DATE: _____ DAY: _____

EXERCISE # 2.1

Q(1-10)

Determine a region of the x-y plane for which the given differential equation would have a unique solution through a point (x_0, y_0) in the region.

1) $\frac{dy}{dx} = y^{2/3}$

Solution

$$f(x,y) = y^{2/3}$$

Taking Partial derivative w.r.t y on b.s

$$\frac{\partial f}{\partial y} = \frac{\partial y^{2/3}}{\partial y}$$

$$\because \frac{dx^n}{dx} = nx^{n-1}\frac{dx}{dx}$$

$$\frac{\partial f}{\partial y} = \frac{2}{3} y^{2/3 - 1} \frac{\partial y}{\partial y}$$

$$= \frac{2}{3} y^{-1/3}$$

$$= \frac{2}{3y^{1/3}}$$

$\Rightarrow$ Given differential equation will have a unique solution in any rectangular region of the plane where $y \neq 0$

i-e; $y > 0$ or $y < 0$

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DAY:

DATE:

3) $x\frac{dy}{dx} = y$

Solution

$$x\frac{dy}{dx} = y$$

$$\frac{dy}{dx} = \frac{y}{x}$$

$$f(x,y) = \frac{y}{x}$$

Taking partial derivative w.r.t y on b.s

$$\frac{\partial f}{\partial y} = \frac{\partial}{\partial y}\left(\frac{y}{x}\right)$$

$$\because \frac{d\,kx}{dx} = k\frac{dx}{dx}$$

where k is constant

$$\Rightarrow \frac{\partial f}{\partial y} = \frac{1}{x}\frac{\partial y}{\partial y}$$

$$= \frac{1}{x}$$

$\Rightarrow$ Given differential equation will have unique solution in any region where $x \neq 0$.

5) $y' = \frac{x^2}{4-y^2}$

Solution

$$\frac{dy}{dx} = \frac{x^2}{4-y^2}$$

$$f(x,y) = \frac{x^2}{4-y^2}$$

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TE: _____ DAY: _____

Taking partial derivative w.r.t y on b.s

$$\frac{\partial f}{\partial y} = \frac{\partial}{\partial y}\left[\frac{x^2}{4-y^2}\right]$$

$$\because \frac{d\,kx}{dx} = k\frac{dx}{dx}$$

$$= x^2 \frac{\partial}{\partial y}(4-y^2)^{-1}$$

$$\because \frac{dx^n}{dx} = nx^{n-1}\frac{dx}{dx}$$

$$\Rightarrow \frac{\partial f}{\partial y} = x^2 \cdot -1(4-y^2)^{-1-1}\frac{\partial}{\partial y}(4-y^2)$$

$$= -x^2(4-y^2)^{-2}\left[\frac{\partial 4}{\partial y} - \frac{\partial y^2}{\partial y}\right]$$

$$= -x^2(4-y^2)^{-2}\left[0 - 2y\right]$$

$$= +\frac{2x^2y}{(4-y^2)^2}$$

Let $(4-y^2)^2 = 0$

$4-y^2 = 0$

$y^2 = 4$

$y = \pm 2$

$\Rightarrow$ Given differential equation will have a unique solution in any region where $y<-2$, $-2<y<2$ or $y>2$

7) $(x^2+y^2)y' = y^2$

Solution

$$(x^2+y^2)y' = y^2$$

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DATE: _____ DAY: _____

$$\Rightarrow y' = \frac{y^2}{(x^2+y^2)}$$

$$f(x,y) = \frac{y^2}{(x^2+y^2)}$$

Taking partial derivative w.r.t to y

$$\frac{\partial f}{\partial y} = \frac{\partial}{\partial y}\left[\frac{y^2}{(x^2+y^2)}\right]$$

$$\because \frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)\frac{d}{dx}f(x) - f(x)\frac{d}{dx}g(x)}{[g(x)]^2}$$

$$\Rightarrow \frac{\partial f}{\partial y} = \frac{(x^2+y^2)\frac{\partial}{\partial y}y^2 - y^2\frac{\partial}{\partial y}(x^2+y^2)}{(x^2+y^2)^2}$$

$$= \frac{(x^2+y^2).2y - y^2\left(0 + \frac{\partial y^2}{\partial y}\right)}{(x^2+y^2)^2}$$

$$= \frac{2x^2y + 2y^3 - 0 - y^2(2y)}{(x^2+y^2)^2}$$

$$= \frac{2x^2y + \cancel{2y^3} - \cancel{2y^3}}{(x^2+y^2)^2}$$

$$= \frac{2x^2y}{(x^2+y^2)^2}$$

$\Rightarrow$ Given differential equation will have unique solution where $x \neq 0$ and $y \neq 0$.

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DATE: DAY:

13) By inspection determine a solution of the non-linear differential equation $y' = y^3$ satisfying $y(0)=0$. Is the solution unique?

Solution

Given IVP is

$$y' = y^3$$

$$\Rightarrow f(x,y) = y^3$$

Talking partial derivation on both sides

$$\frac{\partial f}{\partial y} = \frac{\partial y^3}{\partial y}$$

$$\because \frac{d x^n}{dx} = n x^{n-1} \frac{dx}{dx}$$

$$\frac{\partial f}{\partial y} = 3y^{3-1} \frac{\partial y}{\partial y}$$

$$= 3y^2$$

So, $f(x,y)$ and $\frac{\partial f}{\partial y}$ are continuous on entire xy-plane. Hence given IVP has unique solution at $(0,0)$.

19) Determine whether Theorem 2.1 guarantees that the differential equation $y' = \sqrt{y^2-9}$ possesses a unique solution through the given point. $(2,-3)$

Solution

Given that $\frac{dy}{dx} = \sqrt{y^2-9} \rightarrow (1)$

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DAY:

DATE:

from eq① $f(x,y) = \sqrt{y^2-9}$

Taking partial derivation on b.s

$$\frac{\partial f}{\partial y} = \frac{\partial}{\partial y}(y^2-9)^{\frac{1}{2}}$$

$$\because \frac{dx^n}{dx} = nx^{n-1}\frac{dx}{dx}$$

$$\Rightarrow \frac{\partial f}{\partial y} = \frac{1}{2}(y^2-9)^{\frac{1}{2}-1}\frac{d}{dy}(y^2-9)$$

$$= \frac{1}{2}(y^2-9)^{-\frac{1}{2}}\left[\frac{dy^2}{dy} - \frac{d9}{dy}\right]$$

$$= \frac{1}{2(y^2-9)^{\frac{1}{2}}}[2y-0]$$

$$= \frac{\cancel{2}y}{\cancel{2}\sqrt{y^2-9}}$$

$$= \frac{y}{\sqrt{y^2-9}}$$

So, The differential equation is not guaranteed to have a unique solution at $(2,-3)$, as for unique solution $y<-3$ or $y>3$, while given region is $(2,-3)$ that not lies in the region where differential equation will have a unique solution.

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DATE: _____ DAY: _____

EXERCISE # 2.2

Q(1-40)

Solve the given differential by separation variables.

2) $\frac{dy}{dx} = (x+1)^2$

Solution

$$\frac{dy}{dx} = (x+1)^2$$

$$dy = (x+1)^2 dx$$

Integrating both sides, we get

$$\int dy = \int (x+1)^2 dx$$

$$\because \int [f(x)]^n f'(x) dx = \frac{[f(x)]^{n+1}}{n+1} + c$$

$$y = \frac{(x+1)^{2+1}}{2+1} + c$$

$$y = \frac{1}{3}(x+1)^3 + c$$ Answer

7) $xy' = 4y$

Solution

$$x\frac{dy}{dx} = 4y$$

$$\frac{1}{4y} dy = \frac{1}{x} dx$$

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DATE: _____ DAY: _____

Integrating both sides, we get

$$\int \frac{1}{4y}\,dy = \int \frac{1}{x}\,dx$$

$$\frac{1}{4}\int \frac{1}{y}\,dy = \int \frac{1}{x}\,dx$$

$$\because \int x^{-1}dx = \ln x + c$$

$$\frac{1}{4}\ln|y| = \ln|x| + \ln|c|$$

$$\because \ln|m| + \ln|n| = \ln|mn|$$

$$\because n\ln m = \ln m^n$$

$$\ln|y| = 4\ln|xc|$$

$$\ln|y| = \ln|(xc)^4|$$

$$y = (xc)^4$$

$$\Rightarrow \quad y = cx^4 \quad \text{Answer}$$

13) $\frac{dy}{dx} = e^{3x+2y}$

Solution

$$\frac{dy}{dx} = e^{3x+2y}$$

$$dy = e^{3x}\cdot e^{2y}\,dx$$

$$\frac{1}{e^{2y}}\,dy = e^{3x}\,dx$$

$$e^{-2y}\,dy = e^{3x}\,dx$$

Integrating both sides, we get

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DATE: _____ DAY: _____

$$\int e^{-2y}\,dy = \int e^{3x}\,dx$$

$$-\frac{1}{2}\int e^{-2y}\cdot -2\,dy = \frac{1}{3}\int e^{3x}\cdot 3\,dx$$

$$\because \int e^{f(x)}\, f'(x).\,dx = e^{f(x)} + c$$

$$-\frac{1}{2}\, e^{-2y} = \frac{1}{3}\, e^{3x} + c$$

$$\Rightarrow \quad c = \frac{1}{3} e^{3x} + \frac{1}{2} e^{-2y}$$

$$c = 2e^{3x} + 3e^{-2y}$$ Answer.

19) $$y\ln|x|\frac{dx}{dy} = \left(\frac{y+1}{x}\right)^2$$

Solution

$$y\ln|x|\frac{dx}{dy} = \frac{(y+1)^2}{x^2}$$

$$\because (a+b)^2 = a^2+b^2+2ab$$

$$y\ln|x|\frac{dx}{dy} = \frac{y^2+1^2+2(y)(1)}{x^2}$$

$$x^2\ln|x|\,dx = \left(\frac{y^2+1+2y}{y}\right)dy$$

$$x^2\ln|x|\,dx = \left(y+\frac{1}{y}+2\right)dy$$

Integrating both sides, we get

$$\int x^2\ln|x|\,dx = \int y\,dy + \int \frac{1}{y}\,dy + 2\int 1\,dy$$

$\downarrow$

Apply integration by part, 1st function = $\ln|x|$ and 2nd function = x^2

$$\because \int 1^{st}\text{fun.}\,2^{nd}\text{func} = 1^{st}\int 2^{nd}dx - \int\left[\int 2^{nd}dx\,\frac{d1^{st}}{dx}\right]dx$$

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DATE: _____ DAY: _____

$$\ln|x| \int x^2 dx - \int\left[\int x^2 dx \cdot \frac{d \ln x}{dx}\right] dx = \int y\,dy + \int \frac{1}{y} dy + 2\int dy$$

$$\because \int x^n dx = \frac{x^{n+1}}{n+1} + c$$

$$\because \int \frac{1}{x} dx = \ln|x| + c$$

$$\Rightarrow \ln|x| \cdot \frac{x^3}{3} - \int \frac{x^3}{3} \cdot \frac{1}{x} dx = \frac{y^2}{2} + \ln|y| + 2y + c$$

$$\frac{x^3}{3} \ln|x| - \frac{1}{3} \int x^2 dx = \frac{y^2}{2} + \ln|y| + 2y + c$$

$$\frac{x^3}{3} \ln|x| - \frac{1}{3} \cdot \frac{x^3}{3} = \frac{y^2}{2} + \ln|y| + 2y + c$$

$$\Rightarrow \frac{y^2}{2} + 2y + \ln|y| = \frac{x^3}{3} \ln|x| - \frac{x^3}{9} + c$$

Answer

29) $(e^y+1)^2 e^{-y} dx + (e^x+1)^3 e^{-x} dy = 0$

Solution

$$\frac{(e^y+1)^2}{e^y} dx + \frac{(e^x+1)^3}{e^x} dy = 0$$

$$\frac{(e^y+1)^2}{e^y} dx = -\frac{(e^x+1)^3}{e^x} dy$$

$$\frac{e^x}{(e^x+1)^3} dx = -\frac{e^y}{(e^y+1)^2} dy$$

Integrating both sides, we get

$$\int (e^x+1)^{-3} e^x dx = -\int (e^y+1)^{-2} e^y dy$$

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DATE: _____ DAY: _____

$$\because \int [f(x)]^n f'(x)\,dx = \frac{[f(x)]^{n+1}}{n+1} + c ; n \neq -1$$

$$\frac{(e^x+1)^{-3+1}}{-3+1} = -\frac{(e^y+1)^{-2+1}}{-2+1} + c$$

$$\frac{(e^x+1)^{-2}}{-2} = -\frac{(e^y+1)^{-1}}{-1} + c$$

$$\Rightarrow \quad c = 2(e^y+1)^{-1} + (e^x+1)^{-2}$$ Answer.

37) $x\sqrt{1-y^2}\,dx = dy$

Solution

$$x\sqrt{1-y^2}\,dx = dy$$

$$\Rightarrow \quad x\,dx = \frac{1}{\sqrt{1-y^2}}\,dy$$

Integrating both sides, we get

$$\int x\,dx = \int \frac{1}{\sqrt{1-y^2}}\,dy \rightarrow (1)$$

$$\because \int x^n\,dx = \frac{x^{n+1}}{n+1} + c$$

$$\because \int \frac{1}{\sqrt{1-x^2}}\,dx = \sin^{-1}x + c$$

eq. (1) $\Rightarrow$ $$\int x\,dx = \int \frac{1}{\sqrt{1-y^2}}\,dy$$

$$\frac{x^2}{2} = \sin^{-1}y + c$$

$$\sin^{-1}y = \frac{x^2}{2} + c$$

$$\Rightarrow y = \sin\left(\frac{x^2}{2} + c\right)$$ Answer.

43) Solve the given differential equation to the indicated initial condition.

$y\,dy = 4x(y^2+1)^{1/2}\,dx, \quad y(0) = 1$

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DATE: _____ DAY: _____

Solution

$$\frac{y}{(y^2+1)^{1/2}}\,dy = 4x\,dx$$

Integrating both sides, we get

$$\int \frac{y}{(y^2+1)^{1/2}}\,dy = 4\int x\,dx$$

$$\Rightarrow \frac{1}{2}\int \frac{2y}{(y^2+1)^{1/2}}\,dy = 4\int x\,dx$$

$$\frac{1}{2}\int (y^2+1)^{-1/2}\,2y\,dx = 4\int x\,dx$$

$$\because \int [f(x)]^n f'(x)\,dx = \frac{[f(x)]^{n+1}}{n+1} + c;\ n \neq -1$$

$$\frac{1}{\cancel{2}}\,\frac{(y^2+1)^{-\frac{1}{2}+1}}{\cancel{-\frac{1}{2}+1}} = 2x^2 + c$$

$$(y^2+1)^{1/2} = 2x^2 + c \rightarrow (1)$$

As $y(0) = 1$, Put $(x,y) = (0,1)$ in equation (1)

eq (1) $\Rightarrow$

$$((1)^2+1)^{1/2} = 2(0)^2 + c$$

$$\Rightarrow c = (2)^{1/2}$$

$$c = \sqrt{2}$$

Put value of c in equation (1)

$$(y^2+1)^{1/2} = 2x^2 + \sqrt{2} \quad \text{Ans}$$

53) Compare the solutions of the given initial-value problem.

$$\frac{dy}{dx} = (y-1)^2 ; \quad y(0) = 1$$

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DATE: DAY:

Solution

$$\frac{dy}{dx} = (y-1)^2$$

$$\frac{1}{(y-1)^2}\,dy = dx$$

Taking integral on both sides, we get

$$\int (y-1)^{-2}\,dy = \int dx$$

$$\because \int [f(x)]^n f'(x)\,dx = \frac{[f(x)]^{n+1}}{n+1} + c$$

$$, \; n \neq -1$$

$$\frac{(y-1)^{-2+1}}{-2+1} = x + c$$

$$\frac{(y-1)^{-1}}{-1} = x + c$$

$$\Rightarrow -(y-1)^{-1} = x + c$$

$$\frac{-1}{(y-1)} = x + c \rightarrow (1)$$

$$-1 = (x+c)(y-1)$$

$$-1 = xy - x + cy - c$$

$$-1 = y(x+c) - (x+c)$$

$$\frac{-1}{(x+c)} = y - 1$$

$$\frac{-1}{x+c} + 1 = y$$

$$\frac{-1 + x + c}{x+c} = y \rightarrow (2)$$

From eq (1)

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DATE: _____ DAY: _____

$$c = \frac{-1}{y-1} - x$$

Put $(x,y) = (0,1)$ we get

$$c = \frac{-1}{1-1} - 0$$

$$c = \frac{-1}{0} - 0$$

$$c = \infty.$$

Put value of c in eq ②

$$y = \frac{-1+(x+\infty)}{(x+\infty)}$$

$$= \frac{x+\infty}{x+\infty} - \frac{1}{x+\infty}$$

$$= 1 - \frac{1}{\infty}$$

$$= 1 - 0$$

$$\Rightarrow y = 1$$

So $y = 1$ is the only solution.

61) $\frac{dy}{dx} = 2+\sqrt{y-2x+3} \rightarrow ①$

Solution

Let $u = y - 2x + 3$

$$\frac{du}{dx} = \frac{dy}{dx} - 2\frac{dx}{dx} + \frac{d3}{dx}$$

$$\frac{dy}{dx} = \frac{du}{dx} + 2$$

So equation ① becomes

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DATE: _____ DAY: _____

$$\frac{du}{dx} + \cancel{2} = \cancel{2} + \sqrt{u}$$

$$\frac{du}{dx} = \sqrt{u}$$

$$\frac{1}{\sqrt{u}}\, du = dx$$

$$(u)^{-\frac{1}{2}}\, du = dx$$

Taking integration on b.s., we get

$$\int (u)^{-\frac{1}{2}}\, du = \int dx$$

$$\because \int [f(x)]^n f'(x)\, dx = \frac{[f(x)]^{n+1}}{n+1} + c$$

$$\frac{(u)^{-\frac{1}{2}+1}}{-\frac{1}{2}+1} = x + c$$

$$\frac{(u)^{\frac{1}{2}}}{\frac{1}{2}} = x + c$$

$$2\sqrt{u} = x + c$$

Put value of u in above equation.

$$2\sqrt{y-2x+3} = x + c \qquad \text{Ans}$$

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DATE: DAY:

EXERCISE # 2.3

Q 1-10) Determine whether the given function is homogenous. If so, state the degree of homogeneity

2) $\sqrt{x+y}\,(4x+3y)$

Solution

Given,

$$f(x,y) = \sqrt{x+y}\,(4x+3y) \rightarrow (1)$$

Put $(x,y) = (tx, ty)$ in eq (1)

$$f(tx, ty) = \sqrt{tx+ty}\,(4tx+3ty)$$

$$f(tx, ty) = \sqrt{t}\,(\sqrt{x+y})\; t(4x+3y)$$

$$= t^{\frac{1}{2}+1}\,(\sqrt{x+y}\,(4x+3y))$$

$$f(tx, ty) = t^{3/2}\,(f(x,y))$$

So, the given function is homogenous of degree $\frac{3}{2}$

7) $\ln|x^2| - 2\ln|y|$

Solution

Given

$$f(x,y) = \ln|x^2| - 2\ln|y|$$

$\because \ln|m^n| = n\ln|m|$

$\because \ln|m| - \ln|n| = \ln\left|\frac{m}{n}\right|$

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DATE: DAY:

$$\Rightarrow f(x,y) = 2\ln|x| - 2\ln|y|$$
$$= 2(\ln|x| - \ln|y|)$$
$$= 2\ln\left|\frac{x}{y}\right| \rightarrow (1)$$

Put $(x,y) = (tx, ty)$ in eq(1)

$$f(tx, ty) = 2\ln\left|\frac{tx}{ty}\right|$$
$$= 2\ln\left|\frac{x}{y}\right|$$
$$= t^0\, 2(\ln|x| - \ln|y|)$$
$$= t^0\,(2\ln|x| - 2\ln|y|)$$
$$= t^0\,(\ln|x^2| - 2\ln|y|)$$
$$\Rightarrow f(tx, ty) = t^0\,(f(x,y))$$

So, given function is homogenous of degree 0.

11-30) Solve the given function by using an appropiate substitution

13) $xdx + (y-2x)dy = 0$

Solution

Given differential equation is

$$xdx + (y-2x)dy = 0 \rightarrow (1)$$

From eq(1) we have

$M(x,y) = x$ and $N(x,y) = y-2x$

Let $x = vy \rightarrow (i)$

$dx = v\,dy + y\,dv \rightarrow (ii)$

By using (i) and (ii) in eq(1), we have

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DATE: _____ DAY: _____

$$vy\,dx + (y - 2vy)\,dy = 0$$

$$vy\,(v\,dy + y\,dv) + (y - 2vy)\,dy = 0$$

$$v^2 y\,dy + vy^2 dv + (y - 2vy)dy = 0$$

$$v^2 y\,dy + vy^2 dv + y\,dy - 2vy\,dy = 0$$

$$vy^2 dv + (v^2 y + y - 2vy)\,dy = 0$$

$$(v^2 y + y - 2vy)dy = -vy^2 dv$$

$$y\,(v^2 + 1 - 2v)\,dy = -vy^2 dv$$

$$\frac{y\,(v^2 + 1 - 2v)}{y^2}\,dy = -v\,dv$$

$$\frac{1}{y}\,dy = \frac{-v}{v^2 + 1 - 2v}\,dv$$

Taking integral on b.s

$$\int \frac{1}{y}\,dy = -\int \frac{v}{(v-1)^2}\,dv$$

$$\because \int x^{-1} dx = \ln|x| + c$$

$$\Rightarrow \ln|y| + c = -\int \frac{v}{(v-1)^2}\,dv \rightarrow (2)$$

Take $\frac{v}{(v-1)^2}$ and solve by partial fraction

$$\frac{v}{(v-1)^2} = \frac{A}{v-1} + \frac{B}{(v-1)^2} \rightarrow (a)$$

Xing $(v-1)^2$ on b.s of eq (a)

$$v = A\,(v-1) + B \rightarrow (b)$$

put $v = 1$ in eq (b)

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DATE: _____ DAY: _____

$$\frac{du}{dx} + \cancel{2} = \cancel{2} + \sqrt{u}$$

$$\frac{du}{dx} = \sqrt{u}$$

$$\frac{1}{\sqrt{u}}\, du = dx$$

$$(u)^{-\frac{1}{2}}\, du = dx$$

Taking integration on b.s., we get

$$\int (u)^{-\frac{1}{2}}\, du = \int dx$$

$$\because \int [f(x)]^n f'(x)\, dx = \frac{[f(x)]^{n+1}}{n+1} + c$$

$$\frac{(u)^{-\frac{1}{2}+1}}{-\frac{1}{2}+1} = x + c$$

$$\frac{(u)^{\frac{1}{2}}}{\frac{1}{2}} = x + c$$

$$2\sqrt{u} = x + c$$

Put value of u in above equation.

$$2\sqrt{y-2x+3} = x + c \quad \text{Ans}$$

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DATE: _____ DAY: _____

$$1 = A(1-1) + B$$

$$\Rightarrow \boxed{B = 1}$$

Put $B = 1$ and $v = 0$ in eq (b)

$$0 = A(0-1) + 1$$

$$\Rightarrow 0 = -A + 1$$

$$\Rightarrow \boxed{A = 1}$$

By Putting $A = 1$ and $B = 1$ in eq (a), we get.

$$\frac{v}{(v-1)^2} = \frac{A}{v-1} + \frac{B}{(v-1)^2}$$

$$= \frac{1}{v-1} + \frac{1}{(v-1)^2}$$

Taking $\int$ on b.s

$$\int \frac{v}{(v-1)^2}\,dv = \int \frac{1}{v-1}\,dv + \int (v-1)^{-2}\,dv$$

$$\because \int [f(x)]^n f'(x)\,dx = \frac{[f(x)]^{n+1}}{n+1} + c$$

$$\because \int \frac{f'(x)}{f(x)}\,dx = \ln|f(x)| + c$$

$$\Rightarrow \int \frac{v}{(v-1)^2}\,dv = \ln|v-1| + \frac{(v-1)^{-2+1}}{-2+1} + c$$

$$= \ln|v-1| - \frac{1}{(v-1)} + c$$

so eq (2) becomes

$$\ln|y| + c = -\left[\ln|v-1| - \frac{1}{(v-1)}\right]$$

$$\Rightarrow \ln|y| + c = -\ln|v-1| + \frac{1}{v-1}$$

$$\ln|y| + \ln|v-1| = \frac{1}{v-1} + c$$

Scanned with CamScanner

DATE:

DAY:

$\because \ln|m| + \ln|n| = \ln|mn|$

$$\ln|(y)(v-1)| = \frac{1}{v-1} + c$$

Put $v = \frac{x}{y}$

$$\ln\left|y\cdot\frac{x}{y} - y\right| = \frac{1}{\frac{x}{y}-1} + c$$

$$\ln|x-y| = \frac{1}{\frac{x-y}{y}} + c$$

$$\Rightarrow \ln|x-y| = \frac{y}{x-y} + c$$

$$(x-y)\ln|x-y| = y + c(x-y)$$

$$(x-y)\ln|x-y| - y = c(x-y)$$ Ans.

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