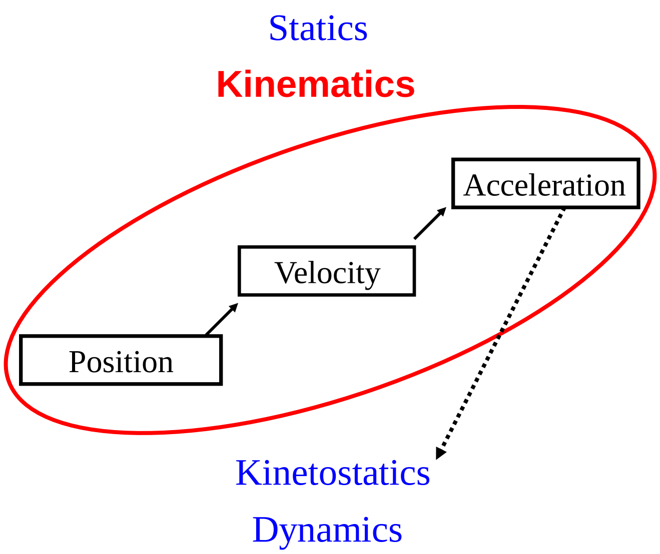
Chapter 3

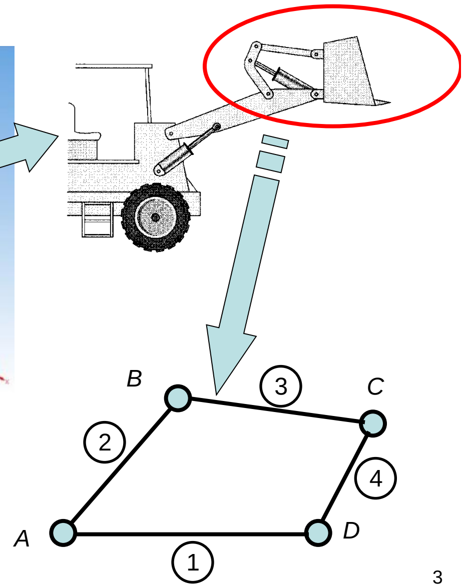
- Kinematic Analysis of Four-bar mechanism
- Kinematic Analysis of Crank-slider mechanism

- Kinetostatic Analysis of Four-bar mechanism
- Kinetostatic Analysis of Crank-slider mechanism

Kinematics analysis roadmap



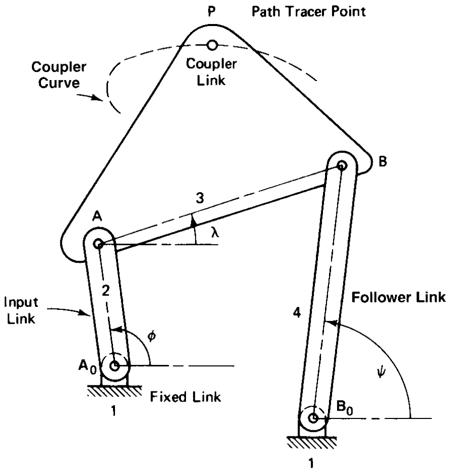
Front loader mechanism

Plane mechanism

Four-Bar Linkage

- Simplest closed-loop linkage; consists of three **moving** links, one **fixed** link (1), and four revolute (pin) joints.
- Primary links are called: the **input** link (connected to power source) denoted by (2), the **output** or **follower** link (4), and **coupler** or **floating** link (3). The latter "couples" the input to the output link.
- Points as *P* on the coupler link generally trace out **sixth order** algebraic coupler curves.

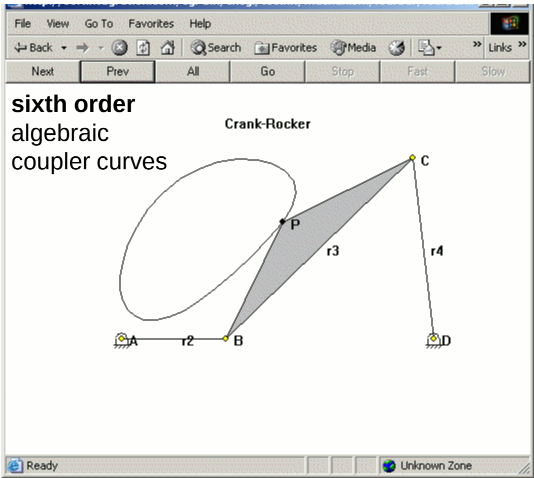


$$M_3 = 3\,m - 2\,l_p - h_p = 3x3 - 2x4 - 0 = 1$$

Number of independent loops: $N = l_p - n + 1 = l_p - m = 4 - 3 = 1$

Observation: *n* is the total number of elements

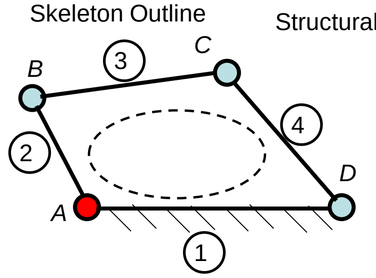
Four-bar mechanism - trajectory of a point on coupler link



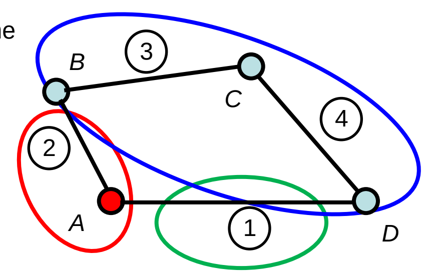
Simulator software: https://www.desmos.com/calculator/iuprdl6sxk

Four-bar mechanism

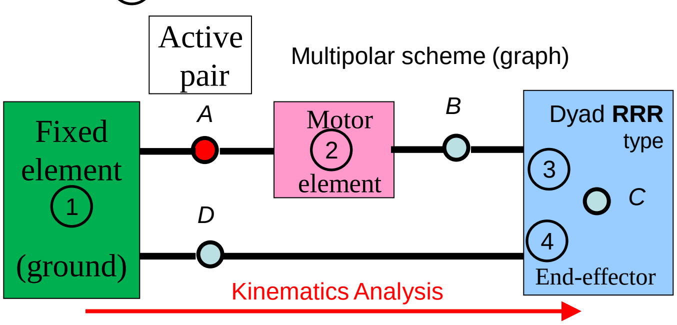
Skeleton Outline



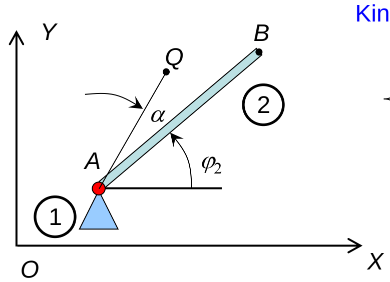
Structural scheme



Multipolar scheme (graph)



Kinematic analysis of motor element



Kinematic parameters of crank φ_2, ω_2 and ε_2 are known

$$\begin{cases} X_B = X_A + AB\cos\varphi_2 \\ Y_B = Y_A + AB\sin\varphi_2 \end{cases} \qquad \begin{cases} v_{BX} = -AB\,\omega_2\sin\varphi_2 \\ v_{BY} = AB\,\omega_2\cos\varphi_2 \end{cases}$$

$$\begin{cases} a_{BX} = -AB\,(\varepsilon_2\sin\varphi_2 + \omega_2^2\cos\varphi_2) \\ a_{BY} = AB\,(\varepsilon_2\cos\varphi_2 - \omega_2^2\sin\varphi_2) \end{cases}$$

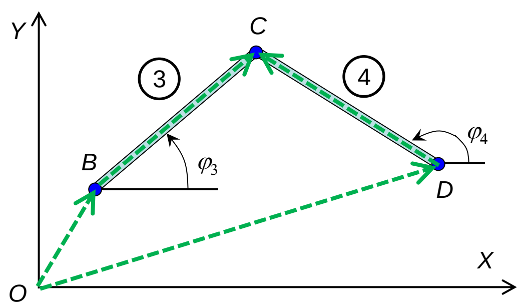
$$\begin{cases} \omega_2 = \text{angular speed of crank ②} \\ \varepsilon_2 = \text{angular acceleration of crank ②} \end{cases}$$

For other point of crank ②
$Q(AQ, \alpha)$, outside
direction AB, we have:

$$\begin{cases} X_Q = X_A + AQ\cos(\varphi_2 + \alpha) \\ Y_Q = Y_A + AQ\sin(\varphi_2 + \alpha) \end{cases} \qquad \begin{cases} v_{QX} = -AQ\,\omega_2\sin(\varphi_2 + \alpha) \\ v_{QY} = AQ\,\omega_2\cos(\varphi_2 + \alpha) \end{cases}$$

$$\begin{cases} a_{QX} = -AQ\,(\varepsilon_2\sin(\varphi_2 + \alpha) + \omega_2^2\cos(\varphi_2 + \alpha)) \\ a_{QY} = AQ\,(\varepsilon_2\cos(\varphi_2 + \alpha) - \omega_2^2\sin(\varphi_2 + \alpha)) \end{cases}$$

Kinematic analysis of Dyad - **RRR** type



Knowns: X_B, Y_B, X_D, Y_D, BC si CD

v_{BX}, v_{BY}, v_{DX}, v_{DY}, a_{BX}, a_{BY}, a_{DX}, a_{DY}

Unknowns: φ_3, φ_4, ω_3, ω_4, ε_3 and ε_4

$$N = l_p - m = 3 - 2 = 1$$

One single vectorial equation:

$$\vec{OB} + \vec{BC} = \vec{OD} + \vec{DC}$$

which is projected on the reference axes resulting 2 analytic equations:

$$\begin{cases} X_B + BC\cos\varphi_3 = X_D + DC\cos\varphi_4 \\ Y_B + BC\sin\varphi_3 = Y_D + DC\sin\varphi_4 \end{cases}$$

non-linear equations ⇒ Numerical Methods

Newton-Raphson Method to solve non-linear equations is based on Taylor series

https://study.com/academy/lesson/newton-raphson-method-for-nonlinear-systems-of-equations.html

Positions (configurations) of Dyad **RRR** type

$$\begin{cases} BC\cos\varphi_3 - DC\cos\varphi_4 + X_B - X_D = 0 \\ BC\sin\varphi_3 - DC\sin\varphi_4 + Y_B - Y_D = 0 \end{cases} \qquad \mathbf{\Phi} = \begin{bmatrix} \varphi_3 \\ \varphi_4 \end{bmatrix} \quad \text{Unknown vector}$$

$$\Rightarrow \mathbf{F}(\mathbf{\Phi}) - \mathbf{B} = 0 \quad \text{where} \quad \mathbf{F}(\mathbf{\Phi}) = \begin{bmatrix} BC\cos\varphi_3 - DC\cos\varphi_4 \\ BC\sin\varphi_3 - DC\sin\varphi_4 \end{bmatrix} \text{ and } \mathbf{B} = \begin{bmatrix} X_D - X_B \\ Y_D - Y_B \end{bmatrix}$$

Newton-Raphson method for solving non-linear equations:

$$\mathbf{J}_{2x2} = \begin{bmatrix} -BC\sin\varphi_3 & DC\sin\varphi_4 \\ BC\cos\varphi_3 & -DC\cos\varphi_4 \end{bmatrix} \Rightarrow \mathbf{J}\,\Delta\mathbf{\Phi} = \mathbf{B} - \mathbf{F} \quad \text{with} \quad \Delta\mathbf{\Phi} = \begin{bmatrix} \Delta\varphi_3 \\ \Delta\varphi_4 \end{bmatrix}$$

$\det(\mathbf{J}) \neq 0$ or $\varphi_3 \neq \varphi_4 + k\pi$

$$\boxed{\Delta\mathbf{\Phi} = \mathbf{J}^{-1}(\mathbf{B} - \mathbf{F})}$$

Initial values: (from drawing !) $\mathbf{\Phi}^{(0)} = \begin{bmatrix} \varphi_3^{(0)} \\ \varphi_4^{(0)} \end{bmatrix} \Rightarrow \Delta\mathbf{\Phi}^{(1)} = \begin{bmatrix} \Delta\varphi_3^{(1)} \\ \Delta\varphi_4^{(1)} \end{bmatrix}$

Solutions are: $\begin{cases} \varphi_3^{(j)} = \varphi_3^{(j-1)} + \Delta\varphi_3^{(j)} \\ \varphi_4^{(j)} = \varphi_4^{(j-1)} + \Delta\varphi_4^{(j)} \end{cases}$ while $\begin{cases} |\Delta\varphi_3^{(j)}| > e_1 \\ |\Delta\varphi_4^{(j)}| > e_2 \end{cases}$ $j \geq 1$ (number of iteration)

Errors $e_1 = e_2 < 0.001$ [radians] !

Velocities and accelerations of **RRR** Dyad

Velocities equations are:

$$\begin{cases} (-BC\sin\varphi_3)\,\omega_3 + (DC\sin\varphi_4)\,\omega_4 + v_{BX} - v_{DX} = 0 \\ (BC\cos\varphi_3)\,\omega_3 + (-DC\cos\varphi_4)\,\omega_4 + v_{BY} - v_{DY} = 0 \end{cases} \quad \text{linear in} \quad \mathbf{\Omega} = \begin{bmatrix} \omega_3 \\ \omega_4 \end{bmatrix}$$

$$\Rightarrow \mathbf{J}\,\mathbf{\Omega} - \mathbf{C} = 0 \quad \text{where} \quad \mathbf{C} = \begin{bmatrix} v_{DX} - v_{BX} \\ v_{DY} - v_{BY} \end{bmatrix} \Rightarrow \mathbf{\Omega} = \mathbf{J}^{-1}\,\mathbf{C} \quad \text{and} \quad \begin{cases} v_{DX} = 0 \\ v_{DY} = 0 \end{cases}$$

$\mathbf{J}^{-1}\,\mathbf{J} = \mathbf{I}_2$ where $\mathbf{I}_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \Rightarrow$ linear equations with 4 equations and 4 unknowns (elements of $\mathbf{J}^{-1}$)

Accelerations equation in matrix form:

$\mathbf{J}\,\mathbf{E} + \mathbf{J}'\,\mathbf{\Omega}^2 - \mathbf{D} = 0$ linear in unknowns $\mathbf{E} = \begin{bmatrix} \varepsilon_3 \\ \varepsilon_4 \end{bmatrix}$

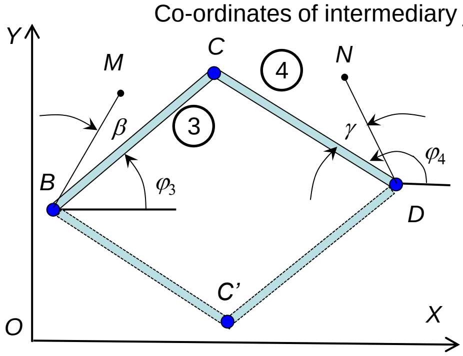
where $\mathbf{J}'_{2x2} = \begin{bmatrix} -BC\cos\varphi_3 & DC\cos\varphi_4 \\ -BC\sin\varphi_3 & DC\sin\varphi_4 \end{bmatrix}$ and $\mathbf{\Omega}^2 = \begin{bmatrix} \omega_3^2 \\ \omega_4^2 \end{bmatrix}$

$$\Rightarrow \mathbf{E} = \mathbf{J}^{-1}(\mathbf{D} - \mathbf{J}'\,\mathbf{\Omega}^2) \quad \text{where} \quad \mathbf{D} = \begin{bmatrix} a_{DX} - a_{BX} \\ a_{DY} - a_{BY} \end{bmatrix} \quad \text{and} \quad \begin{cases} a_{DX} = 0 \\ a_{DY} = 0 \end{cases}$$

Kinematic analysis of Dyad - **RRR** type

Co-ordinates of intermediary joint C are:

$$\begin{cases} X_C = X_B + BC\cos\varphi_3 \\ Y_C = Y_B + BC\sin\varphi_3 \end{cases} \quad \text{or} \quad \begin{cases} X_C = X_D + DC\cos\varphi_4 \\ Y_C = Y_D + DC\sin\varphi_4 \end{cases}$$



$$\begin{cases} v_{CX} = v_{BX} - BC\,\omega_3 \sin\varphi_3 \\ v_{CY} = v_{BY} + BC\,\omega_3 \cos\varphi_3 \end{cases} \quad \text{or} \quad \begin{cases} v_{CX} = v_{DX} - DC\,\omega_4 \sin\varphi_4 \\ v_{CY} = v_{DY} + DC\,\omega_4 \cos\varphi_4 \end{cases}$$

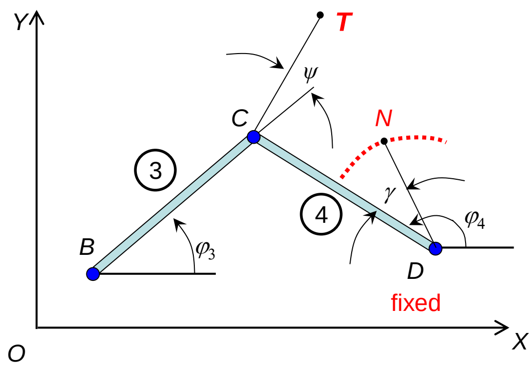
$$\begin{cases} a_{CX} = a_{BX} - BC\,(\varepsilon_3 \sin\varphi_3 + \omega_3^2 \cos\varphi_3) \\ a_{CY} = a_{BY} + BC\,(\varepsilon_3 \cos\varphi_3 - \omega_3^2 \sin\varphi_3) \end{cases} \quad \text{or} \quad \begin{cases} a_{CX} = a_{DX} - DC\,(\varepsilon_4 \sin\varphi_4 + \omega_4^2 \cos\varphi_4) \\ a_{CY} = a_{DY} + DC\,(\varepsilon_4 \cos\varphi_4 - \omega_4^2 \sin\varphi_4) \end{cases}$$

For other point of **RRR** dyad $M(BM, \beta)$ on element ③, outside direction BC we have:

$$\begin{cases} X_M = X_B + BM\cos(\varphi_3 + \beta) \\ Y_M = Y_B + BM\sin(\varphi_3 + \beta) \end{cases}$$

$$\begin{cases} v_{MX} = v_{BX} - BM\,\omega_3 \sin(\varphi_3 + \beta) \\ v_{MY} = v_{BY} + BM\,\omega_3 \cos(\varphi_3 + \beta) \end{cases} \quad \begin{cases} a_{MX} = a_{BX} - BM\,(\varepsilon_3 \sin(\varphi_3 + \beta) + \omega_3^2 \cos(\varphi_3 + \beta)) \\ a_{MY} = a_{BY} + BM\,(\varepsilon_3 \cos(\varphi_3 + \beta) - \omega_3^2 \sin(\varphi_3 + \beta)) \end{cases}$$

Kinematic analysis of Dyad - **RRR** type



For other points of **RRR** dyad $T(CT, \psi)$ of element ③, outside direction BC, and $N(DN, \gamma)$ of element ④, outside direction DC, we have:

$$\begin{cases} X_T = X_C + CT\cos(\varphi_3 + \psi) \\ Y_T = Y_C + CT\sin(\varphi_3 + \psi) \end{cases}$$

$$\begin{cases} X_N = X_D + DN\cos(\varphi_4 - \gamma) \\ Y_N = Y_D + DN\sin(\varphi_4 - \gamma) \end{cases}$$

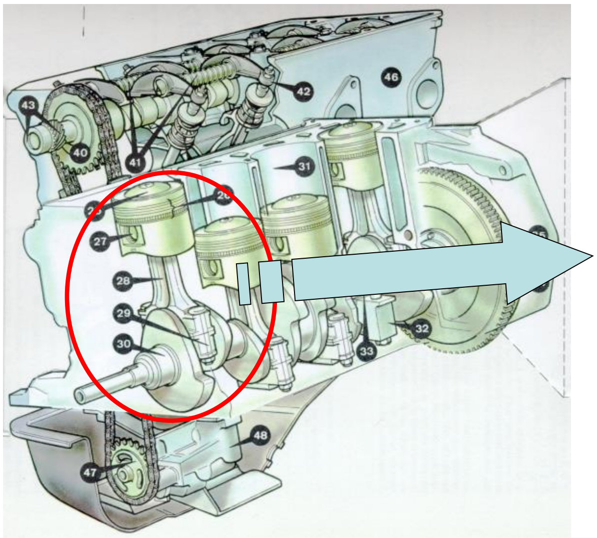
$$\begin{cases} a_{TX} = a_{CX} - CT\,(\varepsilon_3 \sin(\varphi_3 + \psi) + \omega_3^2 \cos(\varphi_3 + \psi)) \\ a_{TY} = a_{CY} + CT\,(\varepsilon_3 \cos(\varphi_3 + \psi) - \omega_3^2 \sin(\varphi_3 + \psi)) \end{cases}$$

$$\begin{cases} v_{TX} = v_{CX} - CT\,\omega_3 \sin(\varphi_3 + \psi) \\ v_{TY} = v_{CY} + CT\,\omega_3 \cos(\varphi_3 + \psi) \end{cases}$$

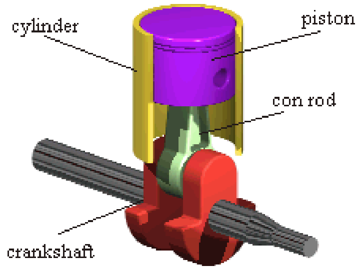
$$\begin{cases} a_{NX} = a_{DX} - DN\,(\varepsilon_4 \sin(\varphi_4 - \gamma) + \omega_4^2 \cos(\varphi_4 - \gamma)) \\ a_{NY} = a_{DY} + DN\,(\varepsilon_4 \cos(\varphi_4 - \gamma) - \omega_4^2 \sin(\varphi_4 - \gamma)) \end{cases}$$

$$\begin{cases} v_{NX} = v_{DX} - DN\,\omega_4 \sin(\varphi_4 - \gamma) \\ v_{NY} = v_{DY} + DN\,\omega_4 \cos(\varphi_4 - \gamma) \end{cases}$$

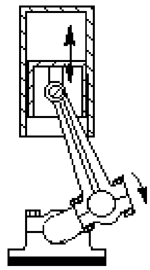
Internal combustion engine



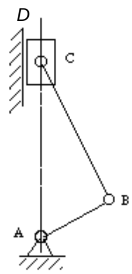
Slider-crank mechanism



CAD model



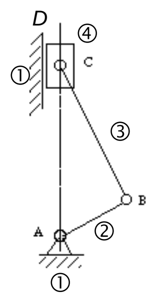
Assembly drawing



skeleton outline

Crank-slider Mechanisms

- Mechanism of four-bar family with a single closed-loop; consists of three **moving** links, one **fixed** link ①, and four inferior joints: three of revolution pair and one prirsmatic pair.
- Input element is the **crank** ②, output elment is the **slider** ④ and coupler-link ③ has a plan-parallel motion (reffered sometime as **connecting rod**).
- Points on the coupler link generally trace out **fourth order** algebraic coupler curves.

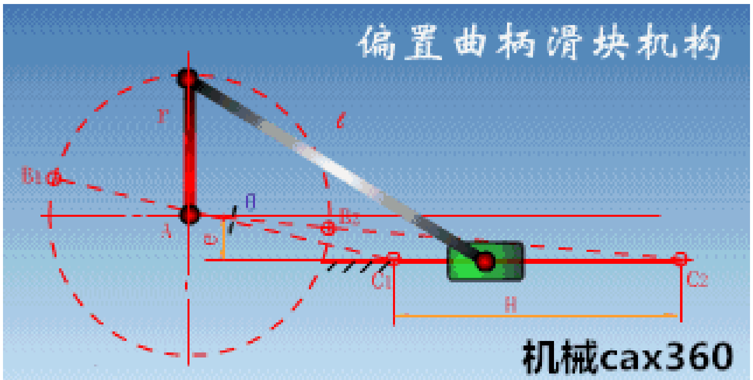


$$M_3 = 3\,m - 2\,l_p - h_p =$$
$$= 3x3 - 2x4 - 0 = 1$$

Number of independent loops:

$$N = l_p - n + 1 = l_p - m = 4 - 3 = 1$$

Off-set Crank-slider mechanism – extreme positions



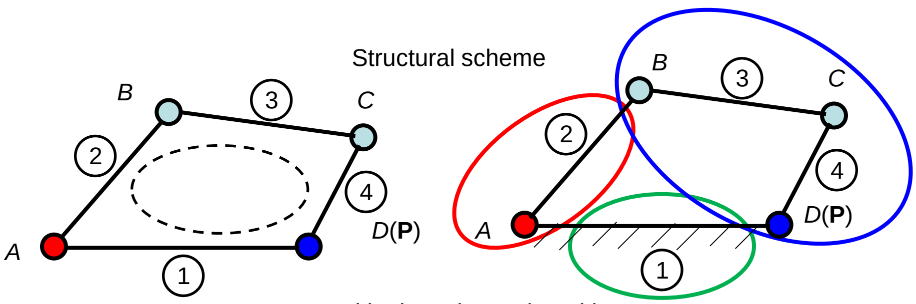
H represents the stroke

Left extreme position (denoted by index 1) is when *A* is between *B* and *C*

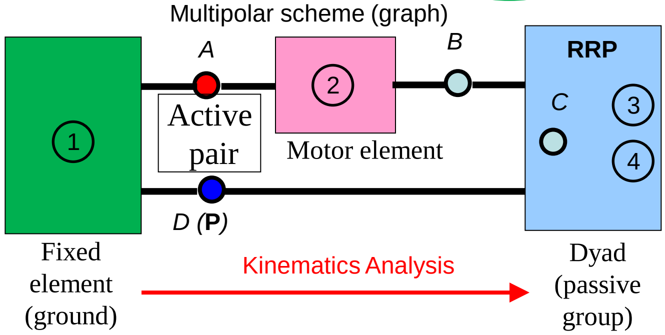
Right extreme position (denoted by index 2) is when *B* is between *A* and *C*

Crank-slider Mechanisms

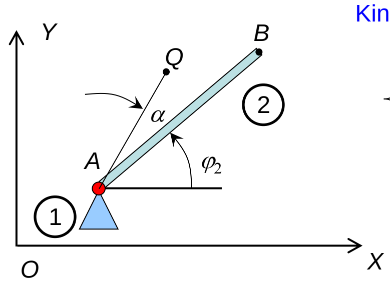
Structural scheme



Multipolar scheme (graph)



Kinematic analysis of motor element



Kinematic parameters of crank φ_2, ω_2 and ε_2 are known

$$\begin{cases} X_B = X_A + AB\cos\varphi_2 \\ Y_B = Y_A + AB\sin\varphi_2 \end{cases} \qquad \begin{cases} v_{BX} = -AB\,\omega_2\sin\varphi_2 \\ v_{BY} = AB\,\omega_2\cos\varphi_2 \end{cases}$$

$$\begin{cases} a_{BX} = -AB\,(\varepsilon_2\sin\varphi_2 + \omega_2^2\cos\varphi_2) \\ a_{BY} = AB\,(\varepsilon_2\cos\varphi_2 - \omega_2^2\sin\varphi_2) \end{cases}$$

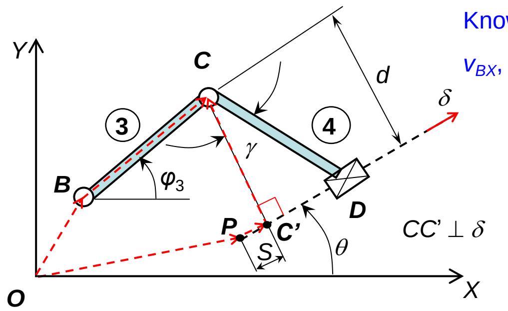
$$\begin{cases} \omega_2 = \text{angular speed of crank ②} \\ \varepsilon_2 = \text{angular acceleration of crank ②} \end{cases}$$

For other point of crank ② $Q(AQ, \alpha)$, outside direction AB, we have:

$$\begin{cases} X_Q = X_A + AQ\cos(\varphi_2 + \alpha) \\ Y_Q = Y_A + AQ\sin(\varphi_2 + \alpha) \end{cases} \qquad \begin{cases} v_{QX} = -AQ\,\omega_2\sin(\varphi_2 + \alpha) \\ v_{QY} = AQ\,\omega_2\cos(\varphi_2 + \alpha) \end{cases}$$

$$\begin{cases} a_{QX} = -AQ\,(\varepsilon_2\sin(\varphi_2 + \alpha) + \omega_2^2\cos(\varphi_2 + \alpha)) \\ a_{QY} = AQ\,(\varepsilon_2\cos(\varphi_2 + \alpha) - \omega_2^2\sin(\varphi_2 + \alpha)) \end{cases}$$

Kinematic analysis of Dyad - **RRP** type



Knowns: X_B, Y_B, X_P, Y_P, θ, BC, d, and v_{BX}, v_{BY}, a_{BX}, a_{BY}, v_{PX}, v_{PY}, a_{PX}, a_{PY}, $\dot{\theta}$, $\ddot{\theta}$

$CC' \perp \delta$

Unknowns: φ_3, ω_3, ε_3, S, $\dot{S}$ and $\ddot{S}$

$$N = l_p - m = 3 - 2 = 1$$

One single vectorial equation:

$$\vec{OB} + \vec{BC} = \vec{OP} + \vec{PC'} + \vec{C'C}$$

$$\begin{cases} X_B + BC\cos\varphi_3 = X_P + S\cos\theta - d\sin\theta \\ Y_B + BC\sin\varphi_3 = Y_P + S\sin\theta + d\cos\theta \end{cases}$$

non-linear ⇒ Numerical Methods

Newton-Raphson Method to solve non-linear equations is based on Taylor series

https://study.com/academy/lesson/newton-raphson-method-for-nonlinear-systems-of-equations.html

Positions (configurations) of Dyad **RRP** type

$$\begin{cases} BC\cos\varphi_3 - S\cos\theta + d\sin\theta + X_B - X_p = 0 \\ BC\sin\varphi_3 - S\sin\theta - d\cos\theta + Y_B - Y_P = 0 \end{cases} \qquad \mathbf{x} = \begin{bmatrix} \varphi_3 \\ S \end{bmatrix} \text{ Unknown vector}$$

$$\Rightarrow \mathbf{F}(\mathbf{x}) - \mathbf{B} = 0 \text{ where } \mathbf{F}(\mathbf{x}) = \begin{bmatrix} BC\cos\varphi_3 - S\cos\theta \\ BC\sin\varphi_3 - S\sin\theta \end{bmatrix} \text{ and } \mathbf{B} = \begin{bmatrix} X_P - X_B - d\sin\theta \\ Y_P - Y_B + d\cos\theta \end{bmatrix}$$

Newton-Raphson method for solving non-linear equations:

$$\mathbf{J}_{2x2} = \begin{bmatrix} -BC\sin\varphi_3 & -\cos\theta \\ BC\cos\varphi_3 & -\sin\theta \end{bmatrix} \Rightarrow \mathbf{J}\,\Delta\mathbf{x} = \mathbf{B} - \mathbf{F} \text{ where } \Delta\mathbf{x} = \begin{bmatrix} \Delta\varphi_3 \\ \Delta S \end{bmatrix}$$

$\det(\mathbf{J}) \neq 0$ or $\varphi_3 \neq \theta + k\,\pi/2$

$\Rightarrow \Delta\mathbf{x} = \mathbf{J}^{-1}(\mathbf{B} - \mathbf{F})$

Initial values: (from a drawing !) $\mathbf{x}^{(0)} = \begin{bmatrix} \varphi_3^{(0)} \\ S^{(0)} \end{bmatrix} \Rightarrow \Delta\mathbf{x}^{(1)} = \begin{bmatrix} \Delta\varphi_3^{(1)} \\ \Delta S^{(1)} \end{bmatrix}$

Solutions are: $\begin{cases} \varphi_3^{(j)} = \varphi_3^{(j-1)} + \Delta\varphi_3^{(j)} \\ S^{(j)} = S^{(j-1)} + \Delta S^{(j)} \end{cases}$ while $\begin{cases} |\Delta\varphi_3^{(j)}| > e_1 \\ |\Delta S^{(j)}| > e_2 \end{cases}$ $j \geq 1$ (number of iteration)

Errors should be $e_1 < 0.001$ [rad] and $e_2 < 0.1$ [mm] respectively !

Velocities and accelerations of **RRP** Dyad

Velocities equations are:

$$\begin{cases} (-BC\sin\varphi_3)\,\omega_3 + (-\cos\theta)\,\dot{S} + (S\sin\theta)\dot{\theta} + v_{BX} - v_{PX} = 0 \\ (BC\cos\varphi_3)\,\omega_3 + (-\sin\theta)\,\dot{S} + (-S\cos\theta)\dot{\theta} + v_{BY} - v_{PY} = 0 \end{cases} \quad \text{linear in } \dot{\mathbf{x}} = \begin{bmatrix} \omega_3 \\ \dot{S} \end{bmatrix}$$

$$\Rightarrow \mathbf{J}\,\dot{\mathbf{x}} - \mathbf{C} = 0 \text{ where } \mathbf{C} = \begin{bmatrix} v_{PX} - v_{BX} - (S\sin\theta)\dot{\theta} \\ v_{PY} - v_{BY} + (S\cos\theta)\dot{\theta} \end{bmatrix} \Rightarrow \dot{\mathbf{x}} = \mathbf{J}^{-1}\,\mathbf{C}$$

$$\mathbf{J}^{-1}\,\mathbf{J} = \mathbf{I}_2 \text{ where } \mathbf{I}_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \Rightarrow$$ linear equations with 4 equations and 4 unknowns (elements of $\mathbf{J}^{-1}$)

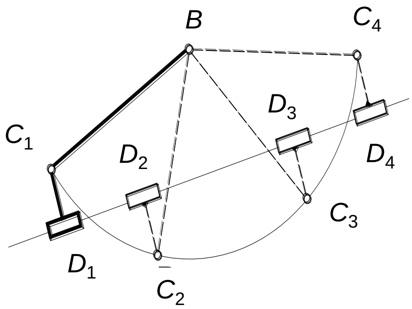
Accelerations equations:

$\mathbf{J}\,\ddot{\mathbf{x}} + \mathbf{J}'\,\dot{\mathbf{x}}^2 - \mathbf{D} = 0$ linear in unknowns $\ddot{\mathbf{x}} = \begin{bmatrix} \varepsilon_3 \\ \ddot{S} \end{bmatrix}$

where $\mathbf{J}'_{2x2} = \begin{bmatrix} -BC\cos\varphi_3 & \sin\theta \\ -BC\sin\varphi_3 & -\cos\theta \end{bmatrix}$ and $\dot{\mathbf{x}}^2 = \begin{bmatrix} \omega_3^2 \\ \dot{S}^2 \end{bmatrix}$

$$\Rightarrow \ddot{\mathbf{x}} = \mathbf{J}^{-1}(\mathbf{D} - \mathbf{J}'\,\dot{\mathbf{x}}^2) \text{ where } \mathbf{D} = \begin{bmatrix} a_{PX} - a_{BX} \\ a_{PY} - a_{BY} \end{bmatrix}$$

Kinematic analysis of Dyad - **RRP** type



From mathematical point of view there are 4 possible positions of intermediary joint *C* ! Kinematic parameters could be computed in two ways:

$$\begin{cases} X_C = X_B + BC\cos\varphi_3 \\ Y_C = Y_B + BC\sin\varphi_3 \end{cases}$$

or

$$\begin{cases} X_C = X_P + S\cos\theta - d\sin\theta \\ Y_C = Y_P + S\sin\theta + d\cos\theta \end{cases}$$

and

$$\begin{cases} v_{CX} = v_{BX} - BC\,\omega_3\sin\varphi_3 \\ v_{CY} = v_{BY} + BC\,\omega_3\cos\varphi_3 \end{cases}$$

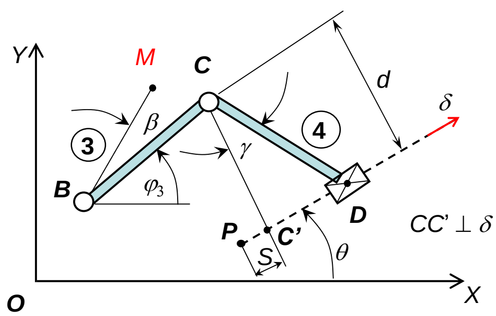
$$\begin{cases} a_{CX} = a_{BX} - BC\,(\varepsilon_3\sin\varphi_3 + \omega_3^2\cos\varphi_3) \\ a_{CY} = a_{BY} + BC\,(\varepsilon_3\cos\varphi_3 - \omega_3^2\sin\varphi_3) \end{cases}$$

or

$$\begin{cases} v_{CX} = v_{PX} + \dot{S}\cos\theta - \dot{\theta}(S\sin\theta + d\cos\theta) \\ v_{CY} = v_{PY} + \dot{S}\sin\theta + \dot{\theta}(S\cos\theta - d\sin\theta) \end{cases}$$

$$\begin{cases} a_{CX} = a_{PX} + \ddot{S}\cos\theta - 2\dot{\theta}\dot{S}\sin\theta - \ddot{\theta}(S\sin\theta + d\cos\theta) - \dot{\theta}^2(S\cos\theta - d\sin\theta) \\ a_{CY} = a_{PY} + \ddot{S}\sin\theta + 2\dot{\theta}\dot{S}\cos\theta + \ddot{\theta}(S\cos\theta - d\sin\theta) - \dot{\theta}^2(S\sin\theta + d\cos\theta) \end{cases}$$

Kinematic analysis of Dyad - **RRP** type



$CC' \perp \delta$

Kinematic parameters of a point *M*(*BM*, β) from connecting rod ③, but outside line *BC*, are:

$$\begin{cases} X_M = X_B + BM\cos(\varphi_3 + \beta) \\ Y_M = Y_B + BM\sin(\varphi_3 + \beta) \end{cases}$$

$$\begin{cases} v_{MX} = v_{BX} - BM\,\omega_3\sin(\varphi_3 + \beta) \\ v_{MY} = v_{BY} + BM\,\omega_3\cos(\varphi_3 + \beta) \end{cases}$$

$$\begin{cases} a_{MX} = a_{BX} - BM\,(\varepsilon_3\sin(\varphi_3 + \beta) + \omega_3^2\cos(\varphi_3 + \beta)) \\ a_{MY} = a_{BY} + BM\,(\varepsilon_3\cos(\varphi_3 + \beta) - \omega_3^2\sin(\varphi_3 + \beta)) \end{cases}$$

If distance *d* and angle γ are known then length $CD = d/\cos\gamma$, $C'D = d\,\mathrm{tg}\gamma$ and:

$$\begin{cases} X_D = X_C + CD\cos(\theta + \gamma + 1.5\,\pi) \\ Y_D = Y_C + CD\sin(\theta + \gamma + 1.5\,\pi) \end{cases}$$

or

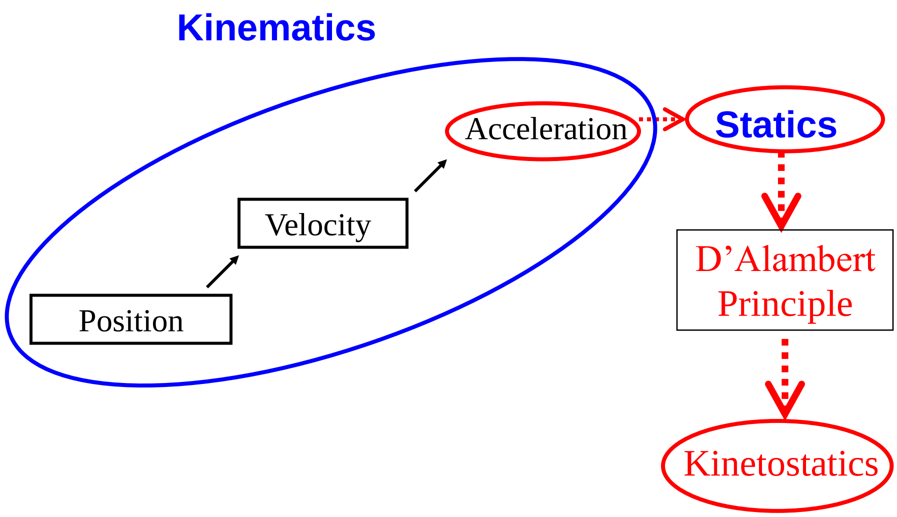
$$\begin{cases} X_D = X_P + (S + C'D)\cos\theta \\ Y_D = Y_P + (S + C'D)\sin\theta \end{cases}$$

and

$$\begin{cases} v_{DX} = v_{CX} \\ v_{DY} = v_{CY} \end{cases}$$

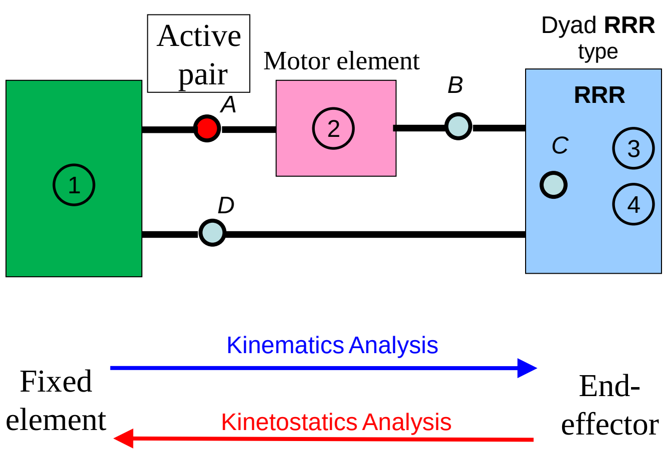
$$\begin{cases} a_{DX} = a_{CX} \\ a_{DY} = a_{CY} \end{cases}$$

Kinetostatic analysis roadmap

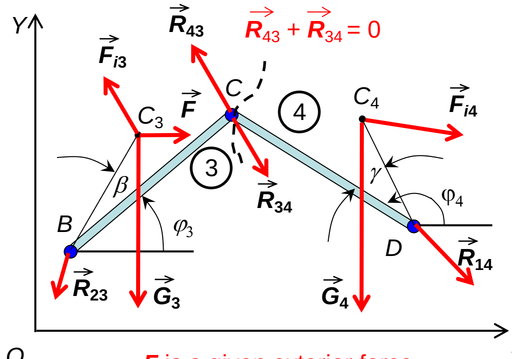


Four-bar mechanism

Multipolar scheme (graph)



Kinetostatic analysis of Dyad - **RRR** type



F is a given exterior force

$$\begin{cases} X_{C_3} = X_B + BC_3 \cos(\varphi_3 + \beta) \\ Y_{C_3} = Y_B + BC_3 \sin(\varphi_3 + \beta) \end{cases}$$

$$\begin{cases} X_{C_4} = X_D + DC_4 \cos(\varphi_4 - \gamma) \\ Y_{C_4} = Y_D + DC_4 \sin(\varphi_4 - \gamma) \end{cases}$$

$$\begin{cases} v_{C_3X} = v_{BX} - BC_3\, \omega_3 \sin(\varphi_3 + \beta) \\ v_{C_3Y} = v_{BY} + BC_3\, \omega_3 \cos(\varphi_3 + \beta) \end{cases}$$

$$\begin{cases} v_{C_4X} = v_{DX} - DC_4\, \omega_4 \sin(\varphi_4 - \gamma) \\ v_{C_4Y} = v_{DY} + DC_4\, \omega_4 \cos(\varphi_4 - \gamma) \end{cases}$$

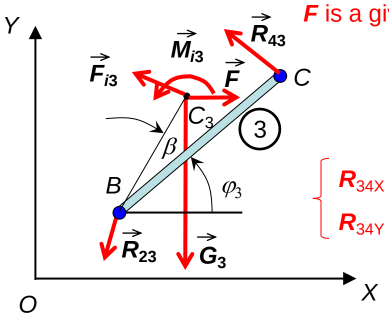
$$\begin{cases} a_{C_3X} = a_{BX} - BC_3\, (\varepsilon_3 \sin(\varphi_3 + \beta) + \omega_3^2 \cos(\varphi_3 + \beta)) \\ a_{C_3Y} = a_{BY} + BC_3\, (\varepsilon_3 \cos(\varphi_3 + \beta) - \omega_3^2 \sin(\varphi_3 + \beta)) \end{cases}$$

$$\begin{cases} a_{C_4X} = a_{DX} - DC_4\, (\varepsilon_4 \sin(\varphi_4 - \gamma) + \omega_4^2 \cos(\varphi_4 - \gamma)) \\ a_{C_4Y} = a_{DY} + DC_4\, (\varepsilon_4 \cos(\varphi_4 - \gamma) - \omega_4^2 \sin(\varphi_4 - \gamma)) \end{cases}$$

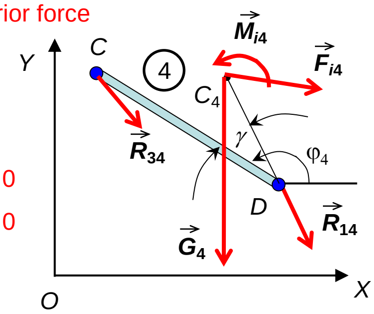
And where are needed too the axial inertial moments with respect to centres of masses of elements: I_2, I_3 and I_4

Kinetostatic analysis of Dyad - **RRR** type

F is a given exterior force



$$\begin{cases} \boldsymbol{R}_{34X} + \boldsymbol{R}_{43X} = 0 \\ \boldsymbol{R}_{34Y} + \boldsymbol{R}_{43Y} = 0 \end{cases}$$



$$\vec{G}_3 + \vec{F} + \vec{R}_{23} + \vec{R}_{43} + \vec{F}_{i3} = 0$$

$$\begin{cases} \boldsymbol{F} + \boldsymbol{R}_{23X} + \boldsymbol{R}_{43X} - m_3\, a_{C3X} = 0 \\ \boldsymbol{R}_{23Y} + \boldsymbol{R}_{43Y} - m_3\, (g + a_{C3Y}) = 0 \end{cases}$$

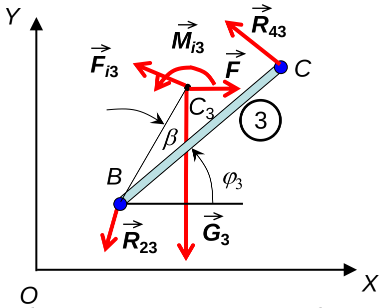
$$\vec{G}_4 + \vec{R}_{14} + \vec{R}_{34} + \vec{F}_{i4} = 0$$

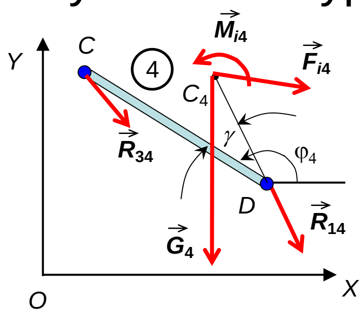
$$\begin{cases} \boldsymbol{R}_{14X} + \boldsymbol{R}_{34X} - m_4\, a_{C4X} = 0 \\ \boldsymbol{R}_{14Y} + \boldsymbol{R}_{34Y} - m_4\, (g + a_{C4Y}) = 0 \end{cases}$$

Equilibrium of moments equations:

$$\begin{cases} \boldsymbol{R}_{23Y}\, (X_B - X_{C3}) - \boldsymbol{R}_{23X}\, (Y_B - Y_{C3}) + \boldsymbol{R}_{43Y}\, (X_C - X_{C3}) - \boldsymbol{R}_{43X}\, (Y_C - Y_{C3}) + \boldsymbol{M}_{i3} = 0 \\ \boldsymbol{R}_{14Y}\, (X_D - X_{C4}) - \boldsymbol{R}_{14X}\, (Y_D - Y_{C4}) + \boldsymbol{R}_{34Y}\, (X_C - X_{C4}) - \boldsymbol{R}_{34X}\, (Y_C - Y_{C4}) + \boldsymbol{M}_{i4} = 0 \end{cases}$$

Kinetostatic analysis of Dyad - **RRR** type





$$\begin{cases} \boldsymbol{R}_{34X} + \boldsymbol{R}_{43X} = 0 \\ \boldsymbol{R}_{34Y} + \boldsymbol{R}_{43Y} = 0 \end{cases} \Rightarrow \begin{cases} \boldsymbol{R}_{34X} = -\boldsymbol{R}_{43X} \\ \boldsymbol{R}_{34Y} = -\boldsymbol{R}_{43Y} \end{cases} \Rightarrow \begin{cases} \boldsymbol{R}_{14X} - \boldsymbol{R}_{43X} - m_4\, a_{C4X} = 0 \\ \boldsymbol{R}_{14Y} - \boldsymbol{R}_{43Y} - m_4\, (g + a_{C4Y}) = 0 \end{cases}$$

$$\begin{cases} \boldsymbol{R}_{23X} + \boldsymbol{R}_{43X} - m_3\, a_{C3X} + \boldsymbol{F} = 0 \\ \boldsymbol{R}_{23Y} + \boldsymbol{R}_{43Y} - m_3\, (g + a_{C3Y}) = 0 \end{cases}$$

$$\Downarrow$$

$$\begin{cases} \boldsymbol{R}_{23Y}\, (X_B - X_{C3}) - \boldsymbol{R}_{23X}\, (Y_B - Y_{C3}) + \boldsymbol{R}_{43Y}\, (X_C - X_{C3}) - \boldsymbol{R}_{43X}\, (Y_C - Y_{C3}) - I_3\, \varepsilon_3 = 0 \\ \boldsymbol{R}_{14Y}\, (X_D - X_{C4}) - \boldsymbol{R}_{14X}\, (Y_D - Y_{C4}) - \boldsymbol{R}_{43Y}\, (X_C - X_{C4}) + \boldsymbol{R}_{43X}\, (Y_C - Y_{C4}) - I_4\, \varepsilon_4 = 0 \end{cases}$$

6 linear equations with 6 unknowns

2021 - L. Ciupitu

27

Kinetostatic analysis of Dyad - **RRR** type

$$\begin{cases} \boldsymbol{R}_{34X} + \boldsymbol{R}_{43X} = 0 \\ \boldsymbol{R}_{34Y} + \boldsymbol{R}_{43Y} = 0 \end{cases} \Rightarrow \begin{cases} \boldsymbol{R}_{34X} = -\boldsymbol{R}_{43X} \\ \boldsymbol{R}_{34Y} = -\boldsymbol{R}_{43Y} \end{cases} \Rightarrow \begin{cases} \boldsymbol{R}_{14X} - \boldsymbol{R}_{43X} - m_4\, a_{C4X} = 0 \\ \boldsymbol{R}_{14Y} - \boldsymbol{R}_{43Y} - m_4\, (g + a_{C4Y}) = 0 \end{cases}$$

$$\Downarrow$$

and

$$\begin{cases} \boldsymbol{R}_{14X} = \boldsymbol{R}_{43X} + m_4\, a_{C4X} \\ \boldsymbol{R}_{14Y} = \boldsymbol{R}_{43Y} + m_4\, (g + a_{C4Y}) \end{cases}$$

$$\begin{cases} \boldsymbol{R}_{23X} + \boldsymbol{R}_{43X} - m_3\, a_{C3X} + \boldsymbol{F} = 0 \\ \boldsymbol{R}_{23Y} + \boldsymbol{R}_{43Y} - m_3\, (g + a_{C3Y}) = 0 \end{cases} \Rightarrow \begin{cases} \boldsymbol{R}_{23X} = \boldsymbol{R}_{43X} + m_3\, a_{C3X} - \boldsymbol{F} = 0 \\ \boldsymbol{R}_{23Y} = \boldsymbol{R}_{43Y} + m_3\, (g + a_{C3Y}) = 0 \end{cases}$$

$$\Downarrow$$

$$\begin{cases} \boldsymbol{R}_{23Y}\, (X_B - X_{C3}) - \boldsymbol{R}_{23X}\, (Y_B - Y_{C3}) + \boldsymbol{R}_{43Y}\, (X_C - X_{C3}) - \boldsymbol{R}_{43X}\, (Y_C - Y_{C3}) - I_3\, \varepsilon_3 = 0 \\ \boldsymbol{R}_{14Y}\, (X_D - X_{C4}) - \boldsymbol{R}_{14X}\, (Y_D - Y_{C4}) - \boldsymbol{R}_{43Y}\, (X_C - X_{C4}) + \boldsymbol{R}_{43X}\, (Y_C - Y_{C4}) - I_4\, \varepsilon_4 = 0 \end{cases}$$

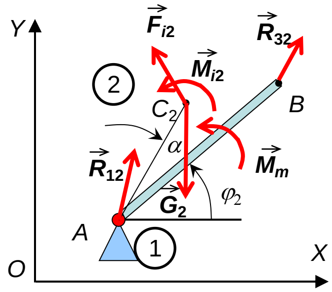
$$\Downarrow$$

$$\boldsymbol{R}_{43X}\,,\ \boldsymbol{R}_{43Y} \Rightarrow \boldsymbol{R}_{34X}\,,\ \boldsymbol{R}_{34Y},\ \boldsymbol{R}_{14X}\,,\ \boldsymbol{R}_{14Y}\,,\ \boldsymbol{R}_{23X}\,,\ \boldsymbol{R}_{23Y}$$

2021 - L. Ciupitu

28

Kinetostatic analysis of motor element



$$\begin{cases} X_{C_2} = X_A + AC_2 \cos(\varphi_2 + \alpha) \\ Y_{C_2} = Y_A + AC_2 \sin(\varphi_2 + \alpha) \end{cases}$$

$$\begin{cases} v_{C_2X} = -AC_2\, \omega_2 \sin(\varphi_2 + \alpha) \\ v_{C_2Y} = AC_2\, \omega_2 \cos(\varphi_2 + \alpha) \end{cases}$$

$$\begin{cases} a_{C_2X} = -AC_2\, (\varepsilon_2 \sin(\varphi_2 + \alpha) + \omega_2^2 \cos(\varphi_2 + \alpha)) \\ a_{C_2Y} = AC_2\, (\varepsilon_2 \cos(\varphi_2 + \alpha) - \omega_2^2 \sin(\varphi_2 + \alpha)) \end{cases}$$

$$\vec{G}_2 + \vec{R}_{12} + \vec{R}_{32} + \vec{F}_{i2} = 0$$

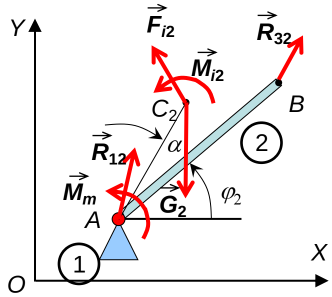
$$\begin{cases} \boldsymbol{R}_{12X} + \boldsymbol{R}_{32X} - m_2\, a_{C2X} = 0 \\ \boldsymbol{R}_{12Y} + \boldsymbol{R}_{32Y} - m_2\, (g + a_{C2Y}) = 0 \end{cases}$$

$$\vec{R}_{32} + \vec{R}_{23} = 0$$

$$\Downarrow$$

$$\begin{cases} \boldsymbol{R}_{32X} + \boldsymbol{R}_{23X} = 0 \\ \boldsymbol{R}_{32Y} + \boldsymbol{R}_{23Y} = 0 \end{cases} \Rightarrow \begin{cases} \boldsymbol{R}_{32X} = -\boldsymbol{R}_{23X} \\ \boldsymbol{R}_{32Y} = -\boldsymbol{R}_{23Y} \end{cases} \Rightarrow \begin{cases} \boldsymbol{R}_{12X} = \boldsymbol{R}_{23X} + m_2\, a_{C2X} \\ \boldsymbol{R}_{12Y} = \boldsymbol{R}_{23Y} + m_2\, (g + a_{C2Y}) \end{cases}$$

Kinetostatic analysis of motor element (motor moment computation)



$\boldsymbol{R}_{23X}$, $\boldsymbol{R}_{23Y}$ are known from previous computation

$$\begin{cases} \boldsymbol{R}_{32X} = -\boldsymbol{R}_{23X} \\ \boldsymbol{R}_{32Y} = -\boldsymbol{R}_{23Y} \end{cases}$$

Equilibrium equation of moments:

$$\boldsymbol{M}_m + \boldsymbol{R}_{32Y}\,(X_B - X_A) - \boldsymbol{R}_{32X}\,(Y_B - Y_A) - m_2\,(a_{C2Y} + g)\,(X_{C2} - X_A) + m_2\, a_{C2X}\,(Y_{C2} - Y_A) + \boldsymbol{M}_{i2}^A = 0$$

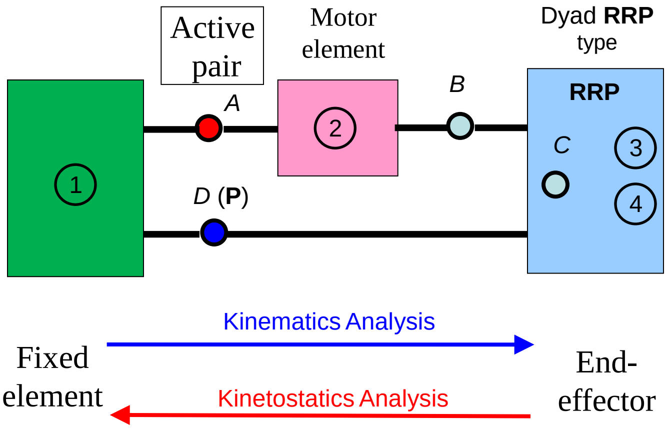
$$\Downarrow$$

$$\boldsymbol{M}_m = \boldsymbol{R}_{32X}\,(Y_B - Y_A) - \boldsymbol{R}_{32Y}\,(X_B - X_A) - m_2\, a_{C2X}\,(Y_{C2} - Y_A) + m_2\,(a_{C2Y} + g)\,(X_{C2} - X_A) + I_2^A\, \boldsymbol{\varepsilon}_2$$

$$I_2^A = I_2 + m_2\, AC_2^2$$

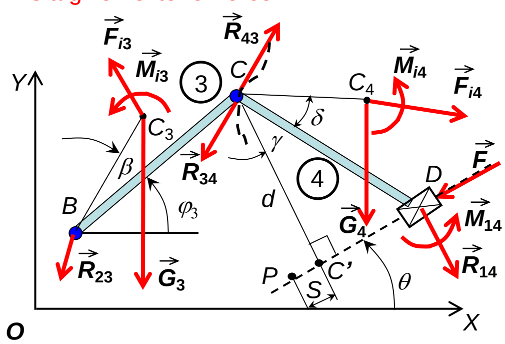
Crank-slider Mechanisms

Multipolar scheme (graph)



Kinetostatic analysis of Dyad **RRP** type

F is a given exterior force



$$\begin{cases} X_{C_3} = X_B + BC_3 \cos(\varphi_3 + \beta) \\ Y_{C_3} = Y_B + BC_3 \sin(\varphi_3 + \beta) \end{cases}$$

$$\begin{cases} X_{C_4} = X_C + CC_4 \cos(\theta + \gamma + \delta + 1.5\pi) \\ Y_{C_4} = Y_C + CC_4 \sin(\theta + \gamma + \delta + 1.5\pi) \end{cases}$$

$$\begin{cases} v_{C_3X} = v_{BX} - BC_3\, \omega_3 \sin(\varphi_3 + \beta) \\ v_{C_3Y} = v_{BY} + BC_3\, \omega_3 \cos(\varphi_3 + \beta) \end{cases}$$

$$v_{C_4X} = v_{CX} - CC_4\, \dot{\theta} \sin(\theta + \gamma + \delta + 1.5\pi)$$

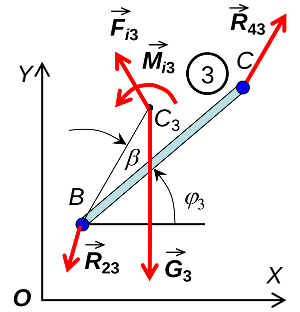
$$v_{C_4Y} = v_{CY} + CC_4\, \dot{\theta} \cos(\theta + \gamma + \delta + 1.5\pi)$$

$$\begin{cases} a_{C_3X} = a_{BX} - BC_3\, (\varepsilon_3 \sin(\varphi_3 + \beta) + \omega_3^2 \cos(\varphi_3 + \beta)) \\ a_{C_3Y} = a_{BY} + BC_3\, (\varepsilon_3 \cos(\varphi_3 + \beta) - \omega_3^2 \sin(\varphi_3 + \beta)) \end{cases}$$

And where are needed too axial inertial moments with respect to centres of masses of elements: I_2, I_3 and I_4

$$\begin{cases} a_{C_4X} = a_{CX} - CC_4\, \ddot{\theta} \sin(\theta + \gamma + \delta + 1.5\pi) - CC_4\, \dot{\theta}^2 \cos(\theta + \gamma + \delta + 1.5\pi) \\ a_{C_4Y} = a_{CY} + CC_4\, \ddot{\theta} \cos(\theta + \gamma + \delta + 1.5\pi) - CC_4\, \dot{\theta}^2 \cos(\theta + \gamma + \delta + 1.5\pi) \end{cases}$$

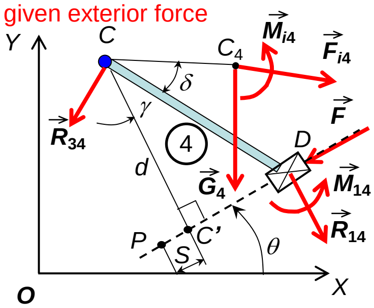
Kinetostatic analysis of Dyad **RRP** type



$\vec{R}_{43} + \vec{R}_{34} = 0$

$$\begin{cases} R_{34X} + R_{43X} = 0 \\ R_{34Y} + R_{43Y} = 0 \end{cases}$$

F is a given exterior force



$$\vec{G}_3 + \vec{R}_{23} + \vec{R}_{43} + \vec{F}_{i3} = 0$$

$$\vec{G}_4 + \vec{R}_{14} + \vec{R}_{34} + \vec{F}_{i4} + \vec{F} = 0$$

$$\begin{cases} R_{23X} + R_{43X} - m_3\, a_{C3X} = 0 \\ R_{23Y} + R_{43Y} - m_3\, (g + a_{C3Y}) = 0 \end{cases}$$

$$\begin{cases} R_{14}\cos(\theta + 1.5\pi) + R_{34X} - m_4\, a_{C4X} + F\cos(\theta + \pi) = 0 \\ R_{14}\sin(\theta + 1.5\pi) + R_{34Y} - m_4\, (g + a_{C4Y}) + F\sin(\theta + \pi) = 0 \end{cases}$$

Equilibrium equations of moments with respect to centers of mass:

$$\begin{cases} R_{23Y}(X_B - X_{C3}) - R_{23X}(Y_B - Y_{C3}) + R_{43Y}(X_C - X_{C3}) - R_{43X}(Y_C - Y_{C3}) + M_{i3} = 0 \\ F\sin(\theta + \pi)(X_D - X_{C4}) - F\cos(\theta + \pi)(Y_D - Y_{C4}) + R_{14}\sin(\theta + 1.5\pi)(X_D - X_{C4}) - \\ R_{14}\cos(\theta + 1.5\pi)(Y_D - Y_{C4}) + R_{34Y}(X_C - X_{C4}) - R_{34X}(Y_C - Y_{C4}) + M_{14} + M_{i4} = 0 \end{cases}$$

Kinetostatic analysis of Dyad **RRP** type

$$\begin{cases} R_{34X} + R_{43X} = 0 \\ R_{34Y} + R_{43Y} = 0 \end{cases} \Rightarrow \begin{cases} R_{34X} = -R_{43X} \\ R_{34Y} = -R_{43Y} \end{cases}$$

$$\begin{cases} R_{23X} + R_{43X} - m_3\, a_{C3X} = 0 \\ R_{23Y} + R_{43Y} - m_3\, (g + a_{C3Y}) = 0 \end{cases}$$

$$\begin{cases} R_{14}\cos(\theta + 1.5\pi) - R_{43X} - m_4\, a_{C4X} + F\cos(\theta + \pi) = 0 \\ R_{14}\sin(\theta + 1.5\pi) - R_{43Y} - m_4\, (g + a_{C4Y}) + F\sin(\theta + \pi) = 0 \end{cases}$$

$$\begin{cases} R_{23Y}(X_B - X_{C3}) - R_{23X}(Y_B - Y_{C3}) + R_{43Y}(X_C - X_{C3}) - R_{43X}(Y_C - Y_{C3}) - I_3\,\varepsilon_3 = 0 \\ F\sin(\theta + \pi)(X_D - X_{C4}) - F\cos(\theta + \pi)(Y_D - Y_{C4}) + R_{14}\sin(\theta + 1.5\pi)(X_D - X_{C4}) - \\ R_{14}\cos(\theta + 1.5\pi)(Y_D - Y_{C4}) - R_{43Y}(X_C - X_{C4}) + R_{43X}(Y_C - Y_{C4}) + M_{14} - I_4\,\ddot{\theta} = 0 \end{cases}$$

Is reduced to a linear equations with 6 equations and 6 unknowns:

$$R_{14},\ M_{14},\ R_{23X},\ R_{23Y},\ R_{43X},\ R_{43Y}$$

Kinetostatic analysis of Dyad **RRP** type

$$\begin{cases} \boldsymbol{R}_{23X} = -\boldsymbol{R}_{43X} + m_3\, a_{C3X} + \boldsymbol{F} \\ \boldsymbol{R}_{23Y} = -\boldsymbol{R}_{43Y} + m_3\,(g + a_{C3Y}) \end{cases} \qquad \begin{cases} \boldsymbol{R}_{43X} = \boldsymbol{R}_{14} \cos(\theta + 1.5\pi) - m_4\, a_{C4X} + \boldsymbol{F} \cos(\theta + \pi) \\ \boldsymbol{R}_{43Y} = \boldsymbol{R}_{14} \sin(\theta + 1.5\pi) - m_4\,(g + a_{C4Y}) + \boldsymbol{F} \sin(\theta + \pi) \end{cases}$$

$$\Downarrow$$

$$\boldsymbol{R}_{23Y}\,(X_B - X_{C3}) - \boldsymbol{R}_{23X}\,(Y_B - Y_{C3}) + \boldsymbol{R}_{43Y}\,(X_C - X_{C3}) - \boldsymbol{R}_{43X}\,(Y_C - Y_{C3}) - I_3\, \varepsilon_3 = 0$$

$$\Downarrow$$

$$\boldsymbol{R}_{14} \Rightarrow \boldsymbol{R}_{43X},\ \boldsymbol{R}_{43Y} \quad \text{si} \quad \boldsymbol{R}_{34X},\ \boldsymbol{R}_{34Y}$$

$$\Downarrow$$

$$\boldsymbol{R}_{23X},\ \boldsymbol{R}_{23Y}$$

$$\boldsymbol{M}_{14} = \boldsymbol{F} \cos(\theta + \pi)\,(Y_D - Y_{C4}) - \boldsymbol{F} \sin(\theta + \pi)\,(X_D - X_{C4}) + \boldsymbol{R}_{14} \cos(\theta + 1.5\pi)\,(Y_D - Y_{C4}) - \boldsymbol{R}_{14} \sin(\theta + 1.5\pi)\,(X_D - X_{C4}) + \boldsymbol{R}_{43Y}\,(X_C - X_{C4}) - \boldsymbol{R}_{43X}\,(Y_C - Y_{C4}) + I_4\, \ddot{\theta}$$

Conclusions

- Kinematic Analysis of a plane mechanism leads to non-linear equations which can be solved by numerical methods
- Kinematic Analysis is used in Kinetostatic Analysis in order to compute the inertial forces in the centres of masses
- Kinetostatic Analysis of plane mechanisms leads to linear equations which can be solved much easier