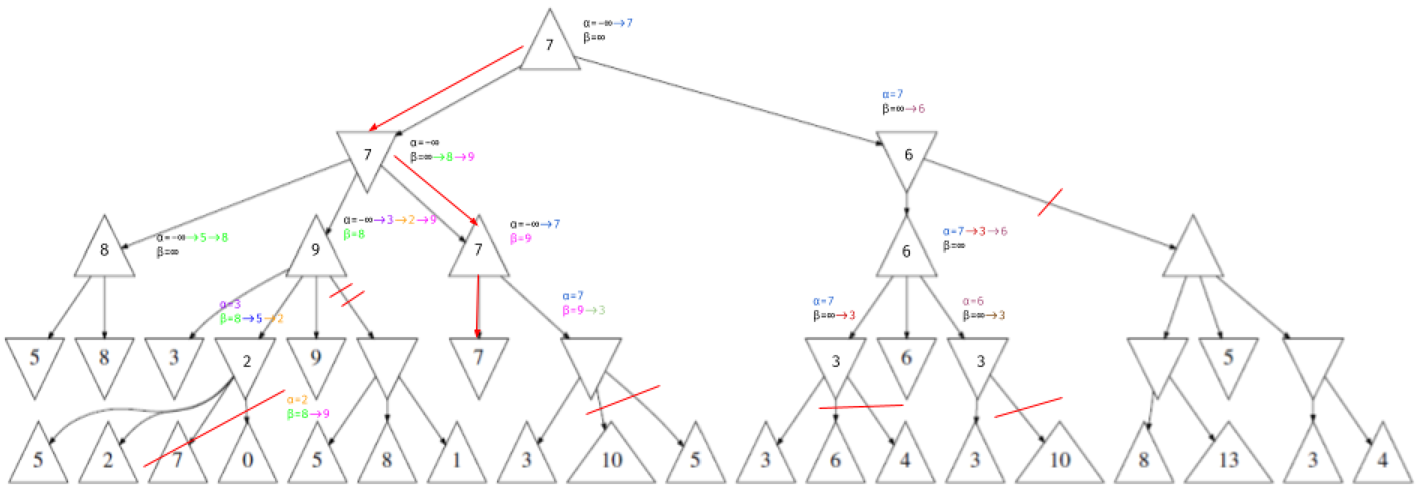

Figure 2: $\alpha - \beta$ pruning

Problem 3. In Minmax search, we used a depth-first exploration through the use of recursion. We know that Minmax gives an optimal solution, however, we also know that depth-first search is suboptimal. Explain why Minmax gives an optimal solution even when it is using a depth-first exploration.

Solution:
MinMax explores the entire game tree, considering all possible moves and counter-moves up to a certain depth or until the game ends.
This means that it does not miss any potential moves, ensuring a complete search of the state space.

Problem 4. Using a truth table, show that the resolution inference rule is valid. Note: valid means "true under all interpretations".

$$\frac{P \vee Q, \neg Q \vee R}{P \vee R}$$

Solution:
The Truth Table is as follows.

P	Q	R	$(P \vee Q)$	$(\neg Q \vee R)$	$(P \vee Q) \wedge (\neg Q \vee R)$	$(P \vee R)$	$(P \vee Q) \wedge (\neg Q \vee R) \rightarrow (P \vee R)$
T	T	T	T	T	T	T	**T**
T	T	F	T	F	F	T	**T**
T	F	T	T	T	T	T	**T**
T	F	F	T	T	T	T	**T**
F	T	T	T	T	T	T	**T**
F	T	F	T	F	F	F	**T**
F	F	T	F	T	F	T	**T**
F	F	F	F	T	F	F	**T**

Table 1: Truth Table

Problem 5. In all of the problems in this section, show each step of the derivation and indicate which law (or other rules) you used: For example, *distributive law, by definition, etc.*
Subproblem 1. Convert $(Q \wedge \neg P) \vee (Q \wedge R) \vee S$ into conjunctive normal form.

Solution:
Converting to CNF is as follows

$$\begin{aligned}(Q \wedge \neg P) \vee (Q \wedge R) \vee S &= (Q \wedge (\neg P \vee R)) \vee S && \text{Distributive Law}\\ &= (Q \vee S) \wedge (\neg P \vee R \vee S) && \text{Distributive Law}\end{aligned}$$

Subproblem 2. Convert $\neg(\neg P \vee Q) \vee R$ into conjunctive normal form.

$$\begin{aligned}\neg(\neg P \vee Q) \vee R &= (\neg\neg P \wedge \neg Q) \vee R && \text{De Morgan Law}\\ &= (P \wedge \neg Q) \vee R && \text{Double Neggation}\\ &= (P \vee R) \wedge (\neg Q \vee R) && \text{Distributive Law}\end{aligned}$$

Subproblem 3. Convert $\neg((\neg P \to Q) \wedge (R \vee S))$ into the disjunctive normal form.

$$\begin{aligned}\neg((\neg P \to Q) \wedge (R \vee S)) &= \neg((\neg\neg P \vee Q) \wedge (R \vee S)) && \text{Implication}\\ &= \neg((P \vee Q) \wedge (R \vee S)) && \text{Double Negation}\\ &= \neg(P \vee Q) \vee \neg(R \vee S) && \text{De Morgan's Law}\\ &= (\neg P \wedge \neg Q) \vee (\neg R \wedge \neg S) && \text{De Morgan's Law}\end{aligned}$$

Problem 6. Using resolution, show that $Q \vee W$ is a logical consequence of the following premises:

1. $R \to W$
2. $R \vee (\neg P \wedge S)$
3. $S \to (P \vee Q)$
4. $S \vee R$

Transform the above problem into a set of clauses (premises and the conclusion), suitable for resolution-based theorem proving.

Solution:
The clauses are as follows:

C1. $\neg R \vee W$

C2. $R \vee \neg P$

C3. $R \vee S$

C4. $\neg S \vee (P \vee Q)$

C5. $S \vee R$

C6. $\neg Q$

C7. $\neg W$

Problem 7. Use resolution to derive False. Show every step. DO NOT USE any other inference rule.

Solution:
C8. Resolve C4 and C6 on Q: $\neg S \vee P$
C9. Resolve C5 and C8 on S: $R \vee P$ C10. Resolve C2 and C9 on P: R
C11. Resolve C1 and C10 on R: W
C12. Resolve C11 and C7 on W: $False$

Problem 8. Convert to prenex normal form

1. $\neg\forall x((\exists y Q(x,y)) \to P(x))$
2. $\forall x \neg(\exists y Q(x,y) \wedge \neg R(x))$
3. $\neg\exists x(\neg(\forall y Q(x,y)) \vee \neg P(x))$

Solution:
For expression 1.

$$\begin{aligned}
\neg\forall x((\exists y Q(x,y)) \to P(x)) &= \neg\forall x(\neg(\exists y Q(x,y)) \vee P(x)) \\
&= \neg\forall x(\forall y \neg Q(x,y) \vee P(x)) \\
&= \exists x \neg(\forall y \neg Q(x,y) \vee P(x)) \\
&= \exists x \exists y \neg(\neg Q(x,y) \vee P(x)) \\
&= \exists x \exists y Q(x,y) \wedge \neg P(x)
\end{aligned}$$

For expression 2.

$$\begin{aligned}
\forall x \neg(\exists y Q(x,y) \wedge \neg R(x)) &= \forall x \forall y \neg(Q(x,y) \wedge \neg R(x)) \\
&= \forall x \forall y \neg Q(x,y) \vee R(x)
\end{aligned}$$

For expression 3.

$$\begin{aligned}
\neg\exists x(\neg(\forall y Q(x,y)) \vee \neg P(x)) &= \neg\exists x(\exists y \neg Q(x,y) \vee \neg P(x)) \\
&= \forall x \neg(\exists y \neg Q(x,y) \vee \neg P(x)) \\
&= \forall x \forall y Q(x,y) \wedge P(x)
\end{aligned}$$

Problem 9. Skolemize the expressions

1. $\exists x P(x)$
2. $\forall x \exists y P(x, y)$
3. $\exists x \exists y \forall z P(x, y) \wedge Q(y, z)$
4. $\forall x \exists y \exists z P(x, y, z) \wedge Q(y, z)$
5. $\forall x \forall y \exists z P(x, y) \wedge Q(x, y, z)$

Solution:
Skolemization is as follows

1. $P(\alpha)$
2. $\forall x P(x, f(x))$
3. $\forall z P(\alpha, \beta) \wedge Q(\beta, z)$
4. $\forall x P(x, f(x), g(x)) \wedge Q(f(x), g(x))$
5. $\forall x \forall y P(x, y) \wedge Q(x, y, f(x, y))$

Problem 10. Convert the following into a standard form (prenex, CNF, skolemization)

$$\forall x[\neg(\exists z(P(z) \wedge Q(x,z))) \rightarrow \exists y R(x,y)]$$

Solution:
Prenex

$$\begin{aligned}\forall x[\neg(\exists z(P(z) \wedge Q(x,z))) \rightarrow \exists y R(x,y)] &= \forall x[\exists z(P(z) \wedge Q(x,z)) \vee \exists y R(x,y)] \\ &= \forall x[(\exists z P(z) \wedge \exists z Q(x,z)) \vee \exists y R(x,y)] \\ &= \forall x \exists z \exists u[(P(z) \wedge Q(x,u)) \vee \exists y R(x,y)] \\ &= \forall x \exists z \exists u \exists y[(P(z) \wedge Q(x,u)) \vee R(x,y)]\end{aligned}$$

CNF

$$\forall x \exists z \exists u \exists y[(P(z) \vee R(x,y)) \wedge (Q(x,u) \vee R(x,y))]$$

Skolemization

$$\forall x[(P(f(x)) \vee R(x,g(x))) \wedge (Q(x,h(x)) \vee R(x,g(x)))]$$

Problem 11. Apply the following substitutions to the expressions

1. Apply $\{x/f(A)\}$ to $P(x,y) \vee Q(x)$.
2. Apply $\{x/A, y/f(z)\}$ to $P(x,y) \vee Q(x)$.
3. Apply $\{y/x\}$ to $P(x,y) \vee Q(x)$.

Solution:
The substitutions are as follows:

1. $P(f(A),y) \wedge Q(f(A))$
2. $P(A,f(z)) \wedge Q(A)$
3. $P(x,x) \wedge Q(x)$

Problem 12. For each of the following, (1) find the unifier, and (2) show the unified expression. For example, given $P(A)$ and $P(x)$, the unifier would be $\{x/A\}$, and the unified expression $P(A)$. If the pair of expressions is not unifiable, indicate so.

1. $P(x, f(x), y)$ and $P(A, f(g(w)), z)$
2. $P(x, A)$ and $P(y, y)$
3. $P(x, f(g(x)), g(A), w)$ and $P(A, f(y), y, y)$
4. $P(x, f(x))$ and $P(A, f(B))$

Solution:
The unifier and unified expression are as follows:

1. $P(x, f(x), y)$ and $P(A, f(g(w)), z)$

    (1) $\{A/x\}$:
    $P(x, f(x), y)$
    $P(x, f(g(w)), z)$

    (2) $\{x/g(w)\}$:
    $P(x, f(g(w)), y)$
    $P(x, f(g(w)), z)$

    (3) $\{y/z\}$:
    $P(x, f(g(w)), z)$
    $P(x, f(g(w)), z)$

    Unifiers: $\{A/x, x/g(w), y/z\}$
    Unified Expression: $P(x, f(g(w)), z)$

2. $P(x, A)$ and $P(y, y)$

   (1) $\{A/x\}$:
   $P(x, x)$
   $P(y, y)$

   (2) $\{x/y\}$:
   $P(y, y)$
   $P(y, y)$

   Unifiers: $\{A/x, x/y\}$
   Unified Expression: $P(y, y)$

3. $P(x, f(g(x)), g(A), w)$ and $P(A, f(y), y, y)$

   (a) $\{A/x\}$:
   $P(x, f(g(x)), g(x), w)$
   $P(x, f(y), y, y)$

   (b) $\{w/y\}$:
   $P(x, f(g(x)), g(x), y)$
   $P(x, f(y), y, y)$

   (c) $\{y/g(x)\}$:
   $P(x, f(g(x)), g(x), g(x))$
   $P(x, f(g(x)), g(x), g(x))$

   Unifiers: $\{A/x, w/y, y/g(x)\}$
   Unified Expression: $P(x, f(g(x)), g(x), g(x))$

4. $P(x, f(x))$ and $P(A, f(B))$

   (1) $\{A/B\}$:
   $P(x, f(x))$
   $P(B, f(B))$

   (2) $\{B/x\}$:
   $P(x, f(x))$
   $P(x, f(x))$

   Unifiers: $\{A/B, B/x\}$
   Unified Expression: $P(x, f(x))$