EXERCISE : 3.2

1. $P(0) = P_0 \rightarrow 1$

$P(5) = 2P_0$

$P(t) = 3P_0 \quad , \quad t = ?$

$P(t) = 4P_0 \quad , \quad t = ?$

$\because \dfrac{dP}{dt} = KP$

$$\frac{dP}{P} = K\,dt$$

$$\int \frac{dP}{P} = K \int dt$$

$$\ln(P) = Kt + \ln C$$

$$= \ln e^{Kt} + \ln C$$

$$\ln(P) = \ln e^{Kt} C$$

$$P(t) = e^{Kt} C \rightarrow 2$$

As $P(0) = P_0$, So, equ 2 $\Rightarrow$

$$P(0) = Ce^{K(0)}$$

$$\boxed{P_0 = C} \rightarrow a$$

So, equ 2 $\Rightarrow$ $P(t) = P_0 e^{Kt} \rightarrow 3$

As, $P(5) = 2P_0$, So

equ 2 $\Rightarrow$ $P(5) = Ce^{K(5)}$

$$2P_0 = Ce^{5K}$$

From eq a $\Rightarrow$ $C = P_0$ So,

$$2\cancel{P_0} = \cancel{P_0} e^{5K}$$

$$2 = e^{5k}$$

$$\ln 2 = \ln e^{5k}$$

$$\ln 2 = 5k$$

$$k = \frac{\ln 2}{5}$$

$$k = 0.139$$

So, equ (3) $\Rightarrow$ $P(t) = P_0 e^{0.139t}$ $\rightarrow$ (4)

if $P(t) = 3P_0$

equ (4) $\Rightarrow$ $3\cancel{P_0} = \cancel{P_0} e^{0.139t}$

$$3 = e^{0.139t}$$

$$\ln 3 = \ln e^{0.139t}$$

$$\ln 3 = 0.139t$$

$$t = 8 \text{ years}$$

if $P(t) = 4P_0$

equ (3) $\Rightarrow$ $4\cancel{P_0} = \cancel{P_0} e^{0.139t}$

$$4 = e^{0.139t}$$

$$\ln 4 = \ln e^{0.139t}$$

$$\ln 4 = 0.139t$$

$$t = 10 \text{ years}$$

(3) $P(0) = P_0 = 500$

$P(t) = P(30) = ?$ if

$P(10) = \frac{15}{100} \times 500 = 75$

So, $P(10) = 75 + 500$
$= 575$

$\because \frac{dP}{dt} = kP$

$$\int \frac{dP}{P} = K\int dt$$

$$\ln P = Kt + \ln C$$

$$\ln P = \ln e^{Kt} + \ln C$$

$$\ln P = \ln Ce^{Kt}$$

$$P = Ce^{Kt} \longrightarrow \text{equ } 1$$

As, $P(0) = P_0 = 500$ So,

$$P(0) = Ce^{K(0)}$$

$$500 = C$$

equ 1 $\Rightarrow$ $P = 500e^{Kt} \longrightarrow 2$

As $P(10) = 500e^{K(10)}$

$$575 = 500e^{10K}$$

$$\frac{23}{20} = e^{10K}$$

$$\ln\frac{23}{20} = \ln e^{10K}$$

$$\ln 1.15 = 10K$$

$$K = 0.0139$$

So, equ 2 $\Rightarrow$ $P(t) = 500e^{0.0139t}$

So, $P(30) = P(t)$

$$P(30) = P_0 e^{Kt}$$

$$= 500e^{(30)\times\frac{1}{10}\ln 1.15}$$

$$= 500e^{\ln(1.15)^3}$$

$$= 500(1.52)$$

$$= 760.43$$

$$P(30) = 760$$

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$A(0) = A_0 = 1$ gram

$A(3.3) = \frac{A_0}{2} = \frac{1}{2}$

$A(t) = 90\%$ decay then $t = ?$

$A(t) = \frac{10}{100} \times 1 = 0.1$ gram remaining

$\frac{dA}{dt} = KA \Rightarrow \int \frac{dA}{A} = K \int dt$

$A(t) = Ce^{Kt} \rightarrow (1)$

As, $A(0) = 1$ So, equ (1) $\Rightarrow$

$A(0) = Ce^{K(0)}$

$1 = C$ So, equ (1) $\Rightarrow A(t) = e^{Kt} \rightarrow (2)$

As, $A(3.3) = \frac{1}{2}$ So, equ (1) $\Rightarrow$

$A(3.3) = Ce^{Kt}$

$\frac{1}{2} = Ce^{K(3.3)}$ $\boxed{\because C = 1}$

$\ln \frac{1}{2} = (1) \ln e^{K(3.3)}$

$\ln \frac{1}{2} = K(3.3)$

$K = -0.21$

equ (2) $\Rightarrow A(t) = e^{-0.21t} \rightarrow (3)$

If $A(t) = 0.1$ then equ (3) $\Rightarrow$

$0.1 = e^{-0.21t}$

$\ln 0.1 = -0.21t$

$t = 11$ hours

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(7) $m(0) = m_0 = 100$ milligram

$m(6) = 3\%$ of $100 = \frac{3}{100} \times 100 = 3$

~~For half life of radioactive~~, $m = 50$

So,

$\frac{dm}{dt} = km$

$\int \frac{dm}{m} = k \int dt$

$\ln m = kt + \ln C$

$\ln m = \ln e^{kt} + \ln C$

$\ln m = \ln e^{kt} C$

$m = Ce^{kt}$

$50 = Ce^{k}$

(7) $m(0) = m_0 = 100$ milligram

$m(6) = 3\%$ of $100 = \frac{3}{100} \times 100 = 3$

So,

$m = \text{Total} - 3$ (Percent)

$m(6) = 100 - 3 = 97$

So,

by using formula $m = m_0 e^{kt} \rightarrow (1)$

$97 = 100\, e^{k(6)}$

$0.97 = e^{6k}$

$\ln 0.97 = 6k$

$k = -0.0051$

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For halflife of radioactive, $m = 50$

So,

equ ① $\Rightarrow$ $50 = 100 e^{-0.0051 t}$

$0.5 = e^{-0.0051 t}$

$\ln 0.5 = -0.0051 t$

$\dfrac{-0.693}{-0.0051} = t$

$135.91 = t$

⑨

Not included

$I(0) = I_0$

$I(3) = 25 I_0$

$I(15) = ?$

$\dfrac{dI}{dt} = kI$

$\int \dfrac{dI}{I} = k \int dt$

$\ln I = kt + C$

$I(t) = Ce^{kt} \rightarrow$ ①

$I(0) = I_0$

$I_0 = Ce^{k(0)}$

$C = I_0$

① $\Rightarrow$ $I(t) = I_0 e^{kt} \rightarrow$ ②

$I(3) = 25 I_0$

equ ② $\Rightarrow$ $25 \cancel{I_0} = \cancel{I_0} e^{kt}$

$I(15)=0.00098\, I_0$

$\ln(25) = kt$

$\ln 25 = k(3)$

$k = 1.07$

(2) $\Rightarrow$ $I(t) = I_0 e^{1.07t} \rightarrow$ (3)

Now for $I(15)$

$\Rightarrow$ $I(15) = I_0 e^{1.07(15)}$

OR

$= I_0 e^{\frac{1}{3}\ln(25) \cdot 15}$

$= I_0 e$

$=$

(11)

$A(0) = A_0 = \frac{1}{1000}$ $\Rightarrow$ $\frac{A_0}{2} = A(5600)$

$A(t) = Ce^{kt} \rightarrow$ (1)

For $A(0)$ equ (1) $\Rightarrow$

$A(0) = Ce^{k(0)}$

$\frac{1}{1000} = C$

equ (1) $\Rightarrow$ $A(t) = \frac{1}{1000} e^{kt}$

By ~~using~~ $\cancel{A = A_0 e^{kt}}$ we have

$\cancel{= \frac{1}{1000} e^{k(\ )}}$

$A(t) =$

$\frac{1}{2}$

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$T_m = 10^\circ C$

$T(0) = 70^\circ C$

$T\left(\frac{1}{2}\right) = 50^\circ C$

$T(1) = ?$

$T(t) = 15^\circ C \quad , \quad t = ?$

$$\frac{dT}{dt} \propto (T - T_m)$$

$$\frac{dT}{dt} = k(T - T_m)$$

$$\frac{dT}{dt} = K(T - 10)$$

$$\int \frac{dT}{T-10} = k\int dt$$

$$\ln(T-10) = kt + \ln C$$

$$\ln T-10 = \ln e^{kt} + \ln C$$

$$T - 10 = Ce^{kt}$$

$$T(t) = Ce^{kt} + 10 \longrightarrow (1)$$

For $T(0)$

$T(0) = 70^\circ C$ equ (1) $\Rightarrow$

$$T(0) = Ce^{k(0)} + 10$$

$$70 = C + 10$$

$$C = 60$$

equ (1) $\Rightarrow$ $T(t) = 10 + 60e^{kt} \longrightarrow (2)$

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For $T(\frac{1}{2}) = 50$ equ ② $\Rightarrow$

$$T(\tfrac{1}{2}) = 10 + 60e^{k(\frac{1}{2})}$$

$$50 = 10 + 60e^{k(\frac{1}{2})}$$

$$\frac{\cancel{40}\,2}{\cancel{60}\,3} = e^{\frac{1}{2}k}$$

$$\ln \frac{2}{3} = \frac{1}{2}k$$

$$k = -0.811$$

equ ① $\Rightarrow$

$$T(t) = 10 + 60e^{-0.811(t)} \rightarrow ③$$

For $T(1) = 10 + 60e^{-0.811(1)}$

$$= 36.66\ °C$$

$T(t) = 15$, $t = ?$ equ ③ $\Rightarrow$

$$15 = 10 + 60e^{-0.811t}$$

$$\frac{5}{60} = e^{-0.811t}$$

$$\ln \frac{5}{60} = -t\,0.811$$

$$t = 3.058 \text{ min}$$

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$E(t) = 30$, $L = 0.1$

$R = 50\,\Omega$, $i(0) = 0$

$i(t) = ?$ if $t \to \infty$

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$$L\frac{di}{dt} + Ri = E(t)$$

$$0.1\frac{di}{dt} + 50\,i = 30$$

$$\frac{di}{dt} + 500\,i = 300 \rightarrow 1$$

$$I.F \Rightarrow e^{\int 500\,dt} \Rightarrow e^{500t}$$

Multiply with equ 1

$$e^{500t}\frac{di}{dt} + 500\,i \cdot e^{500t} = 300\,e^{500t}$$

$$\frac{d}{dt}\left[i e^{500t}\right] = 300\,e^{500t}$$

Integrate

$$i e^{500t} = \frac{300}{\cancel{500}} \int e^{500t}$$

$$= \frac{3\cancel{00}}{5\cancel{00}} e^{500t} + C \rightarrow 2$$

As, $i(t) = 0$ equ 2 $\Rightarrow$

$$0 = \frac{3}{5} e^{500(0)} + C$$

$$C = -3/5$$

equ 2 $\Rightarrow$

$$i(t) = \frac{3}{5} e^{500t} + \frac{3}{5}$$

$\boxed{e^{\infty} = 0}$

if $t \rightarrow \infty$ then

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$$i(t) = \frac{3}{5} - \frac{3}{5}e^{\infty}$$

$$i(t) = \frac{3}{5}$$