Date:

Chp 3

first Order D.E

3.1 Orthagonal Trajectories

perpendicular — path followed by particle.

paths / curves tangent are orthagonal so its called O.T.

→ $f(x, y)$

↓

diff. E of Curve

↓

" " of O.curve

↓

orthogonal Curve.

$m_1 = \frac{dy}{dx}$.

$m_2 = \frac{-1}{m_2}$.

$\int \quad = \int$

→ $m_1 \cdot m_2 = -1$

$m_1 = -\frac{1}{m_2}$

Q $y = c_1 x^3$ $\quad$ c = family of curve

→ D.E of family of curve

$\frac{dy}{dx} = 3c_1 x^2$

· Vertex - curve change their behaviour on some point.

from question $c_1 = y/x^3$

$\frac{dy}{dx} = \frac{3y}{x^3} x^2$

$dy/dx = \frac{3y}{x}$

→ the d.e of orthagonal trajectory is

$\frac{dy}{dx} = -\frac{x}{3y}$ $\left(\frac{-1}{\frac{3y}{x}} \rightarrow -\frac{x}{3y} \right)$

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$$(-3y)\,dy = x\,dx$$

$$\frac{-3y^2}{2} = \frac{x^2}{2} + C$$

Q. $y = (x - c_1)^2 \longrightarrow 1$

D.E of family of curve

$$\frac{dy}{dx} = 2(x - c_1) \longrightarrow (2)$$

$$\sqrt{y} = x - c_1$$

$$c_1 = x - \sqrt{y} \quad \text{use in } 2$$

$$\frac{dy}{dx} = 2(\cancel{x} - \cancel{x} + \sqrt{y})$$

$$\frac{dy}{dx} = 2\sqrt{y}$$

D.E of O.T

$$\frac{dy}{dx} = \frac{-1}{2\sqrt{y}}$$

$$\int dy\,(2\sqrt{y}) = \int -1\,dx$$

$$2\,\frac{y^{3/2}}{3/2} = -x + C$$

$$\frac{4y^{3/2}}{3} = -x + C$$

Q.17 $y = c_1 e^{-x} \longrightarrow 1$

$$\frac{dy}{dx} = -c_1 e^{-x} \rightarrow \text{D.E of family of curve}$$

$$y = c_1 e^{-x} \qquad c_1 = \frac{y}{e^{-x}}$$

$$y' = -\frac{y}{\cancel{e^{-x}}}\,\cancel{e^{-x}}$$

$$y' = -y \longrightarrow y' = \frac{1}{y}$$

Date:

$$\int \frac{dy}{dx} = \int \frac{1}{y}$$

$$\int dy(y) = \int 1\, dx$$

$$\frac{y^2}{2} = x + c$$

2) $y = \frac{1}{\ln c_1 x}$

D.E of family of curve

$$\frac{dy}{dx} = \frac{dy}{dx} \frac{1}{\ln c_1 x}$$

$$= \frac{dy}{dx} (\cancel{\ln c_1 x})^{-1}$$

$$= \frac{\ln c_1 x (0) - 1 (1/c_1 x)}{(\ln c_1 x)^2}$$

$$= \frac{\ln c_1 - 1/c_1 x}{(\ln c_1 x)^2}$$

$$= \frac{-\frac{1}{c_1 x}}{(\ln c_1 x)^2} \quad \Big| \quad \frac{-1}{x(\ln c_1 x)^2}$$

→ value of C

$$\ln_e c_1 x = 1/y_e$$

$$c_1 x = e^{1/y} \qquad c_1 = \frac{e^{1/y}}{x}$$

$$\frac{dy}{dx} = -\frac{1}{x\left(\ln\left[\frac{e^{1/y}}{x} \cdot x\right]\right)^2}$$

$$\frac{dy}{dx} = \frac{-1}{x\left(\frac{1}{y^2}\right)} = \frac{-y^2}{x}$$

Date:

$$\frac{dy}{dx} = \frac{y^2}{x}$$

$$\frac{dy}{dx} = \frac{x}{y^2} \longrightarrow \text{OT}$$

$$\int dy\,(y^2) = \int x\,dx$$

$$y^3/3 = x^2/2$$

Q10 $y^a = c_1 x^b$

D.E of family of curve

$$\cancel{\frac{dy}{dx}} =$$

$$c_1 = \frac{y^a}{x^b}$$

$$a y^{a-1} \frac{dy}{dx} = b c_1 x^{b-1}$$

$$\frac{dy}{dx} = \frac{c_1 b x^{b-1}}{a\, y^{a-1}}$$

$$\frac{dy}{dx} = \frac{\cancel{y^a}}{\cancel{x^b}} \frac{b}{a} \cdot \frac{\cancel{x^b}\, x^{-1}}{\cancel{y^a}\, y^{-1}}$$

$$\frac{dy}{dx} = \frac{b}{a} \frac{x^{-1}}{y^{-1}} = \frac{b}{a} \frac{y}{x}$$

$$\frac{dy}{dx} = -\frac{a}{b} \frac{x}{y}$$

$$b \int y\,dy = -a \int x\,dx$$

$$by^2/2 = -ax^2/2 + c.$$

Q1-4 growth P
Q5,7 decay P

Q11 → Carbon dating
Q12 → 14 → cooling
Q15, 16, 17, 18 → Circuit

3.2

Application of linear Algebra

- Growth & Decay / half life
- Carbon Dating
- Newton law of cooling
- series circuits
- Mixture problems

$$\frac{dy}{dx} + P(x)y = Q(x)$$

$$I.F = e^{\int P(x)dx}$$

$$y = \frac{1}{I.F}\left[\int I.F\,Q(x)\,dx + C\right]$$

→ Growth & Decay

Let $P(t)$ be pop of region at any time t

- $\frac{dP}{dt} \propto P$ → rate of growth

$$\frac{dP}{dt} = kP \longrightarrow \frac{dP}{dt} - kP = 0$$

$$p(t) = -k \quad , \quad Q(t) = 0$$

$$I.F = e^{\int k\,dt} = e^{-kt}$$

$$P(t) = \frac{1}{e^{-kt}}\left[\int e^{-kt} \cdot 0 \cdot dt + C\right]$$

$$P(t) = Ce^{kt}$$

Example 1

Let $n(t)$ be no of bacteria at any time t

$$n(0) = n_0$$

when $n(1) = \frac{3}{2} n_0$

$$\frac{dN}{dt} \propto N$$

$$\frac{dN}{dt} = kN \rightarrow \frac{dN}{dt} - kN = 0$$

$t = ?$ when $N(t) = 3N_0$

after solving the primary

$$N(t) = Ce^{kt}$$

using $N(0) = N_0$

$$N(0) = Ce^{k(0)}$$

$$N_0 = C$$

$$\rightarrow N(t) = N_0 e^{kt}$$

using $N(1) = 3/2\, N_0$

$$N(1) = N_0 e^{k(1)}$$

$$\frac{3}{2}\cancel{N_0} = \cancel{N_0} e^{k}$$

$$e^{k} = 3/2$$

$$k = \ln 3/2 = 0.4055$$

$$\rightarrow N(t) = N_0 e^{0.4055t}$$

so Now

$$3\cancel{N_0} = \cancel{N_0} e^{0.4055t}$$

$$e^{0.4055t} = 3$$

$$0.4055t = \ln 3$$

$$t = \frac{\ln 3}{0.4055}$$

$$t = 2.71h$$

growth $\rightarrow$ k positive value

decay $\rightarrow$ k negative value

Example : 2 (Decay)

Let $A(t)$ be the amount of plutonium present at any time t

$$A(0) = A_0$$

$$A(15) = A_0 - \frac{0.043A_0}{100} = 0.99957A_0$$

$$\frac{dA}{dt} = kA \qquad \frac{dA}{dt} - kA = 0$$

$$A(t) = Ce^{kt} \qquad \therefore C = A_0 \qquad t = ? \text{ when } A(t) = A_0/2$$

$$\Rightarrow A(t) = A_0 e^{kt}$$

using $A(15) = 0.99957A_0$

$$A(15) = A_0 e^{15kt}$$

$$0.99957\cancel{A_0} = \cancel{A_0} e^{15k}$$

$$e^{15k} = 0.99957$$

$$15k = \ln 0.99957$$

$$k = \frac{\ln 0.99957}{15}$$

$$k = -0.00002867$$

$$\Rightarrow A(t) = A_0 e^{-0.00002867t}$$

$$\frac{\cancel{A_0}}{2} = \cancel{A_0} e^{-0.00002867t}$$

$$\ln \frac{1}{2} = -0.00002867t$$

$$t = \frac{\ln 1/2}{-0.00002867} = 24180y$$

Carbon Dating

let $A(t)$ be the amount of C-14 present in the first of any time t

Initially $A(0) = A_0$

$A(5600) = A_0/2$

$$\frac{dA}{dt} - kA = 0$$

$t \to$ where $A(t) = \frac{A_0}{1000}$

P.S. $A(t) = Ce^{kt}$

$A(t) = A_0 e^{kt}$

$\Rightarrow$ using $A(5600) = A_0/2$

$\Rightarrow A(5600) = A_0 e^{kt}$

$$A_0/2 = A_0 e^{kt}$$

$$\frac{1}{2} = e^{kt} \text{ value}$$

$$\ln 1/2 = kt \text{ value}$$

$$\ln 1/2 = k\,5600$$

$$k = \frac{\ln 1/2}{5600} = -0.0012378$$

$$\Rightarrow A(t) = A_0 e^{-0.0012378t}$$

$$\frac{A}{1000} = A_0 e^{-0.0012378t}$$

$$\ln\left(\frac{1}{1000}\right) = -0.0012378t$$

$$t = \frac{\ln(1/1000)}{-0.0012378} \approx 55800 \text{ years}$$

Cooling

$$\frac{dT}{dt} \propto T - T_m$$

T = temp of body

T_m = temp of surrounding

$$\frac{dT}{dt} = k(T - T_m)$$

example : 4

$T(0) = 300°F$

$T(3) = 200°F$

$T_m = 70°F$

$$\frac{dT}{dt} = k(T - T_m)$$

$$\frac{dT}{dt} = k(T - 70)$$

$$\frac{dT}{dt} - kT = -70k$$

$p(t) = -k \qquad Q(t) = -70k$

$I.F = e^{\int k\,dt} = e^{-kt}$

$$y = \frac{1}{I.F}\left[\int I.F\,Q(x)\,d(x) + c\right]$$

$$T(t) = \frac{1}{e^{-kt}}\left[-70k\int -e^{-kt}\,dt + c\right]$$

$$T(t) = e^{kt}\left[-70k\,\frac{e^{-kt}}{-k} + c\right]$$

$T(t) = 70 + Ce^{kt}$

Using Condition

$T(0) = 70 + Ce^{k(0)}$

$300 = 70 + Ce^{0}$

$300 = 70 + C$

$C = 230$

$T(t) = 70 + 230e^{kt}$

$T(3) = 70 + 230e^{k(3)}$

$300 = 70 + 230e^{3k}$

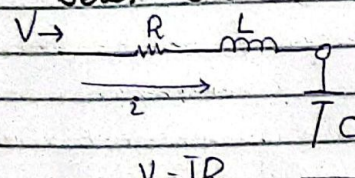
$\frac{130}{230} = e^{3k}$

$3k = \ln 13/23$

$k = \frac{\ln(13/23)}{3} = -0.19018$

$T(t) = 70 + 230e^{-0.19018t}$

Series Circuit



$V = IR$ → ohm's law

$V = iR$

$V = L\frac{di}{dt}$ → second law of Induction

$V = iR + L\frac{di}{dt}$ → kirchoff's current law

$\Rightarrow L\frac{di}{dt} + iR = E(t)/V$

$R\frac{dq}{dt} + \frac{1}{C}q = E(t)(V)$ → Capacitor

Example: 5

$E(t) = 12V$

$L = \frac{1}{2}$ Hery $\quad R = 10\Omega$

$i(t) = ?$ when $i(0) = 0$

$L\frac{di}{dt} + iR = 12$

$\frac{1}{2}\frac{di}{dt} + 10i = 12$

$\frac{di}{dt} + 20i = 24$

$P(t) = 20 \quad Q(t) = 24$

$I.F = e^{\int P(t)dt}$

$= e^{\int 20dt} = e^{20t}$

$i(t) = \frac{1}{e^{20t}}\left[\int e^{20t} \cdot 24dt + C\right]$

$$= \frac{1}{e^{20t}}\left[\frac{e^{20t}}{20}\cdot 24 + C\right]$$

$$i(t) = \frac{24}{20} + Ce^{-20t}$$

$$i(0) = \frac{24}{20} + Ce^{-20(0)}$$

$$0 = 24/20 + C$$

$$C = -24/20$$

$$i(t) = \frac{24}{20} - \frac{24}{20}e^{-20t}$$

Q. $\frac{dP}{dt} \propto P(t)$

$$\frac{dP}{dt} = kP(t) \qquad \frac{dP}{dt} - kP = 0$$

$$P(t) = Ce^{kt}$$

$$P(3) = 10{,}000$$

$$P(0) = P_0$$

$$P(5) = 2P_0$$

$$P_0 = ? \qquad P_{10} = ?$$

$$P(0) = Ce^{k(0)}$$

$$P_0 = C$$

$$P(t) = P_0 e^{kt}$$

$$P(5) = 2P_0$$

$$P(5) = P_0 e^{5k}$$

$$2P_0 = P_0 e^{5k}$$

$$2 = e^{5k}$$

$$k = \frac{\ln 2}{5}$$

$$P(t) = P_0 e^{kt}$$

$$P(t) = P_0 e^{0.13t}$$

$$P(3) = 10{,}000$$

$$10000 = P_0 e^{0.13(3)}$$

Chapter 4 : 15-28 , 33-40

Linear differential eq of higher order

$$a_n(x)\frac{d^n y}{dx^n} + a_{n+1}(x)\frac{d^{n+1}}{dx^{n+1}} + \cdots a(n)\frac{dy}{dx} + a_0(x) = g(x)$$

$$a_1(x)\frac{dy}{dx} + a_0(x)y = g(x)$$

$$\frac{dy}{dx} + \frac{a_0(x)}{a_1(x)}y = \frac{g(x)}{a_1(x)}$$

subject to these conditions is called initial value problem.

$$y(x_0) = y_0$$

If right side of eq is 0
$g(x) = 0$

nth order linear homogeneous equation.

Type of $x \Rightarrow g(x)$ will be absent.

existance of unique solution

If $a(x)$ function exist and conti. on some interval than there exist unique solution.

so $a_1(x) \neq 0$

it will never be undefined

so unique solu exist be $(-\infty, +\infty)$

except 0

so $(-\infty, 0) \cup (0, +\infty)$

→ Example

$y'' - 4y = 12x \qquad y(0) = 4 \cdot y'(0) = 1$

Coefficient are

1, -4, 12

They are continuous

It is not = 0

So it has unique solu on $(-\infty, +\infty)$

Linearly Dependent

on book

$y_1(x) = \sin 2x \qquad , \quad y_2(x) = \sin x \cos x$

$C_1 \sin 2x + C_2 \sin x \cos x = 0$

$2C_1 \sin x + C_2 \sin x \cos x = 0$

$2C_1 = 0$

$C_1 = 0 \qquad C_2 = 0$

Criteria for finding Linear dependent & Independent solution

Wronskein

$$W(f_1(x), f_2(x) \ldots f_n(x)) = \begin{vmatrix} f_1(x) & f_2(x) & \cdots & f_n(x) \\ f_1'(x) & f_2'(x) & \cdots & f_n'(x) \\ f_1''(x) & f_2''(x) & \cdots & f_n''(x) \\ f_1^{(n-1)'}(x) & f_2^{(n-1)'}(x) & \cdots & f_n^{(n-1)'}(x) \end{vmatrix}$$

$W \neq 0$ L. independent

$W = 0$ Linearly Dependent

Previous Sin question

$$W(\sin 2x, \sin x\cos x) = \begin{vmatrix} \sin 2x & \sin x\cos x \\ 2\cos 2x & -\sin^2 x + \cos^2 x \end{vmatrix}$$

$$= \sin 2x(-\sin^2 x + \cos^2 x) - 2\cos 2x(\sin x\cos x)$$

$$= 2\sin x\cos x(\cos^2 x - \sin^2 x) - 2(\cos^2 x - \sin^2 x)(\sin x\cos x)$$

$$= 2\sin x\cos^3 x - 2\sin^3 x\cos x - 2\cos^3 x\sin x + 2\sin^3 x\cos x$$

$$= 0$$

Superposition Principal.

Sum of Linear combinations of solutions is also solution.....

$$y_1 = x^2 \qquad y_2 = x^2 \ln x$$

Let. $y = c_1 y_1 + c_2 y$

$$y = c_1 x^2 + c_2 x^2 \ln x.$$

$$y' = 2c_1 x + c_2 + 2c_2 x \ln x$$

$$y'' = 2c_1 + c_2 + 2c_2 + 2c_2 \ln x.$$

$$y''' = 2c_2/x$$

L.H.S

$$= x^3 \frac{2c_2}{x} - 2c(2c_1 x + c_2 x + 2c_2 x \ln x) + 4(c_1 x^2 + c_2 x^2 \ln$$

$$= 0.$$

Fundamental set of Solution

If $W \neq 0$ they form fundamental set of solution.

General Solution

* Homogenous

$$y = c_1 f(x) + c_2 f_2(x) + \cdots c_n f_n(x)$$

* Non-homogenous. $[g(x) \neq 0]$

(non-homogenous) → particular solution (integral) → y_p or PI

(Homogenous) → general solution (complemtry func) → y_c

→ general solution of non-homogo

$$y = y_p + y_c$$

Superposition:

$$y = c_1 e^{x} + c_2 e^{-x}$$

$y' - y = 0$ $\quad y(0) = 0 \quad y'(0) = 1$

$$y(0) = c_1 e^{0} + c_2 e^{0}$$

$$0 = c_1 + c_2$$

$$y' = c_1 e^{x} - c_2 e^{-x}$$

$$y'(0) = c_1 e^{0} - c_2 e^{-0}$$

$$1 = c_1 - c_2$$

$$c_1 + \cancel{c_2} = 0$$

$$c_1 - \cancel{c_2} = 1$$

$$2c_1 = 1$$

$$c_1 = 1/2$$

$$c_2 = -1/2$$

$$y = \tfrac{1}{2} e^{x} - \tfrac{1}{2} e^{-x}$$

Q19 $y_1(x) = x$ $\quad y_2(x) = x-1 \quad y_3(x) = x+$

$$W(x, x-1, x+1) = \begin{vmatrix} x & x-1 & x+3 \\ 1 & 1 & 1 \\ 0 & 0 & 0 \end{vmatrix} = 0$$

$y_1(x)$, $y_2(x)$, $y_3(x)$ are L.

4.1

Q. $y = c_1 + c_2 x^2$ $\quad y(0) = 0,\ y'(0) = 1$

$y(0) = c_1 + c_2(0)$

$= 0 = c_1$

$y' = 2c_2 x$

$y'(0) = 2c_2(0)$

$1 = 0$ which is impossible

Q7. $y(0) = 0 \quad y(0) = 0$

$y(0) = c_1 + c_2(0)$

$c_1 = 0$

$y' = 2c_2(x)$

$y'(0) = 2c_2(0)$

$0 = 0$

$c_2 \neq 0$

$y = c_2 x^2 \quad c_2$ any real number

$c_2 \neq 0 \in \mathbb{R}$

Qs to 22

Q18 $y_1(x) = \cos 2x,\ y_2(x) = 1 \quad y_3(x) = \cos^2 x$

$$W(\cos 2x, 1, \cos^2 x) = \begin{vmatrix} \cos 2x & 1 & \cos^2 x \\ 2\sin 2x & 0 & -2\cos x \sin x \\ -4\cos 2x & 0 & -\cos 2x \end{vmatrix} = (-\sin 2x)$$

$$= \cos 2x \begin{vmatrix} 0 & -\sin 2x \\ 0 & -\cos 2x \end{vmatrix} - 1 \begin{vmatrix} 2\sin 2x & -\sin 2x \\ -4\cos 2x & -\cos 2x^2 \end{vmatrix} +$$

$$\cos^2 x \begin{vmatrix} -2\sin 2x & 0 \\ -4\cos 2x & 0 \end{vmatrix}$$

$= \cos 2x(0) - 1(4\sin 2x \cos 2x - 4\sin 2x \cos 2x) + \cos^2 x(0-0)$

$= 0 - 1(0) + 0$

$= 0$

Linearly Dependent.

→ for higher matrix

$$\begin{matrix} a_{11} & a_{12} & a_{13} & a_{11} & a_{12} \\ a_{21} & a_{22} & a_{23} & a_{21} & a_{22} \\ a_{31} & a_{32} & a_{33} & a_{31} & a_{32} \end{matrix}$$

$= a_{11}a_{22}a_{33} + a_{12}a_{23}a_{31} + a_{13}a_{21}a_{32}$

$- a_{13}a_{22}a_{31} - a_{11}a_{23}a_{32} - a_{12}a_{21}a_{33}$

Q20 $y_1(x) = 2 + x,\ y_2(x) = 2 + |x|$

$(-\infty, +\infty)$

$$|x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$$

$y_2(x) = 2 - x \quad \text{if } x \in (-\infty, 0)$

$y_2(x) = 2 + x \quad \text{if } (x) \in [0, \infty)$

$$W(2+x, 2-x) = \begin{bmatrix} 2+x & 2-x \\ 1 & -1 \end{bmatrix}$$

$= -2 - \cancel{x} - 2 + \cancel{x}$

$= -4 \rightarrow$ L. Independent

$$W(2+x, 2+x) = \begin{bmatrix} 2+x & 2+x \\ 1 & 1 \end{bmatrix}$$

$= 2+x-2-x$

$= 0 \rightarrow$ L. dependant

Q. $f_1(x) = e^x \quad f_2(x) = e^{-x} \quad f_3(x) = \sinh x$

$$W(e^x, e^{-x}, \sinh x) = \begin{vmatrix} e^x & e^{-x} & \sinh x \\ e^x & -e^{-x} & \cosh x \\ e^x & e^{-x} & \sinh x \end{vmatrix} = 0$$

They are L.D and so e^x, e^{-x}, $\sinh x$ do not make fundamental set.

Ex # 4.3

Homogeneous Linear differential equation with constant coefficient

$$a_n(x)\frac{d^n y}{dx^n} + a_{n-1}(x)\frac{d^{n-1} y}{dx^{n-1}} = g(x)$$

should be constant — should be 0.

$a_i(x) = C_i$ - -

C_i is scalar / constant

$$C_n \frac{dy}{dx^n} + C_{n-1}\frac{d^{n-1} y}{dx^{n-1}} \cdots C_1 \frac{dy}{dx} + C_0 y = 0$$

→ Auxiliary equation / operator fcn

$$(C_n D^n + C_{n-1} D^{n-1} \cdots C_1 D^1 + C_0) y = 0$$

→ Characteristic equation

$$C_n M^n + C_{n-1} M^{n-1} \cdots C_1 M^1 = 0$$

then find roots for characteristic eq.

Real and distinct	Real + Repeated	Complex conjugate	Repeated complex conj
$\alpha_1, \alpha_2, \alpha_3 \cdots \alpha_n$	α, α, α	$\alpha + i\beta$	$\alpha \pm i\beta \quad \alpha \pm i\beta$
$y = C_1 e^{\alpha_1 x} + C_2 e^{\alpha_2 x} \cdots C_n e^{\alpha_n x}$	$y = (C_1 + C_2 x + C_3 x^2 \cdots C_n x^{n-1}) e^{\alpha x}$	$y = e^{\alpha x}(C_1 \sin \beta x - \cos \beta x)$	$y = e^{\alpha x}(C_1 + C_2 x \sin \beta x + (C_3 + C_4) x \cos \beta x)$

$$e^{\alpha x}(C_1 \sin \beta x - \cos \beta x)$$

Ex 4.3 Q1 → 52

Q2 $2y'' - 5y' = 0$

$(2D^2 - 5D)y = 0$

$2m^2 - 5m = 0$

$m(2m - 5) = 0$

$m = 0 \qquad 2m - 5 = 0 \qquad m = 5/2$

$y = C_1 e^{0x} + C_2 e^{5/2 x}$

They are real & distinct

Q $y''' + 3y'' - 4y' - 12y = 0$

$(D^3 + 3D^2 - 4D - 12)y = 0$

$M^3 + 3M^2 - 4M - 12 = 0$

Synthetic division

where $(N = 2)$ is used for S.D

N-2 is the factor

2	1	3	-4	-12
		2	10	12
	1	5	6	0

$m^2 + 5m + 6 = 0$

$m^2 + 3m + 2m + 6 = 0$

$m(m+3) + 2(m+3) = 0$

$(m+2)(m+3) = 0$

$m = -2 \qquad m = -3$

$y = C_1 e^{2x} + C_2 e^{-2x} + C_3 x^{-3x}$

Q 30 $\dfrac{d^4y}{dx^4} - 2\dfrac{d^2y}{dx^2} + y = 0$

$(D^4 - 2D^2 + 1)y = 0$

$m^4 - 2m^2 + 1 = 0$

$(m^2 - 1)^2 = 0$

$(m^2 - 1)(m^2 - 1) = 0$

$m^2 = \pm 1 \qquad m = \pm 1$

These are real and repeated

$y = (C_1 + C_2)(e^{1x}) + (C_3 + C_4)(e^{-1x})$

$y = (C_1 + C_2 x)e^{x} + (C_3 + C_4 x)e^{-x}$

Ex # 4.4

Undetermind Coefficient

Non homogeneous

homogeneous $g(x) = 0$ → previous → y_c = Complementary function → y → general solution → $y = y_c + y_p$

Non-homogeneous $g(x) \neq 0$ → undetermind - coefficient → y_p = Particular integral

$g(x)$	y_p
Any Constant (1)	A
x, x^2 ... $x, x^2, 1$ etc	$Ax^n + Bx^{n-1} + Cx^{n-2} + \dots$
e^x, e^{2x}, e^{-2x}	Ae^{ax}
$\sin x, \cos x$ (Trigonometric) $\sin x + \cos x$	$(A\sin ax + B\cos ax)$
$(x^2+1)e^{-2x}$	$(Ax^2+Bx+C)e^{-2x}$
$x\cos 3x$	$(Ax+B)\sin 3x + (Cx+D)\cos 3x$
$x^2 e^{3x}\sin 2x$	$(Ax^2+Bx+C)e^{3x}\sin 2x + (Dx^2+Ex+F)e^{3x}\cos 2x$

Example : 9

$y'' - 6y' + 9y = 6x + 2 - 12e^{3x} \longrightarrow *$

→ For Complementary function let us consider homogeneous part of eq*.

$y'' - 6y' + 9y = 0$

$(D^2 - 6D + 9)y = 0$

$M^2 - 6M + 9 = 0$

$(m-3)(m-3) = m = 3, 3$

$y_c = (C_1 + C_2 x)e^{3x}$

→ For particular solution let,

$y_p = Ax + B + Cx^2 e^{3x}$

$y_p' = A + 3Cx^2 e^{3x} + 2Cxe^{3x}$

$y_p'' = 9Cx^2 e^{3x} + 6Cxe^{3x} + 6Cxe^{3x} + 2Ce^{3x}$

$= 9Cx^2 e^{3x} + 12Cxe^{3x} + 2Ce^{3x}$

Using in *

$9Cx^2 e^{3x} + 12Cxe^{3x} + 2Ce^{3x} - 6A - 18Cx^2 e^{3x} - 12Cxe^{3x} + 9Ax + 9B + 9Cx^2 e^{3x} = 6x + 2 - 12e^{3x}$

$2Ce^{3x} - 6A + 9Ax + 9B = 6x + 2 - 12e^{3x}$

Comparing Co-efficients

$2C = -12$

$C = -6$

$9A = 6$

$A = 2/3$

$-6A + 9B = 2$

$-4 + 9B = 2$

$9B = 6$

$B = 2/3$

Putting in y_p

$y_p = \frac{2}{3}x + \frac{2}{3} - 6x^2 e^{3x}$

6-Dec-2023

4.4

(18) $y'' + 4y' + 8y = 2x$

$$y'' + 2y' = 2x + 5 - e^{-2x} \quad (*)$$

Non-Homo

$g(x) = 0$ → Homo → y_c

$g(x) \neq 0$ → Non-Homo → y_p

$y = y_c + y_p$

For Complementary function (y_c) let us consider homogeneous part of eq (*)

$y'' + 2y' = 0.$

~~$m^2 + 2m = 0$~~

$m^2 + 2m = 0.$

$m(m+2) = 0$

$m = 0, -2$

$y_c = C_1 e^{0x} + C_2 e^{-2x}$

$y_c = C_1 + C_2 e^{-2x}$

For particular Integral (y_p) consider eq (*).

$y'' + 2y' = 2x + 5 - e^{-2x}$

$y_p = x(Ax + B) + Cxe^{-2x}$

let $y_p = Ax^2 + Bx + Cxe^{-2x}.$

$y_p = A - 2Cxe^{-2x}.$

$y'_p = A + (x(-2e^{-2x}) + Ce^{-2x}$

$y'_p = A - 2Cxe^{-2x} + Ce^{-2x}$

$y'_p = -2Cx(-2e^{-2x}) + (-2C)e^{-2x} - 2Ce^{-2x}$

$y''_p = 4Cxe^{-2x} - 2Ce^{-2x} - 2Ce^{-2x}$

$y''_p = 4Cxe^{-2x} - 4Ce^{-2x}$

Using in eq (*)

$4Cxe^{-2x} - 4Ce^{-2x} + 2(A - 2Cxe^{-2x} + Ce^{-2x}) = 2x + 5 - e^{-2x}$

$4Cx.$

$y'_p = 2Ax + B - 2Cxe^{-2x} + Ce^{-2x}$

$y''_p = 2A - 2Cx(-2e^{-2x}) + (-2C)e^{-2x} - 2Ce^{-2x}$

$y''_p = 2A + 4Cxe^{-2x} - 4Ce^{-2x}$

Using in eq (*)

$$2A + 4Cxe^{-2x} - 4Ce^{-2x} + 2\left[2Ax + B - 2Cxe^{-2x} + Ce^{-2x}\right] = 2x + 5 - e^{-2x}$$

$$2A + \cancel{4Cxe^{-2x}} + \cancel{4Ce^{-2x}} + 4Ax + 2B - \cancel{4Cxe^{-2x}} + \cancel{2Ce^{-2x}}$$

$$2A + 2B - 2Ce^{-2x} + 4Ax = 2x + 5 - e^{-2x}.$$

Comparing coefficients

$$-2C = -1, \quad \boxed{C = 1/2}$$

$$4A = 2 \Rightarrow \boxed{A = 1/2}$$

$$2A + 2B = 5, \quad 1 + 2B = 5$$

$$2B = 4$$

$$\boxed{B = 2}$$

$$y_p = \frac{1}{2}x^2 + 2x + \frac{1}{2}xe^{-2x}$$

$$y = y_c + y_p$$

$$y = c_1 + c_2e^{-2x} + \frac{1}{2}x^2 + 2x + \frac{1}{2}xe^{-2x}$$

[3 step rule

(18) $y'' - 2y' + 2y = e^{x}(\cos x - 3\sin x)$

→ Read
→ Teach
→ Failure gaps

$$y'' - 2y' + 2y = 0$$

$$m^2 - 2m + 2 = 0$$

$$m = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(2)}}{2(1)}$$

$$m = \frac{2 \pm \sqrt{4-8}}{2}$$

$$m = \frac{2 \pm 2i}{2}$$

$$m = 1 \pm i$$

$$y_c = e^{x}(c_1\cos x + c_2\sin x).$$

$$y_p = Ae^{2x}\cos x + Be^{2x}\sin x$$

$$y_p' = -Ae^{2x}\sin x + 2Ae^{2x}\cos x + Be^{2x}\cos x + 2Be^{2x}\sin x$$

$$y_p' = (-A+2B)e^{2x}\sin x + (2A+B)e^{2x}\cos x$$

$$y_p'' = [-A+2B](e^{2x}\cos x + 2e^{2x}\sin x) + (2A+B)[-e^{2x}\sin x + 2e^{2x}\cos x]$$

$$y_p'' = -Ae^{2x}\cos x - 2Ae^{2x}\sin x + 2Be^{2x}\cos x + 4Be^{2x}\sin x - 2Ae^{2x}\sin x + 4Ae^{2x}\cos x - Be^{2x}\sin x + 2Be^{2x}\cos x$$

$$y_p'' = 3Ae^{2x}\cos x - 4Ae^{2x}\sin x + 4Be^{2x}\cos x + 3Be^{2x}\sin x.$$

using in (*).

$$3Ae^{2x}\cos x - 4Ae^{2x}\sin x + 4Be^{2x}\cos x + 3Be^{2x}\sin x - 2(-Ae^{2x}\sin x + 2Ae^{2x}\cos x + 2Be^{2x}\sin x + Be^{2x}\cos x) + 2Ae^{2x}\cos x + 2Be^{2x}\sin x = e^{2x}(\cos x - 3\sin x).$$

$$Ae^{2x}\cos x - 2Ae^{2x}\sin x + 2Be^{2x}\cos x + Be^{2x}\sin x = e^{2x}\cos x - 3e^{2x}\sin x$$

$$(A+2B)e^{2x}\cos x + (-2A+B)e^{2x}\sin x = e^{2x}\cos x - 3e^{2x}\sin x.$$

$$A+2B=1 \quad \text{—(i)}$$
$$-2A+B=3 \quad \text{—(ii)}$$

$2(i)+(ii)$

$$2A+4B=2$$
$$-2A+B=-3$$
$$5B=-1$$
$$B=-1/5 \quad \text{Put in (ii)}$$

$$-2A - 1/5 = -3$$
$$-2A = -3 + 1/5$$
$$-2A = -14/5$$
$$A = 14/10 \Rightarrow 7/5.$$

$$y_p = 7/5\, e^{2x}\cos x - 1/5\, e^{2x}\sin x$$

$$\boxed{y = y_c + y_p}$$

$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $D' \to D$

$y'' + 2y' + y = \sin x + 3\cos 2x$

$(D^2 + 2D + 1)y = \cancel{\sin x + 3\cos 2x}$ 0

$m^2 + 2m + 1 = 0$

$(m+1)^2 = 0$

$(m+1)(m+1) = 0$

$m_1 = -1, \; m_2 = -1$

Real, Repeated

$y = C_1 e^{\alpha x} + x C_2 e^{\alpha x}$

$y_c = C_1 e^{-x} + x C_2 e^{-x}$ — (*)

$y_p = A\sin x + B\cos 2x$

$y'_p = A\cos x + B(-2\sin 2x)$ — (*)

$\frac{3}{4} - \frac{3}{1}$ $3 - \frac{12}{4}$ $-\frac{9}{4}\cos 3x$

$y''_p = -A\sin x - 2B\cos 2x\,(2)$

$y''_p = -A\sin x - 4B\cos 2x$ — (*)

$-A\sin x - 4B\cos 2x + 2(A\cos x - B2\sin x)$

$+ A\sin x + B\cos 2x = \sin x + 3\cos 2x$

$\cancel{-A\sin x} - 4B\cos 2x + B\cos 2x + 2A\cos x - 4B\sin x$

$-3B\cos 2x + 2A\cos x - 4B\sin x = \sin x + 3\cos 2x$

$-3B\cos 2x + 2A\cos x = 3\cos 2x$

$-4B\sin x = \sin x$

$\boxed{B = -\frac{1}{4}}$ $A = -\frac{9}{4}$

$\frac{3}{4}\cos 2x - 3\cos 2x + 2A\cos x =$

8-6-2014

4.7

$y'' + y = \sec x \tan x$

$(D^2+1)y = 0$

$m^2+1 = 0$

$m^2 = -1 \qquad m = \pm\sqrt{-1}$

$m = \pm i$

conjugate complex

$y_c = e^{0x}(c_1 \sin x + c_2 \cos x)$

$y_1 \qquad y_2$

$$W = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} = \begin{vmatrix} \sin x & \cos x \\ \cos x & -\sin x \end{vmatrix}$$

$= -\sin^2 x - \cos^2 x$

$= -(\sin^2 x + \cos^2 x)$

$\sec = \frac{1}{\cos}$

$= -1$

$$W_1 = \begin{vmatrix} 0 & \cos x \\ \sec x \tan & -\sin x \end{vmatrix} = 0 - \sec x \tan x \cos x$$

$$W_2 = \begin{vmatrix} \sin x & 0 \\ \cos x & \sec x \tan x \end{vmatrix} = ?$$

$u_1 = \int \frac{W_1}{W} \qquad u_2 = \int \frac{W_2}{W}$

$y_p = u_1 y_1 + u_2 y_2$

$y = y_c + y_p$

$$e^{3x}\begin{vmatrix} e^{x} & e^{2x} \\ e^{x} & 2e^{2x} \end{vmatrix} = e^{3x}\begin{vmatrix} 2e^{3x} - e^{3x} \end{vmatrix}$$

$$= 2e^{6x} - e^{6x}$$

$$e^{6x}$$

Chapter # 07
Laplace Transform

$$ax^2 + 2hxy + dy^2 + 2gx + 2fy + C = 0$$

- By definition

$$L\{f(t)\} = \int_0^\infty e^{-st} f(t)\, dt$$

put then which you have to find Laplace

$$\lim_{b \to \infty} \int_0^b e^{-st} f(t)\, dt$$

∴ put $b = \infty$

if ans is b/w $-\infty \longrightarrow +\infty$ = ✓

but ∞/∞, $0/0$ = ✗

use L'hopital rule

$$\lim_{b \to \infty} \frac{e^{-st}}{-s}\Big|^{b}$$

$$\lim_{b \to \infty} \frac{e^{-s(b)}}{-s} - \left(\frac{e^{-s(0)}}{-s}\right)$$

$$= \frac{e^{-\infty}}{-s} - \frac{e^{0}}{-s}$$

$$= \frac{1}{s}\left[e^{-s(\infty)} - e^{-s(0)}\right]$$

$$= -\frac{1}{s}\left[\frac{1}{e^{\infty}} - e^{0}\right]$$

$$= -\frac{1}{s}\left[0 - 1\right]$$

$$= \frac{1}{s}$$

so $L\{1\} = 1/s$

Q. $L\{t\} = \int_0^\infty e^{-st}\, t\, dt$

$$\lim_{0 \to \infty} \int_0^b t e^{-st}\, dt$$

$u = t$, $dv = e^{-st}\, dt$

$du = dt$, $v = e^{-st}/-s$

$uv - \int v\, du$

$$-t\frac{e^{-st}}{s} + \frac{1}{s}\int e^{-st}\, dt$$

$$-\frac{t e^{-st}}{s} + \frac{1}{s}\frac{e^{-st}}{-s}$$

$$= -\frac{t e^{-st}}{s} - \frac{1}{s^2} e^{-st}$$

$$= -\lim_{b\to\infty} \left[\frac{t}{st} + \frac{1}{s^2 e^{st}}\right]_0^b$$

L-hopital

$$= -\lim_{b\to\infty} \left[\left(\frac{b}{se^{sb}} + \frac{1}{s^2 e^{sb}}\right) - \left(\frac{0}{se^{s(0)}} + \frac{1}{s^2 e^{s(0)}}\right)\right]$$

$$= -\lim_{b\to\infty} \left[\frac{1/\infty}{s^2 e^{sb}} + \frac{1/\infty}{s^2 e^{sb}}\right] - \left[\frac{0}{se^0} + \frac{1}{s^2 e^0}\right]$$

$$= 0 + 0 + \frac{1}{s^2}$$

$$= 0 + 1/s^2$$

$\rightarrow L\{t^0\} = L\{1\} = 1/s$ $\qquad 0!/s^{0+1}$

$\rightarrow L\{t^1\} = 1/s^2$ $\qquad 1!/s^{1+1}$

$\rightarrow L\{t^2\} = 2/s^3$ $\qquad 2!/s^{2+1}$

$\rightarrow L\{t^3\} = 6/s^4$ $\qquad 3!/s^{3+1}$

$\rightarrow L\{t^n\} = n!/s^{n+1} \rightarrow$ Th. 7.2

L → linear operator to find Laplace transform

$$L\{\alpha f(t) + \beta g(t)\} = \alpha L\{f(t)\} + \beta L\{g(t)\}$$

E.g

$$f(t) = 3t^2 + \tfrac{1}{2}t - 1$$

$$L\{f(t)\} = L\{3t^2 + \tfrac{1}{2}t - 1\}$$

$$= L\{3t^2\} + L\{\tfrac{1}{2}t\} - L\{1\}$$

$$= 3L\{t^2\} + \tfrac{1}{2}L\{t\} - L\{t\}$$

$$= 3\frac{2!}{s^3} + \frac{1}{2}\frac{1}{s^2} - \frac{1}{s}$$

$$= 3 \cdot \frac{2\times 1}{s^3} + \frac{1}{2s^2} - \frac{1}{s}$$

$$= \frac{6}{s^3} + \frac{1}{2s^2} - \frac{1}{s}$$

$$\rightarrow L\{t^{at}\} = \int_0^\infty e^{-st} e^{at}\, dt$$

$$\lim_{t\to\infty} \int_0^b e^{at-st}\, dt$$

$$\lim_{t\to\infty} \int_0^b e^{t(a-s)}\, dt$$

$$\lim_{t\to\infty} \left[\frac{e^{(a-s)t}}{a-s}\right]_0^b$$ (upper − lower)

Practice

Real & distinct

$y = C_1 e^{\alpha x} + C_2 e^{\alpha x}$

Real & Repeated

$y = e^{\alpha x}(C_1 + C_2 x) + e^{\alpha x}(C_3 + C_4 x)$

Complex Conjugate

$y = e^{\alpha x}(C_1 \sin\beta x - \cos\beta x) \rightarrow e^{\alpha x}(C_1 \sin x + C_2 \cos x)$

Repeated + complex Conjugate

$y = e^{\alpha x}(C_1 + C_2 x \sin\beta x) + (C_3 + C_4 x \cos\beta x)$

$y'' + 2y' + y = \sin x + 3\cos 2x$

$(D^2 + 2D + 1)y = 0$

$m^2 + 2m + 1 = 0$

$(m+1)^2 = 0$

$(m+1)(m+1) = 0$

$m = -1 \quad m = -1$

real & repeat

$y = e^{\alpha x}(C_1 + C_2 x)$

$= e^{\alpha x} C_1 + e^{\alpha x} C_2$

$= e^{-x} C_1 + e^{-x} C_2$

$= C_1 e^{-x} + C_2 e^{-x}$

$y = \sin x + 3\cos 2x$

$y_p = A\sin x + b\cos x$

$y_p' = A\cos x + (-B\sin 2x)$

$= A\cos x - 2B\sin 2x$

$y_p'' = -A\sin x - 4B\cos x$

4.7 Q 11-18 & 19-42

$\frac{1}{a-s}\left[\lim_{a-s \to \infty}\left[e^{(a-s)b} - e^{(a-s)0}\right]\right]$

$\frac{1}{a-s}\left[e^{-(a-s)\infty} - 1\right]$

$\frac{1}{a-s}[0 - 1] = -\frac{1}{a-s} = \frac{1}{s-a}$

$\rightarrow$ so $\mathcal{L}\{e^{at}\} = \frac{1}{s-a}$

$\rightarrow$ Q $\mathcal{L}\{\sin kt\} = k/s^2 + k^2$

$\rightarrow \mathcal{L}\{\cos kt\} = \frac{s}{s^2 + k^2}$

Th = 7.2

$\rightarrow \mathcal{L}\{\cosh kt\} = \frac{s}{s^2 - k^2}$

Q 29 $y(t) = (1 + e^{-t})^2$

$\mathcal{L}\{y(t)\} = \mathcal{L}\{1 + e^{-t} + 2e^{-2t}\}$

$= \mathcal{L}\{1\} + \mathcal{L}\{e^{-4t}\} + 2\mathcal{L}\{1 \cdot e^{-2t}\}$

$= \frac{1}{s} + \frac{1}{s-4} + 2\frac{1}{s-2}$

Q 31 $f(t) = e^{-t} \cosh t$

$f(t) = e^{-t}(e^{t} + e^{-t})/2$

$y(t) = \frac{1}{2} + \frac{e^{-2t}}{2}$

$y(t) = \mathcal{L}\{\frac{1}{2}\} + \mathcal{L}\{\frac{e^{-2t}}{2}\}$

$= \frac{1}{2s} + \frac{1}{s+2}$

37) $f(t) = \sin 2t \cos 2t$

$\frac{1}{2}[2\sin 2t \cos 2t] = \frac{1}{2}\sin 4t$

39) $f(t) = \cos t \cdot \cos 2t$

$\frac{1}{2}[2\cos t \cos 2t]$

$= \frac{1}{2}[\cos 3t + \cos t]$

38) $f(t) = \cos^2 t$

$= \frac{1 + \cos 2t}{2} = \frac{1}{2} + \frac{\cos 2t}{s}$

42) $f(t) = \sin^3 t$

$= \sin t \cdot \sin^2 t$

$= \sin t \cdot \frac{1 - \cos 2t}{2}$

Inverse Laplace Transform

Ex#7.2

if $\mathcal{L}\{1\} = \frac{1}{s} \rightarrow \mathcal{L}^{-1}\{1/s\} = 1$

$$\mathcal{L}\left\{\frac{1}{s^n}\right\} \quad \frac{n!}{s^{n+1}}$$

$$\mathcal{L}^{-1}\left\{\frac{n!}{s^{n+1}}\right\} = t^n$$

$$\mathcal{L}^{-1}\left\{\frac{1}{s-a}\right\} = e^{at}$$

$$\mathcal{L}^{-1}\left\{\frac{k}{s^2+k^2}\right\} = \sin kt$$

$$\mathcal{L}^{-1}\left\{\frac{s}{s^2+k^2}\right\} = \cos kt$$

$$\mathcal{L}^{-1}\left\{\frac{k}{s^2-k^2}\right\} = \sinh kt$$

$$\mathcal{L}^{-1}\left\{\frac{s}{s^2-k^2}\right\} = \cosh kt$$

Theorem

$$\mathcal{L}^{-1}\{\alpha F(s) + \beta G(s)\} = \alpha \mathcal{L}^{-1}\{F(s)\} + \beta \mathcal{L}^{-1}\{G(s)\}$$

Q → $\mathcal{L}^{-1}\left\{\frac{1}{s^5}\right\}$ $\quad 4!/s^{4+1}$

$$= \mathcal{L}^{-1}\left\{\frac{1}{s^{4+1}}\right\} = \frac{1}{4!}\mathcal{L}^{-1}\left\{\frac{4!}{s^{4+1}}\right\} = \frac{1}{24}t^4$$

Q $\left\{\frac{1}{(s-1)(s+2)(s+4)}\right\}$

$$\frac{1}{(s-1)(s-2)(s+4)} = \frac{A}{(s-1)} + \frac{B}{(s-2)} + \frac{C}{s+4}$$

$$1 = A(s-2)(s+4) + B(s-1)(s+4) + C(s-2)(s-2)$$

→ if $s=1$

$1 = 15A = A = 1/15$

→ if $s=-1$

$1 = -6B = B = -1/6$

if $s=-4$

$1 = 10C = C = 1/10$

$$\frac{1}{(s-1)(s-2)(s+4)} = \frac{1/15}{s-1} - \frac{1}{6}\frac{1}{s+2} + \frac{1}{10}\frac{1}{s+4}$$

taking Inverse

$$\mathcal{L}^{-1}\left\{\frac{1}{(s-1)(s-2)(s+4)}\right\} = \frac{1}{15}\mathcal{L}^{-1}\left\{\frac{1}{s-1}\right\}$$

Ex # 7.3

Translation theorem & Derivative of L transform

→ When e^{at} is multiplied with other Laplaces

$$L\{e^{at} \cdot t^n\}$$

$$= \frac{n!}{s^{n+1}} \rightarrow \frac{n!}{(s-a)^{n+1}}$$

place s with power of e i.e a

→ Derivative of Transform

find laplace

$$L\{t^n f(t)\} = (-1)^n \frac{d^n}{ds^n} f(s)$$

$L(f(t)) = f(s) \longrightarrow$ laplace

→ Q → $L\{e^{st}\} = (-1)^2 \frac{d^2}{ds^2} \frac{1}{s-3}$

$$= 1 \frac{d}{d} \left(\frac{d}{ds} (s-3)^{-1} \right)$$

$$= \frac{d}{ds} \left[-1 (s-3)^{-2} \right]$$

$$= 2 (s-3)^{-3}$$

$$= \frac{2}{(s-3)^3}$$

7) $L\{e^{st} \sinh 3t\}$

$$\sinh 3t = \frac{3}{s^2-9}$$

$$= \frac{3}{(s-5)^2-9}$$

8) $L^{-1}\left\{\frac{2s+5}{s^2+6s+34}\right\}$

$$= 2L^{-1}\left\{\frac{s}{s^2+6s+34}\right\} + 5L^{-1}\left\{\frac{1}{s^2+6s+34}\right\}$$

$$= 2L^{-1}\left\{\frac{s}{(s)^2+2(s)(3)+9^2+25}\right\} + 5L^{-1}\left\{\frac{1}{s^2+6s+9+25}\right\}$$

$$= 2L^{-1}\left\{\frac{s+3}{(s+3)^2+5^2}\right\} + 5L^{-1}\left\{\frac{1}{(s+3)^2+5^2}\right\}$$

$s+3-3$

$$= 2L^{-1}\left\{\frac{s+3}{(s+3)^2+5^2}\right\} - \frac{3\times 2}{6}L^{-1}\left\{\frac{1}{(s+3)^2+5^2}\right\} + 5L^{-1}$$

$$+ 5L^{-1}\left\{\frac{1}{(s+3)^2+5^2}\right\}$$

$$= 2L^{-1}\left\{\frac{s+3}{(s+3)^2+5^2}\right\} - L^{-1}\left\{\frac{1}{(s+3)^2+5^2}\right\}$$

$$= \quad '' \quad - \frac{L^{-1}}{5}\left\{\frac{5}{(s+3)^2+5^2}\right\}$$

$$= 2e^{-3t}\cos 5t - \frac{1}{5}e^{-3t}\sin 5t$$