

JACOBI ITERATION METHOD:

The Jacobi iterative method is considered as an iterative algorithm which is used for determining the solutions for the system of linear equations in numerical linear algebra, which is diagonally dominant. In this method, an approximate value is filled in for each diagonal element.

EXAMPLE:

Apply the Jacobi method to solve

$$3x_1 - 2x_2 + 4x_3 = 1$$

$$-3x_1 + 9x_2 + x_3 = 2$$

$$2x_1 + x_2 + 7x_3 = 3$$

Continue iterations until two successive approximations are identical to three significant digits.

SOLUTION:

To begin, rewrite the system

$$x_1 = \frac{1}{3} + \frac{2}{3}x_2 - \frac{4}{3}x_3$$

$$x_2 = \frac{2}{9} + \frac{3}{9}x_1 - \frac{1}{3}x_3$$

$$x_3 = \frac{3}{7} - \frac{2}{7}x_1 - \frac{1}{7}x_2$$

Initial guess:

$$x_1=0, x_2=0, x_3=0.$$

The first approximation is

$$x_1^{(1)} = \frac{1}{3} + \frac{2}{3}(0) - \frac{4}{3}(0) = 0.333$$

$$x_2^{(1)} = \frac{2}{9} + \frac{3}{9}(0) - \frac{1}{3}(0) = 0.222$$

$$x_3^{(1)} = \frac{3}{7} - \frac{2}{7}(0) - \frac{1}{7}(0) = 0.428$$

Continue iteration we obtain

n	K=0	K=1	K=2	K=3
$x_1^{(k)}$	0.000	0.333	-0.333	-1
$x_2^{(k)}$	0.000	0.222	-0.222	0.22
$x_3^{(k)}$	0.000	0.428	-0.428	-0.85

