Minesweeper

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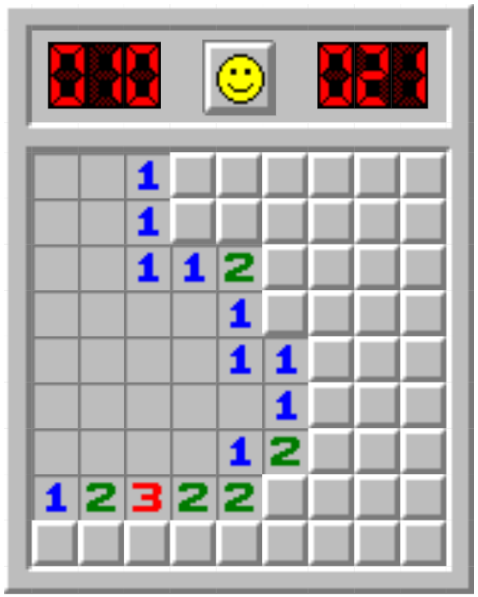
Problem presentation

- Encode a minesweeper game state in a set of SAT formulas.
- Based on the feedback of the SAT solver, identify:
  - Cells that are surely hiding a mine.
  - Cells that are totally safe to click on.



Approach

For each hidden cell h:

- Create board borders: add negative clause for each border cell.
- Go through all visible cells:
  - Add negative clause for the cell itself.
  - Add CNF clauses to account for number of neighbouring mines.
- Add h as positive (negative) clause.
- If the solver evaluates to unsatisfiable h is safe (unsafe).

Remark

Positive clauses: mine.
Negative clauses: no mine.

Cardinality constraint

Exactly k

The constraint "exactly k propositional variables in X are true" can be rephrased as the conjunction of "at least k" and "at most k".

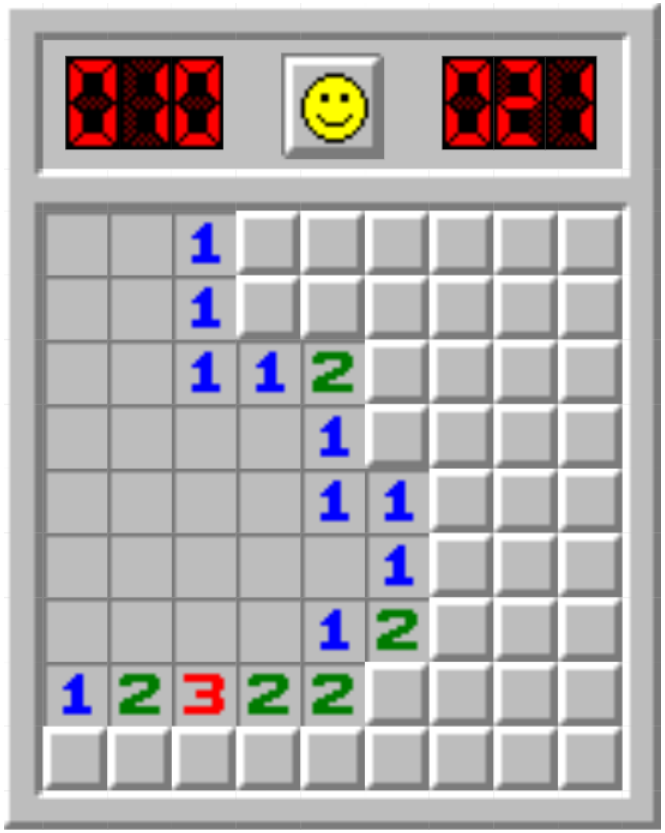
$$\bigwedge_{\substack{I \subseteq [n] \\ |I| = n-k+1}} \bigvee x_i \wedge \bigwedge_{\substack{I \subseteq [n] \\ |I| = k+1}} \bigvee_{i \in I} \neg x_i$$

Example (Exactly 2 among X={a, b, c, d})

$$(a \vee b \vee c) \wedge (a \vee b \vee d) \wedge (b \vee c \vee d) \wedge (\neg a \vee \neg b \vee \neg c) \wedge$$
$$(\neg a \vee \neg b \vee \neg d) \wedge (\neg a \vee \neg c \vee \neg d) \wedge (\neg b \vee \neg c \vee \neg d)$$

Example

Create the first game state.



Find out which cells are safe and unsafe.

row 1 col 4, → No mine
row 2 col 4, → MINE
row 2 col 5, → No mine
row 2 col 6, → No mine
row 3 col 6, → No mine
row 4 col 6, → MINE
row 4 col 7, → No mine
row 5 col 7, → No mine
row 6 col 7, → No mine
row 7 col 7, → MINE
row 8 col 6, → MINE
row 8 col 7, → No mine
row 9 col 1, → No mine
row 9 col 2, → MINE
row 9 col 3, → MINE
row 9 col 4, → MINE
row 9 col 5, → No mine
row 9 col 6, → No mine

Example

Generate next game state by clicking on all safe cells and flagging unsafe cells.

row 1 col 4, → No mine
row 2 col 4, → MINE
row 2 col 5, → No mine
row 2 col 6, → No mine
row 3 col 6, → No mine
row 4 col 6, → MINE
row 4 col 7, → No mine
row 5 col 7, → No mine
row 6 col 7, → No mine
row 7 col 7, → MINE
row 8 col 6, → MINE
row 8 col 7, → No mine
row 9 col 1, → No mine
row 9 col 2, → MINE
row 9 col 3, → MINE
row 9 col 4, → MINE
row 9 col 5, → No mine
row 9 col 6, → No mine

Same as before but this time uncovering the safe hiddens cells leads to the end of the game.

row 1 col 5, → No mine
row 1 col 6, → No mine
row 1 col 7, → No mine
row 2 col 4, → MINE
row 4 col 6, → MINE
row 4 col 8, → No mine
row 7 col 7, → MINE
row 7 col 8, → No mine
row 8 col 6, → MINE
row 8 col 8, → No mine
row 9 col 2, → MINE
row 9 col 3, → MINE
row 9 col 4, → MINE
row 9 col 7, → No mine
row 9 col 8, → No mine

Thank you for your attention