4.1 (1-43)

① $y = c_1 e^{x} + c_2 e^{-x} \rightarrow ①$

$y'' - y = 0 \rightarrow ②$

$y(0) = 0 \quad , \quad y'(0) = 1$

Put $y(0) = 0$ in equ ①

equ ① $\Rightarrow \quad 0 = c_1 + c_2 \rightarrow ③$

For $y'(0) = 1$

equ ① $\Rightarrow \quad y' = c_1 e^{x} - c_2 e^{-x} \rightarrow ④$

equ ④ $\Rightarrow \quad 1 = c_1 - c_2 \rightarrow ⑤$

Add equ ③ and ⑤

$$\begin{aligned} 0 &= c_1 + c_2 \\ 1 &= c_1 - c_2 \\ \hline 1 &= 2c_1 \end{aligned}$$

$c_1 = \frac{1}{2}$.

Put c_1 in equ ③

$0 = +\frac{1}{2} + c_2$

$c_2 = -\frac{1}{2}$

equ ① $\rightarrow$

$y = \frac{1}{2} e^{x} - \frac{1}{2} e^{-x}$

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(3)

$$y = c_1 e^{4x} + c_2 e^{-x} \rightarrow (1)$$

$$y'' - 3y' - 4y = 0 \rightarrow (2)$$

$$y(0) = 1 \quad , \quad y'(0) = 2$$

$$y' = c_1 4e^{4x} - c_2 e^{-x} \rightarrow (3)$$

$$y'' = 16 c_1 e^{4x} + c_2 e^{x}$$

For $y(0) = 1$

equ (1) $\Rightarrow$ $1 = c_1 + c_2 \rightarrow (4)$

For $y'(0) = 2$

equ (3) $\Rightarrow$ $2 = 4c_1 - c_2 \rightarrow (5)$

Adding equ (4) & (5), we get

$$\begin{aligned} 1 &= c_1 + \cancel{c_2} \\ 2 &= 4c_1 - \cancel{c_2} \\ \hline 3 &= 5c_1 \end{aligned}$$

$$c_1 = 3/5$$

Put it in equ (4)

equ (4) $\Rightarrow$ $1 = 3/5 + c_2$

$$c_2 = 2/5$$

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So, equ (1) $\Rightarrow$ $y = \frac{3}{5} e^{4x} + \frac{2}{5} e^{-x}$

(5) $y = c_1 x + c_2 x \ln x \rightarrow (1)$

$x^2 y'' - x y' + y = 0$

$y(1) = 3 \qquad y'(1) = -1$

equ (1) $\Rightarrow$ $y' = c_1 + c_2 \left[\cancel{x} \cdot \frac{1}{\cancel{x}} + \ln x (1) \right]$

$y' = c_1 + c_2 \ln x + c_2 \rightarrow (2)$

For $y(1) = 3$

equ (1) $\Rightarrow$ $3 = c_1 \rightarrow (3)$

For $y'(1) = -1$

equ (2) $\Rightarrow$ $-1 = c_1 + c_2 \rightarrow (4)$

For equ (3) $c_1 = 3$, put it in equ (4),

we get

$-1 = 3 + c_2$

$c_2 = -4$

So,

equ (1) $\Rightarrow$ $y = 3x - 4x \ln x$

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7. $xy'' - y' = 0$

$y = c_1 + c_2 x^2 \rightarrow (1)$

$y(0) = 0 \quad , \quad y'(0) = 0$

equ. (1) $\Rightarrow$

$y' = 0 + 2xc_2$

$y' = 2xc_2 \rightarrow (2)$

For $y(0) = 0$

equ. (1) $\Rightarrow \quad 0 = c_1$

For $y'(0) = 0$

equ (2) $\Rightarrow \quad 0 = 2(0) \cdot c_2 \Rightarrow 0 = 0$

c_2 is arbitary (Any value of 'c' can satisfy the given condition)

So, let $c_2 = 1$ or $c_2 = 2$

equ (1) $\Rightarrow \quad y = x^2$

and $\quad y = 2x^2$

9

$y = c_1 e^x \cos x + c_2 e^x \sin x \rightarrow (1)$

$y'' - 2y' + 2y = 0$

a) $y(0) = 1 \quad , \quad y'(0) = 0$

equ (1) $\Rightarrow$

$y' = c_1[-e^x \sin x + \cos x e^x] + c_2[e^x \cos x + \sin x e^x]$

(2)

For $y(0) = 1$

equ ① $\rightarrow$ $1 = c_1\cos(0) + c_2\sin(0)$ $\Rightarrow$ $\boxed{c_1 = 1}$ ③

For $y'(0) = 0$

equ ② $\Rightarrow$ $0 = -c_1\sin(0) + c_1\cos(0) + c_2\cos(0) + c_2\sin(0)$

$0 = c_1 + c_2$ $\rightarrow$ ④

From equ ③, put $c_1 = 1$ in equ ④

equ ④ $\Rightarrow$ $0 = 1 + c_2$

$c_2 = -1$

equ ① $\Rightarrow$

$y = e^x\cos x - e^x\sin x$

b) $y(0) = 1$, $y(\pi) = -1$

For $y(0) = 1$

equ ① $\Rightarrow$ $1 = c_1$

For $y(\pi) = -1$

equ ② $\Rightarrow$ $-1 = -c_1 e^{\pi}$

So,

condition doesn't satisfy

c) $y(0) = 1$, $y(\pi/2) = 1$

For $y(0) = 1$

equ ① $\Rightarrow$ $\boxed{1 = c_1}$

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For $y(\pi/2) = 1$

equ ① $\Rightarrow$ $1 = c_2 e^{\pi/2}$

$c_2 = e^{-\pi/2}$

So, equ ① $\Rightarrow$

$y = e^x \cos x + e^{\pi/2} e^x \sin x$

d) $y(0) = 0$ $\quad$ $y(\pi) = 0$

For $y(0) = 0$

equ ① $\Rightarrow$ $0 = c_1$

equ ① $\Rightarrow$ $y = c_2 e^x \sin x$ ②

For $y(\pi) = 0$

equ ② $\Rightarrow$ $0 = + c_2 e^{\pi} \sin \pi$

$0 = 0$

$\because c_2$ is arbitary constant

for $c_2 = 1$ and $c_2 = 2$

equ ② $\Rightarrow$

$y = e^x \sin x$

$y = 2e^x \sin x$

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(13) $y = c_1 \cos \lambda x + c_2 \sin \lambda x \Rightarrow (1)$

$y'' + \lambda^2 y = 0$

boundary value problem

$y'' + \lambda^2 y = 0 \qquad y(0) = 0, \quad y(\pi) = 0$

For $y(0) = 0$

equ (1) $\Rightarrow 0 = c_1 \cos \lambda 0 + c_2 \sin \lambda 0$

$0 = c_1$

For $y(\pi) = 0$ equ (1) $\Rightarrow$

$0 = c_1 \cos \lambda \pi + c_2 \sin \lambda \pi$

$= c_1 (-1) + 0$

$c_1 = 0$

put it in equ (1)

equ (1) $\Rightarrow \quad y = c_2 \sin \lambda x$

For $y(\pi) = 0$

$0 = c_2 (0)$

The problem will have non trivial so when $c_2 \neq 0$. Thus we require that $\sin \lambda \pi = 0$ or λ be a non-zero integer. (if $\lambda = 0$, the family of solutions is

$y = c_1 = 0$

$\therefore \lambda = n$, where $n = 1, 2, 3 \ldots$

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Determine funcs are linearly dependent or in "

(15) $f_1(x) = x \quad f_2(x) = x^2$

$f_3(x) = 4x - 3x^2$

$$c_1 f_1(x) + c_2 f_2(x) + c_3 f_3(x) = 0$$

$$x c_1 + c_2 x^2 + c_3 (4x - 3x^2) = 0 \rightarrow (A)$$

Comparing with coeffiet of x

$$c_1 + 4c_3 = 0 \rightarrow (1)$$

Comparing with coeffiet of x^2

$$c_2 - 3c_3 = 0 \rightarrow (2)$$

Multiply equ (1) with 3

equ (1) $\Rightarrow$ $3c_1 + 12c_3 = 0 \rightarrow (3)$

Multiply equ (2) with (4)

equ (4) $\Rightarrow$ $4c_2 - 12c_3 = 0 \rightarrow (4)$

Add equ (3) and (4)

$$3c_1 + 12c_3 = 0$$
$$4c_2 - 12c_3 = 0$$
$$3c_1 + 4c_2 = 0$$

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let $c_3 = 1$

Put it in equ ①

equ ① $\Rightarrow$ $c_1 + 4 = 0$

$c_1 = -4$

Put $c_3 = 1$ in equ ②

equ ② $\Rightarrow$ $c_2 - 3 = 0$

$c_2 = 3$

So, equ Ⓐ $\Rightarrow$ $-4x + 3x^2 + 1(4x - 3x^2) = 0$

(The funcs are linearly dependent)

(19) $f_1(x) = x$ $f_2(x) = x - 1$ $f_3(x) = x + 3$

$c_1 f_1(x) + c_2 f_2(x) + c_3 f_3(x) = 0$

$x c_1 + c_2(x - 1) + c_3(x + 3) = 0$

Comparing coefficient of x, we get

$c_1 + c_2 + c_3 = 0 \rightarrow ①$

Comparing ~~coefficient~~ of constant term

$-c_2 + 3c_3 = 0 \rightarrow ②$

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let $c_3 = 1$

Put it in equ (2)

$$-c_2 + 3 = 0$$

$$c_2 = +3$$

Put $c_3 = 1$ & $c_2 = 3$ in equ (1)

$$c_1 + 3 + 1 = 0$$

$$c_1 = -4$$

equ (A) $\rightarrow$ $-4x + 3(x-1) + 1(x+3) = 0$

(funcs are linearly dependent)

2) $f_1(x) = 1+x$ $f_2(x) = x$ $f_3(x) = x^2$

$$c_1 f(x) + c_2 f_2(x) + c_3 f_3(x) = 0$$

$$c_1(1+x) + c_2 x + c_3 x^2 = 0 \rightarrow (A)$$

Comparing coefficient of x

$$c_1 + c_2 = 0$$

Comparing constant terms.

$$c_1 = 0$$

So, $c_2 = 0$, & $c_3 = 0$ As, $c_1, c_2, c_3 = 0$

So, linearly independent.

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OR

(21) $f_1(x) = 1+x \quad f_2(x) = x \quad f_3(x) = x^2$

By using Wronskian method

$$W = \begin{vmatrix} 1+x & x & x^2 \\ 1 & 1 & 2x \\ 0 & 0 & 2 \end{vmatrix} = 2 \neq 0$$

So, linearly independent

(23) Use wronskia method for find func are linearly dependent or independent.

23. $x^{1/2}, \quad x^2 \quad (0, \infty)$

$$W = \begin{vmatrix} x^{1/2} & x^2 \\ \frac{1}{2}x^{-1/2} & 2x \end{vmatrix}$$

$$W = (x^{1/2})(2x) - (x^2)\left(\frac{1}{2}x^{-1/2}\right)$$

$$= 2x^{3/2} - \frac{1}{2}x^{3/2}$$

$$= \frac{3}{2}x^{3/2} \neq 0$$

So, linearly independent.

$x^{1/2}$

$-\frac{1}{2}+2$

$\frac{-1+4}{2}$

$\frac{1}{2}$

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(27) e^{x}, e^{-x}, e^{4x} $(-\infty, \infty)$

$$W = \begin{vmatrix} e^{x} & e^{-x} & e^{4x} \\ e^{x} & -e^{-x} & 4e^{4x} \\ e^{x} & e^{-x} & 16e^{4x} \end{vmatrix}$$

$$= e^{x}\begin{vmatrix} -e^{-x} & 4e^{4x} \\ e^{-x} & 16e^{4x} \end{vmatrix} - e^{-x}\begin{vmatrix} e^{x} & 4e^{4x} \\ e^{x} & 16e^{4x} \end{vmatrix} + e^{4x}\begin{vmatrix} e^{x} & -e^{-x} \\ e^{x} & e^{-x} \end{vmatrix}$$

$$= e^{x}\left[-16e^{3x} - 4e^{3x}\right] - e^{-x}\left[16e^{5x} - 4e^{5x}\right] + e^{4x}\left(1+1\right)$$

$$= e^{x}\left[-20e^{3x}\right] - e^{-x}\left[12e^{5x}\right] + e^{4x}\left[2\right]$$

$$= -20e^{4x} - 12e^{4x} + 2e^{4x}$$

$$= -30e^{4x} \neq 0$$

So, linearly independent.

(24) $f_1(x) = 2$ and $f_2(x) = e^{x}$

Does f_1 and f_2 are linearly depedent if $1 \cdot f_1(0) - 2 \cdot f_2(0) = 0$, on $x=0$

On $x=0$, it is linearly depedent

On other intervals, independent

On $x=0$, $1 \cdot 2 - 2 \cdot e^{0} = 0$

$2 - 2 = 0$

$0 = 0$

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$$c_1 f(x) + c_2 f_2(x) = 0$$

$$c_1 2 + c_2 e^x = 0$$

By comparing constant term

$$c_1 2 = 0$$

$$c_1 = 0$$

By compare e^x

$$c_2 e^x = 0$$

$$c_2 = 0$$

Therefor $f_1(x)$ & $f_2(x)$ are linearly independent except $x = 0$

(31) b)

$$y = \frac{1}{x}$$

$$y'' = 2y^3 \rightarrow (1)$$

$$y = c/x$$

$$y' = \frac{-c}{x^2}$$

$$y'' = \frac{+2c}{x^3}$$

equ (1) $\Rightarrow$ $\frac{2c}{x^3} = 2\frac{c^3}{x^3}$

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$c^3 = c \rightarrow (A)$

If $c \neq 0, \pm 1$ then c/x is not a solution of (1)

Or equ (A) $\Rightarrow$ $c - c^3 = 0$

$c(1 - c^2) = 0$

$c = 0$ $\quad$ $1 - c^2 = 0$

$c^2 - 1 = 0$

$c = 0$ $\quad$ $c = \pm 1$

a) $y'' = 2y^3 \rightarrow (1)$

$y = 1/x$

$y' = \frac{-1}{x^2}$

$y' = -x^{-2}$

$y'' = +2x^{-3}$

equ (1) $\Rightarrow$ $y'' - 2y^3 = 0$

By putting values, we get

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$$+\frac{2}{x^3} - 2\left(\frac{1}{x}\right)^3 = 0$$

$$\frac{2}{x^3} - \frac{2}{x^3} = 0$$

$$0 = 0$$

So, $y = \frac{1}{x}$ is a solution of equ (1)

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37. $x^2y'' - 6xy' + 12y = 0$ $\quad x^3, x^4 \quad (0, \infty)$

39. $x^3y''' + 6x^2y'' + 4xy' - 4y = 0$ $\quad x, x^{-2}, x^{-2}\ln x \quad (0, \infty)$

---

35. $y'' - 2y' + 5y = 0$ (A), $\quad y_1 = e^x \cos 2x, \quad y_2 = e^x \sin 2x \quad (-\infty, \infty)$

$y_1 = e^x \cos 2x$

$y_1' = e^x 2 - \sin 2x + \cos 2x e^x$

$= -2e^x \sin 2x + \cos 2x e^x$

$y'' = -4e^x \cos 2x - \sin 2x\, 2e^x + \cos 2x e^x + 2e^x \sin 2x$

equ (A) $\Rightarrow$ $-4e^x \cos 2x - 2e^x \sin 2x + \cos 2x e^x$
$-2e^x \sin 2x + 4e^x \sin 2x - 2\cos 2x e^x + 5e^x \cos 2x = 0$

$0 = 0$

For $y = e^x \sin 2x$

$y' = 2e^x \cos 2x + \sin 2x e^x$

$y'' = -4e^x \sin 2x + \cos 2x \cdot 2e^x + 2e^x \cos 2x + \sin 2x e^x$

equ (A) $\Rightarrow$ $-4e^x \sin 2x + 2e^x \cos 2x + 2e^x \cos 2x + \sin 2x e^x$
$-4e^x \cos 2x - 2\sin 2x e^x + 5e^x \sin 2x = 0$

$0 = 0$

So, $y_1 = e^x \cos 2x$ and $e^x \sin 2x$ are the sol of equ (A)

$$W = \begin{vmatrix} e^x \cos 2x & e^x \sin 2x \\ -2e^x \sin 2x + \cos 2x e^x & 2e^x \cos 2x + \sin 2x e^x \end{vmatrix}$$

$= 2e^{2x} \cos(2x)^2 + e^{2x} \cos 2x \sin 2x + 2e^{2x} \sin(2x)^2 - e^{2x} \sin 2x \cos 2x$

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$$2e^{2x}\left[\cos(2x)^2 + \sin(2x)^2\right] \neq 0$$

So, linearly independent.

Hence $e^x\cos 2x$ and $e^x\sin 2x$ are the fundamental set of sol. The general sol is $y = c_1 e^x\cos 2x + c_2 e^x \sin 2x$

37. $x^2y'' - 6xy' + 12y = 0 \rightarrow$ (A) $\quad x^3, x^4$

$y = x^3$

$y' = 3x^2$

$y'' = 6x$

equ (A) $\Rightarrow$

$$6x^3 - 18x^3 + 12x^3 = 0$$

$$0 = 0$$

For $y = x^4$

$y' = 4x^3$

$y'' = 12x^2$

equ (A) $\Rightarrow$

$$12x^4 - 24x^4 + 12x^4 = 0$$

$$0 = 0$$

So, x^3 & x^4 are the sol of equ (A)

Now,

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$$W = \begin{vmatrix} x^3 & x^4 \\ 3x^2 & 4x^3 \end{vmatrix}$$

$$= 4x^6 - 3x^6$$

$$= x^6 \neq 0$$

So, linearly independent

Hence, x^3 and x^4 are the fundamental set of sol.

The general sol is $y = c_1 x^3 + c_2 x^4$

39. $x^3 y''' + 6x^2 y'' + 4xy' - 4y = 0 \rightarrow$ (A)

$x, x^{-2}, x^{-2} \ln x$

For $y = x$

$y' = 1$

$y'' = 0$

$y''' = 0$

equ (A) $\Rightarrow 0 + 0 + 4x - 4x = 0$

$0 = 0$

For $y = x^{-2}$

$y' = -2x^{-3}$

$y'' = +6x^{-4}$

$y''' = -24x^{-5}$

equ (A) $\Rightarrow -24x^{-2} + 36x^{-2} - 8x^{-2} - 4x^{-2} = 0$

$0 = 0$

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For $y = x^{-2}\ln x$

$y' = x^{-2}\cdot\frac{1}{x} + \ln x - 2x^{-3}$

$= x^{-3} - 2x^{-3}\ln x$

$y'' = -3x^{-4} - [2x^{-3}\cdot x^{-1} + \ln x(-6x^{-4})]$

$= -3x^{-4} - 2x^{-4} + 6x^{-4}\ln x$

$y''' = +12x^{-5} + 8x^{-5} + 6x^{-4}x^{-1} + \ln x(-24x^{-5})$

$= 26x^{-5} - 24x^{-5}\ln x$

equ (A) $\Rightarrow$

$26x^{-2} - 24x^{-2}\ln x - 30x^{-2} + 36x^{-2}\ln x + 4x^{-2}$

$8x^{-2}\ln x - 4x^{-2}\ln x = 0$

$0 = 0$

So, x, x^{-2} & $x^{-2}\ln x$ are the solution of equ (A)

Now,

$$W = \begin{vmatrix} x & x^{-2} & x^{-2}\ln x \\ 1 & -2x^{-3} & x^{-3} - 2x^{-3}\ln x \\ 0 & 6x^{-4} & -5x^{-4} + 6x^{-4}\ln x \end{vmatrix}$$

$$= x\begin{vmatrix} -2x^{-3} & x^{-3} - 2x^{-3}\ln x \\ 6x^{-4} & -5x^{-4} + 6x^{-4}\ln x \end{vmatrix} - x^{-2}\begin{vmatrix} 1 & x^{-3} - 2x^{-3}\ln x \\ 0 & -5x^{-4} + 6x^{-4}\ln x \end{vmatrix}$$

$$x^{-2}\ln x\begin{vmatrix} 1 & -2x^{-3} \\ 0 & 6x^{-4} \end{vmatrix}$$

$\Rightarrow x(+10x^{-7} + 12x^{-7}\ln x - 6x^{-7} + 12x^{-7}\ln x) - x^{-2}$
$(-5x^{-4} + 6x^{-4}\ln x) + x^{-2}\ln x(6x^{-4})$

$\Rightarrow 4x^{-6} + 5x^{-6} - 6x^{-6}\ln x + 6x^{-6}\ln x$

$\Rightarrow 9x^{-6} \neq 0$

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So,
linearly independent
Hence, x, x^{-2}, $x^{-2}\ln x$ are the fundamental set of solutions.
The general sol is
$y = c_1 x + c_2 x^{-2} + c_3 x^{-2}\ln x$

41. $y'' - 7y' + 10y = 24e^x \rightarrow (1)$
$y = c_1 e^{2x} + c_2 e^{5x} + 6e^x \rightarrow (2)$ $(-\infty, \infty)$

$y' = 2c_1 e^{2x} + 5c_2 e^{5x} + 6e^x$
$y'' = 4c_1 e^{2x} + 25c_2 e^{5x} + 6e^x$
equ (1) $\Rightarrow$
$4c_1 e^{2x} + 25c_2 e^{5x} + 6e^x - 14c_1 e^{2x} - 35c_2 e^{5x} +$
$42e^x + 10c_1 e^{2x} + 10c_2 e^{5x} + 60e^x = 24e^x$
$66e^x - 42e^x = 24e^x$
$24e^x = 24e^x$

So,
equ (2) is a solution of equ (1)

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43. $y'' - 4y' + 4y = 2e^{2x} + 4x - 12 \rightarrow (1)$

$y = c_1 e^{2x} + c_2 x e^{2x} + x^2 e^{2x} + x - 2 \rightarrow (2)$ $(-\infty, \infty)$

$y' = 2c_1 e^{2x} + c_2 e^{2x} + 2c_2 x e^{2x} + 2xe^{2x} + 2x^2 e^{2x} + 1$

$y'' = 4c_1 e^{2x} + 2c_2 e^{2x} + 2c_2 [2e^{2x} x] + 2c_2 e^{2x} + 4xe^{2x} + 2e^{2x} + 4xe^{2x} + 4x^2 e^{2x}$

equ (1) $\Rightarrow$

$4c_1 e^{2x} + 4c_2 e^{2x} + 4c_2 x e^{2x} + 8xe^{2x} + 2e^{2x} + 4x^2 e^{2x} - 8c_1 e^{2x} - 4c_2 e^{2x} - 8c_2 x e^{2x} - 8xe^{2x} - 8x^2 e^{2x} - 4 + 4c_1 e^{2x} + 4c_2 x e^{2x} + 4x^2 e^{2x} + 4x - 8 =$

$2e^{2x} + 4x - 12$

$2e^{2x} - 12 + 4x = 2e^{2x} + 4x - 12$

So,

equ (2) is a solution of equ (1).

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Ex : 4.2 (1-29)

① $y'' + 5y' = 0 \rightarrow ①$, $y_1 = 1$

$\because \; y_2(x) = y_1(x) \int \frac{e^{-\int p(x)\,dx}}{y^2} \rightarrow Ⓐ$

from equ ①,

$P(x) = 5$ and $y_1(x) = 1$

equ Ⓐ $\Rightarrow$

$$y_2(x) = 1 \int \frac{e^{-\int 5\,dx}}{(1)^2}$$

$$= 1 \int e^{-5x}$$

$$= \frac{1}{-5} e^{-5x}$$

By ignoring $-\frac{1}{5}$, we get

$$y_2(x) = e^{-5x}$$

② $y'' - 4y' + 4y = 0 \rightarrow ①$ $y_1 = e^{2x}$

$\because \; y_2(x) = y_1(x) \int \frac{e^{-\int P(x)\,dx}}{y^2} \rightarrow Ⓐ$

from equ ① $\Rightarrow$ $P(x) = -4$ and $y = e^{2x}$

equ Ⓐ $\Rightarrow$ $y_2(x) = e^{2x} \int \frac{e^{-\int -4\,dx}}{(e^{2x})^2}$

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$$= e^{2x} \int \frac{\cancel{e^{4x}}}{\cancel{e^{4x}}} \, dx$$

$$= e^{2x} \int 1 \, dx$$

$$y_2(x) = x e^{2x}$$

(5) $y'' + 16y = 0 \rightarrow (1)$ $\quad y_1 = \cos 4x$

$$\therefore y_2(x) = y_1(x) \int \frac{e^{-\int P(x)\,dx}}{y^2} \rightarrow (A)$$

from equ (1), $P(x) = 0$ & $y = \cos 4x$

equ (A) $\Rightarrow y_2(x) = \cos 4x \int \frac{e^{-\int 0\,dx}}{(\cos 4x)^2}$

$$= \cos 4x \int \frac{e^{-c}}{\cos^2 4x}$$

$$= \cos 4x \int \sec^2 4x$$

$$= \cos 4x \tan 4x$$

$$y_2(x) = \sin 4x$$

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(7) $y'' - y = 0 \rightarrow (1)$, $y_1 = \cosh x$

$$\therefore y_2(x) = y_1(x) \int \frac{e^{-\int P(x)\,dx}}{y^2}\,dx \rightarrow (A)$$

equ (A) $\Rightarrow$ $$y_2(x) = \cosh x \int \frac{e^{-\int 0\,dx}}{\cosh^2 x}\,dx$$

$$= \cosh x \int \frac{e^{-x}}{\cosh^2 x}$$

$$= \cosh x \int \operatorname{sech}^2 x$$

$$= \cosh x \tanh x$$

$$= \sinh x$$

(9) $9y'' - 12y' + 4y = 0$ $\quad y_1 = e^{2x/3}$

$$y'' - \frac{4}{3}y' + \frac{4}{9}y = 0$$

$$\therefore y_2(x) = y_1(x) \int \frac{e^{-\int P(x)\,dx}}{y^2}\,dx$$

equ (A) $\Rightarrow$ $$y_2(x) = e^{2x/3} \int \frac{e^{-\int -\frac{4}{3}\,dx}}{(e^{2x/3})^2}\,dx$$

$$= e^{2x/3} \int \frac{e^{4x/3}}{e^{4x/3}}$$

$$= e^{2x/3} \int 1\,dx$$

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$= x e^{2x/3}$

(11) $x^2 y'' - 7xy' + 16y = 0$ $\quad y_1 = x^4$

$\therefore y_2(x) = y_1(x) \int \frac{e^{-\int P(x)dx}}{(y)^2} dx$

equ (1) $\Rightarrow$ $y'' - \frac{7y'}{x} + \frac{16}{x^2} y = 0$, So, $P(x) = -7/x$

equ (A) $\Rightarrow$ $= x^4 \int \frac{e^{-\int -7/x \, dx}}{x^8} dx$

$= x^4 \int \frac{e^{\ln x^7}}{x^8} dx$

$\Rightarrow x^4 \int \frac{1}{x} dx$

$x^4 \ln|x|$

So,

$y_2(x) = x^4 \ln|x|$

(13) $xy'' + y' = 0$, $y_1 = \ln x$

$y'' + \frac{1}{x} y' = 0$

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$$\therefore y_2(x) = y_1(x) \int \frac{e^{-\int P(x)\,dx}}{(y_1)^2}\,dx$$

$$= \ln x \int \frac{e^{-\int \frac{1}{x}\,dx}}{(\ln x)^2}\,dx$$

$$= \ln x \int \frac{e^{-\ln|x|}}{(\ln x)^2}$$

$$= \ln x \int \frac{e^{\ln x^{-1}}}{(\ln x)^2}$$

$$= \ln x \int \frac{1}{x(\ln x)^2} \rightarrow (2)$$

let $I = \int \frac{1}{x(\ln x)^2}$

let $\ln x = s$

$$\frac{1}{x} = \frac{ds}{dx}$$

$$\frac{dx}{x} = ds$$

$$I = \int \frac{ds}{s^2}$$

$$= -\frac{1}{s}$$

$$= -\frac{1}{\ln x}$$

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Eqn (2) $\Rightarrow y_2(x) = \ln x \left(\frac{-1}{\ln x} \right)$

$y_2(x) = -1$

(15) $(1-2x-x^2)y'' + 2(1+x)y' - 2y = 0$

$y_1 = x+1$

$y'' + \frac{2(1+x)}{1-2x-x^2} y' - \frac{2}{(1-2x-x^2)} y = 0$

So, $P(x) = \frac{2(1+x)}{1-2x-x^2}$

$\therefore y_2(x) = y_1(x) \int \frac{e^{-\int P(x)\,dx}}{(y)^2}$

$= x+1 \int \frac{e^{-\int \frac{2+2x}{-(-1+2x+x^2)}\,dx}}{(x+1)^2}$

$= x+1 \int \frac{+e^{\ln(-1+2x+x^2)}}{(x+1)^2}\,dx$

$= x+1 \int \frac{-1+2x+x^2}{(x+1)^2}\,dx$

$= x+1 \int \frac{+x^2+2x-1}{x^2+2x+1}$

$= x+1 \int 1 - \frac{2}{(x+1)^2}$

$\cancel{(x+1)}\ \cancel{x^2+2x+1}$

$\cancel{x^2} \oplus \cancel{2x} \oplus 1$

-2

let $x+1 = s$

$dx = ds$

$$= x+1\left[x + \frac{2}{x+1}\right]$$

$$= \cancel{x+1} \cdot \frac{x^2+x+2}{\cancel{x+1}}$$

$$= x^2+x+2$$

$\int \frac{2}{s^2}$

$-\frac{2}{s}$

$-\frac{2}{x+1}$

(17) $x^2y'' - xy' + 2y = 0 \qquad y_1 = x\sin(\ln x)$

$$y'' - \frac{1}{x}y' + \frac{2}{x^2}y = 0$$

$$\Rightarrow y_2(x) = x\sin(\ln x)\int \frac{e^{-\int -\frac{1}{x}dx}}{(x\sin(\ln x))^2}$$

$$= x\sin(\ln x)\int \frac{e^{\ln x}}{x^2\sin^2(\ln x)}$$

$$= x\sin(\ln x)\int \frac{\cancel{x}}{\cancel{x^2}\sin^2\ln x}$$

$$= x\sin(\ln x)\int \frac{\operatorname{cosec}^2\ln x}{x}\,dx$$

$$= x\sin(\ln x)\int \operatorname{cosec}^2(s)\,ds$$

let $\ln x = s$

$\frac{dx}{x} = ds$

$\int ds$

$$= x\sin(\ln x) - \cot s$$

$$= x\sin(\ln x)\left[-\cot(\ln x)\right]$$

$$y_2(x) = -x\cos\ln x$$

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