Artificial Intelligence

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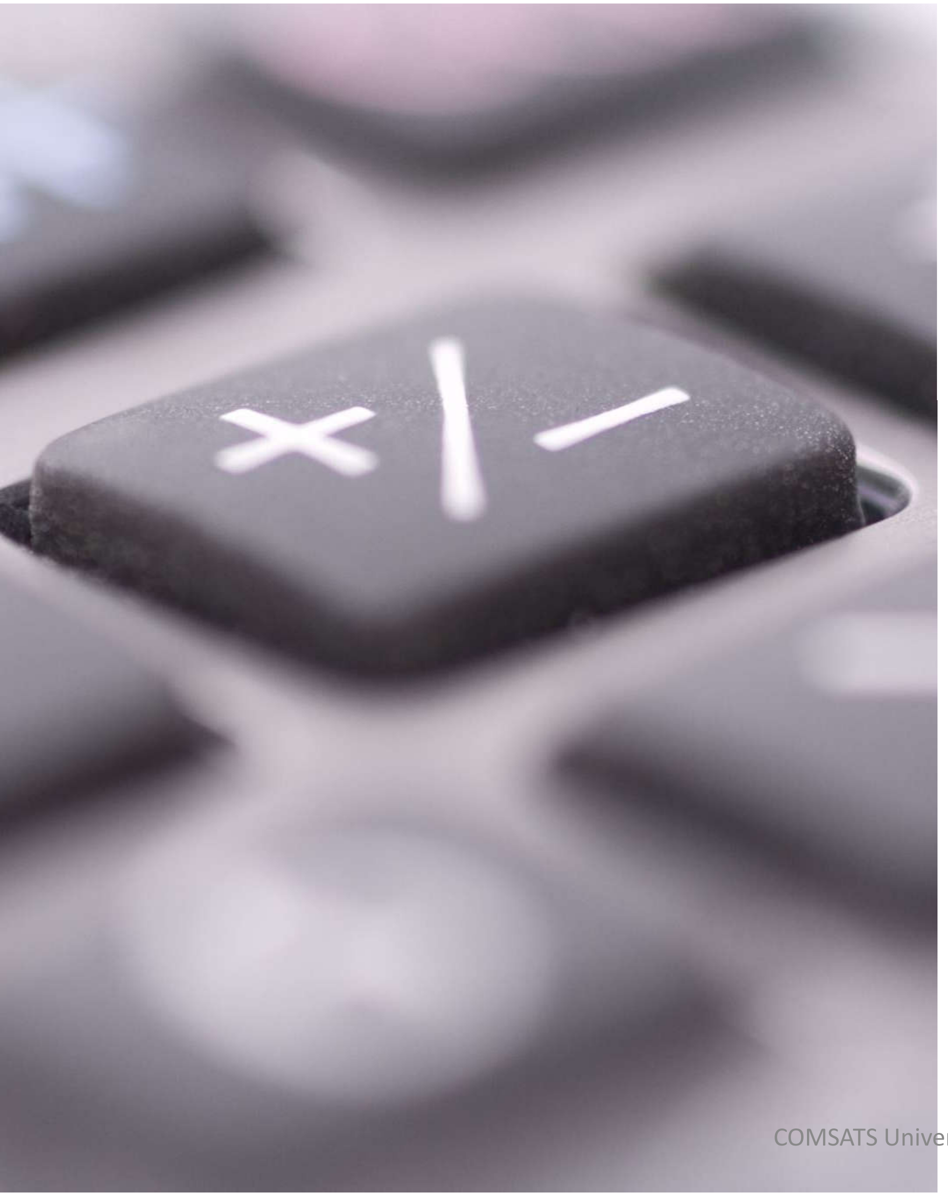


Supervised Learning

- Supervised learning is the type of machine learning in which machines are trained using labeled” training data, and on the basis of that data, machines predict the output. The labeled data means some input data is already tagged with the correct output.
- In supervised learning, the training data provided to the machines work as the supervisor that teaches the machines to predict the output correctly. It applies the same concept as a student learns in the teacher’s supervision.
- Supervised learning is a process of providing input data and correct output data to the machine learning model. The aim of a supervised learning algorithm is to **find a mapping function to map the input variable(x) with the output variable(y)**.

Classification

- Classification algorithms are used when the output variable is categorical, which means there are two classes Yes-No, Male-Female, True-false, etc. Spam Filtering,
- Decision Trees
- Support Vector Machines
- Naive Bayes
- KNN
- Logistic Regression
- Artificial Neural Networks

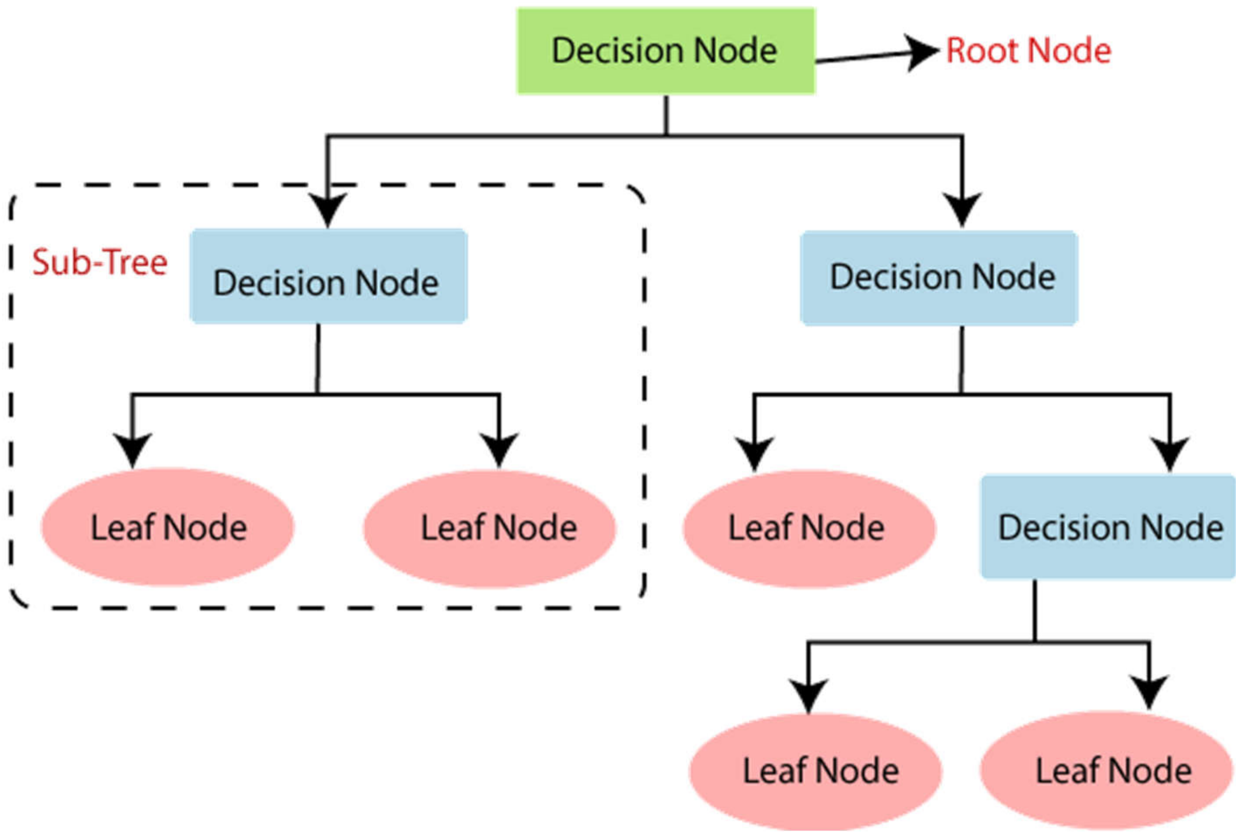
Decision Tree Classification Algorithm

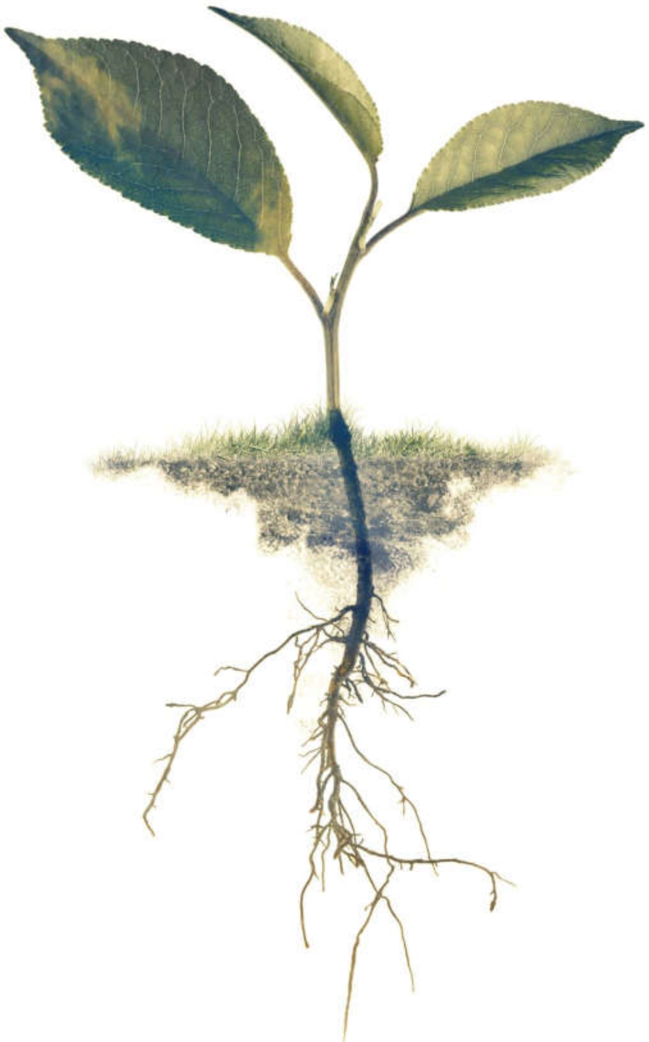
- Decision Tree is a **Supervised learning technique** that can be used for both classification and Regression problems, but mostly it is preferred for solving Classification problems.
- It is a tree-structured classifier, where **internal nodes represent the features of a dataset, branches represent the decision rules** and **each leaf node represents the outcome.**
- In a Decision tree, there are two nodes, which are the **Decision Node** and **Leaf Node.** Decision nodes are used to make any decision and have multiple branches, whereas Leaf nodes are the output of those decisions and do not contain any further branches.

Decision Tree Classification Algorithm

- The decisions or the test are performed on the basis of features of the given dataset.
- Similar to a tree, it starts with the root node, which expands on further branches and constructs a tree-like structure.
- A decision tree simply asks a question, and based on the answer (Yes/No), it further splits the tree into subtrees.

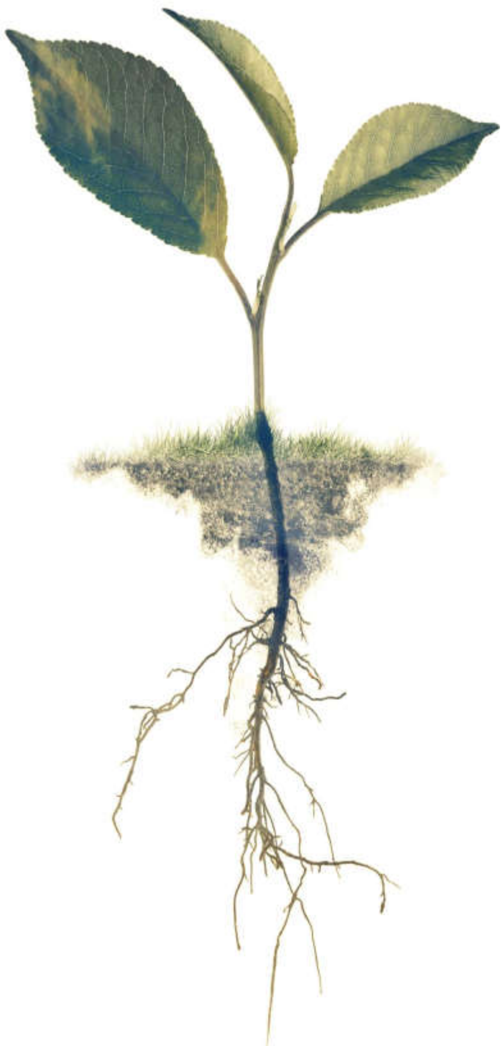
Decision Tree Classification Algorithm





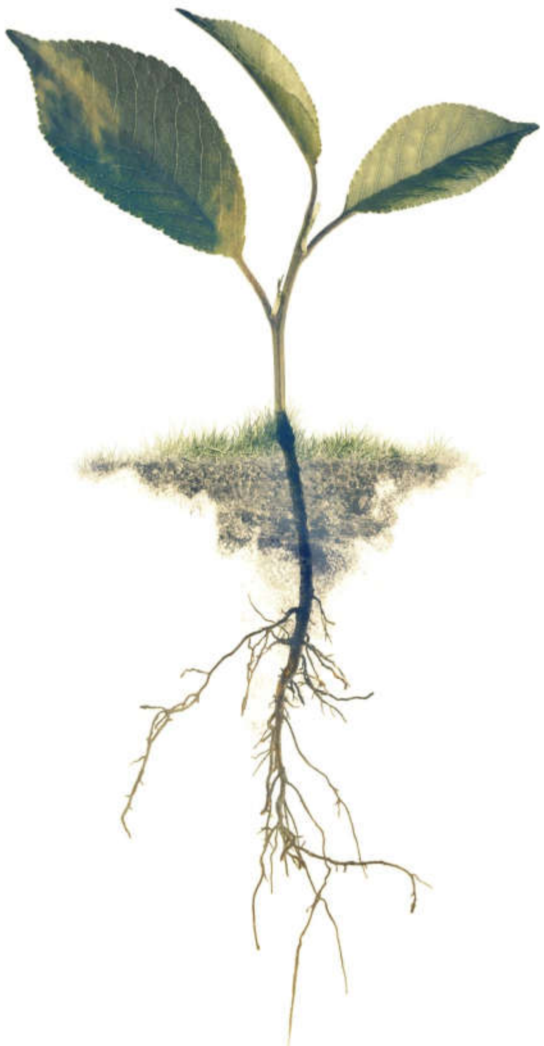
Decision Tree Terminologies

- Root Node: Root node is from where the decision tree starts. It represents the entire dataset, which further gets divided into two or more homogeneous sets.
- Leaf Node: Leaf nodes are the final output node, and the tree cannot be segregated further after getting a leaf node.
- Splitting: Splitting is the process of dividing the decision node/root node into sub-nodes according to the given conditions.
- Branch/Sub Tree: A tree formed by splitting the tree.
- Pruning: Pruning is the process of removing the unwanted branches from the tree.
- Parent/Child node: The root node of the tree is called the parent node, and other nodes are called the child nodes.



How does the Decision Tree algorithm Work?

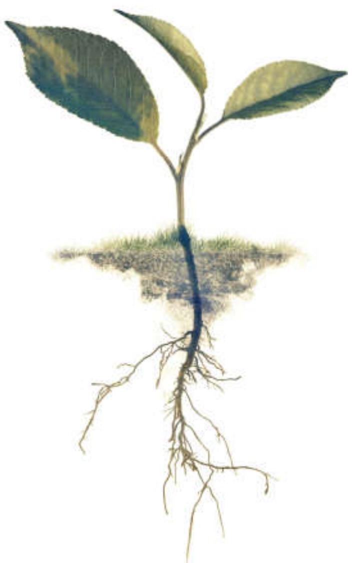
- In a decision tree, for predicting the class of the given dataset, the algorithm starts from the root node of the tree. This algorithm compares the values of the root attribute with the record (real dataset) attribute and, based on the comparison, follows the branch and jumps to the next node.
- For the next node, the algorithm again compares the attribute value with the other sub-nodes and move further. It continues the process until it reaches the leaf node of the tree. The complete process can be better understood using the below algorithm:



How does the Decision Tree algorithm Work?

- **Step-1:** Begin the tree with the root node, says S, which contains the complete dataset.
- **Step-2:** Find the best attribute in the dataset using **Attribute Selection Measure (ASM).**
- **Step-3:** Divide the S into subsets that contains possible values for the best attributes.
- **Step-4:** Generate the decision tree node, which contains the best attribute.
- **Step-5:** Recursively make new decision trees using the subsets of the dataset created in step -3. Continue this process until a stage is reached where you cannot further classify the nodes and called the final node as a leaf node.

Decision Tree ID3 Numerical Example 1



- How to find the Entropy
- Information Gain
- Gini Index
- Splitting Attributes

Instance	a_1	a_2	Target Class
1	T	T	+
2	T	T	+
3	T	F	−
4	F	F	+
5	F	T	−
6	F	T	−
7	F	F	−
8	T	F	+
9	F	T	−

Decision Tree ID3 Numerical Example 1

Instance	a_1	a_2	Target Class
1	T	T	+
2	T	T	+
3	T	F	−
4	F	F	+
5	F	T	−
6	F	T	−
7	F	F	−
8	T	F	+
9	F	T	−

- How to find the Entropy

1. The entropy of the training examples is

$$Entropy(S) = -\sum_{i=1}^{n} p_i \log_2(p_i)$$

$$Entropy(S) = -\frac{4}{9}\log_2\left(\frac{4}{9}\right) - \frac{5}{9}\log_2\left(\frac{5}{9}\right)$$

$$= 0.9911$$

$$\log_2 a = \frac{\log a}{\log 2}$$

Decision Tree ID3 Numerical Example 1

Instance	a_1	a_2	Target Class
1	T	T	+
2	T	T	+
3	T	F	−
4	F	F	+
5	F	T	−
6	F	T	−
7	F	F	−
8	T	F	+
9	F	T	−

- Information Gain

2. What is the information gain of the a1 with respect to the training examples.

$$Entropy(S) = -\sum_{i=1}^{n} p_i \log_2(p_i)$$

$$Entropy(S_T) = -\frac{3}{4}\log_2\left(\frac{3}{4}\right) - \frac{1}{4}\log_2\left(\frac{1}{4}\right)$$

$$= 0.311 + 0.5 = 0.811$$

$$Entropy(S_F) = -\frac{1}{5}\log_2\left(\frac{1}{5}\right) - \frac{4}{5}\log_2\left(\frac{4}{5}\right)$$

$$= 0.4644 + 0.2576 = 0.722$$

$$Gain\ (a_1) = Entropy(S) - \sum_{v\in\{T,F\}} \frac{|S_v|}{|S|} Entropy(S_v)$$

$$Gain\ (a_1) = Entropy(S) - \frac{4}{9}Entropy(S_T) - \frac{5}{9}Entropy(S_F)$$

$$Gain\ (a_1) = 0.9911 - \frac{4}{9} * 0.811 - \frac{5}{9} * 0.722 = 0.2295$$

Decision Tree ID3 Numerical Example 1

Instance	a_1	a_2	Target Class
1	T	T	+
2	T	T	+
3	T	F	−
4	F	F	+
5	F	T	−
6	F	T	−
7	F	F	−
8	T	F	+
9	F	T	−

- Information Gain

2. What is the information gain of the a2 with respect to the training examples.

$$Entropy(S) = -\sum_{i=1}^{n} p_i \log_2(p_i)$$

$$Entropy(S_T) = -\frac{2}{5}\log_2\left(\frac{2}{5}\right) - \frac{3}{5}\log_2\left(\frac{3}{5}\right)$$

$$= 0.5288 + 0.4421 = 0.9709$$

$$Entropy(S_F) = -\frac{2}{4}\log_2\left(\frac{2}{4}\right) - \frac{2}{4}\log_2\left(\frac{2}{4}\right)$$

$$= 0.5 + 0.5 = 1.0$$

$$\boldsymbol{Gain\ (a_2) = Entropy(S) - \sum_{v \in \{T,F\}} \frac{|S_v|}{|S|} Entropy(S_v)}$$

$$\boldsymbol{Gain\ (a_2) = Entropy(S) - \frac{5}{9} Entropy(S_T)}$$

$$\boldsymbol{- \frac{4}{9} Entropy(S_F)}$$

$$\boldsymbol{Gain\ (a_2) = 0.9911 - \frac{5}{9} * 0.9709 - \frac{4}{9} * 1.0 = 0.0072}$$

Decision Tree ID3 Numerical Example 1

Instance	a_1	a_2	Target Class
1	T	T	+
2	T	T	+
3	T	F	−
4	F	F	+
5	F	T	−
6	F	T	−
7	F	F	−
8	T	F	+
9	F	T	−

- Gini Index

3. Compute the Gini Index of the attributes a1.

$$Gini = 1 - \sum_{i=1}^{n} (p_i)^2$$

$$Gini(T) = 1 - \left(\frac{3}{4}\right)^2 - \left(\frac{1}{4}\right)^2 = 0.375$$

$$Gini(F) = 1 - \left(\frac{1}{5}\right)^2 - \left(\frac{4}{5}\right)^2 = 0.32$$

$$GiniIndex(a_1) = \sum_{v \in \{T,F\}} \frac{|S_v|}{|S|} Gini(S_v)$$

$$GiniIndex(a_1) = \left(\frac{4}{9}\right) * Gini(T) + \left(\frac{5}{9}\right) * Gini(F)$$

$$GiniIndex(a_1) = \left(\frac{4}{9}\right) * 0.375 + \left(\frac{5}{9}\right) * 0.32$$

$$GiniIndex(a_1) = 0.3444$$

Decision Tree ID3 Numerical Example 1

Instance	a_1	a_2	Target Class
1	T	T	+
2	T	T	+
3	T	F	−
4	F	F	+
5	F	T	−
6	F	T	−
7	F	F	−
8	T	F	+
9	F	T	−

Compute the Gini Index of the attributes a2.

$$Gini = 1 - \sum_{i=1}^{n} (p_i)^2$$

$$Gini(T) = 1 - \left(\frac{2}{5}\right)^2 - \left(\frac{3}{5}\right)^2 = 0.48$$

$$Gini(F) = 1 - \left(\frac{2}{4}\right)^2 - \left(\frac{2}{4}\right)^2 = 0.5$$

$$GiniIndex(a_2) = \sum_{v \in \{T,F\}} \frac{|S_v|}{|S|} Gini(S_v)$$

$$GiniIndex(a_2) = \left(\frac{5}{9}\right) * Gini(T) + \left(\frac{4}{9}\right) * Gini(F)$$

$$GiniIndex(a_2) = \left(\frac{5}{9}\right) * 0.48 + \left(\frac{4}{9}\right) * 0.5$$

$$GiniIndex(a_2) = 0.4889$$

Decision Tree ID3 Numerical Example 1

Instance	a_1	a_2	Target Class
1	T	T	+
2	T	T	+
3	T	F	−
4	F	F	+
5	F	T	−
6	F	T	−
7	F	F	−
8	T	F	+
9	F	T	−

4. Which is the best splitting attribute between a1 and a2.

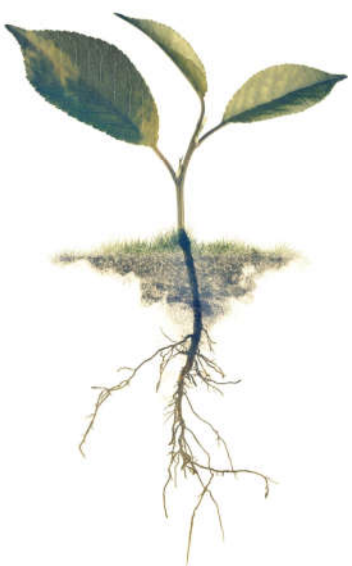
$$GiniIndex(a_1) = 0.3444$$

$$GiniIndex(a_2) = 0.4889$$

Smaller GiniIndex Produces Better Split

Hence, attribute **a1** is the best split attribute

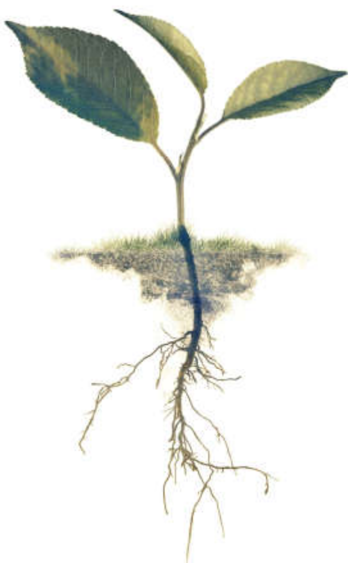
Decision Tree ID3 Numerical Example 2



- Data set
- For instance, the following table informs about decision-making factors for playing tennis outside for the previous 14 days. Play tennis is the target variable.

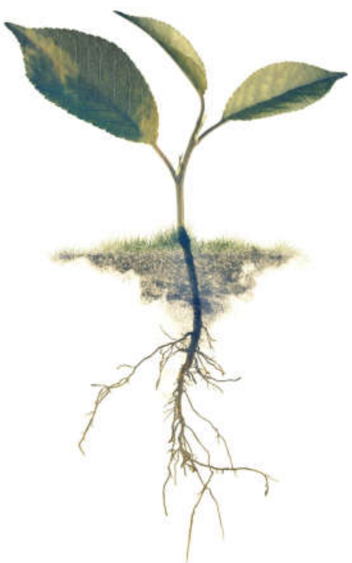
Day	Outlook	Temp	Humidity	Wind	Decision
D1	Sunny	Hot	High	Weak	No
D2	Sunny	Hot	High	Strong	No
D3	Overcast	Hot	High	Weak	Yes
D4	Rain	Mild	High	Weak	Yes
D5	Rain	Cool	Normal	Weak	Yes
D6	Rain	Cool	Normal	Strong	No
D7	Overcast	Cool	Normal	Strong	Yes
D8	Sunny	Mild	High	Weak	No
D9	Sunny	Cool	Normal	Weak	Yes
D10	Rain	Mild	Normal	Weak	Yes
D11	Sunny	Mild	Normal	Strong	Yes
D12	Overcast	Mild	High	Strong	Yes
D13	Overcast	Hot	Normal	Weak	Yes
D14	Rain	Mild	High	Strong	No

Decision Tree ID3 Numerical Example 2



- We have 4 attributes outlook, Temperature, Humidity, and Wind and the decision is yes or no.
- Now we consider the information gained from each attribute.
- Attribute outlook: sunny, forecast, rain.
- To calculate the information gain of every attribute, we need to calculate the entropy of the whole dataset and the entropy of the individual attribute.

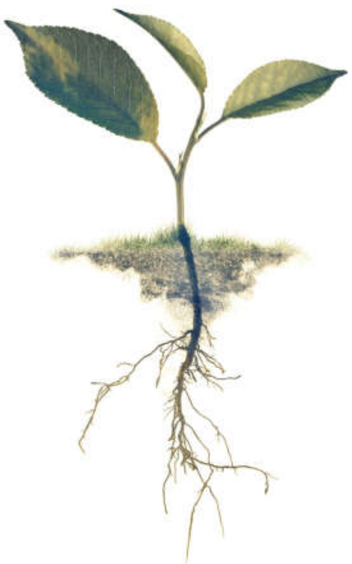
Decision Tree ID3 Numerical Example 2



- **Entropy**
- We need to calculate the entropy first. The decision column consists of 14 instances and includes two labels: yes and no. There are 9 decisions labeled yes, and 5 decisions labeled no.
- **S = [+9, -5]**

Day	Outlook	Temp.	Humidity	Wind	Decision
D1	Sunny	Hot	High	Weak	No
D2	Sunny	Hot	High	Strong	No
D3	Overcast	Hot	High	Weak	Yes
D4	Rain	Mild	High	Weak	Yes
D5	Rain	Cool	Normal	Weak	Yes
D6	Rain	Cool	Normal	Strong	No
D7	Overcast	Cool	Normal	Strong	Yes
D8	Sunny	Mild	High	Weak	No
D9	Sunny	Cool	Normal	Weak	Yes
D10	Rain	Mild	Normal	Weak	Yes
D11	Sunny	Mild	Normal	Strong	Yes
D12	Overcast	Mild	High	Strong	Yes
D13	Overcast	Hot	Normal	Weak	Yes
D14	Rain	Mild	High	Strong	No

Decision Tree ID3 Numerical Example 2



- **Entropy**
- We need to calculate the entropy first. The decision column consists of 14 instances and includes two labels: yes and no. There are 9 decisions labeled yes, and 5 decisions labeled no.

Day	Outlook	Temp.	Humidity	Wind	Decision
D1	Sunny	Hot	High	Weak	No
D2	Sunny	Hot	High	Strong	No
D3	Overcast	Hot	High	Weak	Yes
D4	Rain	Mild	High	Weak	Yes
D5	Rain	Cool	Normal	Weak	Yes
D6	Rain	Cool	Normal	Strong	No
D7	Overcast	Cool	Normal	Strong	Yes
D8	Sunny	Mild	High	Weak	No
D9	Sunny	Cool	Normal	Weak	Yes
D10	Rain	Mild	Normal	Weak	Yes
D11	Sunny	Mild	Normal	Strong	Yes
D12	Overcast	Mild	High	Strong	Yes
D13	Overcast	Hot	Normal	Weak	Yes
D14	Rain	Mild	High	Strong	No

Calculate the entropy of whole dataset and entropy of individual element in the attribute:
If we have same +ve and –ve examples, the entropy is 1. And we have one zero example the entropy is 0.

Day	Outlook	Temp.	Humidity	Wind	Decision
D1	Sunny	Hot	High	Weak	No
D2	Sunny	Hot	High	Strong	No
D3	Overcast	Hot	High	Weak	Yes
D4	Rain	Mild	High	Weak	Yes
D5	Rain	Cool	Normal	Weak	Yes
D6	Rain	Cool	Normal	Strong	No
D7	Overcast	Cool	Normal	Strong	Yes
D8	Sunny	Mild	High	Weak	No
D9	Sunny	Cool	Normal	Weak	Yes
D10	Rain	Mild	Normal	Weak	Yes
D11	Sunny	Mild	Normal	Strong	Yes
D12	Overcast	Mild	High	Strong	Yes
D13	Overcast	Hot	Normal	Weak	Yes
D14	Rain	Mild	High	Strong	No

- **Attribute: outlook**

$$Values\ (Outlook) = Sunny, Overcast, Rain$$

$$S = [9+, 5-] \qquad Entropy(S) = -\frac{9}{14}\log_2\frac{9}{14} - \frac{5}{14}\log_2\frac{5}{14} = 0.94$$

$$S_{Sunny} \leftarrow [2+, 3-] \qquad Entropy(S_{Sunny}) = -\frac{2}{5}\log_2\frac{2}{5} - \frac{3}{5}\log_2\frac{3}{5} = 0.971$$

$$S_{Overcast} \leftarrow [4+, 0-] \qquad Entropy(S_{Overcast}) = -\frac{4}{4}\log_2\frac{4}{4} - \frac{0}{4}\log_2\frac{0}{4} = 0$$

$$S_{Rain} \leftarrow [3+, 2-] \qquad Entropy(S_{Rain}) = -\frac{3}{5}\log_2\frac{3}{5} - \frac{2}{5}\log_2\frac{2}{5} = 0.971$$

$$Gain\ (S, Outlook) = Entropy(S) - \sum_{v \in \{Sunny, Overcast, Rain\}} \frac{|S_v|}{|S|} Entropy(S_v)$$

$$Gain(S, Outlook)$$

$$= Entropy(S) - \frac{5}{14}Entropy(S_{Sunny}) - \frac{4}{14}Entropy(S_{Overcast})$$

$$-\frac{5}{14}Entropy(S_{Rain})$$

$$Gain(S, Outlook) = 0.94 - \frac{5}{14}0.971 - \frac{4}{14}\ 0 - \frac{5}{14}0.971 = 0.2464$$

Calculate the entropy of whole dataset and entropy of individual element in the attribute:
If we have same +ve and –ve examples, the entropy is 1. And we have one zero example the entropy is 0.

Day	Outlook	Temp	Humidity	Wind	Decision
D1	Sunny	Hot	High	Weak	No
D2	Sunny	Hot	High	Strong	No
D3	Overcast	Hot	High	Weak	Yes
D4	Rain	Mild	High	Weak	Yes
D5	Rain	Cool	Normal	Weak	Yes
D6	Rain	Cool	Normal	Strong	No
D7	Overcast	Cool	Normal	Strong	Yes
D8	Sunny	Mild	High	Weak	No
D9	Sunny	Cool	Normal	Weak	Yes
D10	Rain	Mild	Normal	Weak	Yes
D11	Sunny	Mild	Normal	Strong	Yes
D12	Overcast	Mild	High	Strong	Yes
D13	Overcast	Hot	Normal	Weak	Yes
D14	Rain	Mild	High	Strong	No

- **Attribute: Temperature**

$Values\ (Temp) = Hot, Mild, Cool$

$S = [9+, 5-]$ $\quad Entropy(S) = -\frac{9}{14}\log_2\frac{9}{14} - \frac{5}{14}\log_2\frac{5}{14} = 0.94$

$S_{Hot} \leftarrow [2+, 2-]$ $\quad Entropy(S_{Hot}) = -\frac{2}{4}\log_2\frac{2}{4} - \frac{2}{4}\log_2\frac{2}{4} = 1.0$

$S_{Mild} \leftarrow [4+, 2-]$ $\quad Entropy(S_{Mild}) = -\frac{4}{6}\log_2\frac{4}{6} - \frac{2}{6}\log_2\frac{2}{6} = 0.9183$

$S_{Cool} \leftarrow [3+, 1-]$ $\quad Entropy(S_{Cool}) = -\frac{3}{4}\log_2\frac{3}{4} - \frac{1}{4}\log_2\frac{1}{4} = 0.8113$

$$Gain\ (S, Temp) = Entropy(S) - \sum_{v \in \{Hot, Mild, Cool\}} \frac{|S_v|}{|S|} Entropy(S_v)$$

$$Gain(S, Temp)$$

$$= Entropy(S) - \frac{4}{14} Entropy(S_{Hot}) - \frac{6}{14} Entropy(S_{Mild})$$

$$- \frac{4}{14} Entropy(S_{Cool})$$

$= 0.0289$

Calculate the entropy of whole dataset and entropy of individual element in the attribute:
If we have same +ve and –ve examples, the entropy is 1. And we have one zero example the entropy is 0.

Day	Outlook	Temp.	Humidity	Wind	Decision
D1	Sunny	Hot	High	Weak	No
D2	Sunny	Hot	High	Strong	No
D3	Overcast	Hot	High	Weak	Yes
D4	Rain	Mild	High	Weak	Yes
D5	Rain	Cool	Normal	Weak	Yes
D6	Rain	Cool	Normal	Strong	No
D7	Overcast	Cool	Normal	Strong	Yes
D8	Sunny	Mild	High	Weak	No
D9	Sunny	Cool	Normal	Weak	Yes
D10	Rain	Mild	Normal	Weak	Yes
D11	Sunny	Mild	Normal	Strong	Yes
D12	Overcast	Mild	High	Strong	Yes
D13	Overcast	Hot	Normal	Weak	Yes
D14	Rain	Mild	High	Strong	No

- **Attribute: Humidity**

$$Values\ (Humidity) = High, Normal$$

$$S = [9+, 5-] \qquad Entropy(S) = -\frac{9}{14}\log_2\frac{9}{14} - \frac{5}{14}\log_2\frac{5}{14} = 0.94$$

$$S_{High} \leftarrow [3+, 4-] \qquad Entropy(S_{High}) = -\frac{3}{7}\log_2\frac{3}{7} - \frac{4}{7}\log_2\frac{4}{7} = 0.9852$$

$$S_{Normal} \leftarrow [6+, 1-] \qquad Entropy(S_{Normal}) = -\frac{6}{7}\log_2\frac{6}{7} - \frac{1}{7}\log_2\frac{1}{7} = 0.5916$$

$$Gain\ (S, Humidity) = Entropy(S) - \sum_{v \in \{High, Normal\}} \frac{|S_v|}{|S|} Entropy(S_v)$$

$$Gain(S, Humidity)$$

$$= Entropy(S) - \frac{7}{14} Entropy(S_{High}) - \frac{7}{14} Entropy(S_{Normal})$$

$$Gain(S, Humidity) = 0.94 - \frac{7}{14} 0.9852 - \frac{7}{14}\ 0.5916 = 0.1516$$

Calculate the entropy of whole dataset and entropy of individual element in the attribute:
If we have same +ve and –ve examples, the entropy is 1. And we have one zero example the entropy is 0.

Day	Outlook	Temp.	Humidity	Wind	Decision
D1	Sunny	Hot	High	Weak	No
D2	Sunny	Hot	High	Strong	No
D3	Overcast	Hot	High	Weak	Yes
D4	Rain	Mild	High	Weak	Yes
D5	Rain	Cool	Normal	Weak	Yes
D6	Rain	Cool	Normal	Strong	No
D7	Overcast	Cool	Normal	Strong	Yes
D8	Sunny	Mild	High	Weak	No
D9	Sunny	Cool	Normal	Weak	Yes
D10	Rain	Mild	Normal	Weak	Yes
D11	Sunny	Mild	Normal	Strong	Yes
D12	Overcast	Mild	High	Strong	Yes
D13	Overcast	Hot	Normal	Weak	Yes
D14	Rain	Mild	High	Strong	No

- **Attribute: Wind**

$$Values\ (Wind) = Strong, Weak$$

$$S = [9+, 5-] \qquad Entropy(S) = -\frac{9}{14}log_2\frac{9}{14} - \frac{5}{14}log_2\frac{5}{14} = 0.94$$

$$S_{Strong} \leftarrow [3+, 3-] \qquad Entropy(S_{Strong}) = 1.0$$

$$S_{Weak} \leftarrow [6+, 2-] \qquad Entropy(S_{Weak}) = -\frac{6}{8}log_2\frac{6}{8} - \frac{2}{8}log_2\frac{2}{8} = 0.8113$$

$$Gain\ (S, Wind) = Entropy(S) - \sum_{v \in \{Strong, Weak\}} \frac{|S_v|}{|S|} Entropy(S_v)$$

$$Gain(S, Wind) = Entropy(S) - \frac{6}{14}Entropy(S_{Strong}) - \frac{8}{14}Entropy(S_{Weak})$$

$$Gain(S, Wind) = 0.94 - \frac{6}{14}1.0 - \frac{8}{14}0.8113 = 0.0478$$

Outlook has the maximum gain, we consider the outlook as the root node.
So we have 3 possibilities which are Sunny, overcast and rain.

Day	Outlook	Temp.	Humidity	Wind	Decision
D1	Sunny	Hot	High	Weak	No
D2	Sunny	Hot	High	Strong	No
D3	Overcast	Hot	High	Weak	Yes
D4	Rain	Mild	High	Weak	Yes
D5	Rain	Cool	Normal	Weak	Yes
D6	Rain	Cool	Normal	Strong	No
D7	Overcast	Cool	Normal	Strong	Yes
D8	Sunny	Mild	High	Weak	No
D9	Sunny	Cool	Normal	Weak	Yes
D10	Rain	Mild	Normal	Weak	Yes
D11	Sunny	Mild	Normal	Strong	Yes
D12	Overcast	Mild	High	Strong	Yes
D13	Overcast	Hot	Normal	Weak	Yes
D14	Rain	Mild	High	Strong	No

- **Attribute: Gain of all attributes**

$$Gain(S, Outlook) = 0.2464$$

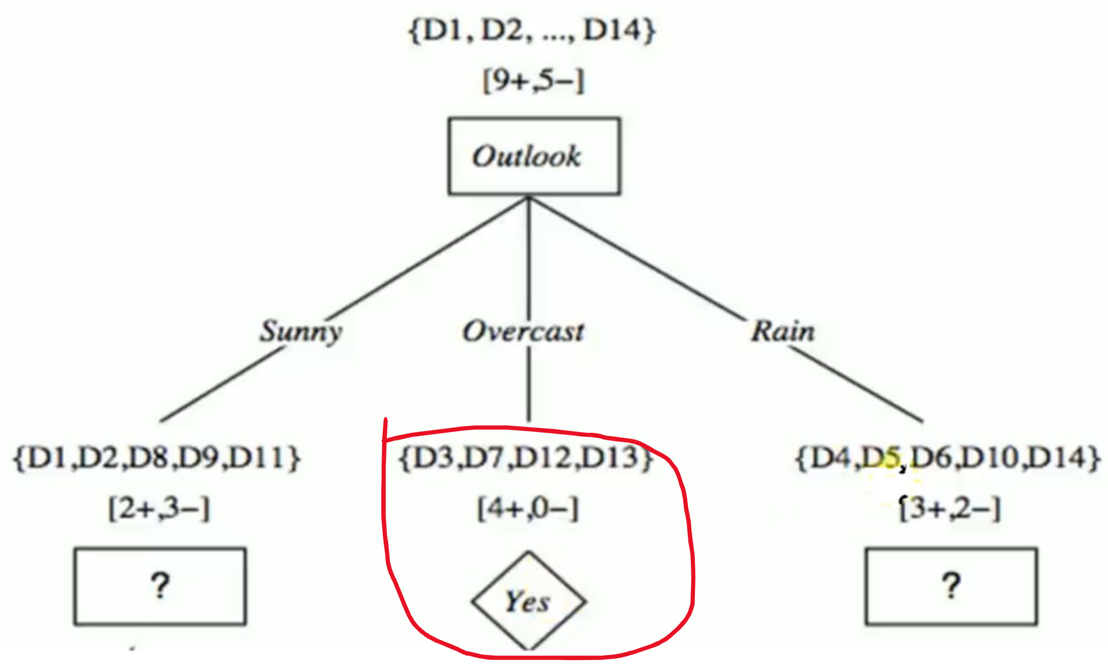
$$Gain(S, Temp) = 0.0289$$

$$Gain(S, Humidity) = 0.1516$$

$$Gain(S, Wind) = 0.0478$$

Outlook is root node, in overcast there all the examples are yes, and in sunny and rain all there are mixed of yes and no, so we cant write yes..

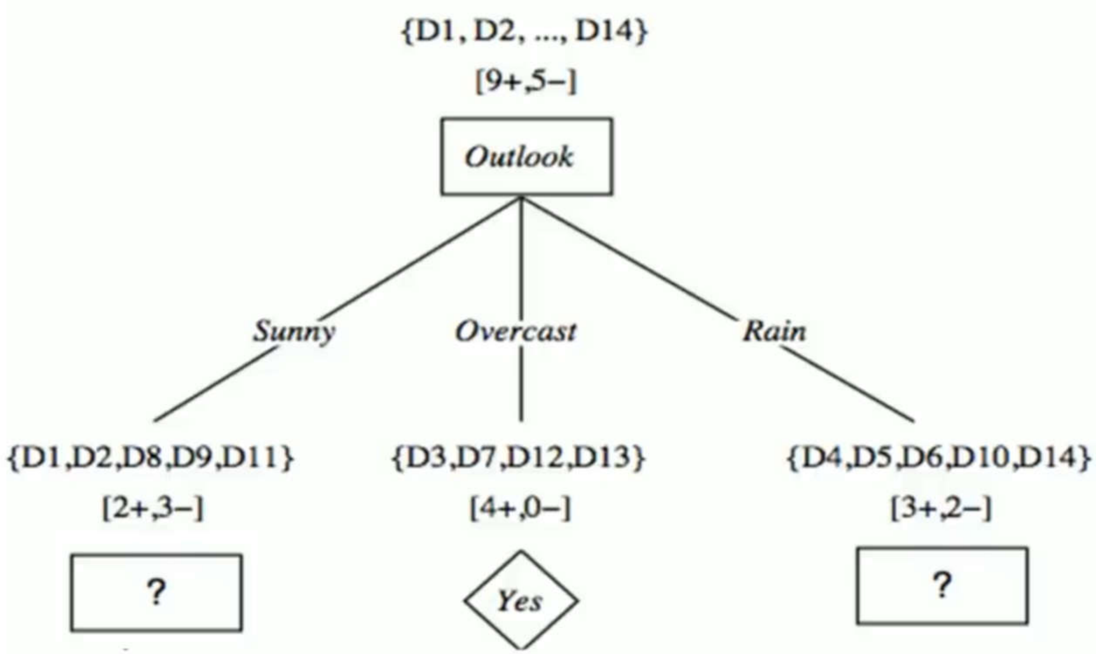
Day	Outlook	Temp.	Humidity	Wind	Decision
D1	Sunny	Hot	High	Weak	No
D2	Sunny	Hot	High	Strong	No
D3	Overcast	Hot	High	Weak	Yes
D4	Rain	Mild	High	Weak	Yes
D5	Rain	Cool	Normal	Weak	Yes
D6	Rain	Cool	Normal	Strong	No
D7	Overcast	Cool	Normal	Strong	Yes
D8	Sunny	Mild	High	Weak	No
D9	Sunny	Cool	Normal	Weak	Yes
D10	Rain	Mild	Normal	Weak	Yes
D11	Sunny	Mild	Normal	Strong	Yes
D12	Overcast	Mild	High	Strong	Yes
D13	Overcast	Hot	Normal	Weak	Yes
D14	Rain	Mild	High	Strong	No



So we don’t consider the outlook attribute from We already considered it.

Day	Temp	Humidity	Wind	Play Tennis
D1	Hot	High	Weak	No
D2	Hot	High	Strong	No
D8	Mild	High	Weak	No
D9	Cool	Normal	Weak	Yes
D11	Mild	Normal	Strong	Yes

Day	Outlook	Temp.	Humidity	Wind	Decision
D1	Sunny	Hot	High	Weak	No
D2	Sunny	Hot	High	Strong	No
D3	Overcast	Hot	High	Weak	Yes
D4	Rain	Mild	High	Weak	Yes
D5	Rain	Cool	Normal	Weak	Yes
D6	Rain	Cool	Normal	Strong	No
D7	Overcast	Cool	Normal	Strong	Yes
D8	Sunny	Mild	High	Weak	No
D9	Sunny	Cool	Normal	Weak	Yes
D10	Rain	Mild	Normal	Weak	Yes
D11	Sunny	Mild	Normal	Strong	Yes
D12	Overcast	Mild	High	Strong	Yes
D13	Overcast	Hot	Normal	Weak	Yes
D14	Rain	Mild	High	Strong	No



Attribute: Temp

So we don't consider the outlook attribute from We already considered it.

Day	Temp	Humidity	Wind	Play Tennis
D1	Hot	High	Weak	No
D2	Hot	High	Strong	No
D8	Mild	High	Weak	No
D9	Cool	Normal	Weak	Yes
D11	Mild	Normal	Strong	Yes

$Values\ (Temp) = Hot, Mild, Cool$

$S_{Sunny} = [2+, 3-]$ $\quad Entropy(S_{Sunny}) = -\frac{2}{5}log_2\frac{2}{5} - \frac{3}{5}log_2\frac{3}{5} = 0.97$

$S_{Hot} \leftarrow [0+, 2-]$ $\quad Entropy(S_{Hot}) = 0.0$

$S_{Mild} \leftarrow [1+, 1-]$ $\quad Entropy(S_{Mild}) = 1.0$

$S_{Cool} \leftarrow [1+, 0-]$ $\quad Entropy(S_{Cool}) = 0.0$

$$Gain\ (S_{Sunny}, Temp) = Entropy(S) - \sum_{v \in \{Hot, Mild, Cool\}} \frac{|S_v|}{|S|} Entropy(S_v)$$

$$Gain(S_{Sunny}, Temp)$$

$$= Entropy(S) - \frac{2}{5}Entropy(S_{Hot}) - \frac{2}{5}Entropy(S_{Mild})$$

$$- \frac{1}{5}Entropy(S_{Cool})$$

$$Gain(S_{sunny}, Temp) = 0.97 - \frac{2}{5}0.0 - \frac{2}{5}1 - \frac{1}{5}0.0 = 0.570$$

Attribute: Humidity

So we don’t consider the outlook attribute from We already considered it.

Day	Temp	Humidity	Wind	Play Tennis
D1	Hot	High	Weak	No
D2	Hot	High	Strong	No
D8	Mild	High	Weak	No
D9	Cool	Normal	Weak	Yes
D11	Mild	Normal	Strong	Yes

$$Values\ (Humidity) = High, Normal$$

$$S_{Sunny} = [2+, 3-] \qquad Entropy(S) = -\frac{2}{5}log_2\frac{2}{5} - \frac{3}{5}log_2\frac{3}{5} = 0.97$$

$$S_{high} \leftarrow [0+, 3-] \qquad Entropy(S_{High}) = 0.0$$

$$S_{Normal} \leftarrow [2+, 0-] \qquad Entropy(S_{Normal}) = 0.0$$

$$Gain\ (S_{Sunny}, Humidity) = Entropy(S) - \sum_{v \in \{High, Normal\}} \frac{|S_v|}{|S|} Entropy(S_v)$$

$$Gain(S_{sunny}, Humidity) = 0.97 - \frac{3}{5}\,0.0 - \frac{2}{5}0.0 = 0.97$$

Attribute: Wind

So we don’t consider the outlook attribute from We already considered it.

Day	Temp	Humidity	Wind	Play Tennis
D1	Hot	High	Weak	No
D2	Hot	High	Strong	No
D8	Mild	High	Weak	No
D9	Cool	Normal	Weak	Yes
D11	Mild	Normal	Strong	Yes

$$Values\ (Wind) = Strong, Weak$$

$$S_{Sunny} = [2+, 3\ -] \qquad Entropy(S) = -\frac{2}{5}log_2\frac{2}{5} - \frac{3}{5}log_2\frac{3}{5} = 0.97$$

$$S_{Strong} \leftarrow [1+, 1-] \qquad Entropy(S_{Strong}) = 1.0$$

$$S_{Weak} \leftarrow [1+, 2-] \qquad Entropy(S_{Weak}) = -\frac{1}{3}log_2\frac{1}{3} - \frac{2}{3}log_2\frac{2}{3} = 0.9183$$

$$Gain\ (S_{Sunny}, Wind) = Entropy(S) - \sum_{v\ \in\{Strong, Weak\}} \frac{|S_v|}{|S|} Entropy(S_v)$$

$$Gain(S_{Sunny}, Wind) = Entropy(S) - \frac{2}{5}Entropy(S_{Strong}) - \frac{3}{5}Entropy(S_{Weak})$$

$$Gain(S_{sunny}, Wind) = 0.97 - \frac{2}{5}1.0 - \frac{3}{5}\ 0.918 = 0.0192$$

Gain of every attribute, we consider the humidity as a node because of higher gain.

Day	Temp	Humidity	Wind	Play Tennis
D1	Hot	High	Weak	No
D2	Hot	High	Strong	No
D8	Mild	High	Weak	No
D9	Cool	Normal	Weak	Yes
D11	Mild	Normal	Strong	Yes

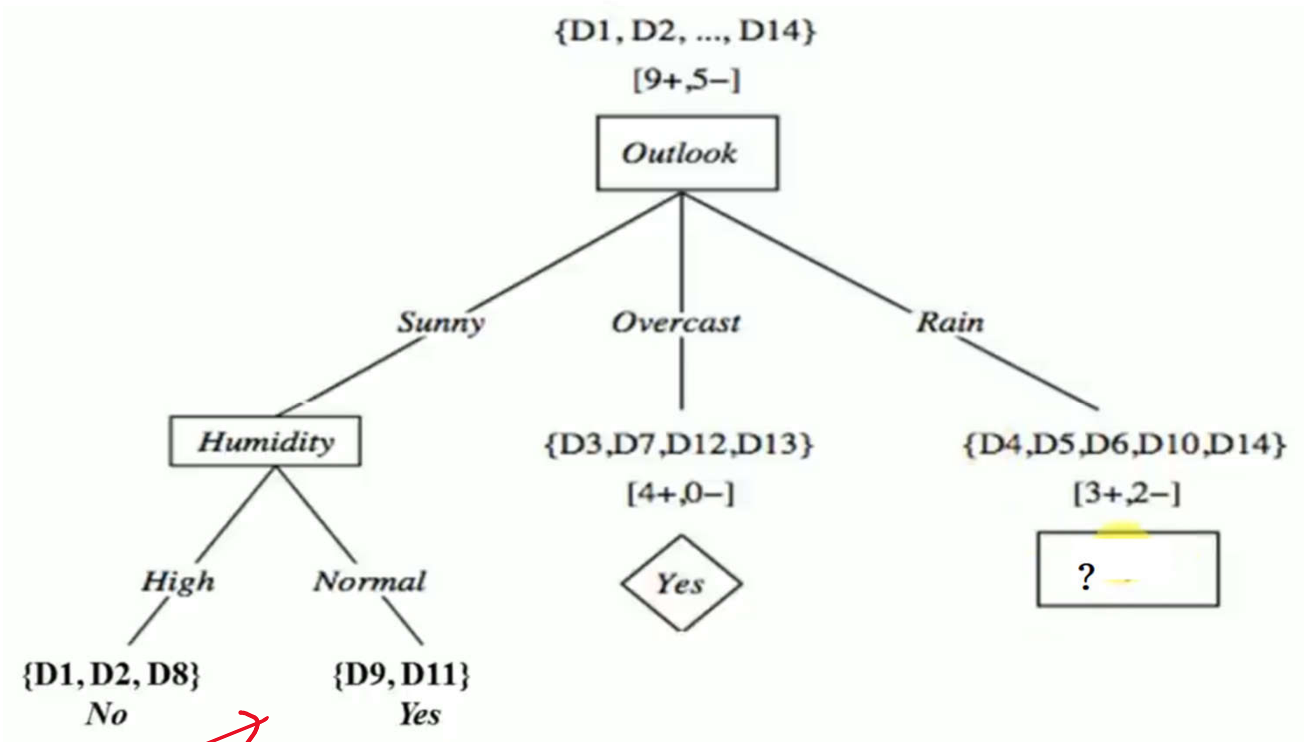
$$Gain(S_{sunny}, Temp) = 0.570$$

$$Gain(S_{sunny}, Humidity) = 0.97$$

$$Gain(S_{sunny}, Wind) = 0.0192$$

Gain of every attribute, we consider the humidity as a node because of higher gain. If we have high, the label is No, and if we have normal, the label is yes.

Day	Temp	Humidity	Wind	Play Tennis
D1	Hot	High	Weak	No
D2	Hot	High	Strong	No
D8	Mild	High	Weak	No
D9	Cool	Normal	Weak	Yes
D11	Mild	Normal	Strong	Yes



Attribute: Temprature

Gain of every attribute, we consider the humidity as a node because of higher gain. If we have high, the label is No, and if we have normal, the label is yes.

Day	Temp	Humidity	Wind	Play Tennis
D4	Mild	High	Weak	Yes
D5	Cool	Normal	Weak	Yes
D6	Cool	Normal	Strong	No
Dl0	Mild	Normal	Weak	Yes
Dl4	Mild	High	Strong	No

$$Values\ (Temp) = Hot, Mild, Cool$$

$$S_{Rain} = [3+, 2-] \qquad Entropy(S_{Rain}) = -\frac{3}{5}log_2\frac{3}{5} - \frac{2}{5}log_2\frac{2}{5} = 0.97$$

$$S_{Hot} \leftarrow [0+, 0-] \qquad Entropy(S_{Hot}) = 0.0$$

$$S_{Mild} \leftarrow [2+, 1-] \qquad Entropy(S_{Mild}) = -\frac{2}{3}log_2\frac{2}{3} - \frac{1}{3}log_2\frac{1}{3} = 0.9183$$

$$S_{Cool} \leftarrow [1+, 1-] \qquad Entropy(S_{Cool}) = 1.0$$

$$Gain\ (S_{Rain}, Temp) = Entropy(S) - \sum_{v \in \{Hot, Mild, Cool\}} \frac{|S_v|}{|S|} Entropy(S_v)$$

$$Gain(S_{Rain}, Temp)$$

$$= Entropy(S) - \frac{0}{5}Entropy(S_{Hot}) - \frac{3}{5}Entropy(S_{Mild})$$

$$-\frac{2}{5}Entropy(S_{Cool})$$

$$Gain(S_{Rain}, Temp) = 0.97 - \frac{0}{5}0.0 - \frac{3}{5}\,0.918 - \frac{2}{5}\,1.0 = 0.0192$$

Attribute: Humidity

Gain of every attribute, we consider the humidity as a node because of higher gain. If we have high, the label is No, and if we have normal, the label is yes.

Day	Temp	Humidity	Wind	Play Tennis
D4	Mild	High	Weak	Yes
D5	Cool	Normal	Weak	Yes
D6	Cool	Normal	Strong	No
Dl0	Mild	Normal	Weak	Yes
Dl4	Mild	High	Strong	No

$$Values\ (Humidity) = High, Normal$$

$$S_{Rain} = [3+, 2-] \qquad Entropy(S_{Rain}) = -\frac{3}{5}log_2\frac{3}{5} - \frac{2}{5}log_2\frac{2}{5} = 0.97$$

$$S_{High} \leftarrow [1+, 1-] \qquad Entropy(S_{High}) = 1.0$$

$$S_{Normal} \leftarrow [2+, 1-] \qquad Entropy(S_{Normal}) = -\frac{2}{3}log_2\frac{2}{3} - \frac{1}{3}log_2\frac{1}{3} = 0.9183$$

$$Gain\ (S_{Rain}, Humidity) = Entropy(S) - \sum_{v \in \{High, Normal\}} \frac{|S_v|}{|S|} Entropy(S_v)$$

$$Gain(S_{Rain}, Humidity) = Entropy(S) - \frac{2}{5}Entropy(S_{High}) - \frac{3}{5}Entropy(S_{Normal})$$

$$Gain(S_{Rain}, Humidity) = 0.97 - \frac{2}{5}\ 1.0 - \frac{3}{5}0.918 = 0.0192$$

Attribute: Wind

Gain of every attribute, we consider the humidity as a node because of higher gain. If we have high, the label is No, and if we have normal, the label is yes.

$$Values\ (wind) = Strong, Weak$$

$S_{Rain} = [3+, 2-]$ $\quad Entropy(S_{Rain}) = -\frac{3}{5}log_2\frac{3}{5} - \frac{2}{5}log_2\frac{2}{5} = 0.97$

$S_{Strong} \leftarrow [0+, 2-]$ $\quad Entropy(S_{Strong}) = 0.0$

$S_{Weak} \leftarrow [3+, 0-]$ $\quad Entropy(S_{weak}) = 0.0$

Day	Temp	Humidity	Wind	Play Tennis
D4	Mild	High	Weak	Yes
D5	Cool	Normal	Weak	Yes
D6	Cool	Normal	Strong	No
DI0	Mild	Normal	Weak	Yes
DI4	Mild	High	Strong	No

$$Gain\ (S_{Rain}, Wind) = Entropy(S) - \sum_{v \in \{Strong, Weak\}} \frac{|S_v|}{|S|} Entropy(S_v)$$

$$Gain(S_{Rain}, Wind) = Entropy(S) - \frac{2}{5}Entropy(S_{Strong}) - \frac{3}{5}Entropy(S_{Weak})$$

Gain of wind is higher so we consider it as a root node.

Day	Temp	Humidity	Wind	Play Tennis
D4	Mild	High	Weak	Yes
D5	Cool	Normal	Weak	Yes
D6	Cool	Normal	Strong	No
Dl0	Mild	Normal	Weak	Yes
Dl4	Mild	High	Strong	No

$$\boldsymbol{Gain(S_{Rain}, Temp) = 0.0192}$$

$$\boldsymbol{Gain(S_{Rain}, Humidity) = 0.0192}$$

$$\boldsymbol{Gain(S_{Rain}, Wind) = 0.97}$$

final Results