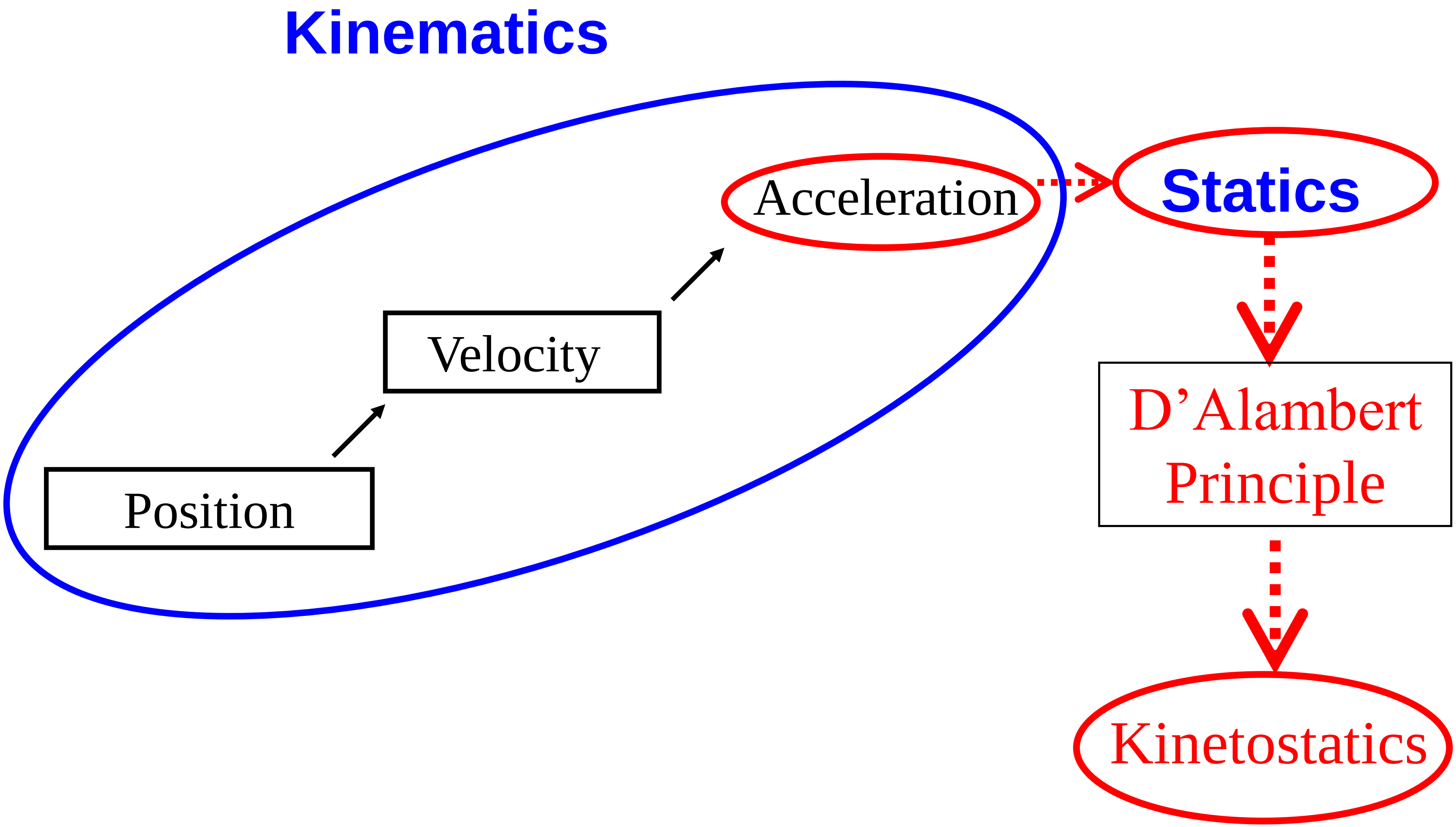
Kinetostatics analysis of Four-bar mechanism

Homework support notes

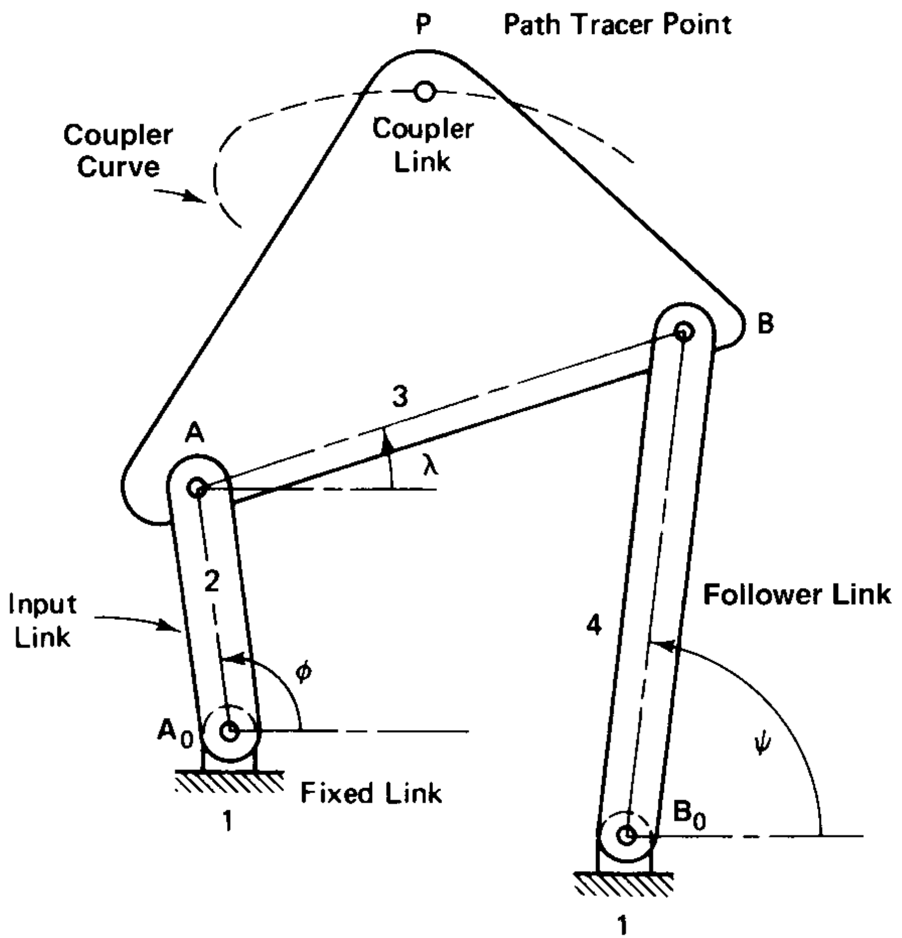
2021-2022

Kinetostatic analysis roadmap



Four-Bar Linkage

- Simplest closed-loop linkage; consists of three **moving** links, one **fixed** link (1), and four revolute (pin) joints.
- Primary links are called: the **input** link (connected to power source) denoted by (2), the **output** or **follower** link (4), and **coupler** or **floating** link (3). The latter "couples" the input to the output link.
- Points as *P* on the coupler link generally trace out **sixth order** algebraic coupler curves.

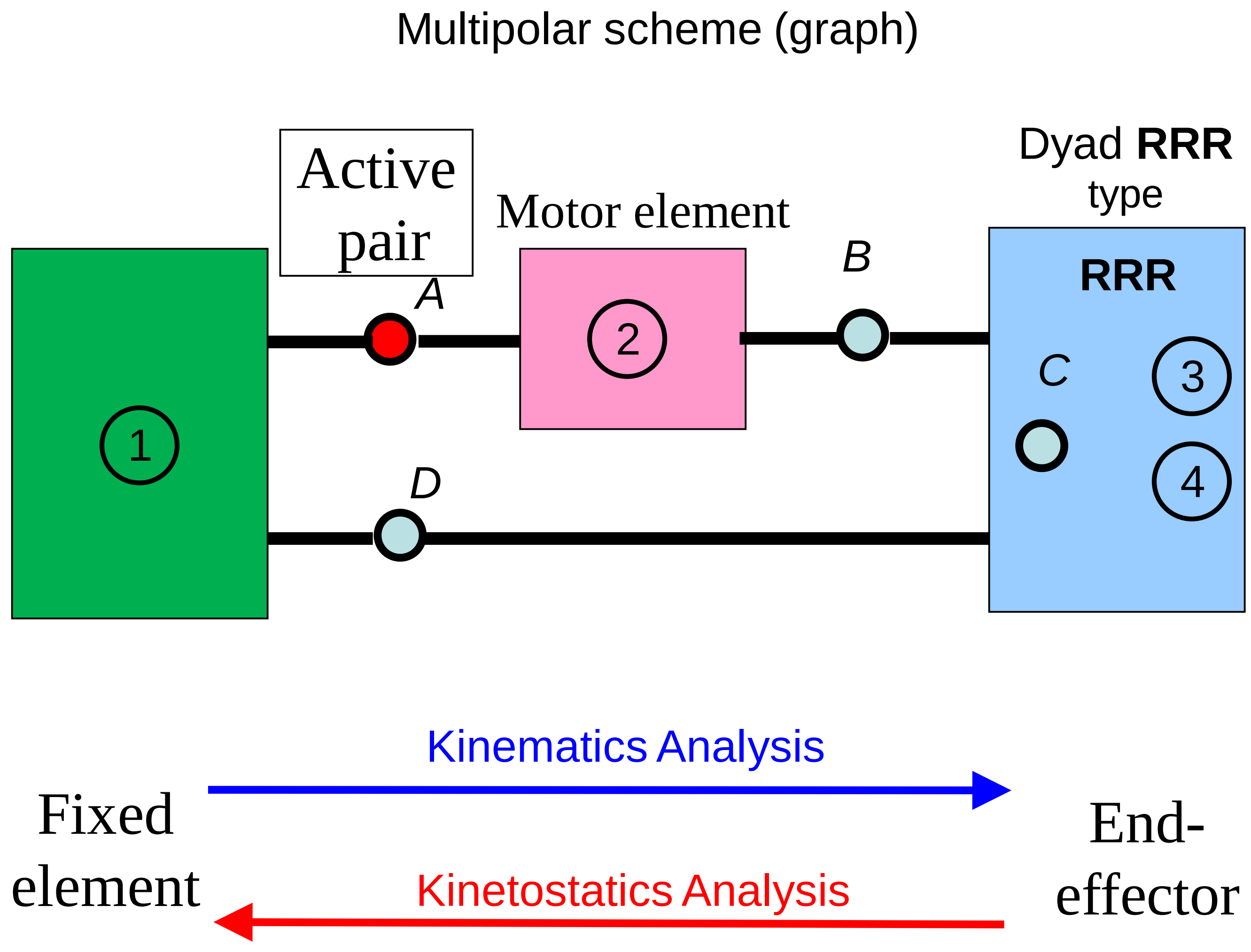


$$M_3 = 3\,m - 2\,l_p - h_p = 3x3 - 2x4 - 0 = 1$$

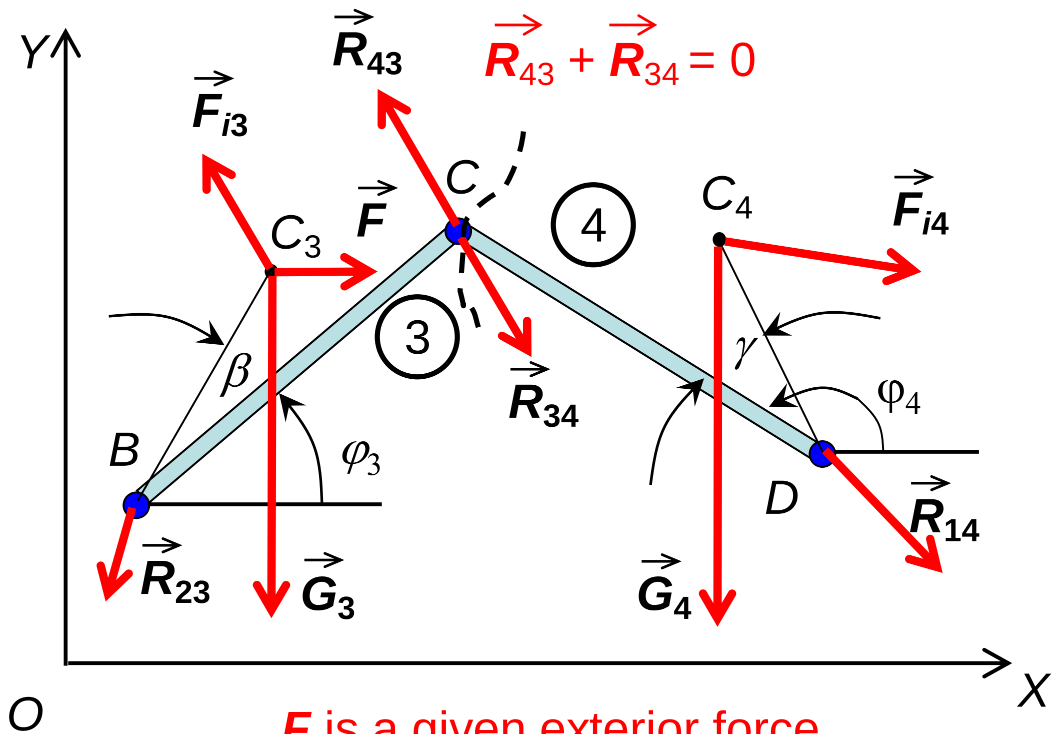
Number of independent loops: $N = l_p - n + 1 = l_p - m = 4 - 3 = 1$

Observation: *n* is the total number of elements

Four-bar mechanism



Kinetostatic analysis of Dyad - **RRR** type



F is a given exterior force

$$\begin{cases} X_{C_3} = X_B + BC_3 \cos(\varphi_3 + \beta) \\ Y_{C_3} = Y_B + BC_3 \sin(\varphi_3 + \beta) \end{cases}$$

$$\begin{cases} X_{C_4} = X_D + DC_4 \cos(\varphi_4 - \gamma) \\ Y_{C_4} = Y_D + DC_4 \sin(\varphi_4 - \gamma) \end{cases}$$

$$\begin{cases} v_{C_3X} = v_{BX} - BC_3\, \omega_3 \sin(\varphi_3 + \beta) \\ v_{C_3Y} = v_{BY} + BC_3\, \omega_3 \cos(\varphi_3 + \beta) \end{cases}$$

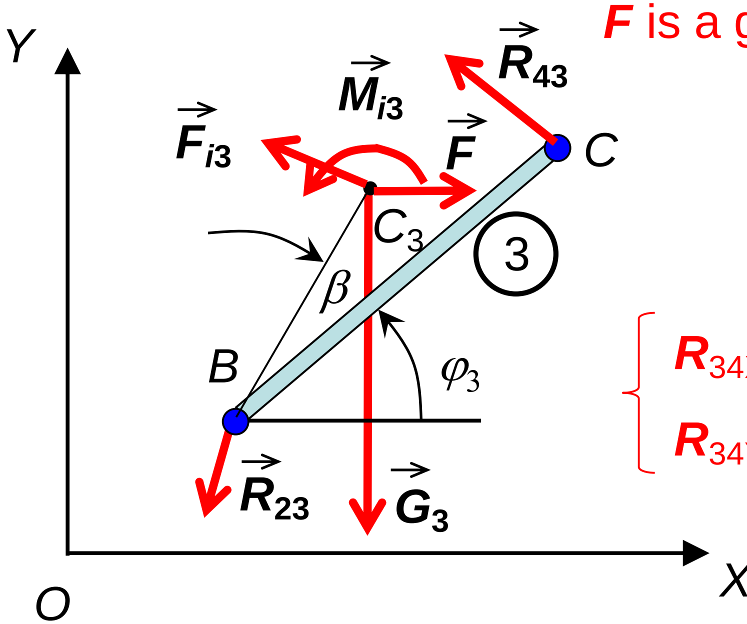
$$\begin{cases} v_{C_4X} = v_{DX} - DC_4\, \omega_4 \sin(\varphi_4 - \gamma) \\ v_{C_4Y} = v_{DY} + DC_4\, \omega_4 \cos(\varphi_4 - \gamma) \end{cases}$$

$$\begin{cases} a_{C_3X} = a_{BX} - BC_3\, (\varepsilon_3 \sin(\varphi_3 + \beta) + \omega_3^2 \cos(\varphi_3 + \beta)) \\ a_{C_3Y} = a_{BY} + BC_3\, (\varepsilon_3 \cos(\varphi_3 + \beta) - \omega_3^2 \sin(\varphi_3 + \beta)) \end{cases}$$

$$\begin{cases} a_{C_4X} = a_{DX} - DC_4\, (\varepsilon_4 \sin(\varphi_4 - \gamma) + \omega_4^2 \cos(\varphi_4 - \gamma)) \\ a_{C_4Y} = a_{DY} + DC_4\, (\varepsilon_4 \cos(\varphi_4 - \gamma) - \omega_4^2 \sin(\varphi_4 - \gamma)) \end{cases}$$

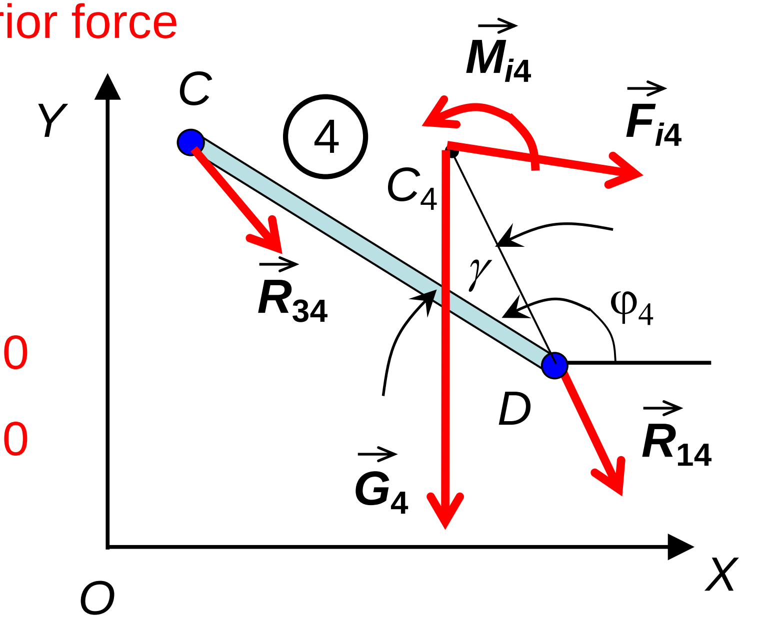
And where are needed too the axial inertial moments with respect to centres of masses of elements: I_2, I_3 and I_4

Kinetostatic analysis of Dyad - **RRR** type



F is a given exterior force

$$\begin{cases} R_{34X} + R_{43X} = 0 \\ R_{34Y} + R_{43Y} = 0 \end{cases}$$



$$\vec{G}_3 + \vec{F} + \vec{R}_{23} + \vec{R}_{43} + \vec{F}_{i3} = 0$$

$$\begin{cases} F + R_{23X} + R_{43X} - m_3\, a_{C_3X} = 0 \\ R_{23Y} + R_{43Y} - m_3\,(g + a_{C_3Y}) = 0 \end{cases}$$

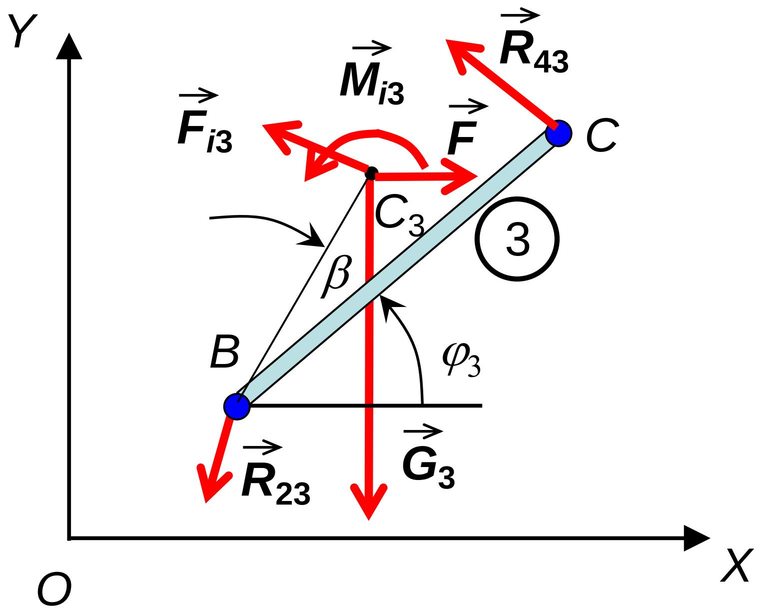
$$\vec{G}_4 + \vec{R}_{14} + \vec{R}_{34} + \vec{F}_{i4} = 0$$

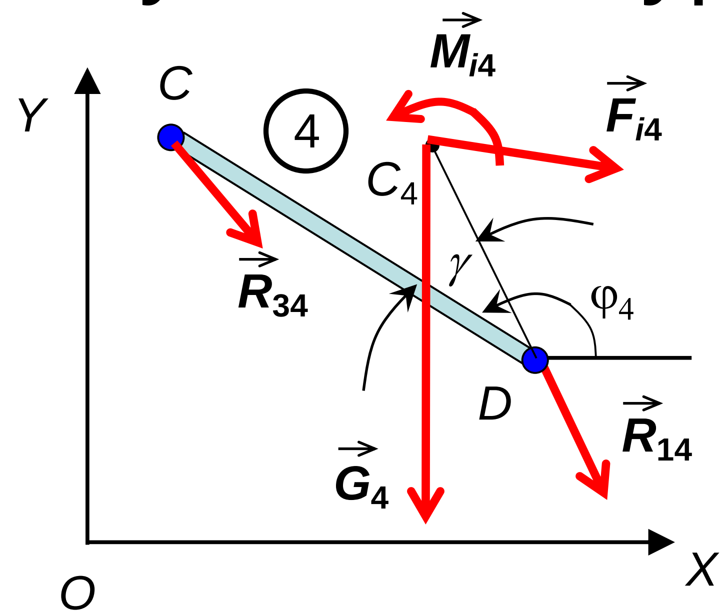
$$\begin{cases} R_{14X} + R_{34X} - m_4\, a_{C_4X} = 0 \\ R_{14Y} + R_{34Y} - m_4\,(g + a_{C_4Y}) = 0 \end{cases}$$

Equilibrium of moments equations:

$$\begin{cases} R_{23Y}\,(X_B - X_{C3}) - R_{23X}\,(Y_B - Y_{C3}) + R_{43Y}\,(X_C - X_{C3}) - R_{43X}\,(Y_C - Y_{C3}) + M_{i3} = 0 \\ R_{14Y}\,(X_D - X_{C4}) - R_{14X}\,(Y_D - Y_{C4}) + R_{34Y}\,(X_C - X_{C4}) - R_{34X}\,(Y_C - Y_{C4}) + M_{i4} = 0 \end{cases}$$

Kinetostatic analysis of Dyad - **RRR** type





$$\begin{cases} \boldsymbol{R}_{34X} + \boldsymbol{R}_{43X} = 0 \\ \boldsymbol{R}_{34Y} + \boldsymbol{R}_{43Y} = 0 \end{cases}$$

$$\begin{cases} \boldsymbol{R}_{34X} = -\boldsymbol{R}_{43X} \\ \boldsymbol{R}_{34Y} = -\boldsymbol{R}_{43Y} \end{cases}$$

$$\begin{cases} \boldsymbol{R}_{14X} - \boldsymbol{R}_{43X} - m_4\, a_{C_4X} = 0 \\ \boldsymbol{R}_{14Y} - \boldsymbol{R}_{43Y} - m_4\,(g + a_{C_4Y}) = 0 \end{cases}$$

$$\begin{cases} \boldsymbol{R}_{23X} + \boldsymbol{R}_{43X} - m_3\, a_{C_3X} + \boldsymbol{F} = 0 \\ \boldsymbol{R}_{23Y} + \boldsymbol{R}_{43Y} - m_3\,(g + a_{C_3Y}) = 0 \end{cases}$$

$$\begin{cases} \boldsymbol{R}_{23Y}\,(X_B - X_{C_3}) - \boldsymbol{R}_{23X}\,(Y_B - Y_{C_3}) + \boldsymbol{R}_{43Y}\,(X_C - X_{C_3}) - \boldsymbol{R}_{43X}\,(Y_C - Y_{C_3}) - I_3\,\varepsilon_3 = 0 \\ \boldsymbol{R}_{14Y}\,(X_D - X_{C_4}) - \boldsymbol{R}_{14X}\,(Y_D - Y_{C_4}) - \boldsymbol{R}_{43Y}\,(X_C - X_{C_4}) + \boldsymbol{R}_{43X}\,(Y_C - Y_{C_4}) - I_4\,\varepsilon_4 = 0 \end{cases}$$

6 linear equations with 6 unknowns

Kinetostatic analysis of Dyad - **RRR** type

$$\begin{cases} \boldsymbol{R}_{34X} + \boldsymbol{R}_{43X} = 0 \\ \boldsymbol{R}_{34Y} + \boldsymbol{R}_{43Y} = 0 \end{cases} \Rightarrow \begin{cases} \boldsymbol{R}_{34X} = -\boldsymbol{R}_{43X} \\ \boldsymbol{R}_{34Y} = -\boldsymbol{R}_{43Y} \end{cases} \Rightarrow \begin{cases} \boldsymbol{R}_{14X} - \boldsymbol{R}_{43X} - m_4\, a_{C_4X} = 0 \\ \boldsymbol{R}_{14Y} - \boldsymbol{R}_{43Y} - m_4\,(g + a_{C_4Y}) = 0 \end{cases}$$

$$\Downarrow$$

and

$$\begin{cases} \boldsymbol{R}_{14X} = \boldsymbol{R}_{43X} + m_4\, a_{C_4X} \\ \boldsymbol{R}_{14Y} = \boldsymbol{R}_{43Y} + m_4\,(g + a_{C_4Y}) \end{cases}$$

$$\begin{cases} \boldsymbol{R}_{23X} + \boldsymbol{R}_{43X} - m_3\, a_{C_3X} + \boldsymbol{F} = 0 \\ \boldsymbol{R}_{23Y} + \boldsymbol{R}_{43Y} - m_3\,(g + a_{C_3Y}) = 0 \end{cases} \Rightarrow \begin{cases} \boldsymbol{R}_{23X} = \boldsymbol{R}_{43X} + m_3\, a_{C_3X} - \boldsymbol{F} = 0 \\ \boldsymbol{R}_{23Y} = \boldsymbol{R}_{43Y} + m_3\,(g + a_{C_3Y}) = 0 \end{cases}$$

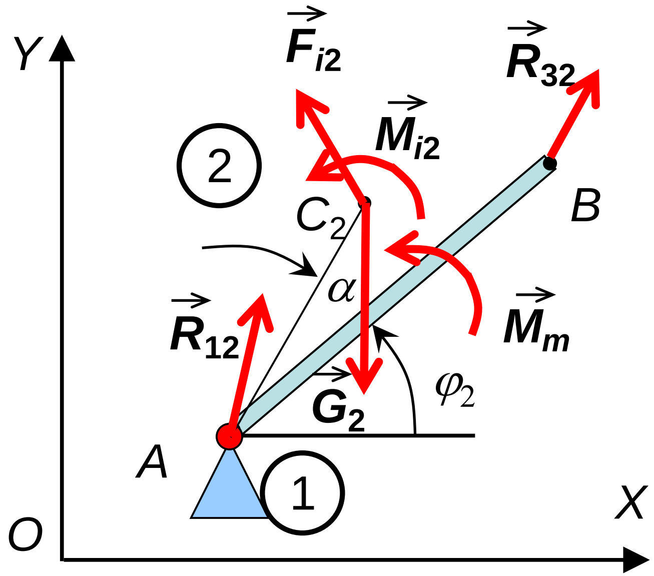
$$\Downarrow$$

$$\begin{cases} \boldsymbol{R}_{23Y}\,(X_B - X_{C_3}) - \boldsymbol{R}_{23X}\,(Y_B - Y_{C_3}) + \boldsymbol{R}_{43Y}\,(X_C - X_{C_3}) - \boldsymbol{R}_{43X}\,(Y_C - Y_{C_3}) - I_3\,\varepsilon_3 = 0 \\ \boldsymbol{R}_{14Y}\,(X_D - X_{C_4}) - \boldsymbol{R}_{14X}\,(Y_D - Y_{C_4}) - \boldsymbol{R}_{43Y}\,(X_C - X_{C_4}) + \boldsymbol{R}_{43X}\,(Y_C - Y_{C_4}) - I_4\,\varepsilon_4 = 0 \end{cases}$$

$$\Downarrow$$

$$\boldsymbol{R}_{43X}\,,\ \boldsymbol{R}_{43Y} \Rightarrow \boldsymbol{R}_{34X}\,,\ \boldsymbol{R}_{34Y},\ \boldsymbol{R}_{14X}\,,\ \boldsymbol{R}_{14Y}\,,\ \boldsymbol{R}_{23X}\,,\ \boldsymbol{R}_{23Y}$$

Kinetostatic analysis of motor element



$$\begin{cases} X_{C_2} = X_A + AC_2 \cos(\varphi_2 + \alpha) \\ Y_{C_2} = Y_A + AC_2 \sin(\varphi_2 + \alpha) \end{cases}$$

$$\begin{cases} v_{C_2X} = - AC_2\, \omega_2 \sin(\varphi_2 + \alpha) \\ v_{C_2Y} = AC_2\, \omega_2 \cos(\varphi_2 + \alpha) \end{cases}$$

$$\begin{cases} a_{C2X} = - AC_2\, \omega_2^2 \cos(\varphi_2 + \alpha) \\ a_{C_2Y} = -AC_2\, \omega_2^2 \sin(\varphi_2 + \alpha)) \end{cases}$$ when $\varepsilon_2 = 0$

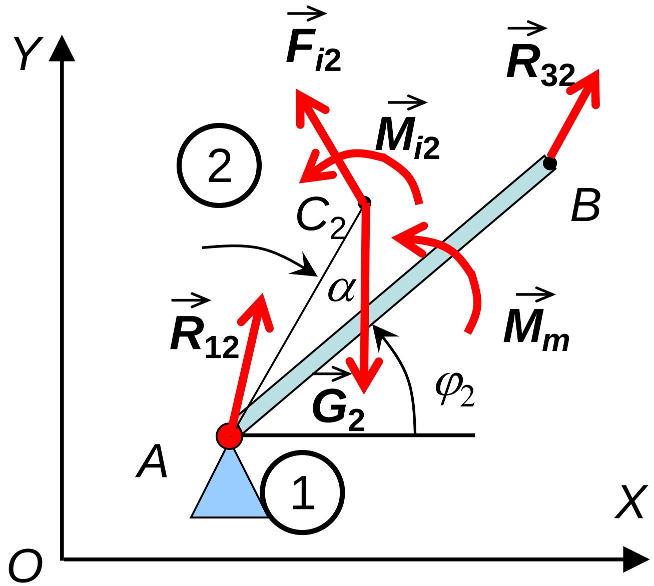
$$\vec{G}_2 + \vec{R}_{12} + \vec{R}_{32} + \vec{F}_{i2} = 0$$

$$\begin{cases} R_{12X} + R_{32X} - m_2\, a_{C2X} = 0 \\ R_{12Y} + R_{32Y} - m_2\, (g + a_{C_2Y}) = 0 \end{cases}$$

$$\vec{R}_{32} + \vec{R}_{23} = 0$$

$$\begin{cases} R_{32X} + R_{23X} = 0 \\ R_{32Y} + R_{23Y} = 0 \end{cases} \Rightarrow \begin{cases} R_{32X} = - R_{23X} \\ R_{32Y} = - R_{23Y} \end{cases} \Rightarrow \begin{cases} R_{12X} = R_{23X} + m_2\, a_{C2X} \\ R_{12Y} = R_{23Y} + m_2\, (g + a_{C_2Y}) \end{cases}$$

Kinetostatic analysis of motor element (motor moment computation)



$\boldsymbol{R}_{23X}$, $\boldsymbol{R}_{23Y}$ are known from previous computation

$$\begin{cases} \boldsymbol{R}_{32X} = -\,\boldsymbol{R}_{23X} \\ \boldsymbol{R}_{32Y} = -\,\boldsymbol{R}_{23Y} \end{cases}$$

Equilibrium equation of moments:

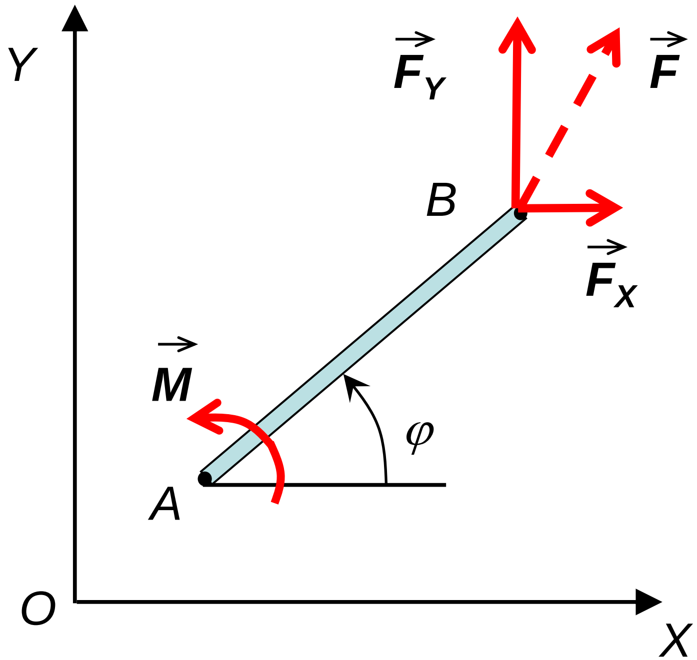
$$\boldsymbol{M}_m + \boldsymbol{R}_{32Y}\,(X_B - X_A) - \boldsymbol{R}_{32X}\,(Y_B - Y_A) - m_2\,(a_{C2Y} + g)\,(X_{C2} - X_A) + m_2\,a_{C2X}\,(Y_{C2} - Y_A) + \boldsymbol{M}_{i2}^{A} = 0$$

$$\Downarrow$$

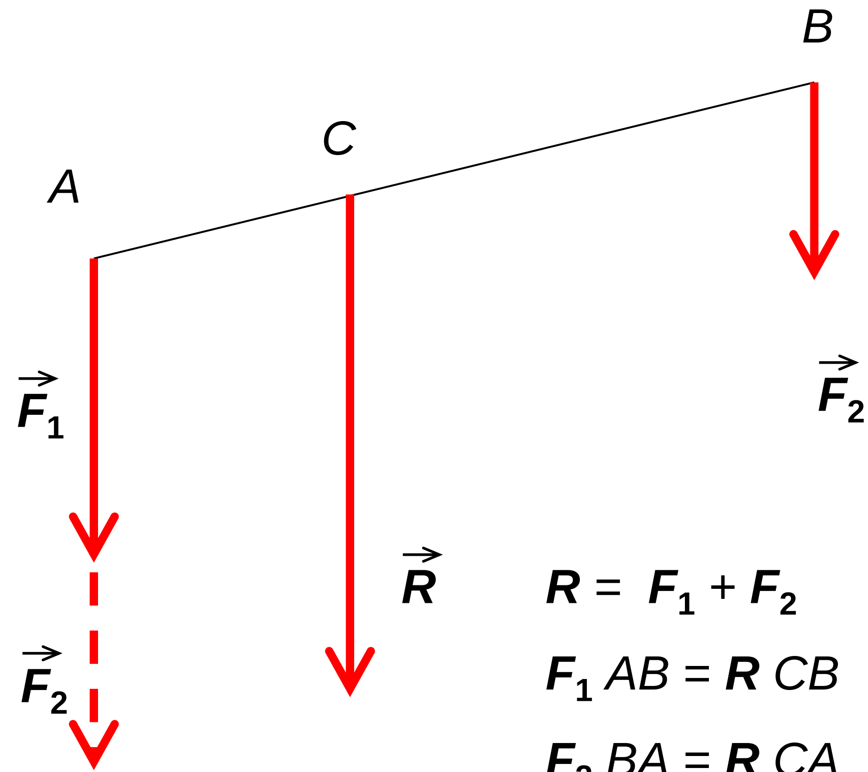
$$\boldsymbol{M}_m = \boldsymbol{R}_{32X}\,(Y_B - Y_A) - \boldsymbol{R}_{32Y}\,(X_B - X_A) - m_2\,a_{C2X}\,(Y_{C2} - Y_A) + m_2\,(a_{C2Y} + g)\,(X_{C2} - X_A) + I_2^{A}\,\varepsilon_2$$

$$I_2^{A} = I_2 + m_2\,AC_2^{\,2}$$

Basic Mechanics



$$\boldsymbol{M_F}^A = \boldsymbol{F}_Y\,(X_B - X_A) - \boldsymbol{F}_X\,(Y_B - Y_A)$$



$$\boldsymbol{R} = \boldsymbol{F_1} + \boldsymbol{F_2}$$

$$\boldsymbol{F_1}\,AB = \boldsymbol{R}\,CB$$

$$\boldsymbol{F_2}\,BA = \boldsymbol{R}\,CA$$

Momentum of a force ***F*** which is acting in a point *B*, with respect to a point *A*, other than origin *O*

Composition in point *C* of parallel forces $\boldsymbol{F}_1$ and $\boldsymbol{F}_2$ which are acting in points *A* and *B* respectively (*A*, *B* and *C* collinear)