Structural Analysis of Four-bar mecanism

Homework support notes

2022-2023

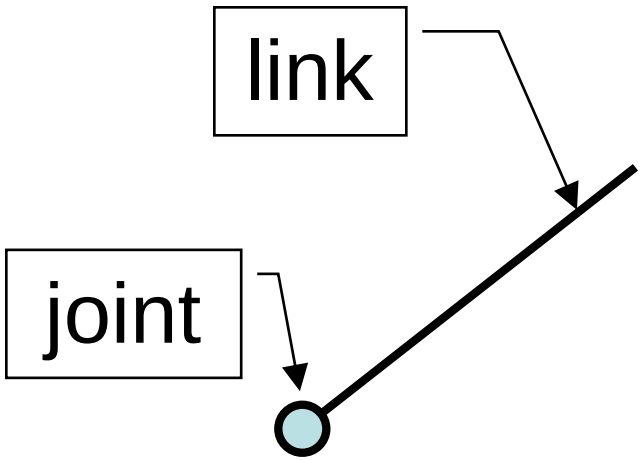
L. Ciupitu

Structure of a mechanism

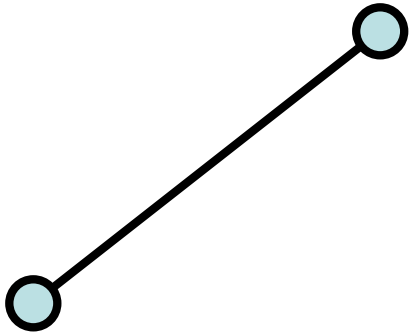
Mechanism = series of links (forming so called kinematic chains) joined together to produce a specific motion

- Links (element, member)
  - links are considered rigid bodies, but bodies with small stiffness (like springs or cables) could be also involved;
  - one link could be formed by one single machine element or by assemblies of machine elements that are forming one rigid system which is transmitting same motion
- Kinematic pair (joint) = functions which express the joining between two links so that the relative motion between these two links is consistent
  - Revolute (turning, pin, hinged, articulated) joint
  - Sliding (prismatic, translational) joint
  - Spherical (globular), planar joint

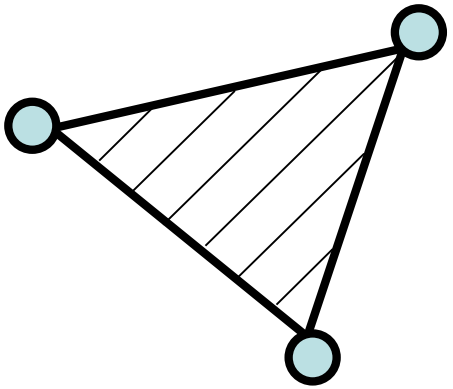
Links



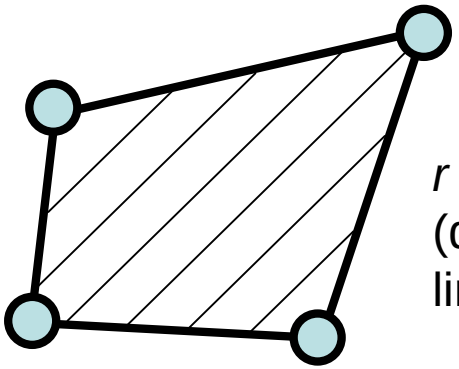
r = 1



r = 2 (binary link)



r = 3 (ternary link)



r = 4 (quaternary link)

Order (or rank) of a link represents the number of link's joints

Kinematic pair (joint)

- Pair between two elements made by direct, mobile and permanent contact
- The constrain of one link to move together with another link between there are relative motions lead to the losing of some degrees of freedom of each link.
- From relative motions point of view and by taking into consideration the number of restricted motions kinematic pairs are classified in five classes C_k (C_1, C_2 , C_3 , C_4 and C_5) where k represents number of relative restricted motions

Definitions

- Degrees of mobility or Mobility (*M*): Number of independent parameters that define the position (configuration) of a multi-body system with respect to a reference system attached to grounded element (Important observation: there is a confusion of terms with *Degrees of Freedom = DOF*).
- Computation formula (Chebychev–Grübler–Kutzbach) for planar mechanisms or for family 3 of mechanisms:

$$M_3 = 3\,m - 2\,l_p - h_p$$

m - number of mobile elements

l_p - number of lower pairs (or of C_5 class)

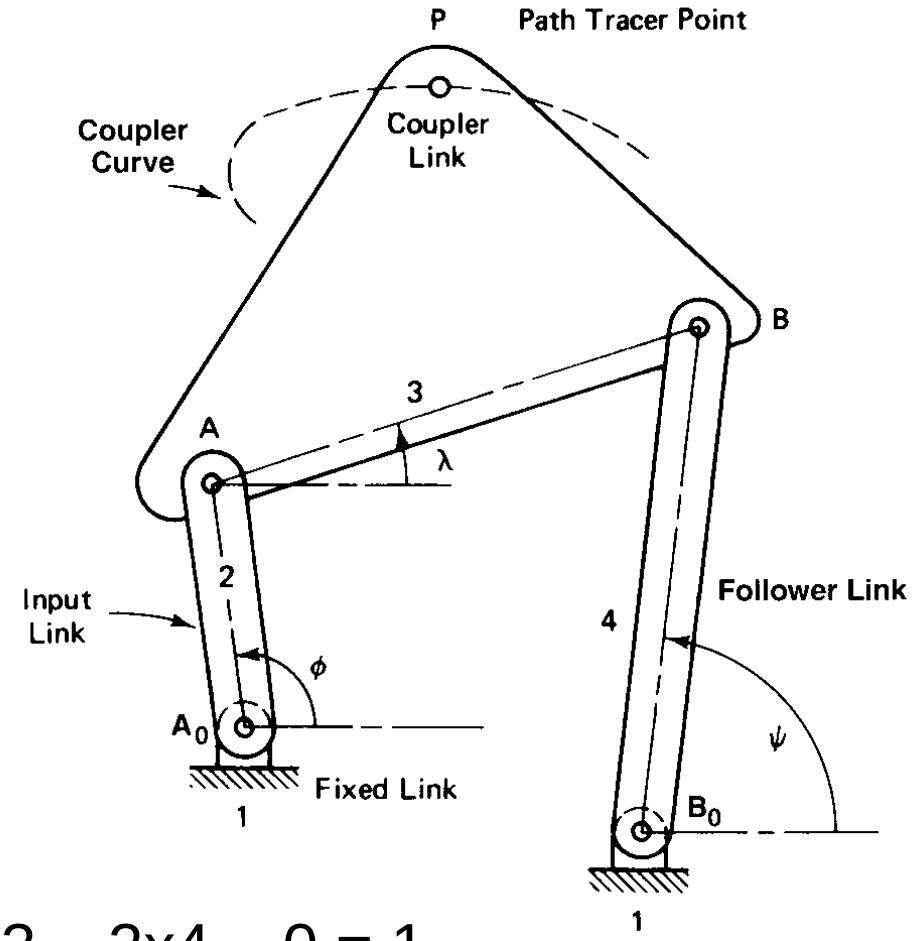
h_p - number of lower pairs (or of C_4 class)

Dictionary

- Crank: link which is making a complete revolution and is pivoted to ground
- Rocker: link which has oscillatory rotation and is pivoted to ground
- Connecting rod (coupler link): link which has a complex motion
- Ground: link fixed (non-moving) with respect to the reference frame

Four-Bar Linkage

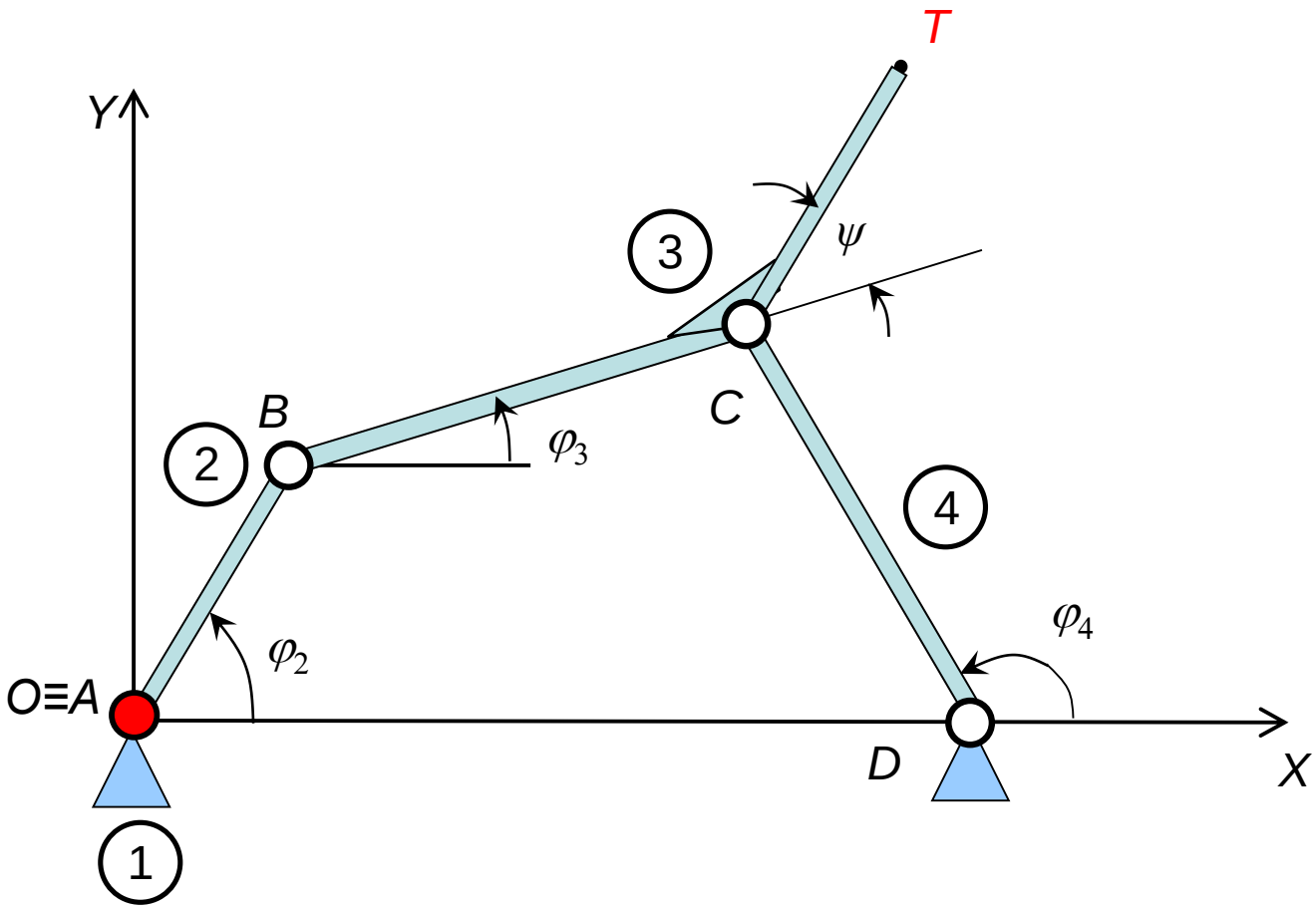
- Simplest closed-loop linkage; consists of three **moving** links, one **fixed** link (1), and four revolute (pin) joints.
- Primary links are called: the **input** link (connected to power source) denoted by (2), the **output** or **follower** link (4), and **coupler** or **floating** link (3). The latter "couples" the input to the output link.
- Points on the coupler link generally trace out **sixth order** algebraic coupler curves.



$$M_3 = 3\,m - 2\,l_p - h_p = 3\text{x}3 - 2\text{x}4 - 0 = 1$$

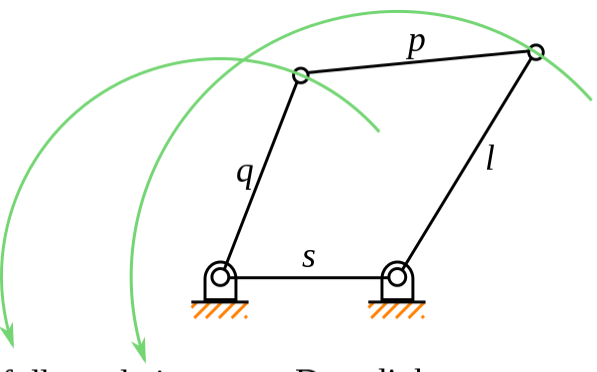
Number of independent loops: $N = l_p - n + 1 = l_p - m = 4 - 3 = 1$

Four-bar mechanism used as a path generator mechanism with a tracing point *T*



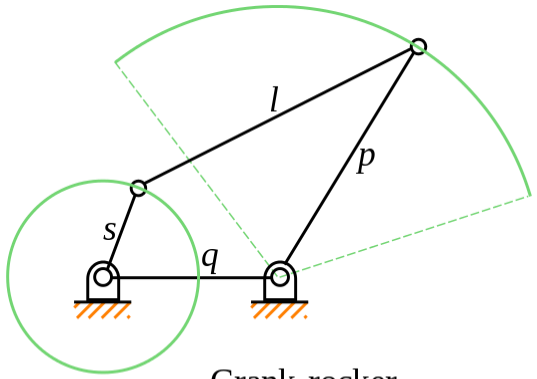
Observation: Cartesian reference system *XOY* is chosen in a particular manner so that origin *O* coincides with active pair from *A*

Grashof condition on four-bar mechanism

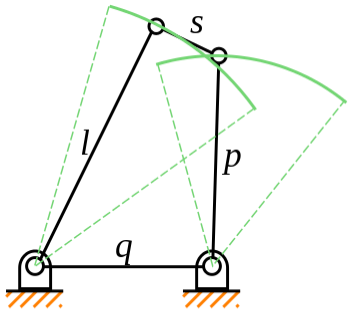


full revolution
both links

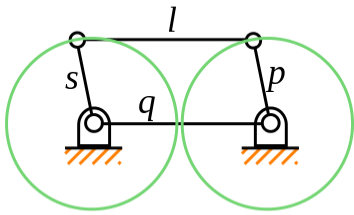
Drag-link
$s+l < p+q$
(continuous motion)



Crank-rocker
$s+l < p+q$
(continuous motion)



Double-rocker
$s+l > p+q$
(no continuous motion)



Parallelogram linkage
$s+l = p+q$
(continuous motion)

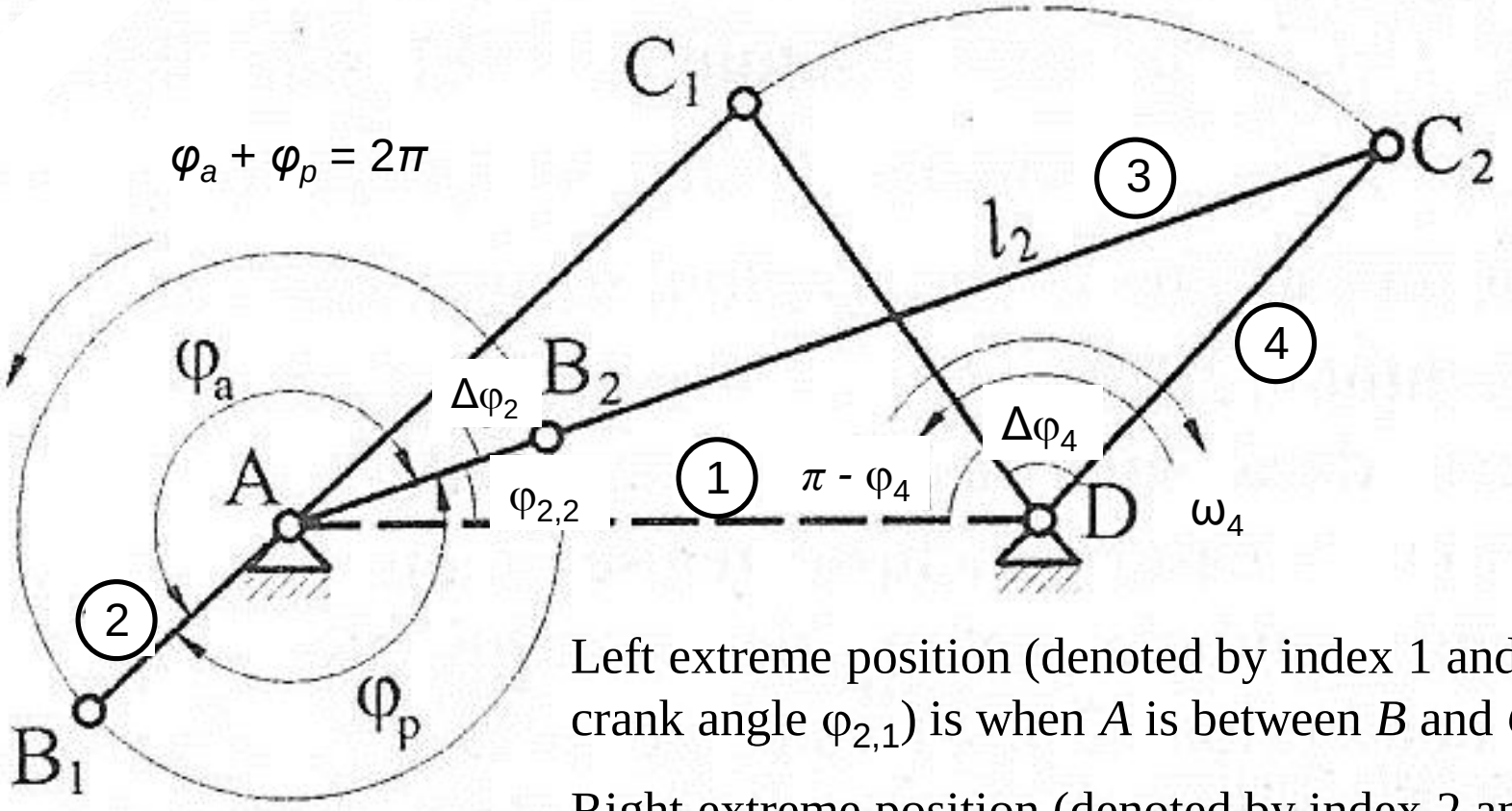
s is the shortest link,
l is the longest link,
p, *q* are the lengths of the other links

$$M_3 = 3x3 - 2x4 - 0 = 1$$

https://en.wikipedia.org/wiki/Four-bar_linkage

Extreme positions of four-bar mechanism (crank-rocker type)

This is happening when joints *A*, *B* and *C* are co-linear in the case of four-bar mechanism with one crank ! Exact values of angles of and crank ② of rocker ④ are obtained by using cosine theorem applied in triangles AC_1D and AC_2D respectively



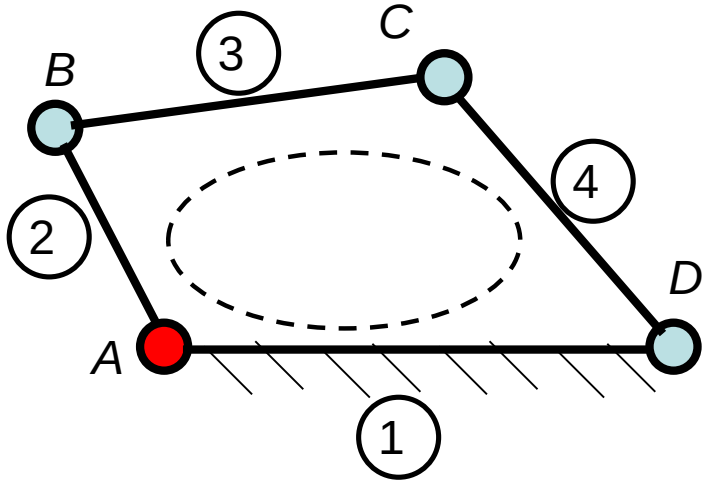
$\varphi_{2,1} = \varphi_{2,2} + \Delta\varphi_2$

Left extreme position (denoted by index 1 and crank angle $\varphi_{2,1}$) is when *A* is between *B* and *C*

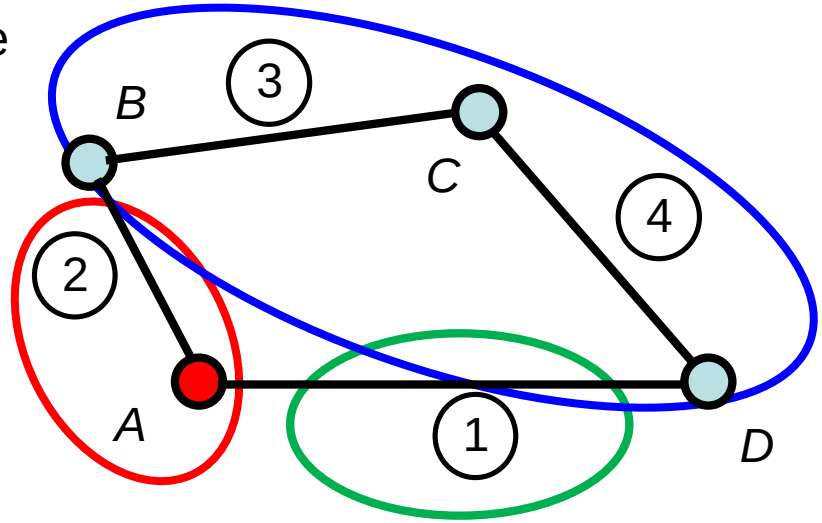
Right extreme position (denoted by index 2 and crank angle $\varphi_{2,2}$) is when *B* is between *A* and *C*

Four-bar mechanism

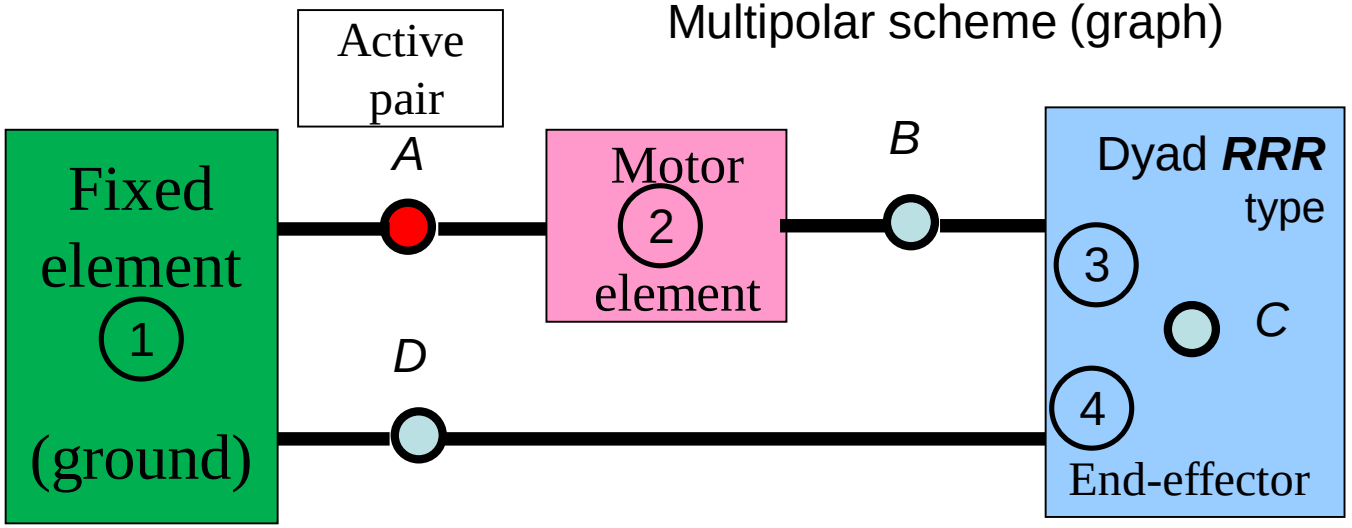
Skeleton Outline



Structural scheme



Multipolar scheme (graph)



Important observation

- Angle to a direction (or to a vector) is measured counter-clockwise from positive X axis, and its value is expressed in radians