2. $I = \left| \int \rho(x', y') e^{-i\pi[(x-x')^2 + (y-y')^2]} dx' dy' \right|^2 = |g(x)|^2$

$$\int \rho(x', y') e^{-i\pi[(x-x')^2 + (y-y')^2]} dx' dy' = g(x, y)$$

$$\rho(x', y') = \begin{cases} 1 & \text{for } -\sqrt{10} \leq x \leq \sqrt{10} \\ 0 & \text{otherwise} \end{cases}$$

By symmetry:

$$g(x, y) \propto \int_{-\sqrt{10}}^{\sqrt{10}} \rho(x', y') e^{-i\pi(x-x')^2} dx' \qquad x - x' = \frac{t}{\sqrt{2}} \qquad dx' = -\frac{1}{\sqrt{2}} dt$$

$$= \frac{1}{\sqrt{2}} \int_{\sqrt{2}(x-\sqrt{10})}^{\sqrt{2}(x+\sqrt{10})} e^{-i\frac{\pi}{2}t^2} dt$$

$$|g(x, y)|^2 = \frac{1}{2} \left| \int_{\sqrt{2}(x-\sqrt{10})}^{\sqrt{2}(x+\sqrt{10})} \cos\left(\frac{\pi}{2}t^2\right) dt + i \int_{\sqrt{2}(x-\sqrt{10})}^{\sqrt{2}(x+\sqrt{10})} \sin\left(\frac{\pi}{2}t^2\right) dt \right|^2$$

$$= \frac{1}{2} \left[\left(\int_0^{\sqrt{2}(x+\sqrt{10})} \cos\left(\frac{\pi}{2}t^2\right) dt - \int_0^{\sqrt{2}(x-\sqrt{10})} \cos\left(\frac{\pi}{2}t^2\right) dt \right)^2 + \dots \right]$$

$$S(z) = \int_0^z \sin\left(\frac{\pi}{2}t^2\right) dt$$

$$C(z) = \int_0^z \cos\left(\frac{\pi}{2}t^2\right) dt$$

$$|g(x, y)|^2 \propto \frac{1}{2} \left[\left| S(\sqrt{2}(x+\sqrt{10})) - S(\sqrt{2}(x-\sqrt{10})) \right|^2 + \left| C(\sqrt{2}(x+\sqrt{10})) - C(\sqrt{2}(x-\sqrt{10})) \right|^2 \right]$$

3. $V = \frac{1}{2}kx^2 + \frac{1}{2}k(x-y)^2 + \frac{1}{2}ky^2$

$= k(x^2 + y^2 - xy)$

$m\ddot{x} = -\frac{\partial V}{\partial x} = -2kx + ky$

$m\ddot{y} = -\frac{\partial V}{\partial y} = kx - 2ky$

$m\ddot{x} = -\omega^2 x m$

$m\ddot{y} = -\omega^2 y m$

$m\omega^2 x = 2kx - ky$

$m\omega^2 y = -kx + 2ky$

$\frac{m\omega^2}{k}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2 & -1 \\ -1 & 2 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \lambda \begin{pmatrix} x \\ y \end{pmatrix} \Rightarrow \lambda = 3, 1$

$\omega = \sqrt{\frac{\lambda k}{m}} = \sqrt{\frac{3k}{m}}$ or $\sqrt{\frac{k}{m}}$