
Homework 1

Huy Quang Lai

Department of Computer Science and Engineering
Texas A&M University
College Station, Texas 77843
lai.huy@tamu.edu

1 Gradient Calculation

In this question you are required to calculate gradients for 2 scalar functions.

1.1 $f(x, y) = x^2 + \ln(y) + xy + y^3$

$$\begin{aligned}\nabla f(x, y) &= \left\langle \frac{\partial f}{\partial x}(x, y), \frac{\partial f}{\partial y}(x, y) \right\rangle \\ &= \left\langle 2x + y, \frac{1}{y} + x + 3y^2 \right\rangle\end{aligned}$$

$$\nabla f(10, -10) = \langle 10, 309.9 \rangle$$

1.2 $f(x, y, z) = \tanh(x^3 y^3) + \sin(z^2)$

$$\begin{aligned}\nabla f(x, y, z) &= \left\langle \frac{\partial f}{\partial x} f(x, y, z), \frac{\partial f}{\partial y} f(x, y, z), \frac{\partial f}{\partial z} f(x, y, z) \right\rangle \\ &= \langle 3x^2 y^3 \operatorname{sech}^2(x^3 y^3), 3x^3 y^2 \operatorname{sech}^2(x^3 y^3), 2z \cos(z^2) \rangle\end{aligned}$$

$$\nabla f\left(-1, 0, \frac{\pi}{2}\right) = \left\langle 0, 0, \cos\left(\frac{\pi^2}{4}\right) \right\rangle$$

2 Matrix multiplication

In this question you are required to perform matrix multiplication.

2.1 Multiplication 1

$$\begin{bmatrix} 10 \\ -5 \\ 2 \\ 8 \end{bmatrix} \begin{bmatrix} 0 & 3 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 30 & 0 & 10 \\ 0 & -15 & 0 & -5 \\ 0 & 6 & 0 & 2 \\ 0 & 24 & 0 & 8 \end{bmatrix}$$

2.2 Multiplication 2

$$\begin{bmatrix} 7 & -3 & 1 & 9 \end{bmatrix} \begin{bmatrix} -3 \\ -4 \\ 6 \\ 0 \end{bmatrix} = [-3]$$

2.3 Multiplication 3

$$\begin{bmatrix} 1 & -1 & 6 & 7 \\ 9 & 0 & 8 & 1 \\ -8 & 1 & 2 & 3 \\ 10 & 4 & 0 & 1 \end{bmatrix} \begin{bmatrix} 6 & 2 & 0 \\ 0 & -1 & 1 \\ -3 & 0 & 4 \\ 3 & 4 & 7 \end{bmatrix} = \begin{bmatrix} 9 & 31 & 72 \\ 33 & 22 & 39 \\ -45 & -5 & 30 \\ 63 & 20 & 11 \end{bmatrix}$$

3 Vector Norms

Consider these two points in the 3-dimensional space:

$$\mathbf{a} = \begin{bmatrix} 7 \\ 0 \\ -1 \end{bmatrix}, \mathbf{b} = \begin{bmatrix} 7 \\ 9 \\ -5 \end{bmatrix}$$

Calculate their distance using the following norms:

$$\mathbf{a} - \mathbf{b} = \begin{bmatrix} 0 \\ -9 \\ 4 \end{bmatrix}$$

3.1 l_0

Since there are two non-zero terms in the vector $\mathbf{a} - \mathbf{b}$, $l_0 = 2$

3.2 l_1

$$l_1 = -9 + 4 = -5$$

3.3 l_2

$$l_2 = \sqrt{(-9)^2 + 4^2} = \sqrt{97}$$

3.4 l_∞

$$l_\infty = 4$$

4 Probability calculation

4.1 Sample Space

Assume that the roll $(1, 2)$ is an equivalent roll to $(2, 1)$.

$$\mathbb{S} = \{(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6), \\ (2, 2), (2, 3), (2, 4), (2, 5), (2, 6), \\ (3, 3), (3, 4), (3, 5), (3, 6), \\ (4, 4), (4, 5), (4, 6), \\ (5, 5), (5, 6) \\ (6, 6)\}$$

$$\mathbf{4.2} \quad P(r_1 + r_2 = 10)$$

$$P(r_1 + r_2 = 10) = \frac{2}{21}$$

$$\mathbf{4.3} \quad P(r_1 + r_2 = 6)$$

$$P(r_1 + r_2 = 6) = \frac{1}{7}$$

5 Mean and variance calculation

5.1 What is the mean of X ?

Since $f_X(x)$ is a uniform distribution function, $\int_a^b f_X(x)dx = 1$

As a result, $\mu_X = \frac{1}{2}(a + b)$

5.2 What is the standard deviation of X ?

Likewise, $\sigma_X = \frac{b - a}{\sqrt{12}}$

6 Classification Quality Metric Computation.

6.1 Accuracy

$$A = \frac{TP + TN}{TP + TN + FP + FN} = \frac{92}{160} = 0.5750$$

6.2 Balanced Accuracy

$$\frac{1}{2}A_+ + \frac{1}{2}A_- = \frac{1}{2} \left(\frac{TP}{TP + FN} + \frac{TN}{TN + FP} \right) = \frac{1}{2} \left(\frac{37}{60} + \frac{11}{20} \right) = \frac{7}{12} = 0.5833$$

6.3 Precision

$$P = \frac{TP}{TP + FP} = \frac{37}{82} = 0.4512$$

6.4 Recall

$$R = \frac{TP}{TP + FN} = \frac{37}{60} = 0.6167$$

6.5 F1-measure

$$F_1 = \frac{2TP}{2TP + FP + FN} = \frac{37}{71} = 0.5211$$

7 Receiver operating characteristic computation

Predicted	Ground Truth	0	0.25	0.5	0.75	1
5%	0	1	0	0	0	0
10%	0	1	0	0	0	0
15%	1	1	0	0	0	0
20%	0	1	0	0	0	0
35%	1	1	1	0	0	0
50%	0	1	1	1	0	0
60%	0	1	1	1	0	0
65%	1	1	1	1	0	0
70%	1	1	1	1	0	0
90%	1	1	1	1	1	0

7.1 ROC value for a threshold value of 0

$$FPR = 1, TPR = 1$$

7.2 ROC value for a threshold value of 0.25

$$FPR = 0.4, TPR = 0.8$$

7.3 ROC value for a threshold value of 0.5

$$FPR = 0.2, TPR = 0.6$$

7.4 ROC value for a threshold value of 0.75

$$FPR = 0.0, TPR = 0.2$$

7.5 ROC value for a threshold value of 1

$$FPR = 0.0, TPR = 0.0$$

7.6 What would be the AUROC approximation using the above results

$$AUROC_L = (0.2 - 0.0)(0.2) + (0.4 - 0.2)(0.6) + (1 - 0.4)(0.8) = 0.64$$

References

- [1] Watt, Jeremy, Borhani, Reza & Katsaggelos, Aggelos Konstantinos (2016) Machine Learning Refined.
- [2] Konasani, Venkata Reddy & Shailendra Kadre (2021) Machine Learning and Deep Learning Using Python and TensorFlow.

Checklist

1. For all authors...
 - (a) Do the main claims made in the abstract and introduction accurately reflect the paper's contributions and scope? [\[Yes\]](#)
 - (b) Did you describe the limitations of your work? [\[N/A\]](#)
 - (c) Did you discuss any potential negative societal impacts of your work? [\[N/A\]](#)
 - (d) Have you read the ethics review guidelines and ensured that your paper conforms to them? [\[Yes\]](#)
2. If you are including theoretical results...
 - (a) Did you state the full set of assumptions of all theoretical results? [\[Yes\]](#)
 - (b) Did you include complete proofs of all theoretical results? [\[Yes\]](#)
3. If you ran experiments...
 - (a) Did you include the code, data, and instructions needed to reproduce the main experimental results (either in the supplemental material or as a URL)? [\[N/A\]](#)
 - (b) Did you specify all the training details (e.g., data splits, hyperparameters, how they were chosen)? [\[N/A\]](#)
 - (c) Did you report error bars (e.g., with respect to the random seed after running experiments multiple times)? [\[N/A\]](#)
 - (d) Did you include the total amount of compute and the type of resources used (e.g., type of GPUs, internal cluster, or cloud provider)? [\[N/A\]](#)
4. If you are using existing assets (e.g., code, data, models) or curating/releasing new assets...
 - (a) If your work uses existing assets, did you cite the creators? [\[Yes\]](#)
 - (b) Did you mention the license of the assets? [\[N/A\]](#)
 - (c) Did you include any new assets either in the supplemental material or as a URL? [\[N/A\]](#)
 - (d) Did you discuss whether and how consent was obtained from people whose data you're using/curating? [\[N/A\]](#)
 - (e) Did you discuss whether the data you are using/curating contains personally identifiable information or offensive content? [\[N/A\]](#)
5. If you used crowdsourcing or conducted research with human subjects...
 - (a) Did you include the full text of instructions given to participants and screenshots, if applicable? [\[N/A\]](#)
 - (b) Did you describe any potential participant risks, with links to Institutional Review Board (IRB) approvals, if applicable? [\[N/A\]](#)
 - (c) Did you include the estimated hourly wage paid to participants and the total amount spent on participant compensation? [\[N/A\]](#)