# BISECTION METHOD:

This method is a root-finding method that applies to any continuous functions with two known values of opposite signs.

# FORMULA:

𝑥𝑚

𝑥𝑙+𝑥𝑢 2

# ADVANTAGES:

=

1. Always convergent.
2. The roots get halved with each iteration.

# DISADVANTAGES:

1. Slow convergence.
2. if one of the initial guesses is close to the root, the convergence is slower.
3. If a function f(x)is such that it just touches the x-axis it will be unable to find the lower and upper guesses.
4. Function changes sign but roots do not exist.

# EXAMPLE:

cosx-x𝑒𝑥=0

F(0) =cos(0)-0𝑒0=0 F(2) =cos (2)-2𝑒2=0

So, the roots are (0,2).

𝑥𝑙=0

𝑥𝑢=2

Now,

𝑥𝑚

=0+2=1

Now,

2

New roots are

𝑥𝑚

=1+2=1.5

New roots are (1.5,2)

2

𝑥𝑚

=1.5+2=1.7

# Error:

2

|𝐸𝑎

|=|𝑥𝑛𝑒𝑤−𝑥𝑜𝑙𝑑|

𝑥𝑛𝑒𝑤

1.7−1.5

=

|

1.7

| × 100

=11%

1. **FIXED POINT METHOD:**

In numerical analysis, fixed-point iteration is a method of computing fixed points of a function. More specifically, given a function f defined on the real numbers with real values and given a point

F(x) = 0 also written in x = Ø(x)

Where <1

**Advantages**:

Operations can be applied on the number just like on integers.

**Disadvantages:**

It requires a starting interval containing a change of sign. Therefore it cannot find repeated roots. It has a fixed rate of convergence, which can be much slower than other methods, requiring more iterations to find the root to a given degree of precision.

**Example:**

***F(0)= -1***

***F(1)=1***

Roots are **(-1,1)**

**Take any initial guess which lies between 1 and -1**

**=0.5 (initial value)**

**=x**

**Derivative**

**Then**

**Also**

**X=**

**Again**

**=Ø(x)**

**Now take the value which is closer to zero like our first iteration**

**Taking to formula for fixed point method:**

**Solving: = = 0.793**

**=**

**=**

**=**

**ERROR:**

=

= = 10%

1. **SECANT METHOD:**

In numerical analysis, the secant method is a root-finding algorithm that uses a succession of roots of secant lines to better approximate a root of a function f. The secant method can be thought of as a finite-difference approximation of Newton's method.

**FORMULA:**

**=**

**ADVANTAGES:**

* It converges at faster than a linear rate, so that it is more rapidly. convergent than the bisection method.
* It does not require use of the derivative of the. Function something that is not available in a number. of applications.
* It requires only one function evaluation per iteration.

**DISADVANTAGES:**

It does not require use of the derivative of the function, something that is not available in a number of applications.

**EXAMPLE:**

F(x) = x-

First we find its roots:

* F(0) = 0-= -1

For second root:

* F(1) = 1-= 0.632

These roots are not close to each other so if I take 0.5 and 0.6.

Roots are (0.5, 0.6)

Now:

And: f () = 0.0511

By using formula for secant method

=

For n=1

=

=

Put value, we get:

=

= 0.5675

Now for n=2

Now for n=3

**Error:**

**Formula for error is**: =

= 0%

1. **NEWTON DIVIDER DIFFERENCE INTERPOLATION**:

Interpolation is an estimation of a value within two known values in a sequence of values. Newton's divided difference interpolation formula is **a interpolation technique used when the interval difference is not same for all sequence of values**.

**Formula:**

Let

Table:

|  |  |  |  |
| --- | --- | --- | --- |
| **x** | **y=** | **First order D.D** | **Second order D.D** |
|  |  |  |  |
|  |  |  |  |
|  |  |  |  |
|  |  |  |  |

**Example:**

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| x | 1 | 3 | 4 | 8 |
| y | 2 | 5 | 9 | 11 |

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| x | y | First order | Second order | Third order |
| 1 | 2 |  |  |  |
|  |  |  |  |  |
| 3 | 5 |  |  |  |
|  |  |  |  |  |
| 4 | 9 |  |  |  |
|  |  |  |  |  |
| 8 | 11 |  |  |  |

Now put in formula

=

After simplification and putting in formula it gives:

**+**

1. **JORDAN METHOD:**

Gauss-Jordan Elimination is **an algorithm that can be used to solve systems of linear equations and to find the inverse of any invertible matrix**. It relies upon three elementary row operations one can use on a matrix: Swap the positions of two of the rows. Multiply one of the rows by a nonzero scalar.

Gaussian Elimination is a structured method of solving a system of linear equations. Thus, it is an algorithm and can easily be programmed to solve a system of linear equations. The main goal of Gauss-Jordan Elimination is:

* to represent a system of linear equations in an **augmented matrix form**
* then performing the 3 row operations on it until the **reduced row echelon form**

**Example:**

**Row operation can be used to express the matrix in reduced rowechelon form**

The matrix now says that x=1,y=4,z=-2

1. **GUASS ELIMINATION METHOD:**

**In matrix form: AX=B**

**=**

**C = [A: B] =**

**Now find echelon form**

C =

And then back substitute:

**Example:**

**AX = B**

By making augmented matrix:

C = [A: B] =

Now equate the equation by echelon method by performing row operations :

The corresponding system of equation:

x+3y+2z=5

y+5z=7

-9z=-9

Solving by back substitution we get;

1. **GUASS SEIDLE METHOD:**

In numerical linear algebra, the **Gauss**–**Seidel method**, also known as the Liebmann **method** or the **method** of successive displacement, is an iterative **method**.

Diagonal dominance property must be satisfies:

Rewriting the equations for x,y,z

For second approximation we get;

For third:

**Example:**

**Solution:**

Magnitude of first row is greater to other and so on so the diagonal property satisfies:

Rewriting the equations.

**Now assume**

=3.1481

=3.5408

=1.9132

=2.4322

=3.5720

=1.9258

=2.4256

=3.5730

=1.9260

**But formula for error is:**

**∞sign means maximum value….**

* **SPLINE CUBIC RULE:**

**Example: construct a cubic spline that passes through the (1,2)(2,3)and(3,5)**

Solution:

[1,2]:

[2,3]:

By making augmented matrices:

S(x)={2+

S(x)={3+)+

1. **SIMPSONS**

**Formula**:

**find solution using this rule:**

|  |  |
| --- | --- |
| **x** | **F(x)** |
| **1.4** | **4.0552** |
| **1.6** | **4.9530** |
| **1.8** | **6.0436** |
| **2.0** | **7.3891** |
| **2.2** | **9.0250** |

**Solution:**

**The value of table for x and y**

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **x** | **1.4** | **1.6** | **1.8** | **2.0** | **2.2** |
| **y** | **4.0552** | **4.9530** | **6.0436** | **7.3891** | **9.0250** |

**By using formula:**

**Solution by Simpsons rule is 4.5636**

1. **TRAPEZOIDAL RULE:**

**Formula:**

**Example:**

**Find solution by this method:**

|  |  |
| --- | --- |
| **x** | **F(x)** |
| **0.0** | **1.0000** |
| **0.1** | **0.9975** |
| **0.2** | **0.9900** |
| **0.3** | **0.9776** |
| **0.4** | **0.8604** |

**Solution:**

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **x** | **0** | **0.1** | **0.2** | **0.3** | **0.4** |
| **y** | **1** | **0.9975** | **0.99** | **0.9776** | **0.8604** |

**Using formula:**

**Solution by trapezoidal rule is:**