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MATH 151-557

In [1]:
```
from sympy import *
from sympy.plotting import (plot, plot_parametric)
```

1a Values of r which solve differential equation

In [2]:
```
x = symbols('x')
r = symbols('r')

y = exp(r * x)
dy = y.diff(x)
ddy = dy.diff(x)
solve(ddy + 5 * dy + 6 * y, r)
```

Out[2]: `[-3, -2]`

1b Values of r which solve differential equation

In [3]:
```
y = exp(r * x)
dy = y.diff(x)
ddy = dy.diff(x)
solve(ddy + 6 * dy + 13 * y, r)
```

Out[3]: `[-3 - 2*I, -3 + 2*I]`

1c Compute left hand side with cosines and sines and conclusion

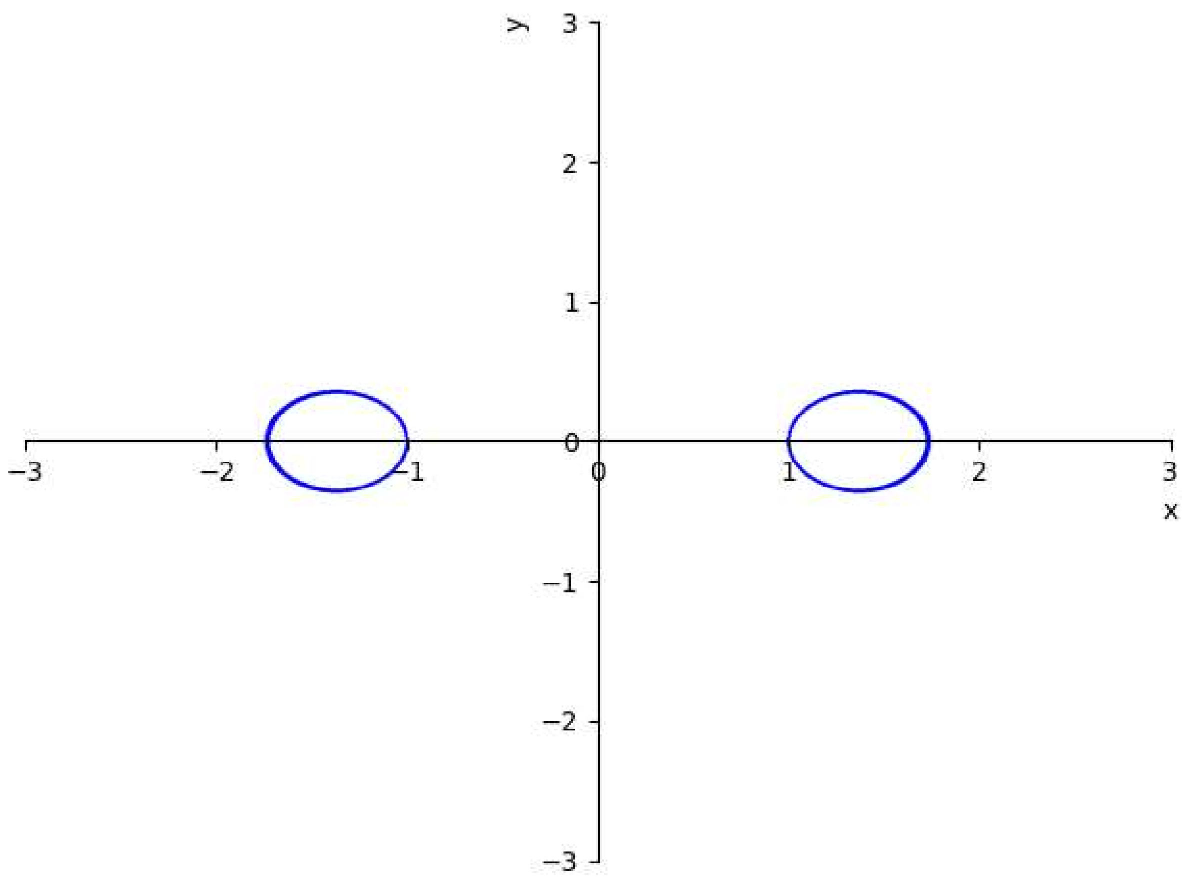
In [4]:
```
y = exp(-3 * x) * (cos(2 * x) + sin(2 * x))
dy = y.diff(x)
ddy = dy.diff(x)
simplify(ddy + 6 * dy + 13 * y)

# e^(-3x)(cos(2x)+sin(2x)) is a solution to the differential equation
```

Out[4]: 0

2a Implicit Plot

In [5]:
```
matplotlib notebook
```

In [6]:
```
x = symbols('x', real=True)
y = symbols('y', real=True)
F = (x**2 + y**2)**2 - (4 * (x**2 - y**2)) + 3
Fplot = plot_implicit(F, (x, -3, 3),(y,-3,3))
```



2b dy/dx

In [7]:
```
dydx = idiff(F, y, x)
print("The derivative of the function is", dydx)
```

```
The derivative of the function is x*(-x**2 - y**2 + 2)/(y*(x**2 + y**2 + 2))
```

2c Horizontal tangent lines

In [8]:
```
num = numer(dydx)
htan = solve([num, F],[x,y])
print("The horizontal tangents are", htan)
```

```
The horizontal tangents are [(-sqrt(30)/4, -sqrt(2)/4), (-sqrt(30)/4, sqrt(2)/4), (sqrt
(30)/4, -sqrt(2)/4), (sqrt(30)/4, sqrt(2)/4)]
```

2d Tangent line equation at (sqrt(2), sqrt(sqrt(17)-4))

In [9]:

```
point = {x:sqrt(2), y:sqrt(sqrt(17) - 4)}
m = dydx.subs(point)
line = m * (x - point[x]) + point[y]
print("The tangent line equation is", line)
```

```
The tangent line equation is -sqrt(34)*sqrt(-4 + sqrt(17))*(x - sqrt(2))/17 + sqrt(-4 +
sqrt(17))
```

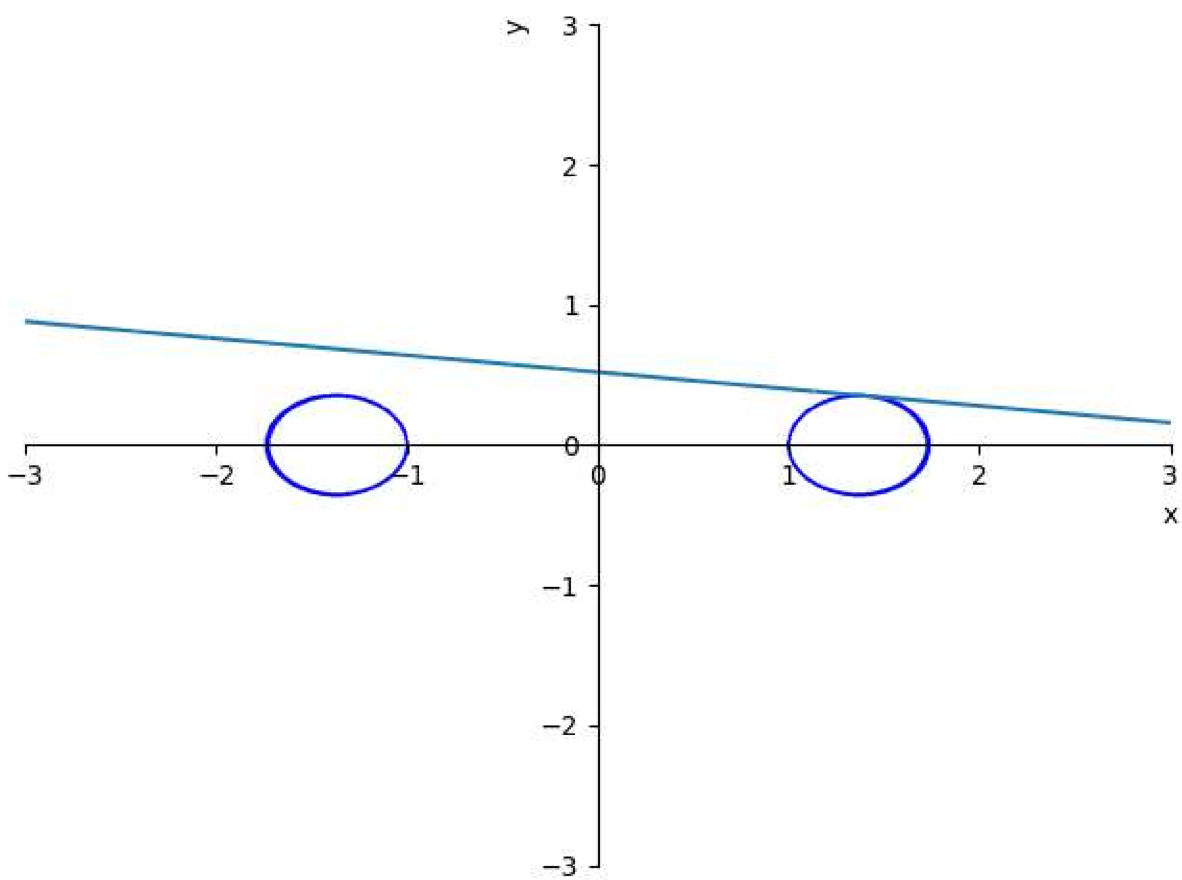
2e Plot equation with tangent line in part (d)

In [10]:

```
matplotlib notebook
```

In [11]:

```
pcurve = plot_implicit(F, (x, -3, 3), (y, -3, 3),show=False)
pcurve.extend(plot(line, show=False))
pcurve.show()
```



3a (n+1)th derivative of x^n ln(x)

In [13]:
```
x = symbols('x')
n = symbols('n')
eq = x ** n * ln(x)
[simplify(eq.subs(n, i).diff(x, i)) for i in range(2, 9)]
```

Out[13]:
```
[2*log(x) + 3,
 6*log(x) + 11,
 24*log(x) + 50,
 120*log(x) + 274,
 720*log(x) + 1764,
 5040*log(x) + 13068,
 40320*log(x) + 109584]
```

3b Conclusion

In [14]:
```
print("n!(ln(x) + 1 + 1/2 + ... + 1/n)")
```

```
n!(ln(x) + 1 + 1/2 + ... + 1/n)
```

4a Logarithmic Differentiation

In [15]:
```
x = symbols('x')
y = symbols('y')
q = sqrt(x) * log(x) - log(y)
simplify(idiff(q, y, x))
```

Out[15]:
$$\frac{y\left(\log\left(x\right)+2\right)}{2\sqrt{x}}$$

4b Direct Differentiation

In [16]:
```
q = x**sqrt(x)
simplify(diff(q, x))
```

Out[16]:
$$\frac{x^{\sqrt{x}-\frac{1}{2}}\left(\log\left(x\right)+2\right)}{2}$$