Application of R-matrix approach in the Analysis of ^{9}Be(p, γ)^{10}B and ^{12}C(p,γ)^{13}N

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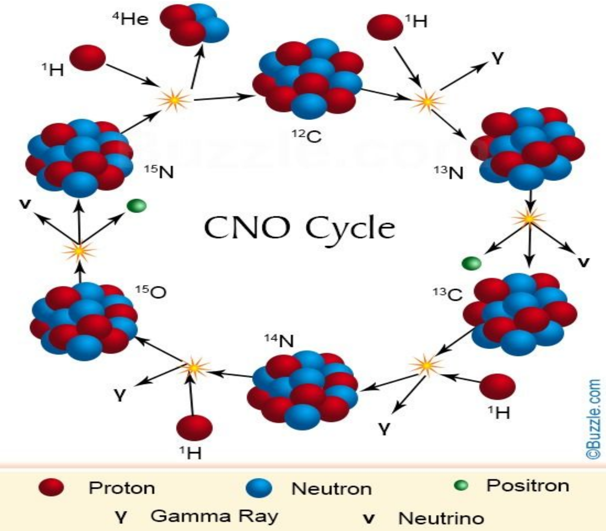
Outlines

Background

- A step-wise conversion takes place because of a step-wise rise in temperature in stars, from lighter nuclei to heavier nuclei. In this process of stellar evolution, the Carbon-Nitrogen-Oxygen (CNO) cycle and the synthesis of low mass nuclei in the $p-p$ chain reactions play an important role in primordial and stellar nucleosynthesis of light elements.
- Stars generate energy by a process called nuclear fusion ($p-p$ chain reaction or CNO cycle).
- The CNO cycle initiate when the core temperature reaches 14×10^6 K and the mass of the star reaches 1.5 $\mathrm{M}_\odot$.
- Elements like Be, B, and Li are synthesized during the non-thermal process called spallation reaction and as well as by the radiative capture processes.
- The ^{9}Be (p,γ) ^{10}B is important for the determination of Li, Be, and B abundances.

Background



^4He
^1H
^1H
γ
^{12}C
^{15}N
^{13}N
v
CNO Cycle
v
^{15}O
^{13}C
^{14}N
γ
γ
^1H
^1H
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Proton
Neutron
Positron
γ Gamma Ray
v Neutrino

Literature Review

We revised the following literature

1. Angulo, C., Arnould, M., Rayet, M., et al.: Nucl. Phys. A656, 3–183(1999)
2. Dubovichenko, S.B., Dzhazairov-Kakhramanov, A., Fessenkov, V.G.:Int. J. Mod. Phys. E21(3), 1–44 (2012)
3. Fowler, W.A.: Nobel Lecture, pp. 172–229 (1983)
4. Fowler, W.A., Caughlan, G.R., Zimmerman, B.A.: Annu. Rev. Astron.Astrophys.5 525–570 (1967)
5. Radiative capture of proton by ^{12}C at low energy.
6. Proton capture cross-section for ^{12}C at low energy.
7. Radiative capture of proton by ^{13}C at low energy.
8. Astrophysical S-factor of proton capture by ^{13}C at low energy.
9. Re-examination of proton capture by ^{9}Be in stellar matter.

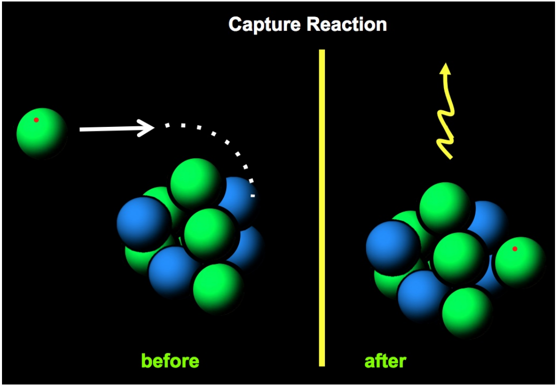
Problem Statement

To calculate and analyze the

- The nuclear cross-section
- The astrophysical S-factor
- The nuclear reaction rates
- The nuclear abundances of ^{9}Be(p, γ)^{10}B and ^{12}C(p,γ)^{13}N

Objective

p–^{Z}X



Motivation

1. This project will help us learn about the mechanism of nuclear reactions of astrophysical significance and all the underlying variables that dictate such reactions both at low and at high energies.
2. Understanding the spectroscopy of nuclei, especially, Beryllium, Boron, and CNO nuclei.
3. Sequences of proton capture reactions on proton-rich unstable nuclei are the power source of the explosions in binary star systems. The determination of these rates is one of the frontiers of nuclear astrophysics.
4. The radiative capture data of experiments can be completely described by the employed $R-$matrix approach.

Limitations

1. The R-matrix approach is applicable for all types of atomic and nuclear resonance reactions.
2. This method fails for the direct capture processes which have low amplitude for the calculation of cross section.
3. For the direct capture process the Potential Model (PM) is applicable.

R-matrix Approach

The resonant cross-section of the radiation capture reaction is computed by the equation

$$\sigma_r(E) = \frac{\pi}{k^2}\omega\frac{\Gamma_p(E)\Gamma_\gamma(E)}{(E-E_R)^2+\Gamma(E)^2/4}, \tag{1}$$

k is the wave number of incoming particle in the center of mass (CM) system, $\Gamma_p(E)$, $\Gamma_\gamma(E)$ and $\Gamma(E)$ are the proton partial width, radiative partial width, and total width, respectively; ω is the statistical term

$$\omega = (1+\delta_{ij})\frac{(2J+1)}{(2I_1+1)(2I_2+1)}, \tag{2}$$

The energy dependent proton width $\Gamma_p(E)$ and radiative capture gamma width $\Gamma_\gamma(E)$ are defined as

$$\Gamma_p(E) = \frac{P_l(E)}{P_l(E_R)} \Gamma_p(E_R), \tag{3}$$

$$\Gamma_\gamma(E) = \left(\frac{E + \varepsilon_f}{E_R + \varepsilon_f}\right)^{2\lambda+1} \Gamma_\gamma(E_R), \tag{4}$$

The penetrability $P_l(E)$ is defined as

$$P_l(E) = \frac{kb}{F_l^2(k,b) + G_l^2(k,b)}, \tag{5}$$

where b, $F_l(k,b)$ and $G_l(k,b)$ are the channel radius, regular and irregular Coulomb functions.

For the interaction of charged particles, the astrophysical S-factor is defined as

$$S(E) = \sigma(E) E \exp(2\pi\eta) \tag{6}$$

The astrophysical S-factor is smooth function over energy. The accompanying equation was used to calculate the astrophysical reaction rates for radiative capture reaction is

$$\begin{aligned} N_A\langle\sigma v\rangle &= N_A \left(\frac{8\pi}{\mu}\right)^{1/2} \frac{1}{(k_B T)^{3/2}} \times \\ &\int_0^\infty \sigma(E) E \exp(-E/k_B T) dE, \end{aligned} \tag{7}$$

Data Gathering and Parameters used

Data Sources

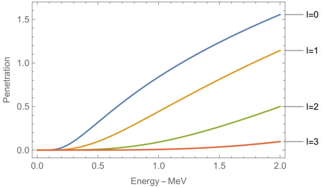
o TUNL Data (online)
`(https://nucldata.tunl.duke.edu)`
o Numerical Data and Functional Relationships in Science and Technology (Book) [Landolt-Bornstein]

Interaction Parameters

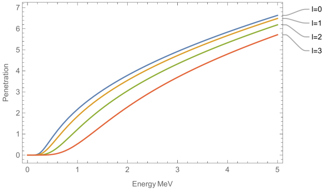
Reaction	E (MeV)	Γ_γ (MeV)	ε_f (MeV)	J^π
^{9}Be (p,γ) ^{10}B	7.750	0.27×10^{-6}	6.5859	$1^- \rightarrow 3^+$
^{12}C (p,γ) ^{13}N	3.547	0.015×10^{-6}	1.94	$\frac{5^+}{2} \rightarrow \frac{1^-}{2}$

Table. 1 The parameters for the calculation of cross-section and S-factor. The first column defines the resonance energy E_p, the second column represents the partial width Γ_p in the CM frame, the third column Γ_γ is the radiative width, the fourth column represents the Q energy ε_f and the fifth column represents the spin J^π

Observed penetration factors at different ℓ

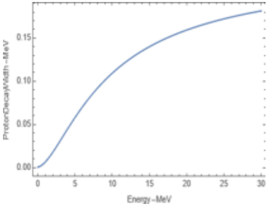


(a) penetration factors of ^{9}Be (p,γ)^{10}B

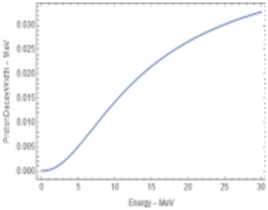


(b) penetration factors of ^{12}C (p,γ)^{13}N

Entry channel width at the resonance energy

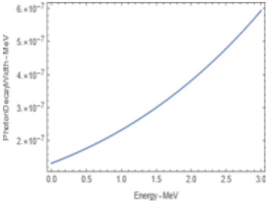


(a) Gamma width (Γ_γ) of ^{9}Be (p,γ)^{10}B

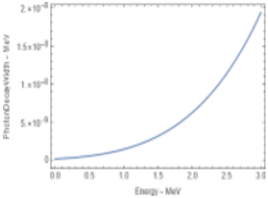


(b) Gamma width (Γ_γ) of ^{12}C (p,γ)^{13}N

Partial (exit channel) width at the resonance energy

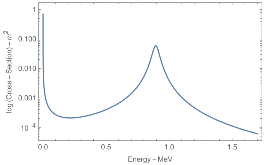


(a) Partial width (Γ_p) of ^{9}Be (p,γ)^{10}B

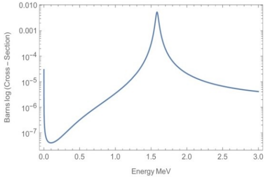


(b) Partial width (Γ_p) of ^{12}C (p,γ)^{13}N

Cross-Sections

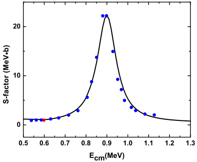


(a) Cross-section of ^{9}Be (p,γ)^{10}

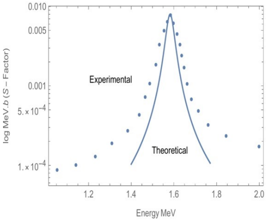


(b) Cross-section of ^{12}C (p,γ)^{13}N

Fitted S-Factor

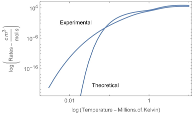


(a) S-Factor of ^{9}Be (p,γ)^{10}

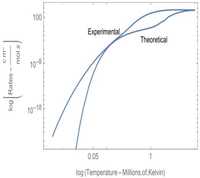


(b) S-Factor of ^{12}C (p,γ)^{13}N

Rates of the reactions

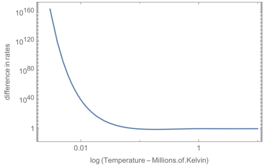


(a) Rate of ^{9}Be (p,γ)^{10}

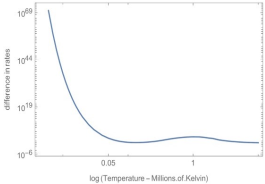


(b) Rate of ^{12}C (p,γ)^{13}N

Difference of the experimental and modeled rates



(a) Difference of Rates of ^{9}Be (p,γ)^{10}



(b) Difference of Rates of ^{12}C (p,γ)^{13}N

Abundance of C-12

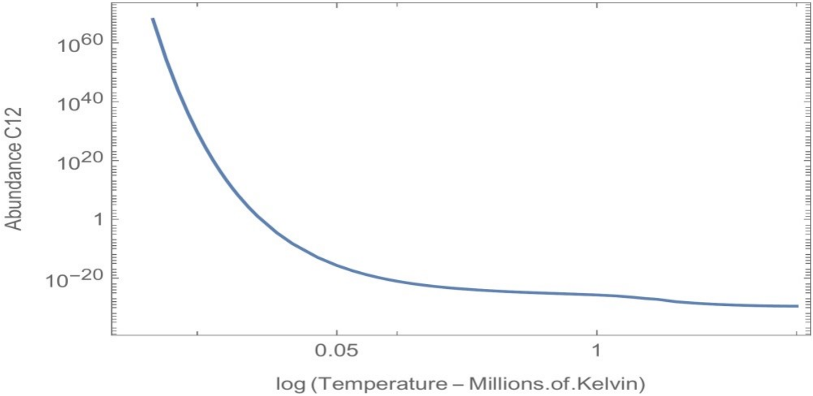


Figure 8 – Rate of ^{9}Be (p,γ)^{10}

Project Timeline

S.No.	Elapsed time since start of the project	Milestone	Deliverable
1.	1.5 Months	Literature Review	Summary of the work done in the relevant field
2.	3 Months	Experimentation with the problem	Simulation of the reaction environment
3.	4.5 Months	Theoretical implementation	Cross-sections for the reaction up to 2 MeV
4.	6 Months	Further theoretical implementations	S-factor and Rates
5.	7.5 Months	Digitized data from graphs	Data in Raw form
6.	9 Months	Research Paper and Thesis Writing	Thesis Publication

Conclusion

1. The studies conclude in the calculation of abundances, rates, fitted S-factors and cross-sections for ^{12}C (p,γ)^{13}N and ^{9}Be (p,γ)^{10}B.
2. The penetrations of charged nucleon for l= 0> l= 1 > l= 2> l= 3> both for ^{12}C (p,γ)^{13}N and ^{9}Be (p,γ)^{10}B processes.
3. The present studies concluded that the R-matrix method regenerated shape form of nuclear cross-section both for the sharp and broad resonance peaks for ^{12}C (p,γ)^{13}N and ^{9}Be (p,γ)^{10}B processes.

Future studies

1. We plan to compute the neutron capture cross-section by heavy nuclei for different types of neutron (slow neutron and for fast neutron).
2. We can also analyze the (^{12}C (p,γ)^{13}N and ^{9}Be (p,γ)^{10}B) by using the Potential model to study for the broad resonances by employing the Morse potential and Woods-Saxon potential.

THANK YOU

publications

1. Radiative capture of proton by ^{9}Be(p, γ)^{10}B A. Kabir, J.U Nabi, S. Sagheer, L. Rashid. **Communications in Theoretical Physics** 74 (2), 025301
2. Re-analysis of radiative capture ^{11}C(p, γ)^{12}N at low energy A Kabir, B.F. Irgaziev, S. Sagheer **Journal of Physics G: Nuclear and Particle Physics** 49 (7), 075101