Differential Equations

Assignment no 1

Name : Nida Shafique
Reg no : SP20 - BSE - 024
Submitted To : MUHAMMAD ILYAS

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In Problem 1-10 state whether the given differential equations are linear or non-linear. Give the order of each equation

1) $(1-x)y'' - 4xy' + 5y = \cos x$

Given differential equation is linear, and is of second order.

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3) $yy' + 2y = 1 + x^2$

Given differential equation is non-linear because it contains product of dependent variable, i-e; yy'

It is of first order.

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5) $x^3 y^{(4)} - x^2 y'' + 4xy' - 3y = 0$

Given differential equation is linear.

This differential equation contains fourth-order.

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7) $\frac{dy}{dx} = \sqrt{1 + \left(\frac{d^2y}{dx^2}\right)^2}$

Taking square on both sides

$$\left(\frac{dy}{dx}\right)^2 = 1 + \left(\frac{d^2y}{dx^2}\right)^2$$

This differential equation is non-linear because of $\left(\frac{d^2y}{dx^2}\right)^2$, degree is greater than 1, which makes it non-linear.

It is of second-order.

In Problems 11-40 verify that the indicated function is a solution of the given differential equation. Where appropriate, c_1 and

c_2 denote constants.

11) $2y' + y = 0 ; \rightarrow (1) \quad y = e^{-x/2}$

$y = e^{-x/2}$

Taking derivative on both side.

$\frac{dy}{dx} = \frac{d}{dx} e^{-x/2}$

$\because \frac{d}{dx} e^{x} = e^{x} \frac{dx}{dx}$

$y' = e^{-x/2} \frac{d}{dx} \frac{-x}{2}$

$\because \frac{d}{dx} kx = k \frac{dx}{dx}$

$y' = e^{-x/2} \cdot \frac{-1}{2} \frac{dx}{dx}$

$y' = \frac{-1}{2} e^{-x/2}$

Put value of y' and y in equation (1)

equation (1) $\Rightarrow 2y' + y = 0$

$\cancel{2}\left(\frac{-1}{\cancel{2}} e^{-x/2}\right) + e^{-x/2} = 0$

$-\cancel{e^{-x/2}} + \cancel{e^{-x/2}} = 0$

$0 = 0$

L.H.S = R.H.S

So, hence proved, given function $y = e^{-x/2}$ is the solution of given differential equation $2y' + y = 0$.

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13) $\frac{dy}{dx} - 2y = e^{3x} ; \rightarrow (1) \quad y = e^{3x} + 10e^{2x}$

$y = e^{3x} + 10 e^{2x}$

Taking derivation on both side

$\frac{dy}{dx} = \frac{d}{dx} e^{3x} + \frac{d}{dx} 10 e^{2x}$

$\because \frac{d}{dx} e^x = e^x \frac{dx}{dx} = e^x$

$\because \frac{d}{dx} kx = k \frac{dx}{dx} = k$

$$\frac{dy}{dx} = e^{3x} \frac{d\,3x}{dx} + 10 \frac{d}{dx} e^{2x}$$

$$= e^{3x} . 3 + 10 . e^{2x} \frac{d\,2x}{dx}$$

$$= 3e^{3x} + 10 . e^{2x} . 2$$

$$\frac{dy}{dx} = 3e^{3x} + 20e^{2x}$$

Put value of $\frac{dy}{dx}$ and y in equation ①

$$\frac{dy}{dx} - 2y = e^{3x}$$

$$\Rightarrow 3e^{3x} + 20e^{2x} - 2(e^{3x} + 10e^{2x}) = e^{3x}$$

$$3e^{3x} + \cancel{20e^{2x}} - 2e^{3x} - \cancel{20e^{2x}} = e^{3x}$$

$$e^{3x} = e^{3x}$$

$$L.H.S = R.H.S$$

Hence proved, so given function $y = e^{3x} + 10e^{2x}$ satisfied the given differential equation $\frac{dy}{dx} - 2y = e^{3x}$, $y = e^{3x} + 10e^{2x}$ is the solution of $\frac{dy}{dx} - 2y = e^{3x}$.

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17) $y' + y = \sin x; \rightarrow$ ① $\qquad y = \frac{1}{2}\sin x - \frac{1}{2}\cos x + 10e^{-x}$

$$y = \frac{1}{2}\sin x - \frac{1}{2}\cos x + 10e^{-x}$$

Taking derivative on both side

$$\frac{dy}{dx} = \frac{d}{dx}\frac{1}{2}\sin x - \frac{d}{dx}\frac{1}{2}\cos x + \frac{d}{dx} 10e^{-x}$$

$\because \frac{d}{dx} kx = k\frac{dx}{dx} = k$

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$$y' = \frac{1}{2}\frac{d}{dx}\sin x - \frac{1}{2}\frac{d}{dx}\cos x + 10\frac{d}{dx}e^{-x}$$

$$\because \frac{d}{dx}\sin x = \cos x, \quad \because \frac{d}{dx}\cos x = -\sin x$$

$$\because \frac{d}{dx}e^{x} = e^{x}\frac{dx}{dx} = e^{x}$$

$$y' = \frac{1}{2}\cos x - \frac{1}{2}(-\sin x) + 10e^{-x}\frac{d}{dx}-x$$

$$= \frac{1}{2}\cos x + \frac{1}{2}\sin x - 10e^{-x}$$

Put value of y' and y in equation ①

$$y' + y = \sin x$$

$$\cancel{\frac{1}{2}\cos x} + \frac{1}{2}\sin x \cancel{-10e^{-x}} + \frac{1}{2}\sin x \cancel{-\frac{1}{2}\cos x} \cancel{+10e^{-x}} = \sin x$$

$$\cancel{\frac{2}{2}}\sin x = \sin x$$

$$\sin x = \sin x$$

$$L.H.S = R.H.S$$

Hence proved, given function satisfied the given differential equation, so $y = \frac{1}{2}\sin x - \frac{1}{2}\cos x + 10e^{-x}$ is the solution of given differential equation

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19) $x^2 dy + 2xy\,dx = 0; \rightarrow ①$ $\qquad y = \frac{-1}{x^2}$

$$y = -\frac{1}{x^2}$$

Taking derivative on both sides

$$\frac{dy}{dx} = \frac{d}{dx} - x^{-2}$$

$$\because \frac{d}{dx}x^n = nx^{n-1}\frac{dx}{dx}$$

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$$\frac{dy}{dx} = -(-2)x^{-2-1}\frac{dx}{dx}$$

$$\frac{dy}{dx} = 2x^{-3}$$

$$\Rightarrow \frac{dy}{dx} = \frac{2}{x^3}$$

Given differential equation is

$$x^2 dy + 2xy\,dx = 0$$

$\div dx$ on both sides

$$\frac{x^2 dy + 2xy\,dx}{dx} = \frac{0}{dx}$$

$$x^2\frac{dy}{dx} + 2xy\frac{\cancel{dx}}{\cancel{dx}} = 0$$

$$x^2\frac{dy}{dx} + 2xy = 0$$

Put value of $\frac{dy}{dx}$ and y

$$\Rightarrow x^2\left(\frac{2}{x^3}\right) + 2x\left(\frac{-1}{x^2}\right) = 0$$

$$\frac{2}{x} - \frac{2}{x} = 0$$

$$0 = 0$$

$$L.H.S = R.H.S$$

Hence proved, given function $y = \frac{-1}{x^2}$ is the solution of given differential equation $x^2 dy + 2xy\,dx = 0$.

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23) $y' - \frac{1}{x}y = 1; \rightarrow (1) \qquad y = x\ln x,\ x > 0$

$$y = x\ln x$$

Taking derivative on both sides

$$\frac{dy}{dx} = \frac{d}{dx} x \ln x$$

$$\because \frac{d}{dx}[f(x_1)][f(x_2)] = f(x_1)\frac{d}{dx}f(x_2) + f(x_2)\frac{d}{dx}f(x_1)$$

$$\frac{dy}{dx} = x\frac{d}{dx}\ln x + \ln x \frac{dx}{dx}$$

$$\because \frac{d}{dx}\ln x = \frac{1}{x}\frac{dx}{dx}$$

$$y' = \cancel{x}\frac{1}{\cancel{x}}\frac{dx}{dx} + \ln x$$

$$y' = 1 + \ln x$$

Put value of y' and y in equation ①

$$1 + \ln x - \frac{1}{\cancel{x}}(\cancel{x}\ln x) = 1$$

$$1 + \cancel{\ln x} - \cancel{\ln x} = 1$$

$$1 = 1$$

$$L.H.S = R.H.S$$

Hence proved, given function $y = x\ln x$ is the solution of given differential equation $y' - \frac{1}{x}y = 1$

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29) $y'' - 6y' + 13y = 0 ; \rightarrow ①$ $\quad y = e^{3x}\cos 2x$

$$y = e^{3x}\cos 2x$$

Taking derivative on both sides

$$\frac{dy}{dx} = \frac{d}{dx} e^{3x}\cos 2x$$

$$\because \frac{d}{dx}[f(x_1)].[f(x_2)] = f(x_1)\frac{d}{dx}f(x_2) + f(x_2)\frac{d}{dx}f(x_1)$$

$$y' = e^{3x}\frac{d}{dx}\cos 2x + \cos 2x \frac{d}{dx}e^{3x}$$

$$\because \frac{d}{dx}\cos x = -\sin x \frac{dx}{dx}$$

$$\because \frac{d}{dx} e^x = e^x \frac{d}{dx} x$$

$$y' = e^{3x} - \sin(2x) \frac{d}{dx} 2x + \cos 2x \cdot e^{3x} \frac{d}{dx} 3x$$

$$y' = -\sin(2x) \cdot e^{3x} \cdot 2 + \cos 2x \, e^{3x} \, 3$$

$$= -2\sin(2x) e^{3x} + 3\cos 2x \, e^{3x}$$

Taking derivation on both sides

$$y'' = \frac{d}{dx}\left(-2\sin(2x) e^{3x} + 3\cos 2x \, e^{3x}\right)$$

$$= \frac{d}{dx} -2\sin(2x) e^{3x} + \frac{d}{dx} 3\cos 2x \, e^{3x}$$

$$\because \frac{d}{dx}[f(x_1) + f(x_2)] = \frac{d}{dx} f(x_1) + \frac{d}{dx} f(x_2)$$

$$= -2 \frac{d}{dx} \sin(2x) e^{3x} + 3 \frac{d}{dx} \cos(2x) e^{3x}$$

$$\because \frac{d}{dx}[f(x_1)][f(x_2)] = f(x_1) \frac{d}{dx} f(x_2) + f(x_2) \frac{d}{dx} f(x_1)$$

$$\Rightarrow y'' = -2\left[\sin(2x) \frac{d}{dx} e^{3x} + e^{3x} \frac{d}{dx} \sin(2x)\right] + 3\left[\cos(2x) \frac{d}{dx} e^{3x} + e^{3x} \frac{d}{dx} \cos(2x)\right]$$

$$= -2\left[\sin(2x) e^{3x} \cdot 3 + e^{3x} \cos(2x) \cdot 2\right] + 3\left[\cos(2x) e^{3x} \cdot 3 + e^{3x}(-\sin 2x) 2\right]$$

$$= -6e^{3x}\sin(2x) - 4e^{3x}\cos(2x) + 9e^{3x}\cos(2x) - 6e^{3x}\sin(2x)$$

$$= -12e^{3x}\sin(2x) + 5e^{3x}\cos(2x)$$

Put value of y'', y' and y in equation ①

$$-12e^{3x}\sin(2x) + 5e^{3x}\cos(2x) - 6\left(-2\sin(2x)e^{3x} + 3\cos(2x)e^{3x}\right) + 13\left(e^{3x}\cos(2x)\right) = 0$$

$$\Rightarrow \cancel{-12e^{3x}\sin(2x)} + 5e^{3x}\cos(2x) + \cancel{12\sin(2x)e^{3x}} - 18\cos(2x)e^{3x} + 13e^{3x}\cos(2x) = 0$$

$$-13\cos(2x)e^{3x} + 13\cos(2x)e^{3x} = 0$$

$0 = 0$

L.H.S = R.H.S

Hence proved, given function satisfied the given differential equation, so $y = e^{3x}\cos 2x$ is it's solution.

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31) $y'' = y$; $\rightarrow$ (1) $\qquad y = \cosh x + \sinh x$

$$y = \cosh x + \sinh x$$

Taking derivative on both sides

$$\frac{dy}{dx} = \frac{d}{dx}(\cosh x + \sinh x)$$

($\because$ derivative addition rule is applied)

$$y' = \frac{d}{dx}\cosh x + \frac{d}{dx}\sinh x$$

$$y' = \sinh x + \cosh x$$

Taking derivative on both sides

$$y'' = \frac{d}{dx}(\sinh x + \cosh x)$$

($\because$ derivative addition rule is applied)

$$\Rightarrow y'' = \frac{d}{dx}\sinh x + \frac{d}{dx}\cosh x$$

$$= \cosh x + \sinh x$$

Put value of y'' and y in equation (1)

$$\cosh x + \sinh x = \cosh x + \sinh x$$

L.H.S = R.H.S

Hence proved, given function satisfied the given differential equation, so $y = \cosh x + \sinh x$ is the solution of $y'' = y$.

37) $x^2y'' - 3xy' + 4y = 0 \rightarrow ①$ $\quad y = x^2 + x^2 \ln x, \; x > 0$

$$y = x^2 + x^2 \ln x$$

Taking derivative on both sides

$$\frac{dy}{dx} = \frac{d}{dx}(x^2 + x^2 \ln x)$$

$$= \frac{d}{dx}x^2 + \frac{d}{dx}x^2 \ln x$$

$$= 2x + \left[x^2 \frac{d}{dx}\ln x + \ln x \frac{d}{dx}x^2\right]$$

$$= 2x + \left[x^2 \frac{1}{x} + \ln x \, 2x\right]$$

$$= 2x + x + \ln x \, 2x$$

$$y' = 3x + \ln x \, 2x$$

Taking derivative on both sides

$$y'' = 3\frac{d}{dx}x + \frac{d}{dx}[\ln x . 2x]$$

$$= 3 + \left[\ln x \frac{d}{dx}2x + 2x \frac{d}{dx}\ln x\right]$$

$$= 3 + \left[2\ln x + 2x \frac{1}{x}\right]$$

$$= 3 + [2\ln x + 2]$$

$$= 3 + 2\ln x + 2$$

$$= 2\ln x + 5$$

Put y'', y', y in equation ①

$$x^2(2\ln x + 5) - 3x(3x + \ln x \, 2x) + 4(x^2 + x^2 \ln x) = 0$$

$$2x^2 \ln x + 5x^2 - 9x^2 - 6x^2 \ln x + 4x^2 + 4x^2 \ln x = 0$$

$$\cancel{-4x^2 \ln x} \cancel{- 4x^2} \cancel{+ 4x^2} \cancel{+ 4x^2 \ln x} = 0$$

$$0 = 0$$

L.H.S = R.H.S

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Hence proved, given function satisfied the given differential equation

$\Rightarrow$ $y = x^2 + x^2 \ln x$ is the solution of

$x^2 y'' - 3xy' + 4y = 0$