CHAPTER # 1

"INTRODUCTION TO DIFFERENTIAL EQUATIONS"

"Exercise 1.1"

Q# 1-10:-

Q# 1:- $(1-x)y'' - 4xy' + 5y = \cos x$

- linear
and - 2nd order.

Q# 2:- $x\dfrac{d^3y}{dx^3} - 2\left(\dfrac{dy}{dx}\right)^4 + y = 0$

- Non-linear
and 3rd order

Q# 3:- $yy' + 2y = 1 + x^2$

(y is dependant variable so, non-linear) - Non-linear
and 1st Order.

Q# 4:- $x^2dy + (y - xy - xe^x)\,dx = 0$

- linear
and - 1st Order.

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Q.5:- $x^3 y^{(4)} - x^2 y'' + 4xy' - 3y = 0$

- linear

and

- 4rth order

Q.6:- $\frac{d^2y}{dx^2} + 9y = \sin y$

- linear

and

- 2nd order

Q.7:- $\frac{dy}{dx} = \sqrt{1 + \left(\frac{d^2y}{dx^2}\right)^2}$

- Non-linear

and

- 2nd Order

Q.8:- $\frac{d^2r}{dt^2} = -k/r^2$

- Non-linear

and

- 2nd order

Q.9:- $(\sin x) y''' - (\cos x) y' = 2$

- linear

and

- 3rd order

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Q.12:- $y' + 4y = 32 \rightarrow (1)$

$y = 8 \rightarrow (2)$

Differentiate eq (2) w.r.t "x", we get,

$y' = 0 \rightarrow (3)$

Put eq (2), eq (3) in eq (1), we get,

$0 + 4(8) = 32$

$\Rightarrow \boxed{32 = 32}$ verified.

Q.13:- $dy/dx - 2y = e^{3x} \rightarrow (1)$

$y = e^{3x} + 10e^{2x} \rightarrow (2)$

Differentiate eq (2) w.r.t "x", we get,

$dy/dx = e^{3x}(3) + 10e^{2x}(2)$

$dy/dx = 3e^{3x} + 20e^{2x} \rightarrow (3)$

Put eq (2), eq (3) in eq (1), we get,

$3e^{3x} + 20e^{2x} - 2(e^{3x} + 10e^{2x}) = e^{3x}$

$\Rightarrow 3e^{3x} + \cancel{20e^{2x}} - 2e^{3x} - \cancel{20e^{2x}} = e^{3x}$

$\Rightarrow \boxed{e^{3x} = e^{3x}}$ Proved.

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Q114:- $\frac{dy}{dt} + 20y = 24$; —— ① ②

$$y = \frac{6}{5} - \frac{6}{5}e^{-20t} \quad \text{—— ①}$$

Differentiate eq ② w.r.t 't', we get

$$\frac{dy}{dt} = 0 - \frac{6}{\cancel{5}}e^{-20t}(-\cancel{20}^{4})$$

$$\frac{dy}{dt} = 24e^{-20t} \rightarrow ③$$

Put eq ② and eq ③ in eq ①, we get

$$24e^{-20t} + 20\left(\frac{6}{5} - \frac{6}{5}e^{-20t}\right) = 24$$

$$\Rightarrow 24e^{-20t} + \cancel{20}^{4}\left(\frac{6}{\cancel{5}}\right) - \cancel{20}^{4}\left(\frac{6}{\cancel{5}}\right)e^{-20t} = 24$$

$$\Rightarrow \cancel{24e^{-20t}} + 24 - \cancel{24e^{-20t}} = 24$$

$$\Rightarrow \boxed{24 = 24}$$

Proved

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Q115:- $y' = 25 + y^2$; → ① $y = 5\tan 5x$ → ②

So Differentiate eq ② w.r.t "x", we get

$$y' = 25\sec^2 5x \rightarrow ③$$

Put eq ②, ③ in eq ①, we get

$$25\sec^2 5x = 25 + (5\tan 5x)^2$$

$$\Rightarrow 25\sec^2 5x = 25 + 25\tan^2 5x$$

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Q.10:- $(1-y^2)\,dx + x\,dy = 0$

- linear
and
- 1st order

Q.11-40:- Verify that the Indicated functions are the solutions of the given differential equations:-

Q.11:- $2y' + y = 0 \quad \rightarrow (1)$

$y = e^{-x/2} \quad \rightarrow (2)$

Differentiate eq (2) w.r.t "x", we get

$y' = e^{-x/2}\left(-\frac{1}{2}\right)$

$\Rightarrow y' = -\frac{1}{2} e^{-x/2} \quad \rightarrow (3)$

Put eq (2) and eq (3) in eq (1), we g

$\cancel{2}\left(-\frac{1}{\cancel{2}} e^{-x/2}\right) + e^{-x/2} = 0$

$\Rightarrow -\cancel{e^{-x/2}} + \cancel{e^{-x/2}} = 0$

$\Rightarrow \boxed{0 = 0}$ Proved.

$\Rightarrow 25 \sec^2 5x = 25(1 + \tan^2 5x)$

$\Rightarrow \boxed{25 \sec^2 5x = 25 \sec^2 5x}$

Proved.

Q11b:- $\frac{dy}{dx} = \sqrt{\frac{y}{x}}$; $\rightarrow$ ①

$y = (\sqrt{x} + c_1)^2$, $x > 0$, $c_1 > 0$ $\rightarrow$ ②

From eq ②;

$\frac{dy}{dx} = \cancel{2}(\sqrt{x} + c_1)\frac{1}{\cancel{2}\sqrt{x}}$

$\Rightarrow \frac{dy}{dx} = \frac{(\sqrt{x} + c_1)}{\sqrt{x}}$ $\rightarrow$ ③

Put eq ② and eq ③ in eq ①, we get;

$\frac{\sqrt{x} + c_1}{\sqrt{x}} = \sqrt{\frac{(\sqrt{x} + c_1)^2}{x}}$

$\Rightarrow \frac{\sqrt{x} + c_1}{\sqrt{x}} = \frac{\sqrt{(\sqrt{x} + c_1)^2}}{\sqrt{x}}$

$\Rightarrow \boxed{\frac{\sqrt{x} + c_1}{\sqrt{x}} = \frac{\sqrt{x} + c_1}{\sqrt{x}}}$

Proved.

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Q117:- $y' + y = \sin x$; $\rightarrow$ ①

$y = \frac{1}{2}\sin x - \frac{1}{2}\cos x + 10e^{-x}$ $\rightarrow$ ②

Differentiate eq ② w.r.t "x", we get

$\frac{dy}{dx} = \frac{1}{2}\cos x - \frac{1}{2}(-\sin x) + 10e^{-x}(-1)$

$\Rightarrow y' = \frac{1}{2}\cos x + \frac{1}{2}\sin x - 10e^{-x}$ $\rightarrow$ ③

Put eq ② and eq ③ in eq ①, we get;

$\left(\cancel{\frac{1}{2}\cos x} + \frac{1}{2}\sin x - \cancel{10e^{-x}}\right) + \frac{1}{2}\sin x - \cancel{\frac{1}{2}\cos x} + \cancel{10e^{-x}} = \sin x$

$\Rightarrow \frac{1}{2}\sin x + \frac{1}{2}\sin x = \sin x$

$\Rightarrow \boxed{\sin x = \sin x}$

Proved.

Q118:- $2xy\,dx + (x^2+2y)\,dy = 0$; $\rightarrow$ ①

$x^2y + y^2 = c_1$ $\rightarrow$ ②

From eq ①,

$2xy\,dx + (x^2+2y)\,dy = 0$

$\Rightarrow 2xy\,dx = -(x^2+2y)\,dy$

$\Rightarrow \frac{dy}{dx} = \frac{-2xy}{x^2+2y}$ $\rightarrow$ ③

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Differentiate eq ①, w.r.t "x" we get

Q.120:-

$$\frac{d}{dx}(x^2y) + \frac{d}{dx}y^2 = \frac{dy}{dx}c_1$$

$$\Rightarrow x^2\frac{dy}{dx} + y\,2x + 2y\frac{dy}{dx} = 0$$

$$\Rightarrow \frac{dy}{dx}(x^2+2y) = -2xy$$

$$\Rightarrow \frac{dy}{dx} = \frac{-2xy}{x^2+2y} \rightarrow ④$$

Comparing eq ③ & eq ④;

$$\boxed{\frac{-2xy}{x^2+2y} = \frac{-2xy}{x^2+2y}}$$

Proved.

Q.119:- $x^2dy + 2xy\,dx = 0$; $\rightarrow$ ① $\quad y = -1/x^2 \rightarrow$ ②

From eq ②;

$$y = -1/x^2$$

$$\Rightarrow (yx^2) = -1$$

Taking Differential on both sides, we get

$$d(yx^2) = d(-1)$$

$$\Rightarrow y(2x\,dx) + x^2dy = 0$$

$$\Rightarrow \boxed{x^2dy + 2xy\,dx = 0}$$ Proved

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Q1201- $(y')^3 + xy' = y \; ; \rightarrow (1)$

$y = x + 1 \rightarrow (2)$

Differentiate eq (2) we get;

$y' = 1 + 0$

$\Rightarrow y' = 1 \rightarrow (3)$

Put eq (2) & eq (3) in eq (1) we get

$(1)^3 + x(1) = x + 1$

$\Rightarrow \boxed{x + 1 = x + 1}$ Proved.

Q1211- $y = 2xy' + y(y')^2 \rightarrow (1)$; $y^2 = c_1(x + \frac{1}{4}c_1) \rightarrow (2)$

From eq (2);

$y^2 = c_1(x + \frac{1}{4}c_1)$

$\Rightarrow y^2 = c_1 x + \frac{c_1^2}{4}$

Differentiate w.r.t x on both sides, we get;

$2y \frac{dy}{dx} = c_1 + 0$

$\Rightarrow \frac{dy}{dx} = \frac{c_1}{2y}$

$\Rightarrow y' = \frac{c_1}{2y} \rightarrow (3)$

Put eq (3) in eq (1) we get

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$$y = \cancel{2}x \frac{c_1}{\cancel{2}y} + \cancel{y} \frac{c_1^2}{4y^{\cancel{2}}}$$

$$= \frac{c_1 x}{y} + \frac{c_1^2}{4y}$$

$$\Rightarrow y = \frac{1}{y}\left(c_1 x + \frac{c_1^2}{4}\right)$$

As, $c_1 x + \frac{c_1^2}{4} = y^2$

So, $y = \frac{1}{y}(y^2)$

$$\Rightarrow \boxed{y = y}$$

Proved.

Qn22:-

$$y' = 2\sqrt{|y|} \rightarrow (1)$$

$$y = x|x| \rightarrow (2)$$

$$|x| = \begin{cases} -x & \text{if } x \leq 0 \\ x & \text{if } x \geq 0 \end{cases}$$

From eq (2), we have two cases;

Case I

If $x < 0$;

Therefore;

$y = x(-x) \rightarrow (a)$

$\Rightarrow y = -x^2 \rightarrow (b)$

From eq (b),

$y' = -2x \rightarrow (c)$

Case II

If $x > 0$;

Therefore;

$y = x(x) \rightarrow (d)$

$\Rightarrow y = x^2 \rightarrow (e)$

So,

$y' = 2x \rightarrow (f)$

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From eq ⓑ;

$$|y| = |-x^2|$$
$$= x^2$$

So $\sqrt{|y|} = \sqrt{x^2}$

$\Rightarrow \sqrt{|y|} = -x$ (for $x<0$)

or $x = -\sqrt{|y|}$

Put in eq ⓒ, we get,

$$y' = -2\left(-\sqrt{|y|}\right)$$

$$\Rightarrow \boxed{y' = 2\sqrt{|y|}}$$

From eq ⓑ

$$|y| = x^2$$

$\Rightarrow \sqrt{|y|} = \sqrt{x^2}$

$\Rightarrow \sqrt{|y|} = +x$ (for $x>0$)

or $x = \sqrt{|y|}$

Put in eq ⓒ, we get;

$$\boxed{y' = 2\sqrt{|y|}}$$

Proved.

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Q1231— $y' - \frac{1}{x}y = 1 \rightarrow ①$

$y = x.\ln \rightarrow ②$ ($x>0$)

From eq ②;

$$y' = x \cdot \frac{1}{x} + \ln \cdot 1$$

$$y' = 1 + \ln \rightarrow ③$$

Put eq ③ in eq ①, we get.

$$(1 + \ln) - \frac{1}{x}(x \ln) = 1$$

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$1 + \cancel{\ln x} - \cancel{\ln x} = 1$

$\Rightarrow \boxed{1 = 1}$ verified

Q.24:- $dP/dt = P(a - bP) \rightarrow (1)$, $P = \dfrac{ac_1e^{at}}{1+bc_1e^{at}} \rightarrow (2)$

From eq (2):

$$dP/dt = \frac{(1+bc_1e^{at})\, d/dt\,(ac_1e^{at}) - ac_1e^{at}\, d/dt\,(1+bc_1e^{at})}{(1+bc_1e^{at})^2}$$

$$= \frac{(1+bc_1e^{at})\, a^2c_1e^{at} - ac_1e^{at}(bc_1e^{at}a)}{(1+bc_1e^{at})^2}$$

$$= \frac{\cancel{(1+bc_1e^{at})}\, a^2c_1e^{at}}{(1+bc_1e^{at})^{\cancel{2}}} - \frac{a^2c_1^2e^{at}be^{at}}{(1+bc_1e^{at})^2}$$

$$dP/dt = \frac{ac_1e^{at}}{1+bc_1e^{at}}\left(a - \frac{ac_1e^{at}b}{1+bc_1e^{at}}\right)$$

As; $P = \dfrac{ac_1e^{at}}{1+bc_1e^{at}}$ (From eq (2))

So;

$$dP/dt = P(a - Pb)$$

$$\boxed{dP/dt = P(a - bP)}$$

Proved

Q:25:- $dx/dt = (2-x)(1-x)$ — (1)

$$\ln\left(\frac{2-x}{1-x}\right) = t \quad \text{— (2)}$$

From eq (2);

$$\ln\left(\frac{2-x}{1-x}\right) = t$$

Take differential on both sides we get;

$$dx\left(\ln\left(\frac{2-x}{1-x}\right)\right) = dt$$

$$\frac{1}{\left(\frac{2-x}{1-x}\right)} dx\left(\frac{2-x}{1-x}\right) = dt$$

$$\left(\frac{1-x}{2-x}\right)\left[\frac{(1-x)(-1)-(2-x)(-1)}{(1-x)^2}\right]dx =$$

$$\Rightarrow \left[\frac{x-1+2-x}{(2-x)(1-x)}\right]dx = dt$$

$$\Rightarrow \frac{(1)\,dx}{(2-x)(1-x)} = dt$$

$$\Rightarrow \boxed{dx/dt = (2-x)(1-x)}$$

Verified

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Q.26:-

$$y' + 2xy = 1 \longrightarrow (1)$$

$$y = e^{-x^2}\int_0^x e^{t^2}\,dt + c_1 e^{-x^2} \longrightarrow (2)$$

From eq (2);

$$y' = \frac{d}{dx}\left[e^{-x^2}\int_0^x e^{t^2}\,dt\right] + \frac{d}{dx}\left(c_1 e^{-x^2}\right)$$

Product rule,

$$\Rightarrow y' = e^{-x^2}\frac{d}{dx}\int_0^x e^{t^2}\,dt + \int_0^x e^{t^2}\,dt \; \frac{d}{dx}e^{-x^2} - 2c_1 e^{-x^2}(x)$$

Now, as we know that;

$$\frac{d}{dx}\int_0^x e^{t^2}\,dt = e^{x^2}$$

So;

$$y' = \cancel{e^{-x^2}}\cdot\cancel{e^{x^2}} - 2xe^{-x^2}\int_0^x e^{t^2}\,dt - 2c_1 x e^{-x^2}$$

$$= 1 - 2x\left[e^{-x^2}\int_0^x e^{t^2}\,dt + c_1 e^{-x^2}\right]$$

$$\Rightarrow y' = 1 - 2xy$$

or $\boxed{y' + 2xy = 1}$

Proved.

Q.27:-

Q127:- $(x^2+y^2)\,dx + (x^2-xy)\,dy = 0$

$$c_1(x+y)^2 = xe^{y/x} \quad \text{--- (2)}$$

From eq (1):

$$(x^2-xy)\,dy = -(x^2+y^2)\,dx$$

or $$\frac{dy}{dx} = \frac{-(x^2+y^2)}{x^2-xy}$$

$$\Rightarrow \frac{dy}{dx} = \frac{x^2+y^2}{xy-x^2} \quad \text{--- (3)}$$

From eq (2):

$$c_1(x+y)^2 = xe^{y/x}$$

Differentiate w.r.t "x" we get,

$$c_1\, 2(x+y)(1+y') = xe^{y/x}\left(\frac{xy'-y}{x^2}\right) + e^{y/x}$$

$$\Rightarrow 2c_1(x+y)(1+y') = e^{y/x}\left(y'-\frac{y}{x}+1\right) \quad \text{--- (4)}$$

Now; From eq (2):

$$c_1 = \frac{xe^{y/x}}{(x+y)^2}$$

Put in eq (4), we get

$$2\left(\frac{xe^{y/x}}{(x+y)^2}\right)(x+y)(1+y') = e^{y/x}\left(\ldots\right)$$

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Q28

$$\frac{2x}{x+y}(1+y') = (1+y') - y/x$$

$$(1+y')\left(\frac{2x}{x+y} - 1\right) = -y/x$$

$$(1+y')\left(\frac{2x-x-y}{x+y}\right) = -y/x$$

$$\Rightarrow (1+y')\left(\frac{x-y}{x+y}\right) = -y/x$$

$$\Rightarrow \frac{x-y}{x+y} + \frac{(x-y)y'}{x+y} = -y/x$$

$$\Rightarrow \left(\frac{x-y}{x+y}\right)y' = -y/x - \frac{x-y}{x+y}$$

$$\Rightarrow \left(\frac{x-y}{\cancel{x+y}}\right)y' = \frac{-y(x+y)-(x-y)x}{x(\cancel{x+y})}$$

$$\Rightarrow y'(x-y) = \frac{\cancel{-xy} - y^2 - x^2 + \cancel{xy}}{x}$$

$$\Rightarrow y' = \frac{-(x^2+y^2)}{x(x-y)}$$

$$\Rightarrow y' = \frac{-(x^2+y^2)}{x^2-xy}$$

$$\Rightarrow \boxed{y' = \frac{x^2+y^2}{xy-x^2}} \longrightarrow (5)$$

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$y'' + y' - 12y = 0 \rightarrow (1)$ $y = c_1 e^{3x} + c_2 e^{-4x} \rightarrow (2)$

From eq (2);

$$\frac{dy}{dx} = c_1 e^{3x}(3) + c_2 e^{-4x}(-4)$$

$$y' = 3c_1 e^{3x} - 4c_2 e^{-4x}$$

Again, differentiate w.r.t "x";

$$y'' = 9c_1 e^{3x} + 16c_2 e^{-4x} \rightarrow (4)$$

Put eq (3) and eq (4) & eq (2) in eq (1), we get;

$$9c_1 e^{3x} + 16c_2 e^{-4x} + 3c_1 e^{3x} - 4c_2 e^{-4x} - 12\left(c_1 e^{3x} + c_2 e^{-4x}\right) = 0$$

$$\Rightarrow \cancel{12c_1 e^{3x}} + \cancel{12c_2 e^{-4x}} - \cancel{12c_1 e^{3x}} + \cancel{12c_2 e^{-4x}} = 0$$

$$\Rightarrow \boxed{0 = 0}$$

Proved

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Q129:- $y'' - 6y' + 13y = 0 \rightarrow (1)$;

$$y = e^{3x} \cos 2x \rightarrow (2)$$

From eq (2);

$$y' = e^{3x}(-2\sin 2x) + \cos 2x\, e^{3x}(3)$$

$$y' = -2e^{3x}\sin 2x + 3\cos 2x\, e^{3x} \rightarrow (3)$$

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Again, differentiate w.r.t x

$$y'' = -2\left[2e^{3x}\cos 2x + 3e^{3x}\sin 2x\right] + 3\left[-2e^{3x}\sin 2x + 3e^{3x}\cos 2x\right]$$

$$= -4e^{3x}\cos 2x - 6e^{3x}\sin 2x - 6e^{3x}\sin 2x + 9e^{3x}\cos 2x$$

$$y'' = 5e^{3x}\cos 2x - 12e^{3x}\sin 2x \rightarrow (4)$$

Put eq (2), (3), (4) in eq (1), we get,

$$\Rightarrow 5e^{3x}\cos 2x - 12e^{3x}\sin 2x - 6(-2e^{3x}\sin 2x + 3e^{3x}\cos 2x) + 13(e^{3x}\cos 2x) = 0$$

$$\Rightarrow 5e^{3x}\cos 2x - \cancel{12e^{3x}\sin 2x} + \cancel{12e^{3x}\sin 2x} - 18e^{3x}\cos 2x + 13e^{3x}\cos 2x = 0$$

$$\Rightarrow 18e^{3x}\cancel{\cos 2x} - \cancel{18e^{3x}\cos 2x} = 0$$

$$\Rightarrow \boxed{0 = 0}$$ Verified

Q130 - $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = 0 \rightarrow (1)$

$$y = e^{2x} + xe^{2x} \rightarrow (2)$$

From eq (2):

$$y = e^{2x} + xe^{2x}$$

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$$\frac{dy}{dx} = 2e^{2x} + 2xe^{2x} + e^{2x}$$

$$= 3e^{2x} + 2xe^{2x} \longrightarrow (3)$$

Again, differentiate w.r.t "x" on both sides; we get

$$\frac{d^2y}{dx^2} = 4e^{2x} + 2\left[2xe^{2x} + e^{2x}\right]$$

$$= 4e^{2x} + 4xe^{2x} + 2e^{2x}$$

$$\Rightarrow \frac{d^2y}{dx^2} = 8e^{2x} + 4xe^{2x} \longrightarrow (4)$$

Put eq (2), (3) & (4) in eq (1), we get;

$$8e^{2x} + 4xe^{2x} - 4\left(3e^{2x} + 2xe^{2x}\right) + 4\left(e^{2x} + xe^{2x}\right) = 0$$

$$\Rightarrow 8e^{2x} + 4xe^{2x} - 12e^{2x} - 8xe^{2x} + 4e^{2x} + 4xe^{2x} = 0$$

$$\Rightarrow \cancel{12e^{2x}} - \cancel{12e^{2x}} + \cancel{8xe^{2x}} - \cancel{8xe^{2x}} = 0$$

$$\Rightarrow \boxed{0 = 0}$$ verified

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Q.31 :-

$$y'' = y \longrightarrow (1)$$

$$y = \cosh x + \sinh x \longrightarrow (2)$$

From eq (2);

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Q138:- $y''' - y'' + 9y' - 9y = 0; \longrightarrow (1)$

$$y = c_1 \sin 3x + c_2 \cos 3x + 4e^x \longrightarrow (2)$$

From eq (2)

$$y' = 3c_1 \cos 3x - 3c_2 \sin 3x + 4e^x \longrightarrow (3)$$

$$y'' = -9c_1 \sin 3x - 9c_2 \cos 3x + 4e^x \longrightarrow (4)$$

$$y''' = -27c_1 \cos 3x + 27c_2 \sin 3x + 4e^x \longrightarrow (5)$$

Put eq (2), (3), (4) and (5) in eq (1)

$$-27c_1 \cos 3x + 27c_2 \sin 3x + 4e^x - (-9c_1 \sin 3x - 9c_2 \cos 3x + 4e^x) + 9(3c_1 \cos 3x - 3c_2 \sin 3x + 4e^x) - 9(c_1 \sin 3x + c_2 \cos 3x + 4e^x) = 0$$

$$\Rightarrow \cancel{-27c_1 \cos 3x} + \cancel{27c_2 \sin 3x} + \cancel{4e^x} + \cancel{9c_1 \sin 3x} + \cancel{9c_2 \cos 3x} - \cancel{4e^x} + \cancel{27c_1 \cos 3x} - \cancel{27c_2 \sin 3x} + \cancel{36e^x} - \cancel{9c_1 \sin 3x} - \cancel{9c_2 \cos 3x} - \cancel{36e^x} = 0$$

$$\Rightarrow \boxed{0 = 0}$$

verified

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Q1-39:- $y''' - 3y'' + 3y' - y = 0; \longrightarrow (1)$

$$y = x^2 e^x \longrightarrow (2)$$

From eq (2)

$$y' = x^2 e^x + e^x 2x \longrightarrow (3)$$

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$$\frac{dy}{dx} = \sinh x + \cosh x$$

$$\Rightarrow \frac{d^2y}{dx^2} = \cosh x + \sinh x \longrightarrow (3)$$

Comparing eq (2), & (3); we get

$$y = \frac{d^2y}{dx^2}$$

or $\boxed{y = y''}$ Proved.

Q132:- $y'' + 25y = 0$; $\longrightarrow (1)$

$$y = c_1 \cos 5x \longrightarrow (2)$$

From eq (2);

$$\frac{dy}{dx} = c_1 - \sin 5x\,(5)$$

$$\Rightarrow y' = -5c_1 \sin 5x$$

Again, differentiate w.r.t "x", we get;

$$y'' = -5c_1 \cos 5x\,(5)$$

$$\Rightarrow y'' = -25c_1 \cos 5x \longrightarrow (3)$$

Put eq (2) and eq (3) in eq (1), we get

$$-25c_1 \cos 5x + 25(c_1 \cos 5x) = 0.$$

$$\Rightarrow \cancel{-25c_1 \cos 5x} + \cancel{25c_1 \cos 5x} = 0$$

$$\Rightarrow \boxed{0 = 0}$$ Proved.

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$y'' = x^2e^x + e^x 2x + e^x(2) + 2xe^x$

$= x^2e^x + e^x 2x + 2e^x + 2xe^x$

$y'' = x^2e^x + 4xe^x + 2e^x \rightarrow (4)$

$y''' = x^2e^x + e^x 2x + 4(xe^x + e^x) + 2e^x$

$= x^2e^x + e^x 2x + 4xe^x + 4e^x + 2e^x$

$\Rightarrow y''' = x^2e^x + 6xe^x + 6e^x \rightarrow (5)$

Put eq (2), (3), (4) & (5) in eq (1), we get;

$\cancel{x^2e^x} + 6xe^x + \cancel{6e^x} - \cancel{3x^2e^x} - 12x^2e^x - \cancel{6e^x}$
$\cancel{+3x^2e^x} + 3e^x 2x - \cancel{x^2e^x} = 0$

$\Rightarrow 12\cancel{xe^x} - 12\cancel{xe^x} = 0$

$\boxed{0 = 0}$ verified

Q.4 (b) $x^3 \frac{d^3y}{dx^3} + 2x^2 \frac{d^2y}{dx^2} - x\frac{dy}{dx} + y = 12x^2; \rightarrow (1)$

$y = c_1 x + c_2 x \ln x + 4x^2 \rightarrow (2) \quad ; x > 0$

From eq (2),

$\frac{dy}{dx} = c_1 + c_2(1 + \ln x) + 8x \rightarrow (3)$

$\frac{d^2y}{dx^2} = 0 + c_2\left(0 + \frac{1}{x}\right) + 8$

$\frac{d^2y}{dx^2} = \frac{c_2}{x} + 8 \rightarrow (4)$

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"Exercise" 2.1 "

①

Q.1-10:- Determine a region of the x-y plane for which the given differential eq would have a unique solution through a point (x_0, y_0) in the region.

Prepared By;
Muhammad Mehran Yousaf
Reg.no:- FA13-BEE-081

Q.1:- $\frac{dy}{dx} = y^{2/3}$

So, $f(x,y) = y^{2/3}$

$\frac{\partial f}{\partial y} = \frac{2}{3} y^{-1/3}$

$= \frac{2}{3y^{1/3}}$

So, the given differential eq will have a unique solution in any rectangular region of the plane where $y \neq 0$

i.e; $y > 0$ or $y < 0$

Q.3:- $x \frac{dy}{dx} = y$

$\Rightarrow \frac{dy}{dx} = y/x$

So, $f(x,y) = y/x$

$\frac{\partial f}{\partial y} = 1/x$

So, the given diff eq will have unique solution in any region where $\boxed{x \neq 0}$

Q.5. $y' = \dfrac{x^2}{4-y^2}$

So, $f(x,y) = \dfrac{x^2}{4-y^2}$

So, $\dfrac{\partial f}{\partial y} = x^2 \dfrac{\partial}{\partial y}\left[\dfrac{1}{4-y^2}\right]$

$= x^2 \left[\dfrac{(4-y^2)(0)-(0}{(4-y^2)^2}\right.$

$= \dfrac{x^2(2y)}{(4-y^2)^2}$

$\Rightarrow \dfrac{\partial f}{\partial y} = \dfrac{2x^2y}{(4-y^2)^2}$

Let, $(4-y^2)^2 = 0$

So, $4-y^2 = 0$

$y = \pm 2$ $(y \neq 2, y \neq -2)$

So, the given differential eq. will have a unique solution in any region where $y < -2$, $-2 < y < 2$ or $y > 2$

Q.7. $(x^2+y^2)y' = y^2$

$\Rightarrow y' = \dfrac{y^2}{x^2+y^2}$

So, $f(x,y) = \dfrac{y^2}{x^2+y^2}$

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So, $\frac{\partial f}{\partial y} = \frac{(x^2+y^2)(2y) - y^2(2y)}{(x^2+y^2)^2}$

$= \frac{2x^2y + \cancel{2y^3} - \cancel{2y^3}}{(x^2+y^2)^2}$

$\Rightarrow \frac{\partial f}{\partial y} = \frac{2x^2y}{(x^2+y^2)^2}$

Let,

$x^2+y^2 = 0$

So, the differential eq will have a unique solution in any region where $x \neq 0$ & $y \neq 0$.

Q19:- $\frac{dy}{dx} = x^3 \cos y$

So, $f(x,y) = x^3 \cos y$

$\frac{\partial f}{\partial y} = -x^3 \sin y$

$(-\infty, +\infty)$

So, the differential eq will have unique solution in entire plane $(-\infty, +\infty)$.

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Q.11:- Determine by inspection at least two solutions of the given initial value problem.

Given that, $y' = 3y^{2/3}$, $y(0) = 0$

let, $y = 0 \rightarrow (1)$

So, $y' = 0$ So, $\boxed{0 = 0}$ proved,

let, $y = x$, Not possible

for $y = x^2$ Not possible

for $y = x^3 \rightarrow (2)$

$y' = 3x^2$ put in main eq, we get

So, $3x^2 = 3(x^3)^{2/3}$

$\Rightarrow 3x^2 = 3x^2$ proved

So, the two solutions of the given differential equation having an initial value problem are;

$\boxed{y = 0}$

and $\boxed{y = x^3}$

Ans

Q.13:- $y' = y^3$, $y(0) = 0$

let $y = 0$, $y' = 0$, $f(x,y) = y^3$

So, (yes) it is a unique solution by uniq. theorem (2.1). $f_y = 3y^2$

"Exercise" 2.2"

Q: 1–40:- Solve the given differential equation by separation of variables :-

Q: 1:- $\frac{dy}{dx} = \sin 5x$

$\Rightarrow dy = \sin 5x \, dx$

Integrating both sides, we get

$$\int dy = \int \sin 5x \, dx$$

$$\Rightarrow y = -\frac{\cos 5x}{5} + c$$

Ans.

Q: 3:- $dx + e^{3x} dy = 0$

$\Rightarrow dx = -e^{3x} dy$

$\Rightarrow -\frac{1}{e^{3x}} dx = dy$

$\Rightarrow -e^{-3x} dx = dy$

Integrating both sides, we get

$$-\int e^{-3x} dx = \int dy$$

$$\Rightarrow +\frac{e^{-3x}}{+3} = y + c$$

$$\Rightarrow y = \frac{1}{3} e^{-3x} + c$$

Ans.

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Q:5:- ~~$(x+1)\frac{dy}{dx} = x+6$~~

$$\Rightarrow (x+1)\,dy = x+6\,dx$$

$$\Rightarrow dy = \frac{x+6}{x+1}\,dx$$

Integrating both sides, we get;

$$\begin{array}{r|l} & 1 \\ x+1 & x+6 \\ & \underline{x\pm 1} \\ & 5 \end{array}$$

$$\int dy = \int \frac{x+6}{x+1}\,dx$$

$$\Rightarrow \int dy = \int 1\,dx + 5\int \frac{1}{x+1}\,dx$$

$$\Rightarrow y = x + 5\ln|x+1| + C$$

Ans.

Q:7:- $xy' = 4y$

$$\Rightarrow x\,\frac{dy}{dx} = 4y$$

$$\Rightarrow \frac{1}{4y}\,dy = \frac{1}{x}\,dx$$

Integrating both sides, we get

$$\frac{1}{4}\int \frac{1}{y}\,dy = \int \frac{1}{x}\,dx$$

$$\Rightarrow \frac{1}{4}\ln|y| = \ln|x| + C$$

$$\Rightarrow \ln|y| = 4\ln|x| + \ln|C|$$

$$\Rightarrow \ln|y| = 4\ln|Cx|$$

Q:9:-

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$\Rightarrow \cancel{\ln|y| = \ln|cx^4|}$ ②

or $\boxed{y = cx^4}$ Ans.

Q.9:- $\frac{dy}{dx} = \frac{y^3}{x^2}$

$\Rightarrow \frac{1}{y^3}\, dy = \frac{1}{x^2}\, dx$

Integrating both sides, we get

$$\int y^{-3}\, dy = \int x^{-2}\, dx$$

$$\Rightarrow \frac{y^{-3+1}}{-3+1} = \frac{x^{-2+1}}{-2+1} + c$$

$$\Rightarrow \frac{-y^{-2}}{2} = \frac{x^{-1}}{-1} + c$$

$$\Rightarrow \frac{1}{2y^2} = \frac{1}{x} + c$$

$$\Rightarrow \boxed{y^{-2} = 2x^{-1} + c}$$

Ans.

Q.11:- $\frac{dx}{dy} = \frac{x^2y^2}{1+x}$

$\Rightarrow (1+x)\, dx = x^2y^2\, dy$

$\Rightarrow \frac{1+x}{x^2}\, dx = y^2\, dy$

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$$\int \frac{1+x}{x^2}\, dx = \int y^2\, dy$$

$$\Rightarrow \int x^{-2}\, dx + \int \frac{1}{x}\, dx = \int y^2\, dy$$

$$\Rightarrow \frac{-1}{x} + \ln|x| = \frac{y^3}{3} + c$$

Xlying both sides by "3x", we get

$$\Rightarrow \boxed{-3 + 3x \ln|x| = xy^3 + c}$$

Ans 2

$$\frac{dy}{dx} = e^{3x+2y}$$

$$\Rightarrow \frac{dy}{dx} = e^{3x} \cdot e^{2y}$$

$$\Rightarrow e^{-2y}\, dy = e^{3x}\, dx$$

Integrating both sides, we get

$$\int e^{-2y}\, dy = \int e^{3x}\, dx$$

$$\Rightarrow \frac{-e^{-2y}}{2} = \frac{e^{3x}}{3} + c$$

Xlying both sides by "6", we get

$$\boxed{-3e^{-2y} = 2e^{3x} + c}$$

Ans 2

$$\cancel{(4y + yx^2)\,dy - (2x + xy^2)\,dx = 0}$$

$$\Rightarrow \quad y\,(4+x^2)\,dy = x\,(2+y^2)\,dx$$

$$\Rightarrow \quad \frac{y}{2+y^2}\,dy = \frac{x}{4+x^2}\,dx$$

Integrating both sides, we get;

$$\frac{1}{2}\int \frac{2y}{2+y^2}\,dy = \frac{1}{2}\int \frac{2x}{4+x^2}$$

$$\Rightarrow \quad \frac{1}{2}\ln|2+y^2| = \frac{1}{2}\ln|4+x^2|$$

$$\Rightarrow \quad \frac{1}{2}\left(\ln|2+y^2| - \ln|4+x^2|\right) = c$$

$$\Rightarrow \quad \ln\left|\frac{2+y^2}{x^2+4}\right| = 2c$$

$$\Rightarrow \quad \ln\left|\frac{2+y^2}{x^2+4}\right| = c$$

Taking exponential on both sides, we get

$$\frac{2+y^2}{x^2+4} = e^c \qquad (\because e^c = c)$$

$$\Rightarrow \quad 2+y^2 = c\,(x^2+4)$$

or $\boxed{y^2+2 = c\,(x^2+4)}$

Ans.

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~~2y(x+1) dy = x dx~~

17:- $\Rightarrow 2y\,dy = \frac{x}{x+1}\,dx$

$\Rightarrow 2y\,dy = \left(1 - \frac{1}{x+1}\right)dx$

$$\begin{array}{r|l} & 1 \\ \hline 1 & x \\ & x+1 \\ \hline & -1 \end{array}$$

Integrating both sides, we get

$2\int y\,dy = \int 1\,dx - \int \frac{1}{x+1}\,dx$

$\Rightarrow \cancel{2}\,\frac{y^2}{\cancel{2}} = x - \ln|x+1| + c$

$\Rightarrow y^2 = x - \ln|x+1| + c$

Ans.

19:- $y \ln x \, \frac{dx}{dy} = \left(\frac{y+1}{x}\right)^2$

$\Rightarrow y \ln x \, \frac{dx}{dy} = \frac{(y+1)^2}{x^2}$

$\Rightarrow x^2 \ln x\, dx = \frac{(y+1)^2}{y}\,dy$

$\Rightarrow x^2 \ln x\, dx = \frac{y^2+1+2y}{y}\,dy$

$\Rightarrow x^2 \ln x\, dx = \left(y + \frac{1}{y} + 2\right)dy$

Integrating both sides, we get;

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$\Rightarrow S = e^{kr} \cdot e^{c}$ $\quad (\because e^{c} = c)$

$\Rightarrow \boxed{S = c e^{kr}}$ Ans.

$$\frac{dp}{dt} = p - p^2$$

$$\Rightarrow \frac{dp}{dt} = p(1-p)$$

$$\Rightarrow \frac{1}{p(1-p)}\, dp = dt$$

Integrating both sides, we get;

$$\int \frac{1}{p(1-p)}\, dp = \int 1\, dt \longrightarrow (1)$$

We take $\frac{1}{p(1-p)}$ and solve by partial fraction;

$$\frac{1}{p(1-p)} = \frac{A}{p} + \frac{B}{1-p}$$

$$\Rightarrow 1 = A(1-p) + Bp \longrightarrow (2)$$

For "A", put "$p = 0$" in eq (2), we get;

$$1 = A(1-0)$$

$$\Rightarrow \boxed{A = 1}$$

For "B", put "$p = 1$" in eq (2), we get;

$$1 = B(1)$$

$$\Rightarrow \boxed{B = 1}$$

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$$\cancel{\int x^2 \ln x \, dx = \int y \, dy + \int \frac{1}{y} \, dy} \quad (9)$$

$$+ 2\int 1 \, dy$$

$$\Rightarrow \ln x \int x^2 dx - \int \frac{d}{dx} \ln x \int x^2 dx \, dx = \frac{y^2}{2} + \ln y + 2y + c$$

$$\Rightarrow \ln x \cdot \frac{x^3}{3} - \int \frac{1}{x} \frac{x^3}{3} dx = \frac{y^2}{2} + \ln y + 2y + c$$

$$\Rightarrow \frac{x^3 \ln x}{3} - \frac{1}{3} \int x^2 dx = \frac{y^2}{2} + \ln y + 2y + c$$

$$\Rightarrow \frac{x^3}{3} \ln x - \frac{1}{9} x^3 = \frac{y^2}{2} + 2y + \ln|y| + c$$

Ans.

Q12:- $\frac{ds}{dr} = kS$

$$\Rightarrow \frac{1}{S} ds = k \, dr$$

Integrating both sides, we get;

$$\int \frac{1}{S} ds = k \int dr$$

$$\Rightarrow \ln|S| = kr + c$$

Taking exponential on both sides, we get;

$$\cancel{e^{\ln|S|}} = e^{kr+c}$$

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$P(1-P) \qquad 1/P \quad + \quad 1/1-P$

So,

Put in eq (1), we get;

$$\int \frac{1}{P}\,dp + \int \frac{1}{1-P}\,dp = \int 1\,dt$$

$$\Rightarrow \quad \ln|P| - \ln|1-P| = t + c$$

$$\Rightarrow \quad \ln\left|\frac{P}{1-P}\right| = t + c$$

Taking exponential on both sides, we get;

$$\cancel{e^{\ln}}\left|\frac{P}{1-P}\right| = e^{t+c}$$

$$\Rightarrow \quad P = (1-P)e^{t+c}$$

$$\Rightarrow \quad P = (1-P)\,e^{t}.e^{c}$$

$$\Rightarrow \quad P = (1-P)\,e^{t}.c$$

$$\Rightarrow \quad P = ce^{t} - cPe^{t}$$

or $\quad P + cPe^{t} = ce^{t}$

$$\Rightarrow \quad P(1 + ce^{t}) = ce^{t}$$

$$\Rightarrow \quad \boxed{P = \frac{ce^{t}}{1+ce^{t}}}$$

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~~$\sec^2 x\, dy + \operatorname{cosec} y\, dx = 0$~~

Q125:-

$$\Rightarrow \frac{1}{\cos^2 x}\, dy = -\frac{1}{\sin y}\, dx$$

$$\Rightarrow \sin y\, dy = -\cos^2 x\, dx$$

$$\Rightarrow \sin y\, dy + \cos^2 x\, dx = 0$$

Integrating both sides, we get,

$$\int \sin y\, dy + \int \frac{1+\cos 2x}{2}\, dx = 0$$

$$\Rightarrow -\cos y + \frac{1}{2}\int 1\, dx + \frac{1}{2}\int \cos 2x\, dx = 0$$

$\cos^2 x = \frac{1+\cos 2x}{2}$

$\sin^2 x = \frac{1-\cos 2x}{2}$

$$\Rightarrow -\cos y + \frac{1}{2}x + \frac{\sin 2x}{2(2)} + c = 0$$

Xlying both sides by "4", we get;

$$\boxed{-4\cos y + 2x + \sin 2x + c = 0}$$

Ans,

Q127:-

$$e^y \sin 2x\, dx + \cos x\,(e^{2y} - y)\, dy = 0$$

$$e^y \sin 2x\, dx = -\cos x\,(e^{2y} - y)\, dy$$

$$\Rightarrow \frac{e^{2y} - y}{e^y}\, dy = -\frac{\sin 2x}{\cos x}\, dx$$

$$\Rightarrow \frac{\sin 2x}{\cos x}\, dx + \frac{e^{2y} - y}{e^y}\, dy = 0$$

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~~Integrating both sides, we get~~ (6)

$$\int \frac{\sin 2x}{\cos x}\,dx + \int \frac{e^{2y} - y}{e^{y}}\,dy = 0$$

$$\int \frac{2\sin x \cancel{\cos x}}{\cancel{\cos x}}\,dx + \int \left(e^{y} - y/e^{y}\right) dy = 0$$

$$\Rightarrow 2\int \sin x\,dx + \int \left(e^{y} - y e^{-y}\right) dy = 0$$

$$\Rightarrow -2\cos x + \int e^{y}\,dy - \int y e^{-y}\,dy = 0$$

$$\Rightarrow -2\cos x + e^{y} - \left[y\int e^{-y}\,dy - \int \left(\tfrac{d}{dy} y \int e^{-y}\right) dy\right] = 0$$

$$\Rightarrow -2\cos x + e^{y} - \left[y e^{-y}(-1) + \int e^{-y}\,dy\right] = 0$$

$$\Rightarrow -2\cos x + e^{y} + y e^{-y} - \frac{e^{-y}}{-1} + c = 0$$

$$\Rightarrow \boxed{-2\cos x + e^{y} + y e^{-y} + e^{-y} = c}$$

Ans.

$$(e^{y}+1)^{2} e^{-y}\,dx + (e^{x}+1)^{3} e^{-x}\,dy = 0$$

$$\Rightarrow \frac{(e^{y}+1)^{2}}{e^{y}}\,dx + \frac{(e^{x}+1)^{3}}{e^{x}}\,dy = 0$$

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$$\cancel{\frac{(e^y+1)^2}{e^y}\,dx = -\frac{(e^x+1)^3}{e^x}\,dy}$$

$$\Rightarrow \frac{e^x}{(e^x+1)^3}\,dx = -\frac{e^y}{(e^y+1)^2}\,dy$$

$$\Rightarrow \frac{e^x}{(e^x+1)^3}\,dx + \frac{e^y}{(e^y+1)^2}\,dy = 0$$

Integrating both sides, we get;

$$\int \frac{e^x}{(e^x+1)^3}\,dx + \int \frac{e^y}{(e^y+1)^2}\,dy = 0$$

$$\Rightarrow \int (e^x+1)^{-3} e^x\,dx + \int (e^y+1)^{-2} e^y\,dy = 0$$

$$\Rightarrow \frac{(e^x+1)^{-3+1}}{-3+1} + \frac{(e^y+1)^{-2+1}}{-2+1} = c$$

$$\Rightarrow \frac{(e^x+1)^{-2}}{-2} - (e^y+1)^{-1} = c$$

$\Rightarrow$ xlying bs by "-2", we get

$$\boxed{(e^x+1)^{-2} + 2(e^y+1)^{-1} = c}$$

Ans.

Q31:- $(y - yx^2)\, dy/dx = (y+1)^2$ (7)

$$\Rightarrow \quad y(1-x^2)\, dy/dx = (y+1)^2$$

$$\Rightarrow \quad y(1-x^2)\, dy = (y+1)^2\, dx$$

Integrating both sides, we get,

$$\Rightarrow \quad \int \frac{y}{(y+1)^2}\, dy = \int \frac{1}{1-x^2}\, dx \longrightarrow (1)$$

We take "$y/(y+1)^2$" and "$1/1-x^2$" and solve by partial fraction.

Firstly,

$$\frac{y}{(y+1)^2} = \frac{A}{y+1} + \frac{B}{(y+1)^2}$$

$$\Rightarrow \quad y = A(y+1) + B \longrightarrow (2)$$

For "B" put "$y=-1$" in eq (2), we get

$$-1 = B$$

$$\Rightarrow \quad \boxed{B = -1}$$

For "A", comparing the coefficients of "y", we get;

$$\boxed{A = 1}$$

So,

$$* \quad \frac{y}{(y+1)^2} = \frac{1}{y+1} - \frac{1}{(y+1)^2}$$

Now,

$$\frac{1}{1-x^2} = \frac{1}{(1+x)(1-x)}$$

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Q. $\frac{dy}{dx} = \frac{xy + 3x - y - 3}{xy - 2x + 4y - 8}$ (8)

$$\Rightarrow \frac{dy}{dx} = \frac{x(y+3) - 1(y+3)}{x(y-2) + 4(y-2)}$$

$$\Rightarrow \frac{dy}{dx} = \frac{(y+3)(x-1)}{(y-2)(x+4)}$$

$$\Rightarrow \frac{y-2}{y+3}\, dy = \frac{x-1}{x+4}\, dx$$

Integrating both sides, we get :

$$\int \frac{y-2}{y+3}\, dy = \int \frac{x-1}{x+4}\, dx$$

$$\Rightarrow \int \frac{y+3-3-2}{y+3}\, dy = \int \frac{x+4-4-1}{x+4}\, dx$$

$$\Rightarrow \int \frac{(y+3)-5}{y+3}\, dy = \int \frac{(x+4)-5}{x+4}\, dx$$

$$\Rightarrow \int 1\, dy - \int \frac{5}{y+3}\, dy = \int 1\, dx - \int \frac{5}{x+4}\, dx$$

$$\Rightarrow y - 5\ln|y+3| = x - 5\ln|x+4| + c$$

or $y - x - 5\ln|y+3| + 5\ln|x+4| = c$

Ans

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$$\frac{1}{(1+x)(1-x)} = \frac{A}{1+x} + \frac{B}{1-x}$$

$$\Rightarrow 1 = A(1-x) + B(1+x) \longrightarrow (3)$$

For "A", put "$x=-1$" in eq (3), we get;

$$1 = A(1-(-1))$$

$$\Rightarrow 1 = A(2)$$

$$\text{or } \boxed{A = 1/2}$$

For "B", put "$x=1$" in eq (3), we get,

$$1 = B(1+1)$$

$$\Rightarrow \boxed{B = 1/2}$$

So, $\star \quad \dfrac{1}{1-x^2} = \dfrac{1}{2(1+x)} + \dfrac{1}{2(1-x)}$

Put all values in eq (1), we get,

$$\int\left(\frac{1}{y+1} + \frac{1}{(y+1)^2}\right) dy = \int\left(\frac{1}{2(1+x)} + \frac{1}{2(1-x)}\right) dx$$

$$\int \frac{1}{1+y}\, dy + \int \frac{1}{(y+1)^2}\, dy = \frac{1}{2}\int \frac{1}{1+x}\, dx + \frac{1}{2}\int \frac{1}{1-x}\, dx$$

$$\Rightarrow \ln|1+y| + \frac{(y+1)^{-2+1}}{-2+1} = \frac{1}{2}\ln|1+x| - \frac{1}{2}\ln|1-x| + c$$

$$\Rightarrow \boxed{(y+1)^{-1} + \ln|1+y| = \frac{1}{2}\ln\left|\frac{1+x}{1-x}\right| + c}$$

Prepared By;
Muhammad Mehran Khan Marwat
Reg.no:- FA13-BEE-031

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Q135:- $\frac{dy}{dx} = \sin x (\cos 2y - \cos^2 y)$

$$\Rightarrow \frac{1}{\cos 2y - \cos^2 y} dy = \sin x \, dx$$

$$\Rightarrow \frac{1}{\cancel{\cos^2 y} - \sin^2 y - \cancel{\cos^2 y}} dy = \sin x \, dx$$

$$\Rightarrow -\operatorname{cosec}^2 y \, dy = \sin x \, dx$$

Integrating both sides, we get,

$$-\int \operatorname{cosec}^2 y \, dy = \int \sin x \, dx$$

$$\Rightarrow -(-\cot y) = -\cos x + C$$

$$\Rightarrow \cot y = -\cos x + C$$

Ans

Q137:- $x\sqrt{1-y^2}\, dx = dy$.

$$\Rightarrow \frac{1}{\sqrt{1-y^2}} dy = x \, dx$$

Integrating both sides, we get;

$$\int \frac{1}{\sqrt{1-y^2}} dy = \int x \, dx$$

$$\Rightarrow \sin^{-1} y = \frac{x^2}{2} + C$$

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$\Rightarrow \boxed{y = \sin\left(\frac{x^2}{2} + c\right)}$ Ans. 2

9

Q139:- $(e^x + e^{-x})\, dy/dx = y^2$

$$\Rightarrow (e^x + e^{-x})\, dy = y^2\, dx$$

$$\Rightarrow \frac{1}{y^2}\, dy = \frac{1}{e^x + e^{-x}}\, dx$$

Integrating both sides, we get,

$$\int y^{-2}\, dy = \int \frac{1}{e^x + e^{-x}}\, dx$$

$$\Rightarrow -y^{-1} = \int \frac{1}{e^x + 1/e^x}\, dx$$

$$\Rightarrow \cancel{-y^{-1}} = \int \frac{1}{\frac{e^{2x}+1}{e^x}}$$

$$\Rightarrow -y^{-1} = \int \frac{e^x}{(e^x)^2 + (1)^2}$$

$x = \int \frac{1}{1+x^2}\, dx$

$$\Rightarrow \boxed{-y^{-1} = \tan^{-1}(e^x) + c}$$ Ans. 5

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Q.41 → 48 :- Solve the given differential equation subject to the indicated initial condition

Q.41:- $(e^{-y}+1)\sin x\,dx = (1+\cos x)\,dy$

$y(0) =$ (Given)

So; $\dfrac{\sin x}{1+\cos x}\,dx = \dfrac{1}{1+e^{-y}}\,dy$

Integrating both sides, we get,

$$\int \frac{\sin x}{1+\cos x}\,dx = \int \frac{1}{1+e^{-y}}\,dy$$

$$= \int \frac{1}{1+\frac{1}{e^{y}}}\,dy$$

$$= \int \frac{1}{\frac{e^{y}+1}{e^{y}}}\,dy$$

$$\Rightarrow -\int \frac{-\sin x}{1+\cos x}\,dx = \int \frac{e^{y}}{e^{y}+1}\,dy$$

$$\Rightarrow -\ln|1+\cos x| = \ln|1+e^{y}|$$

$$\Rightarrow \ln|1+\cos x| + \ln|1+e^{y}| =$$

$$\Rightarrow \ln|(1+\cos x)(1+e^{y})| =$$

Taking exponential on both sides we get;

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$\cancel{(1+\cos x)(1+e^y) = c \rightarrow (1) \; (\because e^0 = 1)}$ (10)

Put $(x, y) = (0, 0)$, we get;

$$(1+\cos 0)(1+e^0) = c$$

$$\Rightarrow (1+1)(1+1) = c$$

$$\Rightarrow \boxed{c = 4}$$ put in eq (1), we get

$$\boxed{(1+\cos x)(1+e^y) = 4}$$

~~Ans~~ 2

Q.431-

$$y\,dy = 4x(y^2+1)^{1/2}\,dx, \quad \left\{ y(0) = 1 \text{ (Given)} \right.$$

$$\frac{y}{(y^2+1)^{1/2}}\,dy = 4x\,dx$$

Integrating both sides, we get,

$$\int \frac{y}{(y^2+1)^{1/2}}\,dy = 4\int x\,dx$$

$$\Rightarrow \frac{1}{2}\int (y^2+1)^{-1/2}\,2y\,dy = \cancel{4}^{2}\frac{x^2}{\cancel{2}} + c$$

$$\Rightarrow \cancel{\frac{1}{2}}\,\frac{(y^2+1)^{-1/2+1}}{\cancel{-1/2+1}} = 2x^2 + c$$

$$\Rightarrow (y^2+1)^{1/2} = 2x^2 + c \rightarrow (1)$$

Put $(x, y) = (0, 1)$, we get;

$$((1)^2+1)^{1/2} = 2(0)^2 + c$$

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$(2)^{1/2} = 0 + c$

$\Rightarrow \boxed{c = \sqrt{2}}$ put in eq ①,

$(y^2+1)^{1/2} = 2x^2 + \sqrt{2}$

$\Rightarrow \boxed{\sqrt{y^2+1} = 2x^2 + \sqrt{2}}$ Ans.

Q.45:- $\frac{dx}{dy} = 4(x^2+1)$, $x(\pi/4) = 1$ (Given)

$\Rightarrow \frac{1}{4(x^2+1)} dx = dy$

Integrating both sides, we get

$\frac{1}{4}\int \frac{1}{x^2+1} dx = \int 1\, dy$

$\Rightarrow \frac{1}{4} \tan^{-1} x = y + c \longrightarrow ①$

Put $(x, y) = (1, \pi/4)$, we get,

$\frac{1}{4} \tan^{-1}(1) = \pi/4 + c$

$\Rightarrow \frac{1}{4}(\pi/4) = \pi/4 + c$

$\Rightarrow \pi/16 = \pi/4 + c$

or $c = \pi/16 - \pi/4$

$\Rightarrow \boxed{c = -3\pi/16}$

Put in eq ①, we get,

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$\cancel{\frac{1}{4}\tan^{-1}x = -3\pi/16 + y}$

$\Rightarrow \tan^{-1}x = -3\pi/4 + 4y$ (Multiplying b.s by "4", we get,)

$\Rightarrow \boxed{x = \tan\left(4y - 3\pi/4\right)}$ Ans

Q.47:- $x^2 y' = y - xy$, $y(-1) = -1$ (Given)

So, $x^2 \, dy/dx = y(1-x)$

$\Rightarrow \frac{1}{y}\, dy = \frac{1-x}{x^2}\, dx$

Integrating both sides, we get

$\int \frac{1}{y}\, dy = \int x^{-2}\, dx - \int \frac{1}{x}\, dx$

$\Rightarrow \cancel{\ln|y|} = -\frac{1}{x} - \ln|x| + c \rightarrow (1)$

Put $(x, y) = (-1, -1)$, we get,

$\ln|-1| = +1/(+1) - \ln|-1| + c$

$\Rightarrow 0 = 1 - 0 + c$

$\Rightarrow \boxed{c = -1}$ put in eq (1) we get

$\ln|y| = -\frac{1}{x} - \ln|x| - 1$

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$\cancel{\ln|y| + \ln|x| = -1/x - 1}$

$\Rightarrow \ln|yx| = -1/x - 1$

Taking exponential on b.s, we get,

$$yx = e^{-1/x - 1}$$

$$\Rightarrow \boxed{xy = e^{-(1+1/x)}}$$ Ans =

Q1- Find Solution of given differentia -l eq that passes through indicated points :-

1:- $dy/dx - y^2 = -9$

$$\Rightarrow dy/dx = y^2 - 9$$

$$\Rightarrow \int \frac{-1}{y^2-9}\, dy = \int 1\, dx \longrightarrow (1)$$

We take "y^2-9" & Solve by partial fraction,

$$1/(y^2-9) = 1/((y+3)(y-3))$$

$$\Rightarrow \frac{1}{(y+3)(y-3)} = \frac{A}{y+3} + B/(y-3)$$

$$\Rightarrow 1 = A(y-3) + B(y+3) \longrightarrow (2)$$

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~~For "A", put "y = -3" in eq (2), we get,~~

$$1 = A(-3-3)$$

$$\Rightarrow \boxed{A = -1/6}$$

For "B", put "y = 3" in eq (2), we get;

$$1 = B(3+3) \Rightarrow \boxed{B = 1/6}$$

So,

$$\frac{1}{y^2-9} = \frac{-1}{6(y+3)} + \frac{1}{6(y-3)}$$

Put in eq (1), we get,

$$\int \left(\frac{-1}{6(y+3)} + \frac{1}{6(y-3)} \right) dy = \int 1\, dx$$

$$\Rightarrow -\frac{1}{6} \int \frac{1}{y+3} dy + \frac{1}{6} \int \frac{1}{y-3} dy = x + C$$

$$\Rightarrow \boxed{-\frac{1}{6} \ln|y+3| + \frac{1}{6} \ln|y-3| = x + C} \rightarrow [*]$$

Now,

(a) At $(0,0)$

$$-\frac{1}{6} \ln|3| + \frac{1}{6} \ln|-3| = 0 + C$$

$$\Rightarrow -0.183 + 0.183 = C$$

$$\Rightarrow \boxed{C = 0}$$

So, Eq[*] $\Rightarrow$ $x = \frac{1}{6} \ln\left|\frac{y-3}{y+3}\right|$

$$\Rightarrow 6x = \ln\left|\frac{y-3}{y+3}\right|$$

Taking exponential on b.s, we get,

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$$e^{6x} = \frac{y-3}{y+3}$$

$$\Rightarrow \quad ye^{6x} + 3e^{6x} = y-3$$

$$\Rightarrow \quad 3e^{6x} + 3 = y - ye^{6x}$$

$$\Rightarrow \quad 3(e^{6x}+1) = y(1-e^{6x})$$

$$\Rightarrow \quad \boxed{y = 3\left(\frac{e^{6x}+1}{-e^{6x}+1}\right)}$$

(b) At $(0,3)$,

$$-\tfrac{1}{6}\ln|6| + \tfrac{1}{6}\ln|0| = 0 + c$$

$$\Rightarrow \quad c =$$

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(c) ~~From eq [*],~~

(13)

$$\frac{1}{6} \ln \left|\frac{y-3}{y+3}\right| = x + c$$

$$\Rightarrow \ln \left|\frac{y-3}{y+3}\right| = 6x + 6c$$

$$\Rightarrow \frac{y-3}{y+3} = ce^{6x}$$

$$\Rightarrow y - 3 = (y+3)ce^{6x}$$

$$\Rightarrow yce^{6x} + 3ce^{6x} = y - 3$$

$$\Rightarrow 3(ce^{6x}+1) = y(1-ce^{6x})$$

$$\Rightarrow y = 3\,\frac{1+ce^{6x}}{1-ce^{6x}} \rightarrow (1)$$

At $(1/3, 1)$

$$1 = 3\left(\frac{1+ce^{2\cancel{6}(1/\cancel{3})}}{1-ce^{2\cancel{6}(1/\cancel{3})}}\right)$$

$$\Rightarrow 1 - ce^2 = 3 + 3e^2c$$

$$\Rightarrow -2 = 4e^2c$$

$$\boxed{c = -1/2e^2}$$

put in eq (1), we get :-

$$y = 3\,\frac{1+\left(\frac{-e^{6x}}{2e^2}\right)}{1+\frac{e^{6x}}{2e^2}}$$

$$y = 3\,\frac{1-\frac{e^{6x-2}}{2}}{1+\frac{e^{6x-2}}{2}}$$

$$\Rightarrow \boxed{y = 3\,\frac{2-e^{6x-2}}{2+e^{6x-2}}}$$

~~Ans~~ =

Q.53-56 :- Compare the solution of the given initial-value problem

$$\frac{dy}{dx} = (y-1)^2 \; ; \qquad y(0) = 1$$

Q.53 :-

$$\Rightarrow \int (y-1)^{-2}\, dy = \int 1\, dx$$

$$\Rightarrow \frac{1}{1-y} = x + c \rightarrow (1)$$

$$\Rightarrow 1 = (x+c)(1-y)$$

$$\Rightarrow 1 = x - xy + c - cy$$

$$\Rightarrow 1 = x + c - y(x+c)$$

So, $y = \dfrac{-(1-x-c)}{x+c}$

$$\Rightarrow y = \frac{-1+x+c}{x+c} \rightarrow (2)$$

Firstly for "c";
From eq (1);

$$c = \frac{1}{1-y} - x$$

Put $(x, y) = (0, 1)$, we get,

$$c = \frac{1}{1-1} - 0 = \frac{1}{0} = \boxed{\infty}$$

Put in eq (2), we get,

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So, $y = \dfrac{-1 + x + c}{x + c}$ (14)

$$= \frac{x+c}{x+c} - \frac{1}{x+c}$$

$$= 1 - \frac{1}{x+c}$$

$$= 1 - \frac{1}{x+\infty}$$

$$y = 1 - 0 = \boxed{1}$$

So, $\boxed{y=1}$ is the only solution.

Q.55:- $\dfrac{dy}{dx} = (y-1)^2 + 0.01, \quad y(0) = 1$

$$\Rightarrow dy = (y-1)^2 + 0.01\, dx$$

$$\Rightarrow \int \frac{1}{(y-1)^2 + 0.01}\, dy = \int 1\, dx$$

$$\Rightarrow \int \frac{1}{(y-1)^2 + (\sqrt{0.01})^2}\, dy = x + c \rightarrow (1)$$

Let,

$$y - 1 = \sqrt{0.01} \tan\theta \rightarrow (2)$$

$$\Rightarrow dy = \sqrt{0.01} \sec^2\theta\, d\theta \rightarrow (3)$$

Put eq (2) and (3) in eq (1), we get,

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$$\cancel{\int \sqrt{0.01}\sec^2\theta\, d\theta = x + c}$$

$$\int \frac{1}{(\sqrt{0.01}\tan\theta)^2 + (\sqrt{0.01})^2}$$

$$\Rightarrow \int \frac{1}{(\sqrt{0.01})^2(1+\tan^2\theta)} \cancel{\sqrt{0.01}}\sec^2\theta\, d\theta = x + c$$

$$\Rightarrow \frac{1}{\sqrt{0.01}} \int \frac{\cancel{\sec^2\theta}}{\cancel{\sec^2\theta}}\, d\theta = x + c$$

$$\Rightarrow \frac{1}{\sqrt{0.01}}\, \theta = x + c$$

From eq ②;

$$\theta = \tan^{-1}\left(\frac{y-1}{\sqrt{0.01}}\right)$$

So, $$\frac{1}{\sqrt{0.01}} \tan^{-1}\left(\frac{y-1}{\sqrt{0.01}}\right) = x + c$$

$$\Rightarrow 10\tan^{-1}10(y-1) = x + c$$

$$\Rightarrow y - 1 = \frac{1}{10} + \tan x/10 + c$$

$$\Rightarrow y - 1 = 1/10 + \tan x/10 + c) \rightarrow ④$$

Now, for "c", put "y(0) = 1";

So, $1 - 1 = \tan(0 + c)$

$\Rightarrow 0 = \tan(c)$

$\Rightarrow c = \tan^{-1}(0) = \boxed{0}$

So, $y - 1 = \frac{1}{10} + \tan\frac{(x)}{10} + 0$

Ans, $\Rightarrow$ $y = \frac{1}{10} + \tan x/10 + 1$

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$$\frac{dy}{dx} = (x+y+1)^2 \longrightarrow (i)$$

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Let,

$$\Rightarrow \quad x+y+1 = u$$

$$\Rightarrow \quad \frac{du}{dx} = 1 + \frac{dy}{dx}$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{du}{dx} - 1$$

$$\Rightarrow \quad \frac{du}{dx} - 1 = u^2$$

put in eq (i), we get,

$$\Rightarrow \quad \frac{du}{dx} = u^2+1$$

$$\Rightarrow \quad \int \frac{1}{1+u^2}\, du = \int 1\, dx$$

$$\Rightarrow \quad \int \frac{1}{1+u^2}\, du = x + c \longrightarrow (2)$$

Let, $u = \tan\theta \Rightarrow (\theta = \tan^{-1} u)$

$du = \sec^2\theta\, d\theta$

So, eq (2) $\Rightarrow \displaystyle\int \frac{1}{1+\tan^2\theta} \sec^2\theta\, d\theta = x+c$

$$\Rightarrow \int \frac{1}{\cancel{\sec^2\theta}} \cancel{\sec^2\theta}\, d\theta = x+c$$

$$\Rightarrow \quad \theta = x + c$$

$$\Rightarrow \quad \tan^{-1}(u) = x + c$$

or $\quad u = \tan(x+c)$

put "$u = x+y+1$", we get,

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$x+y+1 = \tan(x+c)$

$\Rightarrow \boxed{y = \tan(x+c) - x - 1}$ Ans.

Q159:- $\frac{dy}{dx} = \tan^2(x+y) \rightarrow (1)$

Let, $x+y = u$

$\Rightarrow \frac{du}{dx} = 1 + \frac{dy}{dx}$

$\Rightarrow \frac{dy}{dx} = \frac{du}{dx} - 1$

Put in eq (1), we get,

$\frac{du}{dx} - 1 = \tan^2(u)$

$\Rightarrow \frac{du}{dx} = 1 + \tan^2 u$

$\Rightarrow \int \frac{1}{1+\tan^2 u}\, du = \int 1\, dx$

$\frac{1}{1+\tan^2\theta} = \cos^2\theta$

$\Rightarrow \int \cos^2 u\, du = x + c$

* $\cos^2\theta = \frac{1+\cos 2\theta}{2}$

*

$\Rightarrow \frac{1}{2}\int 1 + \cos 2u\, du = x + c$

$\Rightarrow \frac{1}{2}\left(u + \frac{\sin 2u}{2}\right) = x + c$

$\Rightarrow \frac{u}{2} + \frac{\sin 2u}{4} = x + c$

$\Rightarrow 2u + \sin 2u = 4x + c$

$\Rightarrow 2(x+y) + \sin 2(x+y) = 4x + c$

$\Rightarrow 4x - 2x + 2y + \sin 2(x+y) = c$

$2x + 2y + \sin 2(x+y) = c$

Ans.

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$$\cancel{\frac{dy}{dx} = 2 + \sqrt{y - 2x + 3}} \quad \cancel{\rightarrow (1)}$$

Let, $u = y - 2x + 3$

$$\Rightarrow \frac{du}{dx} = \frac{dy}{dx} - 2$$

$$\Rightarrow \frac{dy}{dx} = \frac{du}{dx} + 2$$

So, eq (1) $\Rightarrow$

$$\frac{du}{dx} + \cancel{2} = \cancel{2} + \sqrt{u}$$

$$\Rightarrow \int \frac{1}{\sqrt{u}}\, du = \int dx$$

$$\Rightarrow \frac{(u)^{-1/2+1}}{-1/2+1} = x + c$$

$$\Rightarrow \frac{\sqrt{u}}{1/2} = x + c$$

$$\Rightarrow \frac{2\sqrt{u}}{} = x + c$$

$$\Rightarrow 2\sqrt{y - 2x + 3} = x + c$$

Squaring both sides, we get,

$$\boxed{4(y - 2x + 3) = (x + c)^2}$$

Ans

Find the Singular Solution for the eq in Problem 37.

Q.37 $\Rightarrow$ $x\sqrt{1-y^2}\, dx = dy$

By inspection, put "$y = 1$", we get

$$x\sqrt{1-1^2}\, dx = d(1)$$

$\Rightarrow x\sqrt{0} = 0$

$\Rightarrow \boxed{0 = 0}$ Satisfied.

So,

$\boxed{y = 1}$ is the singular sol

for Q.37.

Written By;

Muhammad Mehran Yousaf

Reg. no:- FA13-BEE-081

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"Exercise 2.3"

P1 1-10:- Determine whether the given is homogeneous. If so, state the degree of homogeneity:-

Q11:- $x^3 + 2xy^2 - y^4/x$

So,

$$f(x,y) = x^3 + 2xy^2 - y^4/x$$

Put "$x = tx$, $y = ty$", we get,

$$f(tx,ty) = (tx)^3 + 2(tx)(ty)^2 - \frac{(ty)^4}{tx}$$

$$= t^3x^3 + 2t^3xy^2 - t^{3\cancel{4}}y^4/\cancel{t}x$$

$$\Rightarrow f(tx,ty) = t^3\left(x^3 + 2xy^2 - y^4/x\right)$$

So, the function is homogeneous of degree 3.

Q13:- Given,

$$f(x,y) = \frac{x^3y - x^2y^2}{(x+8y)^2}$$

Put $(x,y) = (tx, ty)$, we get,

$$f(tx,ty) = \frac{(tx)^3(ty) - (tx)^2(ty)^2}{(tx+8ty)^2}$$

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$t^2 (x+8y)$

$$= \frac{t^{\cancel{4}2}(x^3y - x^2y^2)}{\cancel{t^4}\,(x+8y)^2}$$

$$\Rightarrow f(tx, ty) = t^2 \left(\frac{x^3y - x^2y^2}{(x+8y)^2} \right)$$

So, the function is homogeneous of degree 2.

Given that,

$$f(x,y) = \frac{\cos x^2}{x+y}$$

$$f(tx, ty) = \frac{\cos(tx)^2}{tx+ty}$$

$$\Rightarrow f(tx, ty) = \frac{\cos t^2x^2}{t(x+y)}$$

So, the function is not homogeneo

Given that,

$$f(x,y) = \ln|x^2| - 2\ln|y|$$

$$= 2\ln x - 2\ln y$$

$$\Rightarrow f(x,y) = 2\ln\left[x/y\right]$$

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~~Put $(x,y) = (tx, ty)$, we get,~~ (2)

$$f(tx,ty) = 2 \ln \left| \frac{tx}{ty} \right|$$

$$= 2 (\ln x - \ln y)$$

$$= 2 \ln x - 2 \ln y$$

$$\Rightarrow f(tx,ty) = t^0 \left(\ln |x|^2 - 2 \ln |y| \right)$$

So, it is a homogeneous function of degree "0".

Given that;

$$f(x,y) = (x^{-1} + y^{-1})^2$$

$$= (x^{-2} + y^{-2} + 2x^{-1}y^{-1})$$

put $(x,y) = (tx, ty)$, we get

$$f(tx,ty) = ((tx)^{-2} + (ty)^{-2} + 2(tx)^{-1}(ty)^{-1})$$

$$= (t^{-2}x^{-2} + t^{-2}y^{-2} + 2t^{-2}x^{-1}y^{-1})$$

$$\Rightarrow f(tx,ty) = t^{-2} (x^{-2} + y^{-2} + 2x^{-1}y^{-1})$$

$$= t^{-2} (x^{-1} + y^{-1})^2$$

So, it is a homogeneous function of degree "-2".

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Q.11-30:- Solve the given differential eq using an appropriate substitution.

Q.11:- "The given eq is homogeneous eq b/c M and N have the same degree" (Mention this in all Questions)

Given that,

$$(x-y)\,dx + x\,dy = 0 \longrightarrow (1)$$

Let, $y = ux \longrightarrow (2)$

So, $dy = u\,dx + x\,du \longrightarrow (3)$

Put eq (2) and eq (3) in eq (1), we get,

$$(x-ux)\,dx + x\,(u\,dx + x\,du) = 0$$

$$x\,dx - \cancel{ux\,dx} + \cancel{xu\,dx} + x^2\,du = 0$$

$$x\,dx + x^2\,du = 0$$

$$\Rightarrow x\,dx = -x^2\,du$$

$$\Rightarrow \frac{1}{x}\,dx = -du$$

Integrating both sides, we get,

$$\int \frac{1}{x}\,dx = -\int du$$

$$\Rightarrow \ln|x| = -u + c$$

$$\Rightarrow \ln|x| = -\frac{y}{x} + c \quad (\because u = y/x \text{ from eq (2)})$$

$$\Rightarrow x\ln|x| = -y + cx$$

or $\boxed{x\ln|x| + y = cx}$

Ans =

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So, Eq (4) $\Rightarrow$ $\frac{u-2}{(u-1)^2} = \frac{1}{u-1} - \frac{1}{(u-1)^2}$

$$\int \frac{1}{u-1}\,du - \int \frac{1}{(u-1)^2}\,du = -\ln|x| + c$$

Put in eq (*) we get

$$\Rightarrow \ln|u-1| + \frac{1}{u-1} = -\ln|x| + c$$

Put "$u = y/x$" from eq ②, we get,

$$\ln\left|y/x - 1\right| + \frac{1}{y/x - 1} = -\ln|x| + c$$

$$\ln\left|\frac{y-x}{x}\right| + \frac{x}{y-x} = -\ln|x| + c$$

$$\Rightarrow \ln|y-x| - \cancel{\ln|x|} + \frac{x}{y-x} = -\cancel{\ln|x|}$$

$$\Rightarrow \boxed{\ln|y-x| + \frac{x}{y-x} = c}$$

Ans 2

Given that,

$$(y^2 + yx)\,dx - x^2\,dy = 0 \rightarrow ①$$

Let $y = ux \rightarrow ②$

So, $dy = u\,dx + x\,du \rightarrow ③$

put eq ② and ③ in eq ①, we get

$$(ux)^2 + (ux)x)\,dx - x^2(u\,dx + x\,du)$$

$$(u^2x^2 + ux^2)\,dx - x^2u\,dx - x^3\,du.$$

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$$\cancel{x\,dx + (y-2x)\,dy = 0} \longrightarrow (1)$$

Let $y = ux \longrightarrow (2)$

So, $dy = u\,dx + x\,du \rightarrow (3)$

put (2) & (3) in eq (1), we get,

$$x\,dx + (ux - 2x)(u\,dx + x\,du) = 0$$

$$\Rightarrow x\,dx + x(u-2)(u\,dx + x\,du) = 0$$

$$\Rightarrow \frac{x\,dx}{x} + \frac{x(u-2)}{x}(u\,dx + x\,du) = 0$$

$$\Rightarrow dx + (u-2)u\,dx + (u-2)x\,du = 0$$

$$\Rightarrow dx + u^2dx - 2u\,dx + ux\,du - 2x\,du = 0$$

$$\Rightarrow (1 + u^2 - 2u)\,dx + (u-2)x\,du = 0$$

$$\frac{u-2}{1+u^2-2u}\,du = -\frac{1}{x}\,dx$$

Integrating both sides, we get,

$$\int \frac{u-2}{1+u^2-2u}\,du = \int -\frac{1}{x}\,dx$$

$$\int \frac{u-2}{(u-1)^2}\,du = -\ln|x| + c \longrightarrow [*]$$

We take $\frac{u-2}{(u-1)^2}$ & solve by Partial fraction.

$$\frac{u-2}{(u-1)^2} = \frac{A}{u-1} + \frac{B}{(u-1)^2} \longrightarrow (4)$$

$$\Rightarrow u-2 = A(u-1) + B$$

Comparing the coefficients of u^0 & u^1, we get,

$\boxed{A=1}$; $-2 = -A + B$ (Put $A=1$)

$\Rightarrow \boxed{B=-1}$

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$$\cancel{(u^2+1)\,x\,dx + (u+1)\,x^2\,du = 0}$$

$$\Rightarrow \quad (u^2+1)\cancel{x}\,dx = (-u+1)\cancel{x^2}\,du$$

$$\Rightarrow \quad \frac{1}{x}\,dx + \frac{u+1}{u^2+1}\,du = 0$$

Integrating both sides, we get,

$$\int \frac{1}{x}\,dx + \int \frac{u+1}{u^2+1}\,du = 0$$

$$\Rightarrow \quad \ln|x| + \frac{1}{2}\int \frac{2u}{u^2+1}\,du + \int \frac{1}{u^2+1}\,du = 0$$

$$\Rightarrow \quad \ln|x| + \frac{1}{2}\ln|u^2+1| + \tan^{-1}(u) + c = 0$$

put $u = y/x$, we get,

$$\ln|x| + \frac{1}{2}\ln\left|\frac{y^2}{x^2}+1\right| + \tan^{-1}\left|\frac{y}{x}\right| + c$$

$$\Rightarrow \quad \ln|x| + \frac{1}{2}\ln\left|\frac{x^2+y^2}{x^2}\right| + \tan^{-1}\left|\frac{y}{x}\right| + c =$$

$$\Rightarrow \quad 2\ln|x| + \ln|x^2+y^2| - \ln|x^2| + 2\tan^{-1}\left|\frac{y}{x}\right| + 2c =$$

$$\Rightarrow \quad \cancel{2\ln|x|} + \ln|x^2+y^2| - \cancel{2\ln|x|} + 2\tan^{-1} + c$$

$$\Rightarrow \quad \boxed{\ln|x^2+y^2| + 2\tan^{-1}\left|\frac{y}{x}\right| = c}$$

try 2

$$\cancel{u^2x^2\,dx + ux^2\,dx - ux^2\,dx - x^3\,du = 0}$$

$$u^2x^2\,dx = +x^3\,du$$

$$\Rightarrow \frac{x^2}{x^3}\,dx = \frac{1}{u^2}\,du$$

$$\frac{1}{x}\,dx = u^{-2}\,du$$

Integrating both sides, we get,

$$\int \frac{1}{x}\,dx = \int u^{-2}\,du$$

$$\Rightarrow \ln|x| = -\frac{1}{u} + c$$

put "$u = y/x$" from eq ②, we get,

$$\ln|x| = \frac{1}{-y/x} + c$$

$$\Rightarrow \ln|x| = -\frac{x}{y} + c$$

or $\boxed{y \ln|x| = -x + cy}$

Ans.

Q17- $\cancel{\frac{dy}{dx} = \frac{y-x}{y+x}}$

$$\Rightarrow dy\,(y+x) = (y-x)\,dx \rightarrow ①$$

• Let, $y = ux \rightarrow ②$

$$\Rightarrow dy = u\,dx + x\,du \rightarrow ③$$

put eq ② & ③ in eq ①, we get,

$$(u\,dx + x\,du)(ux + x) = (ux - x)\,dx$$

$$u^2x\,dx + \cancel{ux\,dx} + ux^2\,du + x^2\,du = \cancel{ux\,dx} - x\,dx$$

$$u^2x\,dx + x\,dx + ux^2\,du + x^2\,du = 0$$

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$\Rightarrow \cancel{-2(u)^{-1/2} + \ln|u| + \ln|x| + c = 0}$

put "$u = y/x$", we get,

$$\ln\left|\frac{y}{x}\right| + \ln|x| = 2\left(\frac{y}{x}\right)^{-1/2} + c$$

$$\Rightarrow \ln\left|\frac{y}{x}\cdot x\right| = 2\sqrt{\frac{x}{y}} + c$$

$$\Rightarrow (\ln|y| - c) = \frac{2\sqrt{x}}{\sqrt{y}}$$

$$\Rightarrow (\ln|y| - c)^2 = \frac{4x}{y}$$

or $\boxed{4x = y\,(\ln|y| - c)^2}$

Ans.

Q12) :-

$$2x^2y\,dx = (3x^3 + y^3)\,dy \longrightarrow (1)$$

let $x = vy \longrightarrow (2)$

$\Rightarrow dx = v\,dy + y\,dv \longrightarrow (3)$

put eq (2), (3) in eq (1), we get,

$$2(vy)^2 y\,(v\,dy + y\,dv) = (3(vy)^3 + y^3)\,dy$$

$$\Rightarrow 2v^2y^3(v\,dy + y\,dv) = (3v^3y^3 + y^3)\,dy$$

$$\Rightarrow 2v^2\cancel{y^3}(v\,dy + y\,dv) = (3v^3 + 1)\cancel{y^3}\,dy$$

$$\Rightarrow 2v^3dy + 2v^2y\,dv - (3v^3 + 1)\,dy = 0$$

$$\Rightarrow 2v^3dy + 2v^2y\,dv - 3v^3dy - dy = 0$$

$$\Rightarrow -v^3dy - dy + 2v^2y\,dv = 0$$

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$\Rightarrow \cancel{2v^2 y\, dv - (v^3+1)dy = 0}$

$$\Rightarrow \frac{2v^2}{v^3+1}\, dv - \frac{dy}{y} = 0 \quad (6)$$

Integrating both sides, we get

$$\int \frac{2v^2}{v^3+1}\, dv - \int \frac{1}{y}\, dy = 0$$

$$\Rightarrow \frac{2}{3}\int \frac{3v^2}{v^3+1}\, dv - \ln|y| = 0$$

$$\Rightarrow \frac{2}{3} \ln|v^3+1| - \ln|y| = c$$

$$\Rightarrow \ln (v^3+1)^{2/3} - \ln|y| = c$$

$$\Rightarrow \ln \left|\frac{(v^3+1)^{2/3}}{y}\right| = c$$

Taking exponential on both sides, we get,

$$\frac{(v^3+1)^{2/3}}{y} = e^c$$

$$\Rightarrow (v^3+1)^{2/3} = yc \quad (\because e^c = c) \longrightarrow (4)$$

put "$v = x/y$" from eq (2) in eq (4), we get,

$$\left(\frac{x^3}{y^3}+1\right)^{2/3} = yc$$

$$\frac{(x^3+y^3)^{2/3}}{(y^3)^{2/3}} = yc$$

$$(x^3+y^3)^{2/3} = y^3 c$$

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$\cancel{-y\,dx + (x + \sqrt{xy})\,dy}$

Let $y = ux \rightarrow (2)$

$\Rightarrow dy = u\,dx + x\,du \rightarrow (3)$

put eq (2), (3) in eq (1), we get,

$$-ux\,dx + \left(x + \sqrt{x(ux)}\right)(u\,dx + x\,du) = 0$$

$$\Rightarrow -ux\,dx + \left(x + \sqrt{ux^2}\right)(u\,dx + x\,du) = 0$$

$$\Rightarrow -ux\,dx + ux\,dx + x^2\,du + u^{\frac{1}{2}+1}x\,dx + u^{\frac{1}{2}}x^2\,du = 0$$

$$\Rightarrow x^2\,du + u^{\frac{3}{2}}x\,dx + u^{\frac{1}{2}}x^2\,du = 0$$

$$\Rightarrow x^2 + u^{\frac{1}{2}}x^2\,du = -u^{\frac{3}{2}}x\,dx$$

$$\Rightarrow x^2\left(1 + u^{\frac{1}{2}}\right)du = -u^{\frac{3}{2}}x\,dx$$

$$\Rightarrow \frac{1 + u^{\frac{1}{2}}}{u^{\frac{3}{2}}}\,du = -\frac{\cancel{x}}{x^{\cancel{2}}}\,dx$$

$$\Rightarrow \frac{1 + (u)^{\frac{1}{2}}}{(u)^{\frac{3}{2}}}\,du = -\frac{1}{x}\,dx$$

Integrating b.s, we get,

$$\int \frac{1 + (u)^{\frac{1}{2}}}{(u)^{\frac{3}{2}}}\,du = \int -\frac{1}{x}\,dx$$

$$\Rightarrow \int (u)^{-\frac{3}{2}}\,du + \int (u)^{-1}\,du = -\ln|x| + c$$

$$\frac{(u)^{-\frac{1}{2}}}{-\frac{1}{2}} + \ln|u| = -\ln|x| + c$$

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$$\Rightarrow \boxed{(x^3+y^3)^2 = y^9 c}$$

Q123:-

$$\frac{dy}{dx} = \frac{y}{x} + \frac{x}{y} \longrightarrow (1)$$

Let, $y = ux \longrightarrow (2)$

$\Rightarrow dy = u\,dx + x\,du \longrightarrow (3)$

Put eq (2) and (3) in eq (1), we get,

$$\frac{u\,dx + x\,du}{dx} = \frac{ux}{x} + \frac{x}{ux}$$

$$\Rightarrow \frac{u\,dx + x\,du}{dx} = \frac{u^2x^2 + x^2}{ux^2}$$

$$\Rightarrow (u^2x^2 + x^2)\,dx = ux^2\,(u\,dx + x\,du)$$

$$\Rightarrow \cancel{u^2x^2\,dx} + x^2\,dx - \cancel{u^2x^2\,dx} - ux^3\,du = 0$$

$$\Rightarrow x^2\,dx - ux^3\,du = 0$$

$$\Rightarrow x^2\,(dx - ux\,du) = 0$$

$$\Rightarrow dx - ux\,du = 0$$

$$\Rightarrow \frac{dx}{x} - u\,du = 0$$

Integrating both sides, we get,

$$\int \frac{1}{x}\,dx - \int u\,du = 0$$

$$\ln|x| - \frac{u^2}{2} + c = 0$$

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$$\cancel{\frac{e^{\frac{2x}{y}}}{2} = 4\ln|y| + c}$$

$$\Rightarrow \boxed{e^{2x/y} - 8\ln|y| = c}$$

Ans.

Q.27:- $\left(y + x\cot y/x\right)dx - x\,dy = 0 \rightarrow (1)$

Let, $y = ux \rightarrow (2)$

$\Rightarrow dy = u\,dx + x\,du \rightarrow (3)$

put eq (2) and (3) in eq (1), we get

$$\left(ux + x\cot\left(\frac{u\cancel{x}}{\cancel{x}}\right)\right)dx - x(u\,dx + x\,du) = 0$$

$$\Rightarrow \cancel{ux\,dx} + x\cot u\,dx - \cancel{xu\,dx} - x^2du = 0$$

$$\Rightarrow \frac{\cancel{x}}{x^{\cancel{2}}}dx = -\frac{1}{\cot u}du$$

$$\Rightarrow \frac{1}{x}dx = -\frac{1}{\cot u}du$$

Integrating both sides, we get

$$\int \frac{1}{x}dx = \int \tan u\,du \quad (\because 1/\cot = \tan$$

$$\Rightarrow \ln|x| = -\ln|\cos u| + c$$

$$\Rightarrow \ln|x| + \ln|\cos u| = c$$

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~~Put "$u = y/x$" from eq ②, we get,~~ ⑦

$$\ln|x| - \frac{y^2/x^2}{2} + c = 0$$

$$\Rightarrow \boxed{2\ln|x| - \left(y/x\right)^2 = c}$$

Ans

Q.125.-

$$y\,dx/dy = x + 4e^{-2x/y} \longrightarrow ①$$

Let, $x = vy \longrightarrow ②$

$\Rightarrow dx = v\,dy + y\,dv \longrightarrow ③$

(sum of highest derivate = degree)

put eq ② and ③ in eq ①, we get,

$$y(v\,dy + y\,dv) = \left(vy + 4e^{-2vy/y}\right)dy$$

$$\Rightarrow \cancel{vy\,dy} + y^2\,dv = \cancel{vy\,dy} + 4ye^{-2v}\,dy$$

$$\Rightarrow y^2\,dv = 4ye^{-2v}\,dy$$

$$\Rightarrow e^{2v}\,dv = \frac{4y}{y^2}\,dy$$

Integrating both sides, we get,

$$\int e^{2v}\,dv = 4\int 1/y\,dy$$

$$\Rightarrow \frac{e^{2v}}{2} = 4\ln|y| + c$$

Put "$v = x/y$" from eq ②, we get,

Prepared By;
Muhammad Mehran Yousaf
Reg. no:- FA13-BEE-081

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$$\ln|x| + \ln\left|\cos y/x\right| = c$$

$$\Rightarrow \ln\left|x.\cos y/x\right| = c$$

$$\Rightarrow e^{\ln|x.\cos y/x|} = e^{c}$$

or $\boxed{x \cos y/x = c}$ $(\because e^{c} = c)$ Ans.

$$(x^2 + xy - y^2)\,dx + xy\,dy = 0 \rightarrow (1)$$

Let $y = ux \rightarrow (2)$

$\Rightarrow dy = u\,dx + x\,du \rightarrow (3)$

put eq (2) & (3) in eq (1), we get,

$$\left(x^2 + x(ux) - (ux)^2\right) dx + x(ux)(u\,dx + x\,du) = 0$$

$$\Rightarrow (x^2 + x^2u - u^2x^2)\,dx + x^2u\,(u\,dx + x\,du) = 0$$

$$\Rightarrow x^2dx + ux^2dx - \cancel{u^2x^2dx} + \cancel{u^2x^2}dx + ux^3du = 0$$

$$\Rightarrow x^2dx\,(1+u) + ux^3du = 0$$

$$\Rightarrow x^2dx\,(1+u) = -ux^3du$$

$$\Rightarrow \frac{\cancel{x^2}}{x^{\cancel{3}}}dx = \frac{-u}{1+u}du$$

$$\Rightarrow \frac{1}{x}dx + \frac{u}{1+u}du = 0$$

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~~Integrating both sides, we get,~~

$$\int \frac{1}{x}\,dx + \int \frac{u}{1+u}\,du = 0$$

$$\Rightarrow \quad \ln|x| + \int \left(1 - \frac{1}{u+1}\right) du = 0$$

$$\begin{array}{r|l} & 1 \\ u+1 & \cancel{u} \\ & \underline{\cancel{u}+1} \\ & -1 \end{array}$$

$$\Rightarrow \quad \ln|x| + \int 1\,du - \int \frac{1}{u+1}\,du = 0$$

$$\Rightarrow \quad \ln|x| + u - \ln|u+1| = c$$

$$\Rightarrow \quad \ln\left|\frac{u+1}{x}\right| = -u + c$$

$$\Rightarrow \quad \frac{u+1}{x} = e^{-u+c}$$

$$\Rightarrow \quad \frac{u+1}{x} = e^{-u} \cdot c$$

$$\Rightarrow \quad \frac{y/x + 1}{x} = e^{-y/x} \cdot c$$

$$\Rightarrow \quad y/x + 1 = e^{-y/x}\, xc$$

$$\Rightarrow \quad \boxed{y + x = x^2 e^{-y/x} c}$$

Ans 2

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Q131-441- Solve the give differential equation subject to the indicated initial condition:-

$$xy^2 \frac{dy}{dx} = y^3 - x^3 \rightarrow (1) \quad ; \quad y(1) = 2 \text{ (Given)}$$

Q131:-

Let, $y = ux \rightarrow (2)$

$\Rightarrow dy = u\,dx + x\,du \rightarrow (3)$

put eq (2) & (3) in eq (1),

we get,

$$x(ux)^2\left(\frac{u\,dx + x\,du}{dx}\right) = ((ux)^3 - x^3)$$

$$\Rightarrow x u^2 x^2 (u\,dx + x\,du) = (u^3x^3 - x^3)\,dx$$

$$\Rightarrow u^2x^3(u\,dx + x\,du) = u^3x^3\,dx - x^3\,dx$$

$$\Rightarrow \cancel{u^3x^3}\,dx + u^2x^4\,du = \cancel{u^3x^3}\,dx - x^3\,dx$$

$$\Rightarrow u^2x^4\,du = -x^3\,dx$$

$$\Rightarrow u^2\,du = -\frac{x^3}{x^4}\,dx$$

$$\Rightarrow u^2\,du + \frac{1}{x}\,dx = 0$$

Integrating both sides, we get,

$$\int u^2\,du + \int \frac{1}{x}\,dx = 0$$

$$\Rightarrow \frac{u^3}{3} + \ln|x| + c = 0$$

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$$\frac{y^3}{3x^3} + \ln|x| + c = 0$$

$$\Rightarrow \quad y^3 + 3x^3 \ln|x| + 3x^3 c = 0$$

$$\Rightarrow \quad y^3 + 3x^3 \ln|x| = x^3 c \quad (\because 3c = c) \rightarrow (4)$$

Now, put, $y(1) = 2$ in eq (4), we get,

i.e; $(x, y) = (1, 2)$

$$(2)^3 + 3(1)^3 \ln|1| = (1)^3 c$$

$$\Rightarrow \quad 8 + 0 = c$$

$$\Rightarrow \boxed{c = 8}$$

put in eq (4), we get,

$$\boxed{y^3 + 3x^3 \ln|x| = 8x^3}$$

Ans 2

$$2x^2 \frac{dy}{dx} = 3xy + y^2 \rightarrow (1) \quad ; \quad y(1) = -2 \text{ (Given)}$$

Let, $y = ux \rightarrow (2)$

$\Rightarrow dy = u\,dx + x\,du \rightarrow (3)$

put eq (2) and (3) in eq (1), we get,

$$2x^2 \frac{(u\,dx + x\,du)}{dx} = 3x(ux) + (ux)^2$$

$$\Rightarrow 2x^2 (u\,dx + x\,du) = (3ux^2 + u^2x^2)\,dx$$

$$\Rightarrow 2ux^2\,dx + 2x^3\,du = 3ux^2\,dx + u^2x^2\,dx$$

$$\Rightarrow 2x^3\,du = ux^2\,du + u^2x^2\,dx$$

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$$\cancel{\ln\left|\frac{u^2}{(u+1)^2}\right| - \ln|x| = c}$$

$$\ln\left|\frac{u^2}{x(u+1)^2}\right| = c$$

$$\Rightarrow \frac{u^2}{x(u+1)^2} = C$$

Put "$u = y/x$" from eq ②, we get,

$$\frac{y^2/x^2}{x\left(\frac{y}{x}+1\right)^2} = C$$

$$\Rightarrow \frac{y^2/x^2}{x\left(\frac{y+x}{x}\right)^2} = c$$

$$\Rightarrow \frac{y^2/x^2}{x\,\frac{(y+x)^2}{x^2}} = c$$

$$\Rightarrow \frac{y^2}{x^2} \times \frac{x}{(y+x)^2} = c$$

$$\Rightarrow \frac{y^2}{x(y+x)^2} = c \rightarrow ⑤$$

Put "$y(1) = -2$" in eq ⑤, we get,

ie; $x = 1,\ y = -2$

$$\frac{(-2)^2}{1(-2+1)^2} = c$$

$$\Rightarrow c = \frac{4}{(-1)^2} = \boxed{4}$$

Put in eq ⑤, we get,

$$\boxed{y^2 = 4x(x+y)^2}$$

Ans.

$$\Rightarrow \cancel{2u^3\,du = x^2\,dx\,(u+u^2)}$$

$$\Rightarrow \frac{2}{u+u^2}\,du = \frac{x^2}{x^3}\,dx$$

$$\Rightarrow \frac{2}{u(1+u)}\,du = \frac{1}{x}\,dx$$

Integrating both sides, we get,

$$\int \frac{2}{u(1+u)}\,du = \int \frac{1}{x}\,dx \rightarrow (4)$$

We take only " $\frac{2}{u(1+u)}$ " & solve by partial fraction.

Let, $\frac{2}{u(1+u)} = \frac{A}{u} + \frac{B}{1+u} \rightarrow (*)$

$$\Rightarrow 2 = A(u+1) + Bu \rightarrow (5)$$

for "A", put "u=0" in eq (5), we get,

$$\boxed{2 = A}$$

for "B", put "u=-1" in eq (5), we get,

$$2 = B(-1)$$

$$\Rightarrow \boxed{B = -2}$$

put in eq (*), we get,

$$\frac{2}{u(1+u)} = \frac{2}{u} - \frac{2}{1+u}$$

Put in eq (4), we get,

$$\int \left(\frac{2}{u} - \frac{2}{1+u}\right) du = \int \frac{1}{x}\,dx$$

$$\Rightarrow 2\ln|u| - 2\ln|1+u| = \ln|x| + c$$

$$2\ln\left|\frac{u}{u+1}\right|^2 = \ln|x| + c$$

$$\Rightarrow \ln\left|\frac{u}{u+1}\right|^2 = \ln|x| + c$$

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$$\cancel{(y^2+3xy)\,dx = (4x^2+xy)\,dy, \quad y(1)=1} \rightarrow ①$$

(Given)

Let, $y = ux \rightarrow ②$

$\Rightarrow dy = u\,dx + x\,du \rightarrow ③$

Put eq ② & ③ in eq ①, we get,

$$(u^2x^2 + 3ux^2)\,dx = (4x^2 + x^2u)(u\,dx + x\,du)$$

$$\Rightarrow \cancel{u^2x^2\,dx} + 3ux^2\,dx = 4ux^2\,dx + 4x^3\,du + \cancel{u^2x^2\,dx} + ux^3\,du$$

$$\Rightarrow ux^2\,dx + 4x^3\,du + ux^3\,du = 0$$

$$\Rightarrow ux^2\,dx + x^3\,du\,(4+u) = 0$$

$$\Rightarrow u\cancel{x^2}\,dx = -x^{\cancel{3}}\,du\,(4+u)$$

$$\Rightarrow \frac{4+u}{u}\,du + \frac{1}{x}\,dx = 0$$

Integrating both sides, we get,

$$4\int \frac{1}{u}\,du + \int 1\,du + \int \frac{1}{x}\,dx = 0$$

$$\Rightarrow 4\ln|u| + u + \ln|x| = C$$

put "$u = y/x$" from eq ②, we get,

$$4\ln|y/x| + y/x + \ln|x| = C$$

$$\Rightarrow \ln\left|\frac{y^4}{x^4}\right| + \ln|x| + y/x = C$$

$$\Rightarrow \ln\left|\frac{y^4}{x^3}\cdot x\right| + y/x = C$$

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$$\cancel{(x + ye^{y/x})\,dx - xe^{y/x}\,dy = 0}, \quad y(1) = 0 \rightarrow (1)$$

Sol:-

Let, $y = ux \longrightarrow (2)$

$\Rightarrow dy = u\,dx + x\,du \rightarrow (3)$

put eq (2) and (3) in eq (1), we

$$\left(x + (ux)e^{\frac{ux}{x}}\right)dx - xe^{\frac{ux}{x}}(u\,dx + x\,du) = 0$$

$$\Rightarrow (x + uxe^{u})\,dx - xe^{u}(u\,dx + x\,du) = 0$$

$$\Rightarrow x\,dx + \cancel{uxe^{u}\,dx} - \cancel{xe^{u}u\,dx} - x^2e^{u}\,du = 0$$

$$\Rightarrow \cancel{x}\,dx = x^{\cancel{2}}e^{u}\,du$$

$$\Rightarrow 1/x\,dx = e^{u}\,du$$

Integrating both sides, we get,

$$\int \frac{1}{x}\,dx = \int e^{u}\,du$$

$$\Rightarrow \ln|x| = e^{u} + c$$

put "$u = y/x$" from eq (2), we get,

$$\ln|x| = e^{y/x} + C \longrightarrow (4)$$

~~Put~~ $y(1) = 0$ i.e; "$x = 1, y = 0$" in eq (4), we get,

$$\ln(1) = e^{0/1} + c$$

$$\Rightarrow 0 = 1 + c$$

$$\Rightarrow \boxed{c = -1}$$

put in eq (4), we get,

$$\boxed{\ln|x| = e^{y/x} - 1}$$

Ans

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$\Rightarrow$ ~~$x^2\,du + x\,du + x^2\sqrt{u}\,du = 0$~~

$\Rightarrow$ $x^2(1+\sqrt{u})\,du + x\,dx = 0$

Dividing each term by "x^2", we get,

$\Rightarrow$ $(1+\sqrt{u})\,du + \frac{1}{x}\,dx = 0$

Integarating both sides, we get,

$$\int 1\,du + \int (u)^{1/2}\,du + \int \frac{1}{x}\,dx = 0$$

$$u + \frac{2}{3}u^{3/2} + \ln|x| = c$$

put "$u = y/x$" from eq ②, we get,

$$y/x + \frac{2}{3}\left(y/x\right)^{3/2} + \ln|x| = c$$

$$\Rightarrow \quad y/x + \frac{2}{3}\frac{(y)^{3/2}}{(x)^{3/2}} + \ln|x| = c \rightarrow ④$$

put "$y(1) = 1$" i.e, $(x,y) = (1,1)$ in eq ④, we get,

$$\frac{1}{1} + \frac{2}{3}\frac{(1)^{3/2}}{(1)^{3/2}} + \ln(1) = c$$

$$\Rightarrow \quad 1 + \frac{2}{3} + 0 = c$$

$$\Rightarrow \quad \frac{5}{3} = c \quad \Rightarrow 3c = 5$$

or $\boxed{c = 5/3}$

Put in eq ④, we get,

$$y/x + \frac{2}{3}\frac{(y)^{3/2}}{(x)^{3/2}} + \ln|x| = \frac{5}{3}$$

~~Mlyng~~ both sides by $3(x)^{3/2}$, we get,

$$\boxed{3(x)^{1/2}y + 2(y)^{3/2} + 3(x)^{3/2}\ln|x| = 5(x)^{3/2}}$$

Ans.

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$$\cancel{\ln\left|\frac{y^4}{x^3}\right| = c - y/x}$$

$$\frac{y^4}{x^3} = e^{c - y/x}$$

$$\Rightarrow \quad y^4 = x^3 e^c \cdot e^{-y/x}$$

$$\Rightarrow \quad y^4 = c x^3 e^{-y/x} \longrightarrow (4)$$

put "$y(1) = 1$"; i.e; $(x, y) = (1, 1)$ in eq (4), we get,

$$(1)^4 = c(1)^3 e^{-1/1}$$

$$\Rightarrow \quad 1 = c e^{-1}$$

$$\Rightarrow \quad \boxed{c = e}$$

put in eq (4), we get,

$$\boxed{y^4 = e x^3 e^{-y/x}}$$

Ans :

7c-

$$(x + \sqrt{xy})\, dy/dx + (x - y) = x^{-1/2} y^{3/2}, \quad y(1) = 1 \longrightarrow (1)$$

(Given)

Let, $y = ux \rightarrow (2)$

$$\Rightarrow dy = u\,dx + x\,du \rightarrow (3)$$

Put eq (2) & (3) in eq (1), we get,

$$(x + \sqrt{x(ux)})\left(\frac{u\,dx + x\,du}{dx}\right) + (x - ux) = x^{-1/2} (ux)^{3/2}$$

$$\Rightarrow \quad (x + x\sqrt{u})(u\,dx + x\,du) + (x - ux)\,dx = x^{-1/2} u^{3/2} x^{3/2}\,dx$$

$$\cancel{xu\,dx} + x^2 du + \cancel{u^{3/2} x\,dx} + x^2 \sqrt{u}\,du + x\,dx - \cancel{ux\,dx} = \cancel{u^{3/2} x\,dx}$$

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$\cancel{y^2 dx + (x^2 + xy + y^2) dy = 0 \rightarrow (1)}$

$y(0) = 1$ (Given)

Let, $x = vy \rightarrow (2)$

$\Rightarrow dx = v dy + y dv \rightarrow (3)$

put eq (2) & (3) in eq (1), we get,

$$y^2 (v dy + y dv) + (v^2 y^2 + v y^2 + y^2) dy = 0$$

$$\Rightarrow v y^2 dy + y^3 dv + v^2 y^2 dy + v y^2 dy + y^2 dy = 0$$

$$\Rightarrow (v + v^2 + v + 1) y^2 dy + y^3 dv = 0$$

$$\Rightarrow (v^2 + 2v + 1) y^2 dy + y^3 dv = 0$$

$$\Rightarrow (v+1)^2 y^2 dy + y^3 dv = 0$$

$$\Rightarrow (v+1)^2 y^2 dy = -y^3 dv$$

or $\frac{1}{(v+1)^2} dv + \frac{1}{y} dy = 0$

Integrating both sides, we get,

$$\int \frac{1}{(v+1)^2} dv + \int \frac{1}{y} dy = 0$$

$$\frac{-1}{v+1} + \ln|y| = C.$$

put $v = x/y$ from eq (2), we get,

$$\frac{-1}{x/y + 1} + \ln|y| = C$$

$$\frac{-y}{x+y} + \ln|y| = C \rightarrow (4)$$

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~~put "$y(0)=1$" or $(x,y)=(0,1)$ in eq (4), we get,~~

$$\frac{-1}{1} + \ln(1) = c$$

$$\Rightarrow \boxed{c=-1}$$

Put in eq (4), we get,

$$\ln|y| - \frac{y}{x+y} = -1$$

$$\Rightarrow \ln|y|(x+y) - \cancel{y} = -x - \cancel{y}$$

$$\Rightarrow \boxed{(x+y)\ln|y| = -x}$$

Ans.

1- $\left(x + \sqrt{y^2 - xy}\right) dy/dx = y$, $y(1/2) = 1$ → (1)

Let, $x = vy$ → (2)

$\Rightarrow dx = v\,dy + y\,dv$ → (3)

Put eq (2) and (3) in eq (1), we get,

$$vy + \sqrt{y^2 - vy^2}\, dy = y(v\,dy + y\,dv)$$

$$\Rightarrow \left(vy + y\sqrt{1-v}\right) dy = y(v\,dy + y\,dv)$$

$$\Rightarrow \left(vy + y\sqrt{(1-v)}\right) dy = vy\,dy + y^2 dv$$

$$\Rightarrow \cancel{vy\,dy} + y\sqrt{1-v}\,dy = \cancel{vy\,dy} + y^2 dv$$

$$\Rightarrow \frac{1}{y}\,dy - \frac{1}{\sqrt{1-v}}\,dv = 0$$

Integrating both sides, we get,

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Comparing eq ③ and eq ⑤,

we get,

$$3y+7 = h'(y)$$

Integrate w.r.t. y, we get,

$$h(y) = \frac{3y^2}{2} + 7y + c$$

put in eq ④, we get,

$$\boxed{f(x,y) = x^2 - x + \frac{3y^2}{2} + 7y + c}$$

$f(x,y)=c$

Ans.

Q13:- $(5x+4y)\,dx + (4x-8y^3)\,dy = 0$ → ①

$M(x,y) = 5x+4y$

and $N(x,y) = 4x-8y^3$

Now, $\frac{\partial M}{\partial y} = 0+4 = 4$

and $\partial N/\partial x = 4$

Since, $\boxed{\frac{\partial M}{\partial y} = \partial N/\partial x}$

Therefore, the given eq is an exact differential equation

Now, $\frac{\partial f}{\partial x} = M = 5x+4y$ → ②

$= -2y^2 + c$

Exercise "2.4"

①

Q12-34:- Determine whether the given differential eq is exact; if exact solve it.

Q12:- $(2x-1)\,dx + (3y+7)\,dy = 0 \longrightarrow ①$

$M(x,y) = 2x-1$

$N(x,y) = 3y+7$

$(x-\text{comp}) \leftarrow \dfrac{\partial M}{\partial y} = 0$

$(y-\text{comp}) \leftarrow \dfrac{\partial N}{\partial x} = 0$

$\therefore \dfrac{\partial M}{\partial y} = \dfrac{\partial N}{\partial x}$

Therefore, the given eq ① is an exact differential equation

Now, $\dfrac{\partial f}{\partial x} = M = 2x-1 \longrightarrow ②$

and $\dfrac{\partial f}{\partial y} = N = 3y+7 \longrightarrow ③$

Integrate eq ② partially w.r.t "x", we get,

$$f(x,y) = \frac{2x^2}{2} - x + h(y)$$

$$= x^2 - x + h(y) \longrightarrow ④$$

Now, from eq ④, (differentiate partially w.r.t "y")

$$\frac{\partial f}{\partial y} = h'(y) \longrightarrow ⑤$$

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Since $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$

$\therefore$ eq ① is an exact differential equation

Now, $\frac{\partial f}{\partial x} = M = y^3 - y^2 \sin x - x \rightarrow ②$

and $\frac{\partial f}{\partial y} = N = 3xy^2 + 2y\cos x \rightarrow ③$

Integrate eq ② partially w.r.t "x", we get,

$$f(x,y) = xy^3 + y^2\cos x - \frac{x^2}{2} + h(y) \rightarrow ④$$

Differentiate eq ④ w.r.t "y", we get

$$\frac{\partial f}{\partial y} = 3xy^2 + 2y\cos x + h'(y) \rightarrow ⑤$$

Comparing eq ③ and eq ⑤, we get;

$$\cancel{3xy^2} + \cancel{2y\cos x} = \cancel{3xy^2} + \cancel{2y\cos x} + h'(y)$$

$$\Rightarrow \int h'(y) = \int 0$$

$$\Rightarrow h(y) = c$$ put in eq ④, we get,

$$\Rightarrow f(x,y) = xy^3 + y^2\cos x - \frac{x^2}{2} + c$$ Ans.

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$2xy + 4 = 2yx^2 + h'(y)$ ②

$\Rightarrow h'(y) = 4$

Integrate w.r.t "y", we get,

$h(y) = 4y + c$ put in eq ①

we get

$f(x,y) = x^2y^2 - 3x + 4y + c$

$f(x,y) = 0$

Ans.

Q17.– $(x^2 - y^2)\,dx + x(x-2y)\,dy = 0 \rightarrow ①$

$M(x,y) = x^2 - y^2$

$N(x,y) = x(x-2y) = x^2 - 2xy$

$\frac{\partial M}{\partial y} = -2y$

$\frac{\partial N}{\partial x} = 2x - 2x = 0$

Since, $\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$

So, the given equation can not be solved since it is not exact differential equation

Q19.– $(y^3 - y^2 \sin x - x)\,dx + (3xy^2 + 2y\cos x)\,dy = 0$ ①

$M(x,y) = y^3 - y^2 \sin x - x$

$N(x,y) = 3xy^2 + 2y\cos x$

$\frac{\partial M}{\partial y} = 3y^2 - \sin x\, 2y$

$= 3y^2 - 2y \sin x$

$\frac{\partial N}{\partial x} = 3y^2 - 2y \sin x$

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Q11:- $(y\ln y - e^{-xy})\,dx + (1/y + x\ln y)\,dy = 0$

$M(x,y) = y\ln y - e^{-xy}$

$N(x,y) = 1/y + x\ln y$

$\frac{\partial M}{\partial y} = 1 + \ln y - e^{-xy}(-x)$

$= 1 + \ln y + xe^{-xy}$

$\frac{\partial N}{\partial x} = 1/y + x\ln y$

$= \ln y$

Since, $\frac{\partial M}{\partial y} \neq \partial N/\partial x$

So, the given equation is not exact differential eq. and cannot be solved.

Q13:-

$x\frac{dy}{dx} = 2xe^{x} - y + 6x^2 \rightarrow (1)$

$\Rightarrow -x\,dy + (2xe^{x} - y + 6x^2)\,dx = 0$

So,

$M(x,y) = 2xe^{x} - y + 6x^2$

and $N(x,y) = -x$

$\frac{\partial M}{\partial y} = -1$

& $\partial N/\partial x = -1$

Since $\frac{\partial M}{\partial y} = \partial N/\partial x$

So eq (1) is an exact differential

Now $\frac{\partial f}{\partial x} = M(x,y) = 2xe^{x} - y + 6x^2 \longrightarrow (2)$

and $\frac{\partial f}{\partial y} = N(x,y) = -x \longrightarrow (3)$

Integrate eq (2) partially w.r.t "x", we get,

$$f(x,y) = 2\int_x xe^{x} - xy + \frac{6x^3}{3}$$

$$= 2xe^{x} - 2\int e^{x}(1)dx - xy + 2x^3 + h(y)$$

$$f(x,y) = 2xe^{x} - 2e^{x} - xy + 2x^3 + h(y) \longrightarrow (4)$$

Differentiate eq (4) partially w.r.t "y", we get,

$$\frac{\partial f}{\partial y} = 0 - 0 - x + 0 + h'(y)$$

$$= -x + h'(y) \longrightarrow (5)$$

Comparing eq (3) and eq (5), we get,

$$-x = -x + h'(y)$$

$$\Rightarrow h'(y) = 0$$

Integrate w.r.t "y", we get

$h(y) = c$ put in eq (4), we get

$$f(x,y) = 2xe^{x} - 2e^{x} - xy + 2x^3 + c$$

Ans.

Q.15:- $\left(1-3/x+y\right)dx + \left(1-3/y+x\right)dy = 0$ ③

So,

$M(x,y) = 1-3/x+y$ → ①

and $N(x,y) = 1-3/y+x$

So, $\frac{\partial M}{\partial y} = 1$

$\partial N/\partial x = 1$

Since,

$$\boxed{\frac{\partial M}{\partial y} = \partial N/\partial x}$$

So, eq ③ is an exact differential equation.

Now,

$\frac{\partial f}{\partial x} = M(x,y) = 1-3/x+y$ → ②

and $\frac{\partial f}{\partial y} = N(x,y) = 1-3/y+x$ → ③

Integrate eq ② partially w.r.t "x", we get

$f(x,y) = x - 3\ln|x| + yx + h(y)$ → ④

Differentiate eq ④ partially w.r.t "y", we get,

$\frac{\partial f}{\partial y} = x + h'(y)$ → ⑤

Comparing eq ③ and eq ⑤, we get

$1 - \frac{3}{y} + x = x + h'(y)$

Integrate w.r.t "y", we get,

$h(y) = y - 3\ln|y| + C$

put in eq ④, we get,

$f(x,y) = x + y - 3\ln x + xy - 3\ln y + c$

put $f(x,y) = 0$

So, $C = -3\ln|xy| + xy - x + y$

Ans.

Q.17)- $\left(x^2y^3 - \frac{1}{1+9x^2}\right)dx + x^3y^2\,dy = 0$ → ①

$M(x,y) = x^2y^3 - \frac{1}{1+9x^2}$

and $N(x,y) = x^3y^2$

So, $\frac{\partial M}{\partial y} = 3x^2y^2$

$\frac{\partial N}{\partial x} = 3x^2y^2$

Now,

$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$

So, eq ① is an exact differential equation.

So, $\frac{\partial \Phi}{\partial x} = M(x,y) = x^2y^3 - \frac{1}{1+9x^2}$ → ②

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$$\frac{\partial f}{\partial y} = N(x,y) = x^3 y^2 \longrightarrow (3)$$

So, Integrate eq (2) w.r.t "x", we partially get,

$$f(x,y) = \frac{x^3 y^3}{3} - \frac{1}{3}\tan^{-1}(3x) + h(y) \longrightarrow (4)$$

Differentiate eq (4) w.r.t "y", we get

$$\frac{\partial f}{\partial y} = \frac{\cancel{3}x^3 y^2}{\cancel{3}} + h'(y)$$

$$= x^3 y^2 + h'(y) \longrightarrow (5)$$

Comparing eq (3) and eq (5), we get,

$$\cancel{x^3 y^2} = \cancel{x^3 y^2} + h'(y)$$

$$h'(y) = 0$$

or $h(y) = c$

Put in eq (4), we get,

$$f(x,y) = \frac{x^3 y^3}{3} - \frac{1}{3}\tan^{-1}(3x) + c$$

Put $f(x,y) = c^3$, we get,

So, $$\boxed{c = \frac{x^3 y^3}{3} - \frac{1}{3}\tan^{-1}(3x)}$$

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Q.19:- $(\tan x - \sin x \sin y)\,dx + \cos x \cos y\,dy = 0$ ... $\rightarrow$ (1)

If,

$M(x,y) = \tan x - \sin x \sin y$

$N(x,y) = \cos x \cos y$

So,

$$\frac{\partial M}{\partial y} = -\sin x \cos y$$

$$\frac{\partial N}{\partial x} = -\sin x \cos y$$

Since, $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$

So eq (1) is an exact differential equation.

So,

$$\frac{\partial f}{\partial x} = M(x,y) = \tan x - \sin x \sin y \rightarrow (2)$$

$$\frac{\partial f}{\partial y} = N(x,y) = \cos x \cos y \rightarrow (3)$$

Integrate eq (2) partially w.r.t "x", we get,

$$f(x,y) = \ln|\sec x| + \cos x \sin y + h(y) \rightarrow (4)$$

Differentiate eq (4) partially w.r.t "y", we get,

$$\frac{\partial f}{\partial y} = \cos y \cos x + h'(y) \rightarrow (5)$$

Comparing eq (3) and eq (5) we get,

$\cancel{\cos x \cos y} = \cancel{\cos y \cos x} + h'(y)$

$\therefore h'(y) = 0$

So, Eq (4) $\Rightarrow$ $f(x,y) = \ln|\sin x| + \cot x \ln y + C$ (4)

Put $f(x,y) = 0$,

So, $\boxed{C = \ln|\sin x| + \cot x \ln y}$

Ans.

Q12) $(4x^3 + 4xy)\,dx - (1 - 2x^2 - 2y)\,dy = 0$ $\rightarrow$ (1)

$M(x,y) = 4x^3 + 4xy$

and $N(x,y) = -(1 - 2x^2 - 2y)$

$= 2x^2 + 2y - 1$

So, $\frac{\partial M}{\partial y} = 4x$

$\partial N / \partial x = 4x$

Since, $\frac{\partial M}{\partial y} = \partial N / \partial x$

So, It is an exact differential equation

Now, $\frac{\partial f}{\partial x} = M = 4x^3 + 4xy$ $\rightarrow$ (2)

and $\frac{\partial f}{\partial y} = N = 2x^2 + 2y - 1$ $\rightarrow$ (3)

So, integrate eq (2) partially w.r.t. x, we get

$$f(x,y) = \frac{4x^4}{4} + \frac{2(4)x^2 y}{2}$$

$$\Rightarrow f(x,y) = x^4 + 2x^2 y + h(y) \rightarrow (4)$$

Differentiate eq (4) partially w.r.t "y", we get,

$$\frac{\partial f}{\partial y} = 2x^2 + h'(y) \rightarrow (5)$$

Comparing eq (3) and eq (5), we get

$$\cancel{2x^2} + h'(y) = \cancel{2x^2} + 2y - 1$$

$$\Rightarrow h'(y) = 2y - 1$$

So, $h(y) = \frac{\cancel{2}y^2}{\cancel{2}} - y$

$$= y^2 - y + c$$

put in eq (4), we get,

$$f(x,y) = x^4 + 2x^2 y + y^2 - y + c$$

put $f(x,y) = 0$

So: $\boxed{c = x^4 + 2x^2 y + y^2 - y}$

*

Ans 2

Q12:- $(4x^3 y - 15x^2 - y)\,dx + (x^4 + 3y^2 - x)\,dy = c \rightarrow (1)$

$$M(x,y) = 4x^3 y - 15x^2 - y$$

$$N(x,y) = x^4 + 3y^2 - x$$

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$$\frac{\partial f}{\partial y} = N = 2xy + x^2 - 1 \longrightarrow (3) \quad (5)$$

Integrate eq (2) partially w.r.t "x", we get,

$$f(x,y) = \frac{x^3}{3} + xy^2 + \frac{2x^2}{2} \cdot y + h(y)$$

$$= x^3/3 + xy^2 + x^2 y + h(y) \longrightarrow (4)$$

Differentiate eq (4) partially w.r.t "y", we get,

$$\frac{\partial f}{\partial y} = 2xy + x^2 + h'(y) \longrightarrow (5)$$

Comparing eq (3) and eq (5), we get,

$$\cancel{2xy} + \cancel{x^2} - 1 = \cancel{2xy} + \cancel{x^2} + h'(y)$$

$$\Rightarrow h'(y) = -1$$

So, $h(y) = -y + c$

put in eq (4), we get

$$f(x,y) = x^3/3 + xy^2 + x^2 y + c - y$$

Put $f(x,y) = 0$, we get,

So,

$$c = x^3/3 + xy^2 + x^2 y - y \longrightarrow (6)$$

put $y(1) = 1$ in eq (6), we get,

$$c = 4/3$$

So, $x^3/3 + xy^2 + x^2 y - y = 4/3$

Ans

$h'(y) = 3y^2$

So, $h(y) = \frac{3y^3}{3} + c$

$h(y) = y^3 + c$ put in eq (4), we get,

$f(x,y) = yx^4 - 5x^3 - yx + y^3 + c$

put $f(x,y) = 0$,

So, $\boxed{c = yx^4 - 5x^3 - yx + y^3}$

$\frac{dy}{dx} =$

Q: 25-30:- Solve the given differential eq subject to the indicated initial condition.

$y(1) = 1$

Q:25:- $(x+y)^2\,dx + (2xy + x^2 - 1)\,dy = 0 \rightarrow (1)$

Now, $M(x,y) = (x+y)^2$

$N = 2xy + x^2 - 1$

$\frac{\partial M}{\partial y} = 2(x+y) = 2x + 2y$

$\partial N/\partial x = 2y + 2x$

Since, $\frac{\partial M}{\partial y} = \partial N/\partial x$

So, the eq (1) is an exact differential eq.

Now, $\frac{\partial f}{\partial x} = M = (x+y)^2 = x^2 + y^2 + 2xy \rightarrow (2)$

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$$\frac{\partial M}{\partial y} = 4x^3 - 1$$

$$\frac{\partial N}{\partial x} = 4x^3 - 1$$

Since, $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$

i.e. it is an exact differential eq.

Now,

$$\frac{\partial f}{\partial x} = M = 4x^3y - 15x^2 - y \rightarrow (2)$$

and $$\frac{\partial f}{\partial y} = N = x^4 + 3y^2 - x \rightarrow (3)$$

Integrate eq (2) partially w.r.t "x", we get

$$f(x,y) = \frac{4x^4}{4}y - \frac{15x^3}{3} - yx + h(y)$$

$$= x^4y - 5x^3 - yx + h(y) \rightarrow (4)$$

Differentiate eq (4) partially w.r.t "y", we get,

$$\frac{\partial f}{\partial y} = x^4 - x + h'(y) \rightarrow (5)$$

Comparing eq (5) and (3), we get,

$$\cancel{x^4} + 3y^2 - \cancel{x} = \cancel{x^4} - \cancel{x} + h'(y)$$

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$$(4y+2x-5)\,dx + (6y+4x-1)\,dy = 0 \rightarrow (1)$$

Q.27:- $(4y+2x-5)\,dx + (6y+4x-1)\,dy = 0$

$M(x,y) = 4y+2x-5$

$N(x,y) = 6y+4x-1$

So, $\dfrac{\partial M}{\partial y} = 4+0-0 = 4$

$\dfrac{\partial N}{\partial x} = 0+4-0 = 4$

Since, $\dfrac{\partial M}{\partial y} = \dfrac{\partial N}{\partial x}$

So, eq (1) is an exact differential equation

Now, $\dfrac{\partial f}{\partial x} = M = 4y+2x-5 \rightarrow (2)$

and $\dfrac{\partial f}{\partial y} = N = 6y+4x-1 \rightarrow (3)$

Integrate eq (2) partially w.r.t "x", we get

$$f(x,y) = 4xy + \frac{2x^2}{2} - 5x$$

$$= 4xy + x^2 - 5x + h(y) \rightarrow (4)$$

Differentiate eq (4) partially w.r.t "y", we get

$$\frac{\partial f}{\partial y} = 4x + h'(y) \rightarrow (5)$$

Comparing eq (3) & (5), we get

$$6y + 4x - 1 = 4x + h'(y)$$

Since, $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$

So, eq ① is an exact differential eq.

Now,

$$\frac{\partial f}{\partial x} = M = y^2 \cos x - 3x^2 y - 2x \longrightarrow ②$$

$$\frac{\partial f}{\partial y} = N = 2y \sin x - x^3 + \log y \longrightarrow ③$$

Integrate eq ② partially w.r.t "x", we get,

$$f(x, y) = y^2 \sin x - \frac{\cancel{3}x^3 y}{\cancel{3}} - \frac{\cancel{2}x^2}{\cancel{2}} + h(y)$$

$$= y^2 \sin x - x^3 y - x^2 + h(y) \longrightarrow ④$$

Differentiate eq ④ partially w.r.t "y", we get,

$$\frac{\partial f}{\partial y} = 2y \sin x - x^3 + h'(y) \longrightarrow ⑤$$

Comparing eq ③ and eq ⑤ we get

$$\cancel{2y \sin x} - \cancel{x^3} + \log y = \cancel{2y \sin x} - \cancel{x^3} + h'(y)$$

$$h'(y) = \log y$$

So $h(y) = y \log y - y + c$ (integrate by parts)

Put in eq ④ we get

$$f(x, y) = y^2 \sin x - x^3 y - x^2 + y \log y - y + c$$

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$\Rightarrow h'(y) = 6y - 1$

$\Rightarrow h(y) = 3y^2 - y + c$

put in eq (4), we get,

$f(x, y) = 4xy + x^2 - 5x + 3y^2 - y + c$

put $f(x, y) = 0$, we get,

$c = 4xy + x^2 - 5x + 3y^2 - y \rightarrow (6)$

put $y(-1) = 2$ i.e; $x = -1$, $y = 2$

in eq (6), we get,

$C = 4(-1)(2) + (-1)^2 - 5(-1) + 3(2)^2 - 2$

$= -8 + 1 + 5 + 12 - 2$

$= 6 - 8 + 12$

$= 6 + 2$

$\boxed{C = 8}$

put in eq (6), we get

$\boxed{4xy + x^2 - 5x + 3y^2 - y = 8}$

Ans,

Q.12.- $(y^2 \cos x - 3x^2 y - 2x)\,dx + (2y \sin x - x^3 + \ln y)\,dy = 0$, $y(0) = e$ $\rightarrow (1)$

So, $M(x, y) = y^2 \cos x - 3x^2 y - 2x$

$N(x, y) = 2y \sin x - x^3 + \ln y$

$\frac{\partial M}{\partial y} = 2y \cos x - 3x^2$

$\partial N / \partial x = 2y \cos x - 3x^2$

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Put $f(x,y) = c$, we get, ⑥

$$c = y^2 \sin x - x^3 y - x^2 + y \ln y - y$$

Put "$y(0) = e$", we get, ⑥

$$c = (e)^2 \sin 0 - (0)^3 (e) - (0)^2 + e \ln e - e$$

$$= 0 - 0 - 0 + e(1) - e$$

$$\boxed{c = 0}$$ Put in eq ⑥, we get,

$$\boxed{y^2 \sin x - x^3 y - x^2 + y \ln y - y = 0}$$

Ans

Q131-34:- Find the values of "k" so that the given differential eq is exact.

Q131:- $(y^3 + kxy^4 - 2x)\,dx + (3xy^2 + 20x^2y^3)\,dy = 0 \rightarrow ①$

Since, given that, eq ① is an exact differential eq.

So, $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$

$$\Rightarrow \cancel{3}y^2 + \cancel{4}y^3 kx = \cancel{3}y^2 + \overset{10}{\cancel{40}}x y^3$$

$$\Rightarrow y^3 kx = 10xy^3$$

$$\Rightarrow \boxed{k = 10}$$

Ans

Integrate eq (2) partially w.r.t "y", we get,

$$\int \frac{\partial M}{\partial y}\,\partial y = \int \left(e^{xy} + xye^{xy} + 2y - \frac{1}{x^2}\right)\partial y$$

$$M = \frac{e^{xy}}{x} + x\left[y\int e^{xy}\,\partial y - \int\left[\frac{\partial y}{\partial x}\int e^{xy}\,\partial x\right]\partial x\right] + \frac{2y^2}{2} - \frac{y}{x^2} + g(x)$$

$$M = \frac{e^{xy}}{x} + x\left[\frac{ye^{xy}}{x} - \frac{e^{xy}}{x^2}\right] + y^2 - \frac{y}{x^2} + g(x)$$

$$= \frac{e^{xy}}{x} + \frac{xye^{xy}}{x} - \frac{xe^{xy}}{x^2} + y^2 - \frac{y}{x^2} + g(x)$$

$$\boxed{M(x,y) = ye^{xy} + y^2 - \frac{y}{x^2} + g(x)}$$

Ans.

Prepared By;
Muhammad Mehran Yousaf
Reg.no:- FA13-BCE-081

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$(2xy^2 + ye^x)\,dx + (2x^2y + ke^x - 1)\,dy = 0 \rightarrow (1)$

Q.33:- $M(x,y) = 2xy^2 + ye^x$

$N(x,y) = 2x^2y + ke^x - 1$

Since, given that eq (1) is an exact differential eq.

So,

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

$$\Rightarrow 4xy + e^x = 4xy + ke^x$$

$$\Rightarrow \boxed{k = 1}$$

Q.35:- Determine $M(x,y)$ so that the following diff eq is exact.

Given eq is,

$$M(x,y)\,dx + \left(xe^{xy} + 2xy + \frac{1}{x}\right)dy = 0 \rightarrow (1)$$

$$M(x,y) = xe^{xy} + 2xy + \frac{1}{x}$$

As, given that eq (1) is an exact diff equation

So,

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

So,

$$\frac{\partial M}{\partial y} = \left(e^{xy}(1) + xe^{xy}y\right) + 2y - \frac{1}{x^2}$$

$$= e^{xy} + yxe^{xy} + 2y - \frac{1}{x^2}$$

$$\rightarrow (1)$$

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Now $\frac{\partial f}{\partial x} = M = -x^2y^2 \sin x + 2xy^2 \cos x \rightarrow (2)$

and

$\frac{\partial f}{\partial y} = N = 2x^2y \cos x \rightarrow (3)$

Integrate eq (2) partially w.r.t "x", we get,

$$f(x,y) = \int \left(-x^2y^2 \sin x + 2xy^2 \cos x\right) dx$$

$$= -y^2 \left[x^2(-\cos x) + \int 2x \cos x \, dx\right] + y^2 \int 2x \cos x \, dx$$

$$= x^2y^2 \cos x - \cancel{y^2 \int 2x \cos x \, dx} + \cancel{y^2 \int 2x \cos x \, dx} + h(y)$$

$$f(x,y) = x^2y^2 \cos x + h(y) \rightarrow (4)$$

Differentiate eq (4) partially w.r.t "y", we get,

$$\frac{\partial f}{\partial y} = 2x^2y \cos x + h'(y) \rightarrow (5)$$

Comparing eq (3) and eq (5), we get

$$\cancel{2x^2y \cos x} = \cancel{2x^2y \cos x} + h'(y)$$

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Q.37-42:- Solve the given diff eq by verifying that the indicated function $\mu(x,y)$ is an indicated integrating factor.

Q.37:- Given that,

$$6xy\,dx + (4y + 9x^2)\,dy = 0 \quad \mu(x,y) = y \rightarrow (1)$$

$$M = 6xy \qquad N = 4y + 9x^2$$

So,

$$\frac{\partial M}{\partial y} = 6x$$

$$\frac{\partial N}{\partial x} = 18x$$

Since,

$$\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$$

So, given eq is not an exact diff eq.

To make it exact,

Xlying eq (1) by $\mu(x,y)$, we get,

$$6xy^3\,dx + (4y^3 + 9x^2y^2)\,dy = 0$$

Now,

$$\frac{\partial M}{\partial y} = 18xy^2$$

$$\frac{\partial N}{\partial x} = 18xy^2$$

So

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x} \text{ (exact)}$$

Verified

Q.171- $\cos x \frac{dy}{dx} + y \sin x = 1$

$$\frac{dy}{dx} + y \frac{\sin x}{\cos x} = \frac{1}{\cos x}$$

$$\Rightarrow \frac{dy}{dx} + y \tan x = \sec x \rightarrow (1)$$

So, $p(x) = \tan x$

$$\mu(x) = e^{\int \tan x \, dx}$$

$$= e^{\log|\sec x|}$$

$$\Rightarrow \mu(x) = \sec x$$

Multiplying eq (1) both sides by "$\mu(x)$", we get,

$$\sec x \frac{dy}{dx} + y \tan x \sec x = \sec^2 x$$

$$\Rightarrow \frac{1}{\cos x} \frac{dy}{dx} + y \frac{\sin x}{\cos^2 x} = \sec^2 x$$

$$\Rightarrow \frac{d}{dx}\left(\frac{1}{\cos x} y\right) = \sec^2 x$$

Integrating both sides, we get,

$$\frac{y}{\cos x} = \int \sec^2 x$$

$$\Rightarrow \frac{y}{\cos x} = \tan x + c$$

or $y = \frac{\cancel{\cos x} \cdot \sin x}{\cancel{\cos x}} + c \cos x$

$$\Rightarrow \boxed{y = \sin x + c \cos x}$$

Ans.

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$$\frac{d}{dx}\left(yx^2e^x\right) = e^{2x} \cdot \frac{x^2}{x^2}$$ (4)

$$\Rightarrow \frac{d}{dx}\left(yx^2e^x\right) = e^{2x}$$

Integrating both, we get,

$$\int \frac{d}{dx}\left(yx^2e^x\right)dx = \int e^{2x}\,dx$$

$$\Rightarrow yx^2e^x = \frac{e^{2x}}{2} + c$$

$$\Rightarrow y = \frac{e^{2x}}{2x^2e^x} + \frac{c}{x^2e^x}$$

$$\Rightarrow \boxed{y = \frac{e^x}{2x^2} + \frac{ce^{-x}}{x^2}}$$

Ans

Interval :-

Q.23:- $\cos^2 x \sin x\, dy + (y\cos^3 x - 1)\,dx = 0$

$$\frac{dy}{dx} + \frac{y\cos^3 x - 1}{\cos^2 x \sin x} = 0$$

$$\frac{dy}{dx} + \frac{y\cos^3 x}{\cos^2 x \sin x} - \frac{1}{\cos^2 x \sin x} = 0$$

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Solve differential equations

Exercise 2.5

(1)

Q: 1-40:- Find the general solution of the given differential equation. State the interval on which the general solution is defined.

Q.1:-

$$\frac{dy}{dx} = 5y$$

$$\Rightarrow \frac{dy}{dx} - 5y = 0 \longrightarrow (1)$$

So, $p(x) = -5$

$$f(x) = e^{\int p(x)\,dx}$$

$$= e^{-5\int dx} = e^{-5x}$$

Multiplying eq (1) by "$f(x)$", we get,

$$e^{-5x}\frac{dy}{dx} - 5ye^{-5x} = 0$$

$$\frac{d}{dx}\left(e^{-5x}y\right) = 0$$

Integrating both sides, we get,

$$\int \frac{d}{dx}\left(e^{-5x}y\right)dx = \int 0\,dx$$

$$e^{-5x}y = C$$

$$\Rightarrow * \boxed{y = Ce^{5x}} \text{ Ans.}$$

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TOMLATS Unit-Rd-Nizam Abad

Interval :-

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Q1:- $x\frac{dy}{dx} + (3x+1)y = e^{-3x}$ (5)

$$\frac{dy}{dx} + \left(\frac{3x+1}{x}\right)y = \frac{e^{-3x}}{x}$$

$$\frac{dy}{dx} + \left(3 + \frac{1}{x}\right)y = \frac{e^{-3x}}{x} \rightarrow (1)$$

So,

$$P(x) = 3 + \frac{1}{x}$$

$$\mu(x) = e^{\int P(x)\,dx}$$

$$= e^{\int (3+1/x)\,dx}$$

$$= e^{3x + \ln|x|}$$

$$= e^{3x} \cdot \cancel{e^{\ln x}}$$

$$\Rightarrow \mu(x) = x e^{3x}$$

Multiplying eq (1) by $\mu(x)$, we get,

$$x e^{3x}\frac{dy}{dx} + \left(3 + \frac{1}{\cancel{x}}\right)y = \frac{\cancel{e^{-3x}} \times \cancel{x} \times \cancel{e^{3x}}}{\cancel{x}}$$

$$x e^{3x}\frac{dy}{dx} + \frac{(3x+1)}{x}y = 1$$

$$\frac{d}{dx}\left(e^{3x}xy\right) = 1$$

Integrating b.s, we get,

$$\Rightarrow e^{3x} \cdot xy = x + c$$

$$\Rightarrow y = \frac{\cancel{x}}{e^{3x}\cancel{x}} + \frac{c}{e^{3x}x}$$

$$\Rightarrow y = e^{-3x} + \frac{c}{e^{3x}x}$$

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Q.31 :- $$\frac{dy}{dx} + y = \frac{1-e^{-2x}}{e^{x}+e^{-x}} \rightarrow (1)$$

$P(x) = 1$

So, $f(x) = e^{\int 1\,dx} = e^{x}$

Multiplying eq (1) b.s by $f(x)$, we get,

$$e^{x}\frac{dy}{dx} + e^{x}y = \frac{e^{x}(1-e^{-2x})}{e^{x}+e^{-x}}$$

$$\frac{d}{dx}\left(e^{x}y\right) = \frac{e^{x}-e^{-x}}{e^{x}+e^{-x}}$$

Integrating both sides, we get,

$$e^{x}\cdot y = \int \frac{e^{x}-e^{-x}}{e^{x}+e^{-x}}\,dx$$

$$e^{x}\cdot y = \ln|e^{x}+e^{-x}| + c$$

$$\Rightarrow \boxed{y = \frac{\ln|e^{x}+e^{-x}|}{e^{x}} + \frac{c}{e^{x}}}$$

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Q129.- $y\,dx - 4(x+y^6)\,dy = 0$

$$\frac{y}{4(x+y^6)} - \frac{dy}{dx} = 0$$

$$\frac{dy}{dx} - \frac{y}{4(x+y^6)} = 0$$

$$\Rightarrow \frac{dx}{dy} - \frac{4(x+y^6)}{y} = 0$$

$$\Rightarrow \frac{dx}{dy} - \frac{4x}{y} - \frac{4y^{\cancel{6}5}}{\cancel{y}} = 0$$

$$\Rightarrow dx/dy - \frac{4x}{y} = 4y^5 \longrightarrow (1)$$

So, $P(y) = -4/y$

So, $\mu(y) = e^{-4\int \frac{1}{y}dy}$

$$= e^{-4\ln|y|}$$

$$= e^{\ln(1/y^4)}$$

$$\Rightarrow \mu(y) = 1/y^4$$

Multiplying eq (1) b.s by "$\mu(y)$", we get

$$\frac{1}{y^4}\frac{dx}{dy} - \frac{4x}{y}\frac{1}{y^4} = \frac{4y^5}{y^4}$$

$$\Rightarrow \frac{d}{dy}\left(x/y^4\right) = 4y$$

Integrating both sides we get,

$$x/y^4 = \frac{\cancel{4}2y^2}{\cancel{2}} + c$$

$$\Rightarrow \boxed{x = 2y^6 + cy^4}$$

$y\,dx + (x + 2xy^2 - 2y)\,dy = 0$

Q133 :- $\Rightarrow \dfrac{dy}{dx} + \dfrac{y}{x + 2xy^2 - 2y} = 0$

$$\Rightarrow \frac{dx}{dy} + \frac{x + 2xy^2 - 2y}{y} = 0$$

$$\Rightarrow \frac{dx}{dy} + x/y + 2xy - 2 = 0$$

$$\Rightarrow \frac{dx}{dy} + x\left(\frac{1}{y} + 2y\right) = 2 \rightarrow (1)$$

So,

$$P(y) = 1/y + 2y$$

$$f(y) = e^{\int (1/y + 2y)\,dy}$$

$$= e^{\ln y + 2\frac{y^2}{2}}$$

$$= e^{\ln y} \cdot e^{y^2}$$

$$\Rightarrow f(y) = y e^{y^2}$$

Mlying eq (1) b.s by "f(y)", we get,

$$y e^{y^2} \frac{dx}{dy} + x\left(\frac{1}{y} + 2y\right) y e^{y^2} = 2y e^{y^2}$$

$$\frac{d}{dy}\left(x y e^{y^2}\right) = 2y e^{y^2}$$

Integrating b.s, we get,

$$x y e^{y^2} = 2\int y e^{y^2}\,dy \rightarrow (2)$$

Let, $y^2 = t \Rightarrow 2y\,dy = dt$

$\Rightarrow y\,dy = dt/2$

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$$\int \frac{d}{du}(xy)\,du = \int \frac{1}{u}\,du$$

$$\Rightarrow xy = \ln|u|$$

$$\Rightarrow \boxed{y = \frac{1}{x}\ln|u| + \frac{c}{x}}$$

Ans

Interval :-

Q.III:- $(x + 4y^2)\,dy + 2y\,dx = 0$

$$x + 4y^2 + 2y\,\frac{dx}{dy} = 0$$

$$x + 2y\,\frac{dx}{dy} = -4y^2$$

$$\text{or} \quad \frac{dx}{dy} + \frac{x}{2y} = -2y \quad \longrightarrow (1)$$

$$P(y) = \frac{1}{2y}$$

$$\mu(y) = e^{\int P(y)\,dy} = e^{\frac{1}{2}\int \frac{1}{y}\,dy}$$

$$= e^{\frac{1}{2}\ln y}$$

$$= e^{\ln y^{\frac{1}{2}}}$$

$$\mu(y) = y^{\frac{1}{2}}$$

Multiplying eq (1) by $\mu(y)$

Q.15:- $(1+e^x)\dfrac{dy}{dx} + e^x y = 0$ (3)

$$\Rightarrow \frac{dy}{dx} + \left(\frac{e^x}{1+e^x}\right) y = 0 \rightarrow (1)$$

So, $P(x) = \dfrac{e^x}{1+e^x}$

$$\mu(x) = e^{\int \frac{e^x}{1+e^x} dx}$$

$$= e^{\ln|1+e^x|}$$

$$\Rightarrow \mu(x) = 1+e^x$$

Multiplying eq (1) by "$\mu(x)$", we get,

$$(1+e^x)\frac{dy}{dx} + \frac{e^x \cancel{(1+e^x)}}{\cancel{1+e^x}} y = 0$$

$$\Rightarrow (1+e^x)\, dy/dx + e^x y = 0$$

$$\Rightarrow \frac{d}{dx}\left((1+e^x)y\right) = 0$$

Integrating both sides we get,

$$(1+e^x)y = c$$

or $\boxed{y = \dfrac{c}{1+e^x}}$ Ans

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Interval :-

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Q.9:-

$$x^2 y' + xy = 1$$

$$\Rightarrow x^2 \frac{dy}{dx} + xy = 1$$

$$\Rightarrow \frac{dy}{dx} + \frac{xy}{x^2} = \frac{1}{x^2}$$

$$\Rightarrow \frac{dy}{dx} + \frac{y}{x} = \frac{1}{x^2} \longrightarrow (1)$$

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$$P(x) = \frac{1}{x}$$

So,

$$\mu(x) = e^{\int P(x)dx}$$

$$= e^{\int \frac{1}{x} dx}$$

$$= e^{\ln x} = x$$

Multiplying eq (1) by "$\mu(x)$", we get,

$$x \frac{dy}{dx} + \frac{xy}{x} = \frac{x}{x^2}$$

$$\Rightarrow \frac{d}{dx}(x \cdot y) = \frac{1}{x}$$

Integrating both sides, we get

$f(x) = e^{x^3}$

Multiplying eq ① by e^{x^3}, we get

$$\frac{dy}{dx} e^{x^3} + 3x^2 y e^{x^3} = x^2$$

$$\Rightarrow \frac{d}{dx}\left(e^{x^3} \cdot y\right) = x^2 e^{x^3}$$

Integrating both sides, we get,

$$\int \frac{d}{dx}\left(e^{x^3} \cdot y\right) = \int x^2 e^{x^3} dx$$

$$e^{x^3} \cdot y = \int x^2 e^{x^3} dx \rightarrow ②$$

$\int x^2 e^{x^3} dx \rightarrow (*)$ Let, $t = x^3 \longrightarrow ①$

So, $\frac{dt}{dx} = 3x^2$

$$\Rightarrow dt/3 = x^2 dx \rightarrow (ii)$$

put eq ① (ii) in eq (*), we get,

$$\int x^2 e^{x^3} dx = \int \frac{dt}{3} \cdot e^t$$

$$= \frac{1}{3} e^t + c$$

$$\int x^2 e^{x^3} dx = \frac{1}{3} e^{x^3} + c$$

$$e^{x^3} \cdot y = \frac{1}{3} e^{x^3} + c$$

Put in eq ② we get

or $\boxed{y = \frac{1}{3} + c e^{-x^3}}$ Ans.

Q15:- $\frac{dy}{dx} + y = e^{3x} \longrightarrow (1)$

$P(x) = 1$

So, $\mu(x) = e^{\int P(x)dx}$

$= e^{\int 1 dx}$

$= e^{x}$

Multiplying eq (1) by "$\mu(x)$", we get

$$e^{x}\frac{dy}{dx} + ye^{x} = e^{x} \cdot e^{3x}$$

$$\Rightarrow \frac{d}{dx}(e^{x} \cdot y) = e^{4x}$$

Integrating both sides, we get:

$$\int \frac{d}{dx}(e^{x} \cdot y) = \int e^{4x}\, dx$$

$$\Rightarrow e^{x} \cdot y = \frac{e^{4x}}{4} + c$$

$$\Rightarrow y = \frac{e^{3x}}{4e^{x}} + \frac{c}{e^{x}}$$

$$\Rightarrow \boxed{y = \frac{1}{4}e^{3x} + \frac{c}{e^{x}}}$$

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Q17:- $\frac{dy}{dx} + 3x^{2}y = x^{2} \longrightarrow (1)$

$P(x) = 3x^{2}$

So,

$\mu(x) = e^{\int P(x)dx}$ $\quad 3\int x^{2}dx$

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Q.3:- $3\frac{dy}{dx} + 12y = 4$

$\Rightarrow \frac{dy}{dx} + 4y = 4/3 \rightarrow (1)$

So, $P(x) = 4$

$\mu(x) = e^{\int P(x)\,dx}$

$= e^{4\int 1\,dx}$

$= e^{4x}$

Multiplying eq (1) by "$\mu(x)$", we get,

$$e^{4x}\frac{dy}{dx} + 4ye^{4x} = 4/3\, e^{4x}$$

$$\Rightarrow \frac{d}{dx}\left(e^{4x} \cdot y\right) = 4/3\, e^{4x}$$

Taking Integration on both sides, we get,

$$\int \frac{d}{dx}\left(e^{4x} \cdot y\right) dx = 4/3 \int e^{4x}\, dx$$

$$\Rightarrow e^{4x} \cdot y = 4/3\, \frac{e^{4x}}{4} + c$$

$$\Rightarrow y = 1/3\, \frac{e^{4x}}{e^{4x}} + \frac{c}{e^{4x}}$$

or

$$\boxed{y = 1/3 + c/e^{4x}}$$

Interval :-

Q12

$$\Rightarrow \frac{dy}{dx} + y \cot x = \sec^2 x \operatorname{cosec} x$$

So, $p(x) = \cot x$

$$\mu(x) = e^{\int \cot x \, dx}$$

$$= e^{\ln|\sin x|}$$

$$\Rightarrow \mu(x) = \sin x$$

Multiplying eq ① by "$\mu(x)$", we get,

$$\sin x \frac{dy}{dx} + y \cot x \sin x = \sec^2 x \operatorname{cosec} x \sin x$$

$$\Rightarrow \sin x \frac{dy}{dx} + y \frac{\cos x}{\sin x} \sin x = \sec^2 x$$

$$\Rightarrow \frac{d}{dx}(\sin x \cdot y) = \sec^2 x$$

Integrating both sides, we get,

$$\sin x \cdot y = \int \sec^2 x \, dx$$

$$\Rightarrow \sin x \, y = \tan x + c$$

$$y = \frac{\sin x}{\cos x} \cdot \frac{1}{\sin x} + \frac{c}{\sin x}$$

$$\Rightarrow \boxed{y = \frac{1}{\cos x} + \frac{c}{\sin x}} \quad \text{Ans.}$$

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(8)

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x} \quad \text{(exact)}$$

verified.

So, $f(x,y)$ is an integrating factor of eq (1).

Now,

$$\frac{\partial f}{\partial x} = M = 2xy^2 + 3x^2 \rightarrow (2)$$

and

$$\frac{\partial f}{\partial y} = N = 2x^2y \rightarrow (3)$$

Integrate eq (2) partially w.r.t "x", we get,

$$f(x,y) = \frac{\cancel{2}x^2y^2}{\cancel{2}} + \frac{\cancel{3}x^3}{\cancel{3}} + h(y)$$

$$= x^2y^2 + x^3 + h(y) \rightarrow (4)$$

Differentiate eq (4) partially w.r.t "y" we get,

$$\frac{\partial f}{\partial y} = 2x^2y + h'(y) \rightarrow (5)$$

Comparing eq (3) and eq (5) we get

$$2x^2y = 2x^2y + h'(y)$$

$$\Rightarrow h'(y) = 0$$

or $h(y) = c$ put in eq (4) we get

$$f(x,y) = x^2y^2 + x^3 + c$$

Put $f(x,y) = 0$, we get

$$\boxed{x^2y^2 + x^3 = c}$$

Ans =

$\Rightarrow h'(y) = 0$

So, $h(y) = c$ put in eq (4), we get,

$f(x,y) = x^2y^2 \cos x + c$

Put $f(x,y) = 0$, we get,

$c = x^2y^2 \cos x$ Ans

Q.41:- $(2y^2 + 3x)\,dx + 2xy\,dy = 0$; $\rightarrow$ (1)

$\mu(x,y) = x$

Here, $M = 2y^2 + 3x$

$N = 2xy$

So, $\frac{\partial M}{\partial y} = 4y$

$\frac{\partial N}{\partial x} = 2y$

Since, $\frac{\partial M}{\partial y} \neq \partial N/\partial x$

So, eq (1) is not an exact diff eq.

To make it exact, multiplying eq (1) by $\mu(x,y)$, we get,

$(2xy^2 + 3x^2)\,dx + 2x^2y\,dy = 0$

Now,

$M = 2xy^2 + 3x^2$

$N = 2x^2y$

$\frac{\partial M}{\partial y} = 4xy$

and $\partial N/\partial x = 4xy$

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