Linear => ① Power of dependent variable & all its derivative is one

② The coefficient of dependent variable & it derivative depends only on independent ".

③ ~~g(x) only depends upon independent variable.~~

Chap #1

Q. Given ques are linear or non-linear. and find order

1. $(1-x)y'' - 4xy' + 5y = \cos x$

Given equ is linear equ ~~having~~ & order of the equ is 2.

2. $x\dfrac{d^3y}{dx^3} - 2\left(\dfrac{dy}{dx}\right)^4 + y = 0$

Not linear equ. Order = 3

3. $yy' + 2y = 1 + x^2$

Non-linear , Order = 1st

4. $x^2dy + (y - xy - xe^x)\,dx = 0$

5. $x^3y^{(4)} - x^2y'' + 4xy' - 3y = 0$

linear , Order = fourth

6. $\frac{d^2y}{dx^2} + 9y = \sin y$

Non-linear , Order = 2

7. $\frac{dy}{dx} = \sqrt{1 + \left(\frac{d^2y}{dx^2}\right)^2}$

Non-linear , Order = 2

8. $\frac{d^2r}{dt^2} = \frac{-k}{r^2}$

Non-linear , Order = 2

9. $(\sin x)y''' - (\cos x)y' = 2$

linear , Order = 3

10. $(1 - y^2)\,dx + x\,dy = 0$

Non-linear , Order = 1

11. $2y' + y = 0 \rightarrow (1)$ $\quad y = e^{-x/2} \rightarrow (2)$

$$y = e^{-x/2}$$

$$y' = e^{-x/2} \frac{d}{dx}\left(-\frac{x}{2}\right)$$

$$y' = e^{-x/2} \frac{1}{2}(-1)$$

$$y' = -\frac{1}{2} e^{-x/2}$$

So, equ (1) $\Rightarrow$

$$\cancel{2}\left(-\frac{1}{\cancel{2}} e^{-x/2}\right) + e^{-x/2} = 0$$

$$\Rightarrow -e^{-x/2} + e^{-x/2} = 0$$

$$\Rightarrow 0 = 0$$

So,

equ (2) is a sol of equ (1)

12. $y' + 4y = 32 \rightarrow (1)$ $\quad y = 8 \rightarrow (2)$

$$y' = \frac{d}{dx} 8$$

$$y' = 0$$

equ ① $\Rightarrow$ $0 + 4(8) = 32$

$32 = 32$

So,

equ ② is a sol of equ ①

13. $\frac{dy}{dx} - 2y = e^{3x} \rightarrow ①$ $\quad y = e^{3x} + 10e^{2x} \rightarrow ②$

$y' = \frac{d}{dx} e^{3x} + \frac{d}{dx} 10 e^{2x}$

$y' = 3e^{3x} + 10(2e^{2x})$

$y' = 3e^{3x} + 20e^{2x}$

equ ① $\Rightarrow$ $3e^{3x} + \cancel{20}e^{2x} - 2e^{3x} - \cancel{20}e^{2x} = e^{3x}$

$\Rightarrow e^{3x} = e^{3x}$

So,

equ ② is a sol of equ ①

14. $\frac{dy}{dt} + 20y = 24 \rightarrow (1)$ $\quad y = \frac{6}{5} - \frac{6}{5} e^{-20x} \rightarrow (2)$

$y' = 0 - \frac{6}{5}(-\overset{4}{\cancel{20}}) e^{-20x}$

$y' = 24 e^{-20x}$

equ (1) $\Rightarrow$

$24e^{-20x} + 20\left(\frac{6}{5} - \frac{6}{5} e^{-20x}\right) = 24$

$24e^{-20x} + \overset{4}{\cancel{20}} \cdot \frac{6}{\cancel{5}}\left(1 - e^{-20x}\right) = 24$

$24e^{-20x} + 24 - 24e^{-20x} = 24$

$24 = 24$

So, equ (2) is a sol of equ (1)

15. $y' = 25 + y^2 \rightarrow (1)$; $\quad y = 5 \tan 5x \rightarrow (2)$

$y' = 5 \frac{d}{dx} \tan 5x$

$y' = 5 \sec^2 5x \cdot 5$

$y' = 25 \sec^2 5x$

equ (1) ⇒

$$25\sec^2 5x = 25 + (5\tan 5x)^2$$

$$25\sec^2 5x = 25 + 25\tan^2 5x$$

$$25\sec^2 5x = 25(1+\tan^2 5x)$$

$$(\because 1+\tan^2 5x = \sec^2 5x)$$

$$25\sec^2 5x = 25\sec^2 5x$$

16. $\frac{dy}{dx} = \sqrt{\frac{y}{x}}$; $y = (\sqrt{x}+c_1)^2$, $x>0$, $c_1>0$ → (1) → (2)

equ (2) ⇒

$$y' = \frac{d}{dx}(\sqrt{x}+c_1)^2$$

$$= 2(\sqrt{x}+c_1)\frac{d}{dx}(\sqrt{x}+c_1)$$

$$= \cancel{2}(\sqrt{x}+c_1)\frac{1}{\cancel{2}}\frac{1}{\sqrt{x}}$$

$$= \frac{\sqrt{x}+c_1}{\sqrt{x}}$$

equ (1) ⇒

$$\frac{\sqrt{x}+c_1}{\sqrt{x}} = \sqrt{\frac{(\sqrt{x}+c_1)^2}{x}}$$

$$\frac{\sqrt{x}+C_1}{\sqrt{x}} = \frac{\sqrt{x}+C_1}{\sqrt{x}}$$

So, equ (2) is a sol of equ (1)

17. $y' + y = \sin x \rightarrow (1)$ $\quad y = \frac{1}{2}\sin x - \frac{1}{2}\cos x + 10e^{-x} \rightarrow (2)$

$$y' = \frac{1}{2}\cos x + \frac{1}{2}\sin x - 10e^{-x}$$

Put in equ (1)

$$\cancel{\frac{1}{2}\cos x} + \frac{1}{2}\sin x - \cancel{10e^{-x}} + \frac{1}{2}\sin x - \cancel{\frac{1}{2}\cos x} + \cancel{10e^{-x}}$$

$$= \sin x$$

$$\Rightarrow \quad + \frac{1}{2}\sin x + \frac{1}{2}\sin x = \sin x$$

$$\frac{\cancel{2}}{\cancel{2}}\sin x = \sin x$$

$$\sin x = \sin x$$

So,
equ (2) is a sol of equ (1)

18. $2xy\,dx + (x^2 + 2y)\,dy = 0 \rightarrow (1)$

$x^2y + y^2 = C_1 \rightarrow (2)$

equ (2) $\Rightarrow \frac{d}{dx}(x^2y) + \frac{d}{dx}y^2 = C_1$

$\Rightarrow x^2\frac{dy}{dx} + y\frac{d}{dx}x^2 + \frac{d}{dx}y^2 = \frac{d}{dx}C_1$

$\Rightarrow x^2\frac{dy}{dx} + y\,2x + 2y\frac{dy}{dx} = 0$

$\Rightarrow \frac{dy}{dx}(x^2 + 2y) = -2xy$

$\frac{dy}{dx} = \frac{-2xy}{x^2 + 2y}$

equ (1) $\Rightarrow 2xy\frac{dx}{dx} + (x^2 + 2y)\frac{dy}{dx} = 0$

$2xy + \cancel{x^2 + 2y}\left(\frac{-2xy}{\cancel{x^2 + 2y}}\right) = 0$

$2xy - 2xy = 0$

$0 = 0$

So,

equ (2) is a sol of equ (1)

19. $x^2\,dy + 2xy\,dx = 0 \rightarrow (1)$ $\qquad y = -\frac{1}{x^2} \rightarrow (2)$

equ (2) $\Rightarrow$ $y' = \frac{d}{dx}\left(\frac{-1}{x^2}\right)$

$$y' = \frac{x^2(0) - (-1)(2x)}{(x^2)^2}$$

$$y' = \frac{+2x}{x^4}$$

$$y' = \frac{2}{x^3}$$

equ (1) $\Rightarrow$ $x^2 \frac{dy}{dx} + 2xy \frac{dx}{dx} = 0$

$$x^2\left(\frac{2}{x^3}\right) + 2xy = 0$$

$$\frac{2}{x} + 2\cancel{x}\left(\frac{-1}{x^{\cancel{2}}x}\right) = 0$$

$$\frac{2}{x} - \frac{2}{x} = 0$$

$$0 = 0$$

So, equ (2) is a sol of equ (1)

20. $(y')^3 + xy' = y \rightarrow (1)$, $y = x+1 \rightarrow (2)$

equ (2) $\Rightarrow$ $y' = \frac{d}{dx}(x+1)$

$y' = 1$

equ (1) $\Rightarrow$ $(1)^3 + x(1) = x+1$

$1 + x = x+1$

or

$1+x = 1+x$

So,

equ (2) is a sol of equ (1)

21. $y = 2xy' + y(y')^2 \rightarrow (1)$

$y^2 = c_1\left(x + \frac{1}{4}c_1\right) \rightarrow (2)$

equ (2) $\Rightarrow$ $\frac{d}{dx}y^2 = \frac{d}{dx} c_1\left(x + \frac{1}{4}c_1\right)$

$2y\frac{dy}{dx} = c_1(1+0)$

$\frac{dy}{dx} = \frac{c_1}{2y}$

$\frac{dy}{dx} = \frac{c_1}{2\sqrt{c_1\left(x+\frac{1}{4}c_1\right)}}$ (from equ (2))

equ ① $\Rightarrow$ $y = \cancel{2}x\left(\frac{c_1}{\cancel{2}y}\right) + y\left(\frac{c_1}{2y}\right)^2$

$$y = \frac{xc_1}{y} + y\,\frac{c_1^2}{2y^2}$$

$$y = c_1\left[\frac{x}{y} + \frac{c_1}{2}\right]$$

From equ ② $y = \sqrt{c_1\left(x + \frac{1}{2}c_1\right)}$

$$\sqrt{c_1\left(x + \tfrac{1}{2}c_1\right)} = c_1\left[\frac{x}{\sqrt{c_1\left(x+\frac{1}{2}c_1\right)}} + \frac{c_1}{2}\right]$$

$$= c_1\left[\frac{2x + c_1\left(\sqrt{c_1\left(x+\frac{1}{2}c_1\right)}\right)}{2\sqrt{c_1\left(x+\frac{1}{2}c_1\right)}}\right]$$

equ ① $\Rightarrow$

$$\sqrt{c_1\left(x+\frac{1}{4}c_1\right)} = \cancel{2}x\left[\frac{c_1}{\cancel{2}\sqrt{c_1\left(x+\frac{1}{4}c_1\right)}}\right. +$$

$$\cancel{\sqrt{c_1}}\sqrt{x+\cancel{\tfrac{1}{4}c_1}} \cdot \left[\frac{c_1}{2\cancel{\sqrt{c_1}}\sqrt{\left(x+\frac{1}{4}c_1\right)}}\right]^2$$

$$\sqrt{c_1\left(x+\frac{1}{4}c_1\right)} = \frac{x\,c_1}{\sqrt{c_1\left(x+\frac{1}{4}c_1\right)}} + \frac{c_1^2}{4\cdot\sqrt{c_1}\sqrt{x+\frac{1}{4}c_1}}$$

$$\sqrt{c_1}\sqrt{x+\frac{1}{4}c_1} = \frac{4x\,c_1 + c_1^2}{4\sqrt{c_1}\sqrt{x+\frac{1}{4}c_1}}$$

$$= \frac{c_1(4x + c_1)}{4\sqrt{c_1}\sqrt{x+\frac{1}{4}c_1}}$$

$$= \frac{\sqrt{c_1}(4x+c_1)}{4\sqrt{x+\frac{1}{4}c_1}}$$

$$= \frac{\cancel{4}\sqrt{c_1}\left(x+\frac{1}{4}c_1\right)}{\cancel{4}\sqrt{x+\frac{1}{4}c_1}}$$

$$\sqrt{c_1}\sqrt{x+\frac{1}{4}c_1} = \sqrt{c_1}\sqrt{x+\frac{1}{4}c_1}$$

So, equ (2) is a sol of equ (1)

22. $y' = 2\sqrt{|y|} \rightarrow (1)$ $\quad y = x|x| \rightarrow (2)$

$$\because |x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$$

For $x \geq 0$

equ (2) $\Rightarrow$ $y = x(x)$

$$y = x^2$$

$$y' = 2x$$

equ (1) $\Rightarrow$

$$2x = 2\sqrt{|x^2|}$$

$$2x = 2x$$

So, equ (2) is a sol of equ (1)

For $x < 0$

equ (2) $\Rightarrow$ $y = x(-x)$

$$y = -x^2$$

$$y' = -2x$$

equ (1) $\Rightarrow$ $-2x = 2\sqrt{|-x^2|}$

$$-2x = 2\sqrt{x^2}$$

$$-2x = 2(x)$$ as x is negative So.

$$-2x = -2x$$

So,

equ (2) is a sol of equ (1)

23. $y' - \frac{1}{x} y = 1 \rightarrow ①$ $\quad y = x \ln x, \; x > 0 \rightarrow ②$

equ ② $\Rightarrow$

$$y' = x \frac{d}{dx} \ln x + \ln x \frac{d}{dx} x$$

$$= x \frac{1}{x} + \ln x$$

$$y' = 1 + \ln x$$

equ ① $\Rightarrow$ $1 + \ln x - \frac{1}{x}(x \ln x) = 1$

$$\Rightarrow 1 + \cancel{\ln x} - \cancel{\ln x} = 1$$

$$\Rightarrow 1 = 1$$

So, ~~solution~~ equ ② is a sol of equ ①

24. $\frac{dP}{dt} = P(a - bP) \rightarrow ①$ $\quad P = \frac{a c_1 e^{at}}{1 + b c_1 e^{at}} \rightarrow ②$

equ ② $\Rightarrow$ $P' = \frac{d}{dt}\left(\frac{a c_1 e^{at}}{1 + b c_1 e^{at}}\right)$

$$P' = \frac{1 + b c_1 e^{at} \frac{d}{dt} a c_1 e^{at} - a c_1 e^{at} \frac{d}{dt}(1 + b c_1 e^{at})}{(1 + b c_1 e^{at})^2}$$

$$P' = \frac{1+bc_1e^{at}\left(a^2c_1e^{at}\right) - ac_1e^{at}\left(0 + abc_1e^{at}\right)}{\left(1+bc_1e^{at}\right)^2}$$

$$= \frac{a^2c_1e^{at} + \cancel{a^2bc_1^2(e^{at})^2} - \cancel{a^2c_1^2e^{at}b}}{\left(1+bc_1e^{at}\right)^2}$$

$$P' = \frac{a^2c_1e^{at}}{\left(1+bc_1e^{at}\right)^2}$$

equ (1) $\Rightarrow$ $\dfrac{a^2c_1e^{at}}{\left(1+bc_1e^{at}\right)^2} = \dfrac{ac_1e^{at}}{1+bc_1e^{at}}\left[a - b\left(\dfrac{ac_1e^{at}}{1+bc_1e^{at}}\right)\right]$

$$\frac{a^2c_1e^{at}}{\left(1+bc_1e^{at}\right)^2} = \frac{ac_1e^{at}}{1+bc_1e^{at}}\left[\frac{a+\cancel{abc_1e^{at}} - \cancel{abc_1e^{at}}}{1+bc_1e^{at}}\right]$$

$$\frac{a^2c_1e^{at}}{\left(1+bc_1e^{at}\right)^2} = \frac{a^2c_1e^{at}}{\left(1+bc_1e^{at}\right)^2}$$

So,

equ (2) is a sol of equ (1)

25. $\frac{dx}{dt} = (2-x)(1-x) \rightarrow ①$; $\ln \frac{2-x}{1-x} = t \rightarrow ②$

equ ② $\Rightarrow$ $\frac{dt}{dx} = \frac{dt}{dx} \ln \frac{2-x}{1-x}$

$$\frac{dt}{dx} = \frac{\frac{1}{\frac{2-x}{1-x}} \, 1-x \, \frac{d}{dx}(2-x) - (2-x)\frac{d}{dt}(1-x)}{(1-x)^2}$$

$$\frac{dt}{dx} = \frac{1-x}{2-x} \left(\right.$$

$$t = \ln \frac{2-x}{1-x}$$

$$t = \ln(2-x) - \ln(1-x)$$

$$\frac{dt}{dx} = \frac{d}{dx} \ln(2-x) - \frac{d}{dx} \ln(1-x)$$

$$= \frac{1}{2-x}(0-1) - \frac{1}{1-x}(0-1)$$

$$= -\frac{1}{2-x} + \frac{1}{1-x}$$

$$= \frac{-1(1-X) + 1(2-X)}{(2-X)(1-X)}$$

$$= \frac{-1 + \cancel{X} + 2 - \cancel{X}}{(2-X)(1-X)}$$

$$\frac{dt}{dX} = \frac{1}{(2-X)(1-X)}$$

So

$$\frac{dX}{dt} = (2-X)(1-X)$$

equ ① $\Rightarrow$

$$(2-X)(1-X) = (2-X)(1-X)$$

So,

equ ② is a sol of equ ①

26. $y' + 2xy = 1$

$$y = e^{-x^2} \cdot \int_0^x e^{t^2}\,dt + c_1 e^{-x^2}$$

$$y' = e^{-x^2} \frac{d}{dx} \int_0^x e^{t^2}\,dt + \int_0^x e^{t^2}\,dt \frac{d}{dx} e^{-x^2} + \frac{d}{dx} c_1 e^{-x^2}$$

$$y' = e^{-x^2}\left[e^{t^2} x(1) - e^{t^2}(0)\right] + \int_0^x e^{t^2}\,dt\; e^{-x^2}(-2x) + c_1 - 2x e^{-x^2}$$

$$\frac{d}{dx}\int_{g_1(x)}^{g_2(x)} f(t)\,dt = f\,g_2(x)\,g_2'(x) - f\,g_1(x)\cdot g_1'(x)$$

$\because g_1(x) = 0 \Rightarrow g_2(x) = x$

$\Rightarrow f(t) = e^{t^2}$

$f(x) = e^{x^2}$

$$y' = e^{-x^2}\left[e^{x^2} - 0\right] + \int_0^x e^{t^2}\,dt\,(-2xe^{-x^2}) - 2xe^{-x^2}c_1$$

$$= e^{-x+x^2} + \int_0^x e^{t^2}\,dt\,(-2xe^{-x^2}) - 2xe^{-x^2}c_1$$

$$= 1 - 2x\left[e^{-x^2}\int_0^x e^{t^2}\,dt + e^{-x^2}c_1\right]$$

$\Rightarrow$ from equ (2)

$\Rightarrow 1 - 2xy$

So, equ (1) $\Rightarrow 1 - 2\cancel{xy} + 2\cancel{xy} = 1$

$1 = 1$

So, equ (2) is a sol. of equ (1)

27. $(x^2+y^2)\,dx + (x^2 - xy)\,dy = 0 \rightarrow (1)$;

$c_1(x+y)^2 = xe^{y/x} \rightarrow (2)$

equ (2) $\Rightarrow \frac{d}{dx}\, c_1(x+y)^2 = \frac{d}{dx}\left(xe^{y/x}\right)$

$$\Rightarrow 2c_1(x+y)\frac{d}{dx}(x+y) = x\frac{d}{dx}e^{y/x} + e^{y/x}\frac{dx}{dx}$$

$$\Rightarrow 2c_1(x+y)\left(1+\frac{dy}{dx}\right) = x e^{y/x}\left(\frac{d}{dx}\frac{y}{x} + e^{y/x}(1)\right)$$

$$2c_1(x+y)\left(1+\frac{dy}{dx}\right) = x e^{y/x}\left[\frac{x\frac{dy}{dx} - y(1)}{x^2}\right] + e^{y/x}$$

$$2c_1(x+y) + \frac{dy}{dx}2c_1(x+y) = x e^{y/x}\left(\frac{x\frac{dy}{dx} - y}{x^2}\right) + e^{y/x}$$

$$= x e^{y/x}\left[\frac{\cancel{x}\frac{dy}{dx}}{x^{\cancel{2}}x} - \frac{y}{x^2}\right] + e^{y/x}$$

$$= x \cdot \frac{1}{x} e^{y/x}\left[\frac{dy}{dx} - \frac{y}{x}\right] + e^{y/x}$$

$$2c_1(x+y) + \frac{dy}{dx}2c_1(x+y) = e^{y/x}\frac{dy}{dx} - e^{y/x}\frac{y}{x} + e^{y/x}$$

$$\frac{dy}{dx}2c_1(x+y) - e^{y/x}\frac{dy}{dx} = -2c_1(x+y) - e^{y/x}\frac{y}{x} + e^{y/x}$$

from equ ② $e^{y/x} = \frac{c_1(x+y)^2}{x}$

$$\frac{dy}{dx}2c_1(x+y) - \frac{c_1(x+y)^2}{x}\frac{dy}{dx} = -2c_1(x+y) - \frac{c_1(x+y)^2}{x}\frac{y}{x} + \frac{c_1(x+y)^2}{x}$$

$$\frac{dy}{dx} c_1(x+y)\left[2-\frac{(x+y)}{x}\right] = c_1(x+y)\left[-2-\frac{(x+y)y}{x^2}+\frac{(x+y)}{x}\right]$$

$$\frac{dy}{dx} c_1(x+y)\left[\frac{2x-x-y}{x}\right] = c_1(x+y)\left[\frac{-2x^2-(x+y)y+x(x+y)}{x^2}\right]$$

$$\frac{dy}{dx} c_1(x+y)\left[\frac{x-y}{x}\right] = c_1(x+y)\left[\frac{-2x^2-\cancel{xy}-y^2+x^2+\cancel{xy}}{x^2}\right]$$

$$\frac{dy}{dx} c_1(x+y)\left[\frac{x-y}{x}\right] = c_1(x+y)\left[\frac{-x^2-y^2}{x^2}\right]$$

$$\frac{dy}{dx} = \frac{c_1\cancel{(x+y)}\left[\frac{-x^2-y^2}{x^2}\right]}{c_1\cancel{(x+y)}\left[\frac{x-y}{x}\right]}$$

$$= \frac{-x^2-y^2}{x\cancel{x^2}} \times \frac{\cancel{x}}{x-y}$$

$$\frac{dy}{dx} = \frac{-x^2-y^2}{x(x-y)}$$

So, equ ① $\Rightarrow$ $(x^2+y^2)\frac{dx}{dx} + (x^2-xy)\frac{dy}{dx} = 0$

$$x^2 + y^2 + (x^2 - xy)\frac{dy}{dx} = 0$$

$$x^2 + y^2 + x^2 - xy\left(\frac{-x^2 - y^2}{x(x-y)}\right) = 0$$

$$x^2 + y^2 + \cancel{x^2 - xy} \times \frac{-x^2 - y^2}{\cancel{x^2 - xy}} = 0$$

$$\cancel{x^2} + \cancel{y^2} - \cancel{x^2} - \cancel{y^2} = 0$$

$$0 = 0$$

So,
equ (2) is a sol of equ (1)

28. $y'' + y' - 12y = 0 \rightarrow (1)$, $y = c_1 e^{3x} + c_2 e^{-4x}$ equ (2)

equ (2) $\Rightarrow y' = \frac{d}{dx} c_1 e^{3x} + \frac{d}{dx} c_2 e^{-4x}$

$$\Rightarrow y' = c_1 3e^{3x} + (-4)c_2 e^{-4x}$$

$$y' = 3c_1 e^{3x} - 4c_2 e^{-4x}$$

$$y'' = 3c_1 e^{3x}(3) - 4c_2 e^{-4x}(-4)$$

$$y'' = 9c_1 e^{3x} + 16c_2 e^{-4x}$$

equ ② ⇒

$$9c_1e^{3x} + 16c_2e^{-4x} + 3c_1e^{3x} - 4c_1e^{-4x} - 12(c_1e^{3x} + c_2e^{-4x}) = 0$$

$$12c_1e^{3x} + 12c_2e^{-4x} - 12c_1e^{3x} - 12c_2e^{-4x} = 0$$

$$0 = 0$$

So,
equ ② is a sol of equ ①

29. $y'' - 6y' + 13y = 0$ → ① ; $y = e^{3x}\cos 2x$, ②

equ ② ⇒ $y' = e^{3x}\frac{d}{dx}\cos 2x + \cos 2x \frac{d}{dx}e^{3x}$

$$y' = -e^{3x}\, 2\sin 2x + \cos 2x\,(3e^{3x})$$

$$y' = -2e^{3x}\sin 2x + 3e^{3x}\cos 2x$$

$$y'' = -2\left[e^{3x}\frac{d}{dx}\sin 2x + \sin 2x\frac{d}{dx}e^{3x}\right] + 3\left[e^{3x}\frac{d}{dx}\cos 2x + \cos 2x\frac{d}{dx}e^{3x}\right]$$

$$y'' = -2\left[2e^{3x}\cos 2x + \sin 2x\,(3e^{3x})\right] + 3\left[e^{3x} - 2\sin 2x + \cos 2x\,(3e^{3x})\right]$$

$y'' = -4e^{3x}\cos 2x - 6e^{3x}\sin 2x - 6e^{3x}\sin 2x + 9e^{3x}\cos 2x$

$y'' = 5e^{3x}\cos 2x - 12e^{3x}\sin 2x$

equ ① $\Rightarrow$ $5e^{3x}\cos 2x - 12e^{3x}\sin 2x - 6(-2e^{3x}\sin 2x + 3e^{3x}\cos 2x) + 13(e^{3x}\cos 2x) = 0$

$\Rightarrow$ $5e^{3x}\cos 2x - \cancel{12e^{3x}\sin 2x} + \cancel{12e^{3x}\sin 2x} - 18e^{3x}\cos 2x + 13e^{3x}\cos 2x = 0$

$\Rightarrow$ $-13e^{3x}\cos 2x + 13e^{3x}\cos 2x = 0$

$0 = 0$

So,
equ ② is a sol of equ ①

30. $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = 0 \rightarrow ①$; $y = e^{2x} + xe^{2x} \rightarrow ②$

equ ② $\Rightarrow$ $y' = e^{2x}(2) + (xe^{2x}2 + e^{2x}(1))$

$y' = 2e^{2x} + 2xe^{2x} + e^{2x}$

$y' = 3e^{2x} + 2xe^{2x}$

$y'' = 6e^{2x} + 2[x(2e^{2x}) + e^{2x}(1)]$

$y'' = 6e^{2x} + 4xe^{2x} + 2e^{2x}$

$= 8e^{2x} + 4xe^{2x}$

equ ① $\Rightarrow 8e^{2x} + 4xe^{2x} - 4(3e^{2x} + 2xe^{2x}) + 4(e^{2x} + xe^{2x}) = 0$

$\Rightarrow 8e^{2x} + 4xe^{2x} - 12e^{2x} - 8xe^{2x} + 4e^{2x} + 4xe^{2x} = 0$

$\Rightarrow -4e^{2x} + 4e^{2x} - 4xe^{2x} + 4xe^{2x} = 0$

$0 = 0$

So,

equ ② is a sol of equ ①

31. $y'' = y \rightarrow ①$; $y = \cosh x + \sinh x \rightarrow ②$

equ ② $\Rightarrow y' = +\sinh x + \cosh x$

$y'' = +\cosh x + \sinh x$

equ ① $\Rightarrow +\cosh x + \sinh x = \cosh x + \sinh x$

So, equ ② is a sol of equ ①

32. $y'' + 25y = 0 \rightarrow (1)$; $y = c_1 \cos 5x \rightarrow (2)$.

equ (2) $\Rightarrow$ $y' = -c_1 5 \sin 5x$

$y'' = -5c_1 5 \cos 5x$

$y'' = -25c_1 \cos 5x$

equ (1) $\Rightarrow$ $-25c_1 \cos 5x + 25(c_1 \cos 5x) = 0$

$\Rightarrow -25c_1 \cos 5x + 25c_1 \cos 5x = 0$

$\Rightarrow 0 = 0$

So, equ (2) is a sol of equ (1)

33. $y'' + (y')^2 = 0 \rightarrow (1)$ $y = \ln|x + c_1| + c_2 \rightarrow (2)$

$y = \ln|x + c_1| + c_2$

$y' = \frac{1}{x + c_1} \frac{d}{dx}(x + c_1) + 0$

$= \frac{1}{x + c_1}(1 + 0)$

$y' = \frac{1}{x + c_1}$