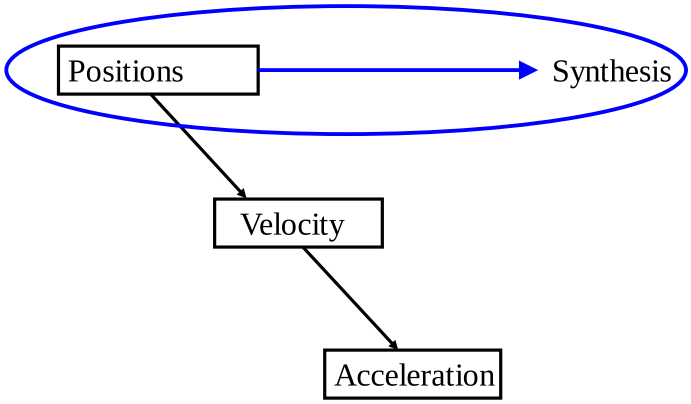
Lecture 4

- Kinematic Synthesis of four-bar mechanism: path generator mechanisms
- Kinematic Synthesis of intermittent rotary mechanisms: ratchet mechanism and Geneva mechanism

Categories of Application

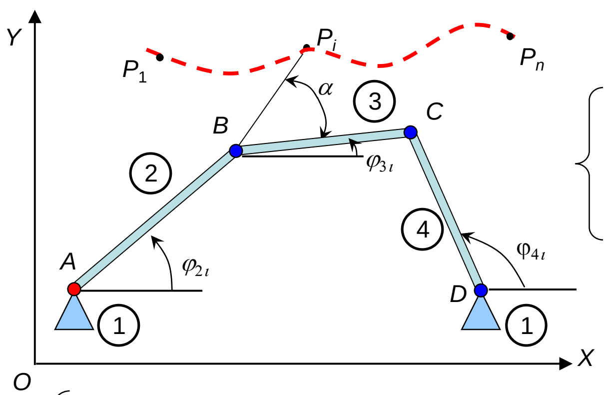
- **Function Generation:** linkage in which the relative motion between links connected to the ground is of interest
- **Path Generation:** concerned only with the path of a tracer point and not with the rotation of the coupler link (ex. Cebyshev four-bar mechanism)
- **Motion Generation:** entire motion of coupler link is of concern (ex. cam-mechanisms)

Kinematics Synthesis roadmap



Observation: Synthesis problem starts from requirements of the application where the mechanism should be used while it's dimensions are unknown; only it's structure is known !

Kinematic synthesis of four-bar mechanism used to approximate a curve given by *n* points



X_{Pi}, Y_{Pi} , i = 1, 2, … n are known

X_A, Y_A, X_D, Y_D, AB, BC, CD, BP and α are unknown (9 unknowns)

φ_{2i}, φ_{3i}, φ_{4i}, i = 1, 2, … n are unknown (3n unknowns)

$$\vec{OA} + \vec{AB} + \vec{BC} = \vec{OD} + \vec{DC}$$

$$\vec{OA} + \vec{AB} + \vec{BP} = \vec{OP_i}$$

$$\begin{cases} AB \cos \varphi_{2i} + BC \cos \varphi_{3i} - DC \cos \varphi_{4i} + X_A - X_D = 0 \\ AB \sin \varphi_{2i} + BC \sin \varphi_{3i} - DC \sin \varphi_{4i} + Y_A - Y_D = 0 \\ AB \cos \varphi_{2i} + BP \cos(\varphi_{3i} + \alpha) + X_A - X_{Pi} = 0 \\ AB \sin \varphi_{2i} + BC \sin(\varphi_{3i} + \alpha) + Y_A - Y_{Pi} = 0 \end{cases}$$

Compatibility equation:

$4n = 3n + 9$

$n = 9$

Kinematic synthesis of four-bar mechanism

$$\begin{cases} AB\cos\varphi_{2i} + BC\cos\varphi_{3i} - DC\cos\varphi_{4i} + X_A - X_D = 0 \\ AB\sin\varphi_{2i} + BC\sin\varphi_{3i} - DC\sin\varphi_{4i} + Y_A - Y_D = 0 \\ AB\cos\varphi_{2i} + BP\cos(\varphi_{3i} + \alpha) + X_A - X_{P_i} = 0 \\ AB\sin\varphi_{2i} + BP\sin(\varphi_{3i} + \alpha) + Y_A - Y_{P_i} = 0 \end{cases}$$

$\Rightarrow$ $\mathbf{F}(\mathbf{X}) = 0$ (in matrix form)

$$\mathbf{X} = \begin{pmatrix} X_A \\ Y_A \\ X_D \\ Y_D \\ AB \\ BC \\ CD \\ BP \\ \alpha \\ \varphi_{2i} \\ \varphi_{3i} \\ \varphi_{4i} \end{pmatrix}$$

Non-linear system with 36 equations and 36 unknown:

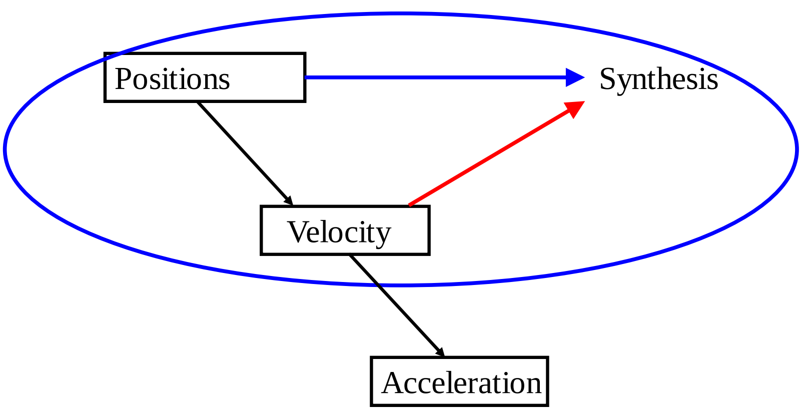
X_A, Y_A, X_D, Y_D, AB, BC, CD, BP, α si φ_{2i}, φ_{3i}, φ_{4i}, where i = 1, 2, …, 9

$$\left.\begin{array}{l} \mathbf{J}\,\Delta\mathbf{X} = -\mathbf{F} \\ \det(\mathbf{J}) \neq 0 \end{array}\right\} \Rightarrow \Delta\mathbf{X} = \mathbf{J}^{-1}(-\mathbf{F})$$

Intermediary solutions: $x_j^{(k)} = x_j^{(k-1)} + \Delta x_j^{(k)}$, where j = 1, 2, …, 36 and $k \geq 1$ (number of iterations)

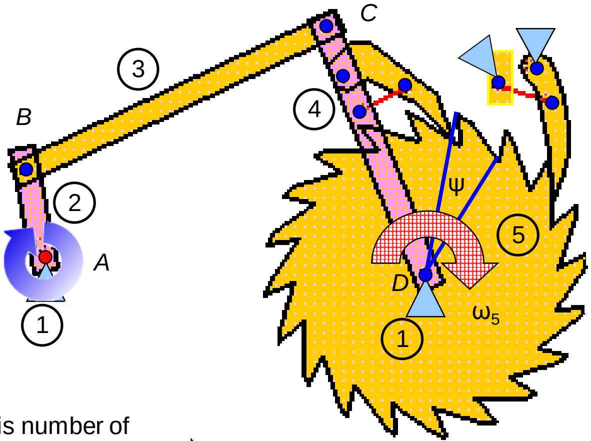
while $|\Delta x_j^{(k)}| > e_j$, j = 1, 2, …, 36

Kinematics Synthesis roadmap



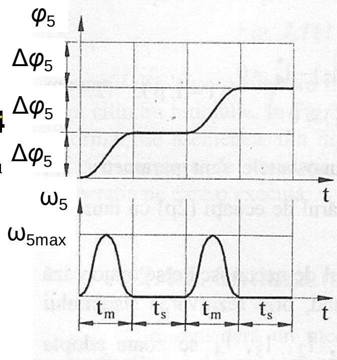
Sometimes the application where the mechanism should be used requires certain velocities or accelerations of some elements (usually the output element !) so that the synthesis problem should take into mathematical model the velocity and accelerations equations too.

Synthesis of a four-bar mechanism which is driving a given Ratchet mechanism



Functioning coefficient of ratchet mechanism:

$$\boxed{k_t = \frac{t_m}{t_s}}.$$



z is number of teeth of ratchet wheel ⇒ $\psi = 2\pi / z$

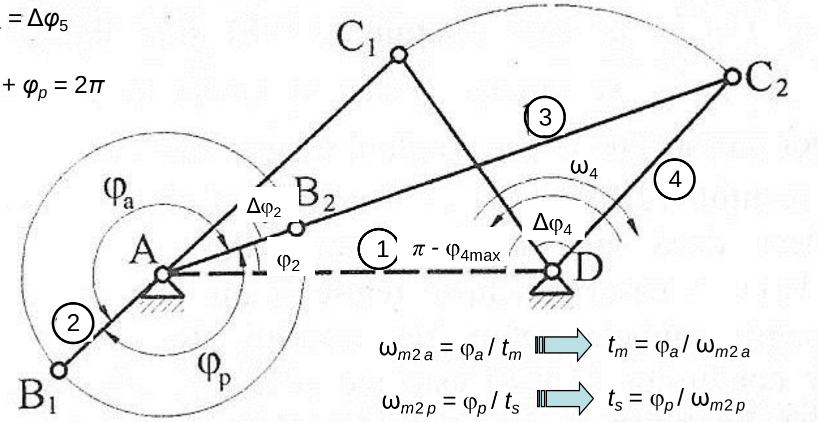
$\Delta\varphi_5 = N\,\psi$

Number *N* is giving the required angular step $\Delta\varphi_5$ which is known (as *z* is known too)

Extreme positions of four-bar mechanism

$\Delta\varphi_4 = \Delta\varphi_5$

$\varphi_a + \varphi_p = 2\pi$



$\omega_{m2\,a} = \varphi_a / t_m$ ⇒ $t_m = \varphi_a / \omega_{m2\,a}$

$\omega_{m2\,p} = \varphi_p / t_s$ ⇒ $t_s = \varphi_p / \omega_{m2\,p}$

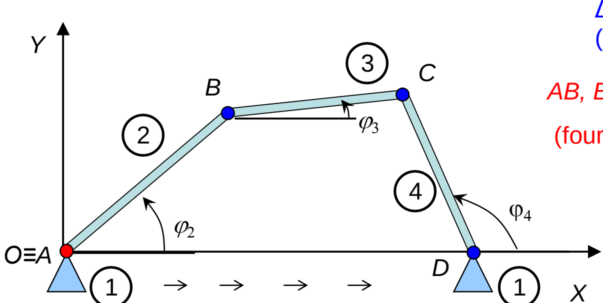
$k_t = t_m / t_s = \varphi_a / \varphi_p \;\; \omega_{m2\,p} / \omega_{m2\,a}$ ⇒ $\boxed{k_t = \varphi_a / \varphi_p}$

$\omega_{m2\,a} = \omega_{m2\,p} = \omega_2 = \text{ct.}$

$\varphi_a = 2\pi\,(k_t / (k_t + 1))$

$\varphi_p = 2\pi\,(1 / (k_t + 1))$

Kinematic synthesis of four-bar mechanism



$\vec{AB} + \vec{BC} = \vec{AD} + \vec{DC}$

$\Delta\varphi_5$, ω_{5max}, z and k_t are known
(ratchet mechanism is known)

AB, BC, CD, AD and ω_2 are unknown

(four-bar mechanism is unknown)

$$\begin{cases} \Delta\varphi_4 = \Delta\varphi_5 \\ \omega_{4\max} = \omega_{5\max} \end{cases}$$

$$\begin{cases} AB \cos \varphi_2 + BC \cos \varphi_3 - DC \cos \varphi_4 - AD = 0 \\ AB \sin \varphi_2 + BC \sin \varphi_3 - DC \sin \varphi_4 = 0 \end{cases}$$

Position equations

$$\begin{cases} -\omega_2 AB \sin \varphi_2 - \omega_3 BC \sin \varphi_3 + \omega_4 DC \sin \varphi_4 = 0 \\ \omega_2 AB \cos \varphi_2 + \omega_3 BC \cos \varphi_3 - \omega_4 DC \cos \varphi_4 = 0 \end{cases}$$

Velocity equations

Function Generation Mechanisms Graphical Solution

Two position synthesis – Design a four-bar crank and rocker mechanism to give for example 30º of rocker rotation with equal time forward and back (k_t = 1), from a constant speed ω_2 motor input.

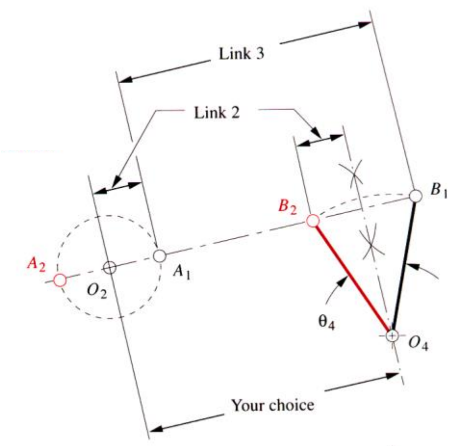
1 – Draw the rocker O_4B in both extreme positions, B_1 and B_2 in any convenient location with angle $\theta_4 = 30^o$.

2 – select a convenient point O_2 on line B_1B_2 extended.

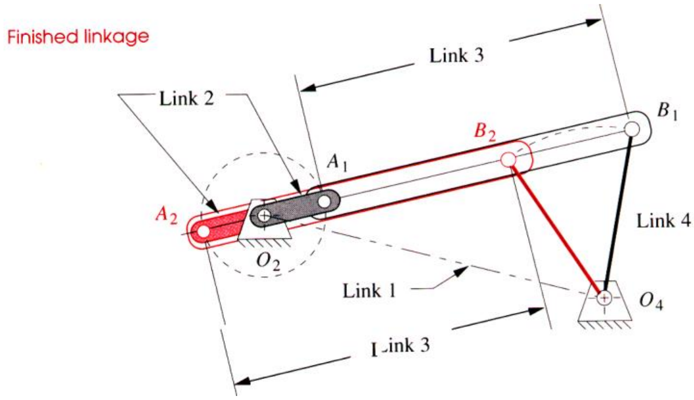
3 – Bisect line B_1B_2 , and draw a circle with that radius about O_2.

4 – Label the two intersection of the circle with B_1B_2 extended, A_1 and A_2.

5 – Measure O_2A (crank, link 2) and *AB* (coupler, link 3).

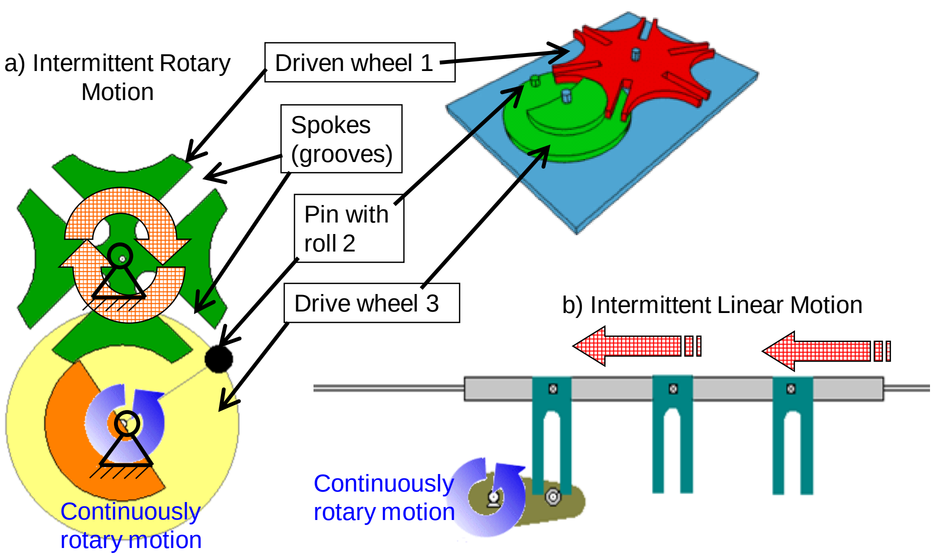


Function Generation Mechanisms
Graphical Solution



Observation: Rocker is represented in simplified form as a line like in skeleton outline.

Geneva or Maltese Cross Mechanisms

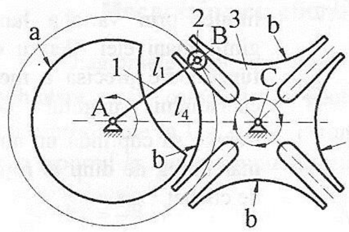


https://en.wikipedia.org/wiki/Geneva_drive

Rotary Geneva Mechanisms

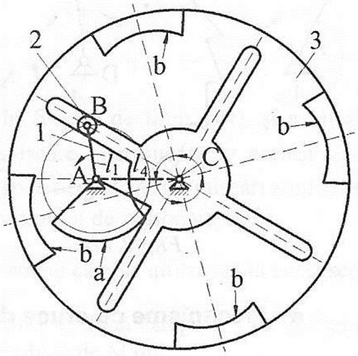
$$2\varphi_3 = \ \psi = \frac{2\pi}{z} = \frac{360^0}{z}.\ z = \text{number of grooves}$$

$$\varphi_1 = \frac{\pi}{2} - \frac{\pi}{z} \qquad \Delta\varphi_{1m} = 2\,\varphi_1 = \pi\,(1 - \frac{2}{z})$$



a) External-groove Geneva

$$\Delta\varphi_{1s} = 2\,\pi - \Delta\varphi_{1m} = \pi\,(1 + \frac{2}{z})$$



b) Internal-groove Geneva

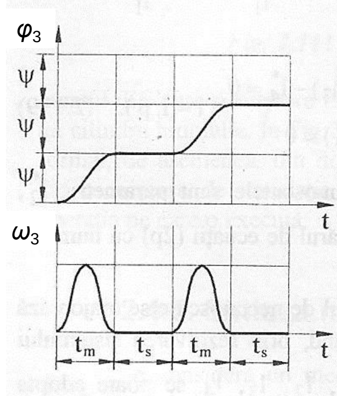
Geneva or Maltese Cross Mechanisms

Functioning coefficient:

$$k_t = \frac{t_m}{t_s} = \frac{z-2}{z+2}$$

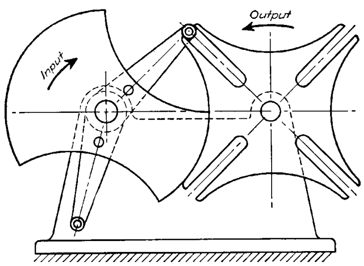
$$t_m = \frac{\Delta\varphi_m}{\omega_1} = \frac{2\varphi_1}{\omega_1}$$

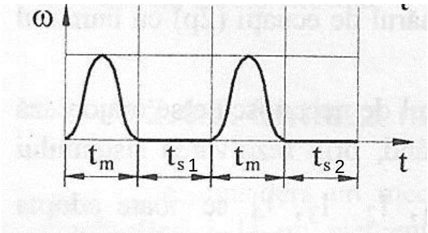
$$t_s = \frac{\Delta\varphi_s}{\omega_1} = \frac{2\pi}{\omega_1} - t_m$$



Geneva Mechanisms

Irregular Geneva drive mechanism





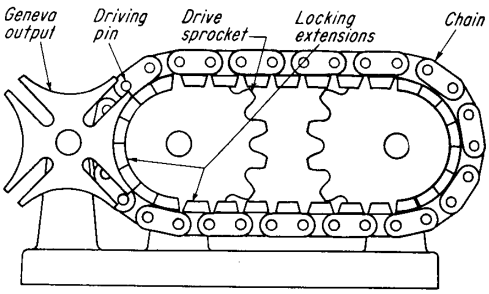
The duration of the dwell periods is changed by arranging the driving rollers asymmetrically around the input shaft.

$t_{s1} \neq t_{s2}$

Which period is changed: motion time t_m or stationary time t_s ?

Geneva Mechanisms

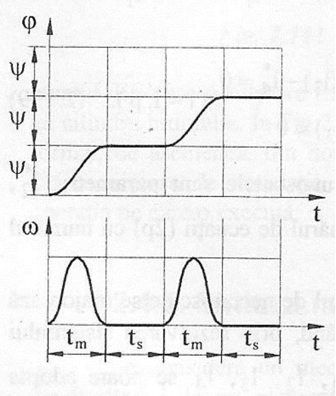
Long-dwell Geneva mechanism



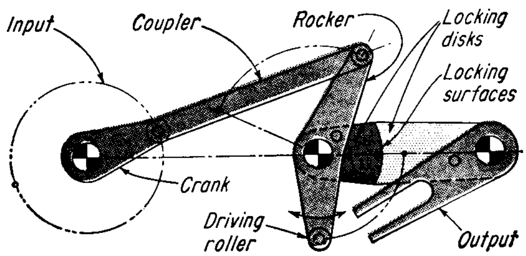
The spacing between the sprockets determines the length of dwell

Functioning coefficient:

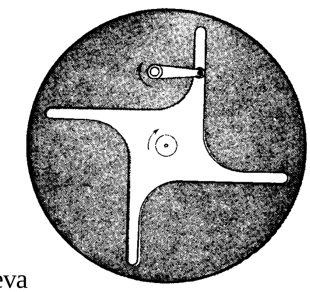
$$k_t = \frac{t_m}{t_s}.$$



Geneva Mechanisms

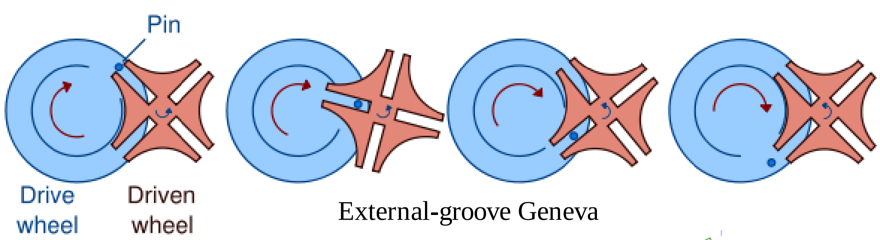


Four-bar Geneva

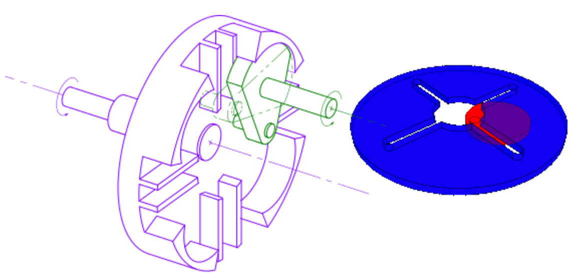


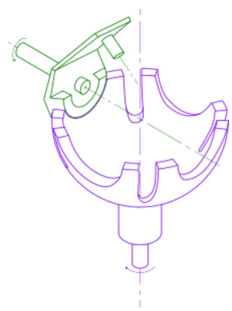
Internal-groove Geneva

Geneva Mechanisms or Maltese Cross



External-groove Geneva



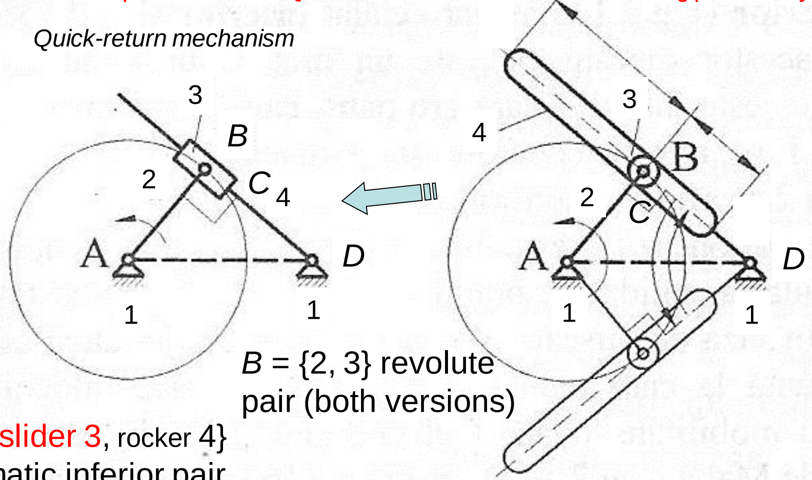


Internal-groove Geneva

Spatial Geneva Mechanism

Equivalence of Rotary Geneva Mechanisms

Observation: Equivalence with Quick-return mechanism is on moving phase only.



B = {2, 3} revolute pair (both versions)

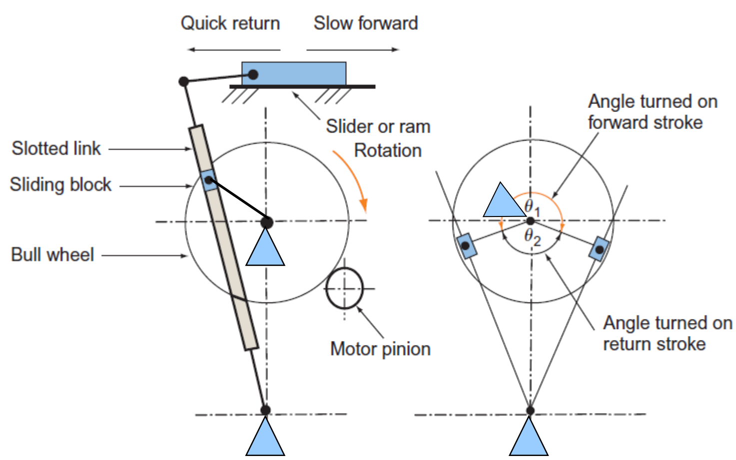
C = {slider 3, rocker 4} prismatic inferior pair

C = {roll 3, rocker 4} higher pair

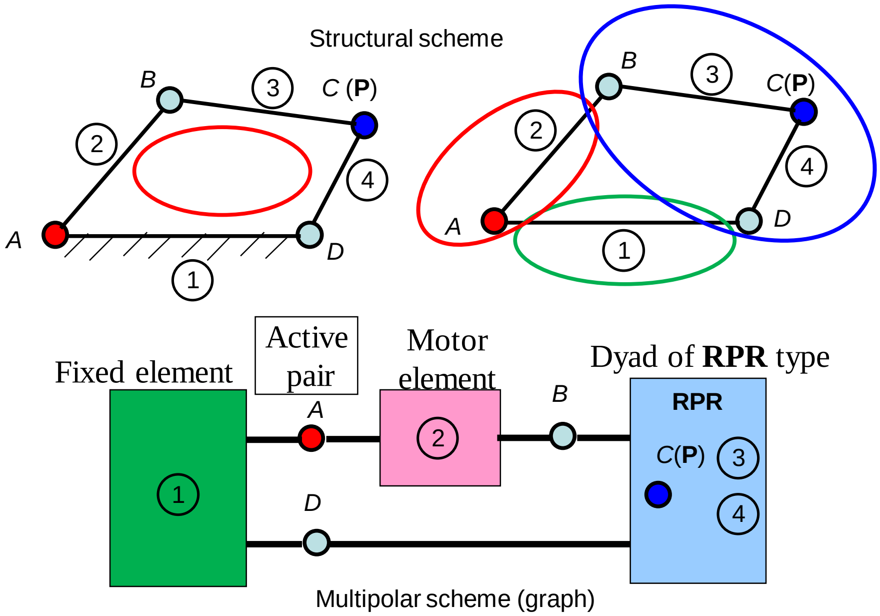
A = {1, 2} revolute pair (both versions)

D = {1, 4} revolute pair (both versions)

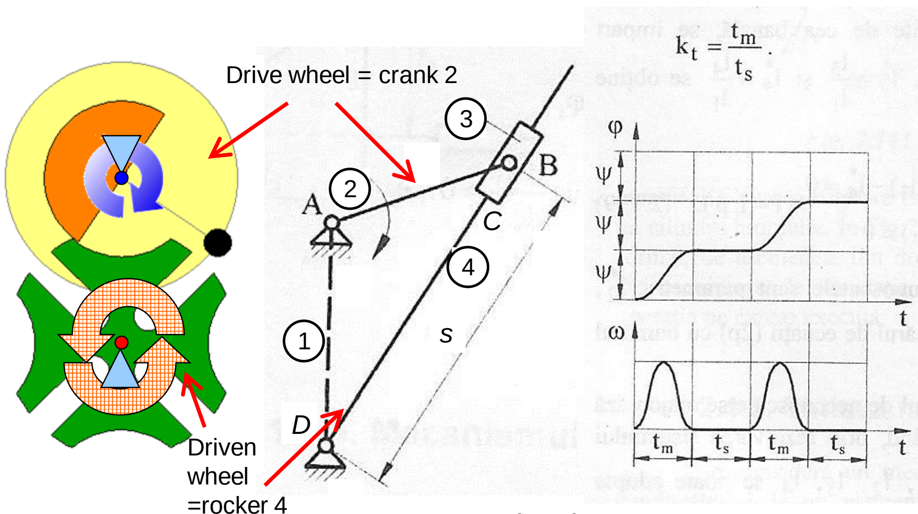
Quick-return mechanism (Crank-slider-rocker mechanism)



Crank-slider-rocker mechanism



Geneva Mechanisms - equivalence mechanism

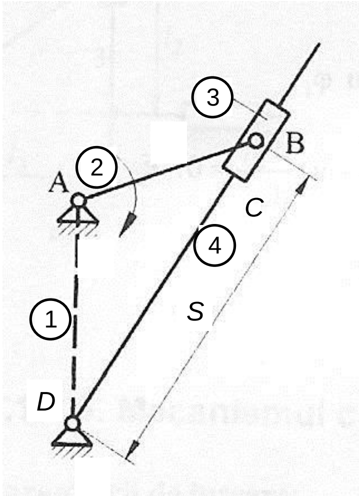


$$k_t = \frac{t_m}{t_s}.$$

$$M_3 = 3\,m - 2\,l_p - h_p = 3x3 - 2x4 - 0 = 1$$

$$N = l_p - n + 1 = l_p - m = 4 - 3 = 1$$

Crank-slider-rocker mechanism



$\Delta\varphi_4$, ω_{4max} , z and k_t are known (Geneva mechanism)

AB, AD and ω_2 are unknown (crank-slider-rocker mechanism)

$$\overrightarrow{AB} = \overrightarrow{AD} + \overrightarrow{s}$$

$$\begin{cases} AB \cos \varphi_2 - S \cos \varphi_4 = 0 \\ AB \sin \varphi_2 - S \sin \varphi_4 - AD = 0 \end{cases}$$

$$\begin{cases} S\,\omega_4 \sin \varphi_4 - AB\,\omega_2 \sin \varphi_2 = 0 \\ AB\,\omega_2 \cos \varphi_2 - S\,\omega_4 \cos \varphi_4 = 0 \end{cases}$$

Attention: Find all 4 extreme positions of this mechanism ! Extreme positions of slider 3 (along the sliding way) and extreme positions of rocker 4.

Conclusions

- For kinematic synthesis of a four-bar mechanism in order to describe a specific curve, only the position equations were used
- For kinematic synthesis of ratchet and Geneva mechanisms both position and velocity equations were used
- Mathematical model expressed by non-linear equations in a great number with many unknowns requires numerical methods in order to solve it.