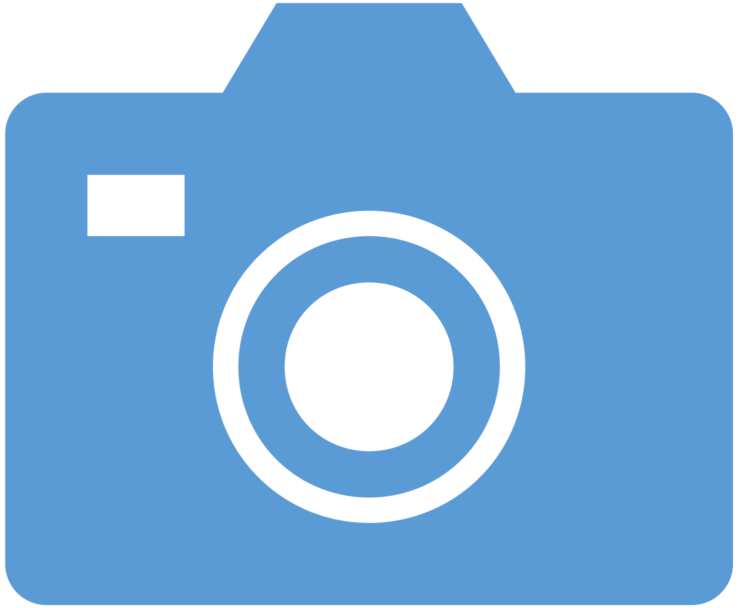

Digital Image Processing

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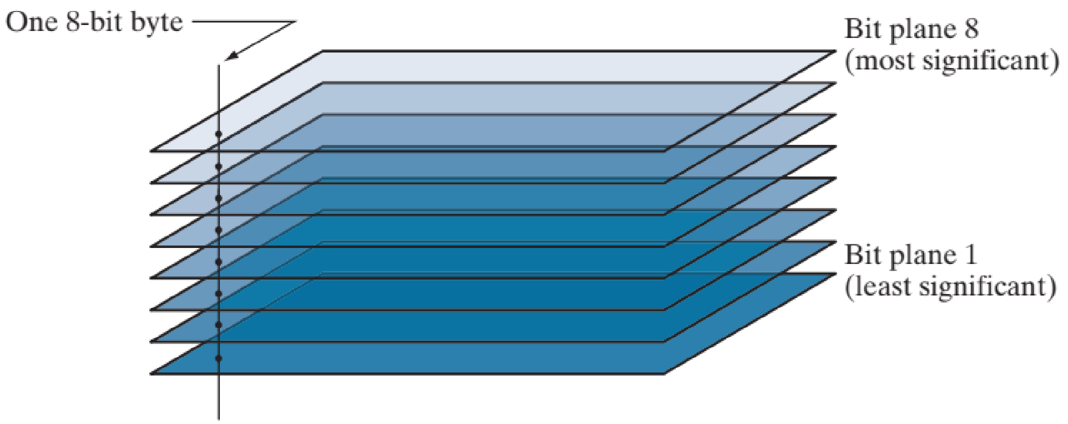
Bit-Plane Slicing

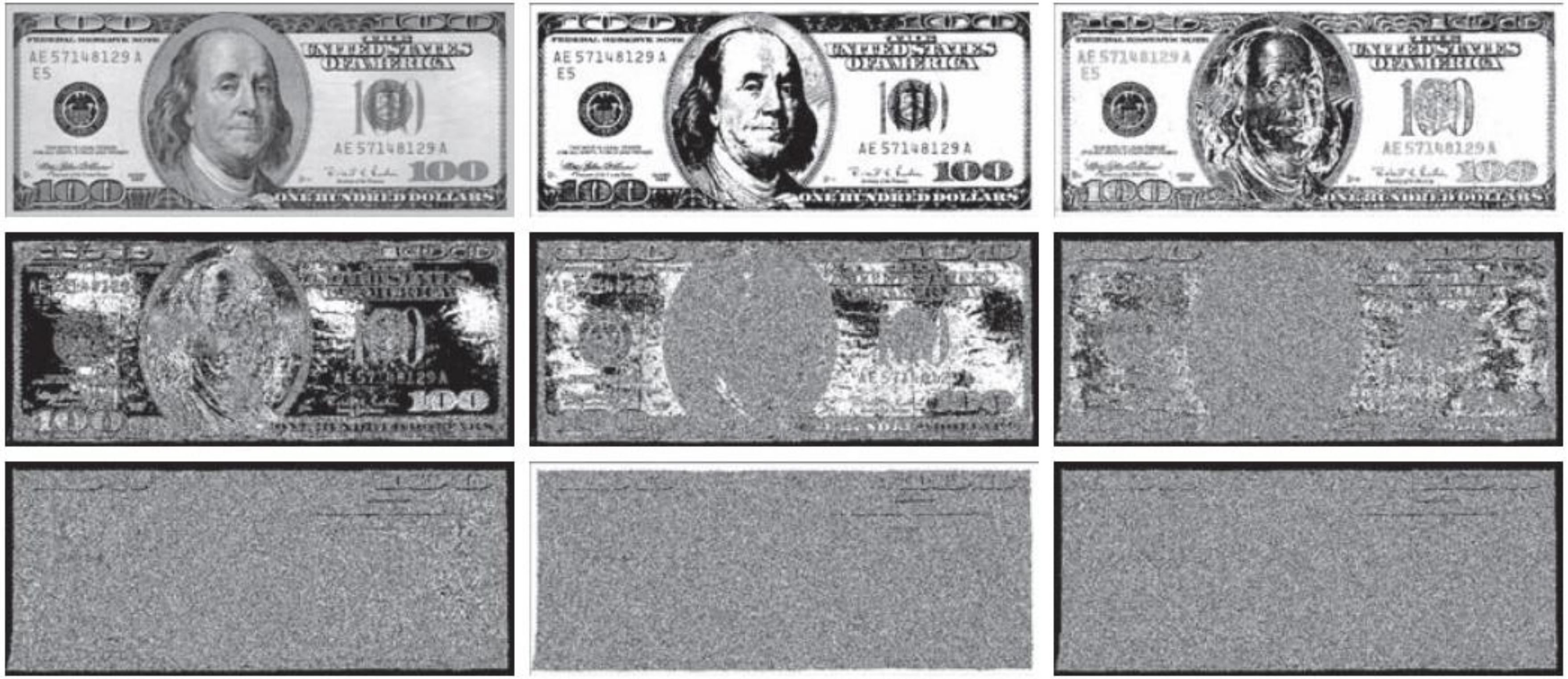
Bit-plane slicing is a method used in digital image processing to decompose an image into its constituent binary layers, or bit planes.

- Pixel values are integers composed of bits. For example, values in a 256-level grayscale image are composed of 8 bits (one byte). Instead of highlighting intensity-level ranges, as 3.3, we could highlight the contribution made to total image appearance by specific bits. As Fig. 3.13 illustrates, an 8-bit image may be considered as being composed of eight one-bit planes, with plane 1 containing the lowest-order bit of all pixels in the image, and plane 8 all the highest-order bits.

FIGURE 3.13 Bit-planes of an 8-bit image.



Bit-Plane Slicing



a	b	c
d	e	f
g	h	i

FIGURE 3.14 (a) An 8-bit gray-scale image of size 550×1192 pixels. (b) through (i) Bit planes 8 through 1, with bit plane 1 corresponding to the least significant bit. Each bit plane is a binary image..

Bit-Plane Slicing

a b c **FIGURE 3.15** Image reconstructed from bit planes: (a) 8 and 7; (b) 8, 7, and 6; (c) 8, 7, 6, and 5.

is not surprising, because two planes can produce only four distinct intensity levels. Adding plane 6 to the reconstruction helped the situation, as Fig. 3.15(b) shows. Note that the background of this image has perceptible false contouring. This effect is reduced significantly by adding the 5th plane to the reconstruction, as Fig. 3.15(c) illustrates. Using more planes in the reconstruction would not contribute significantly to the appearance of this image. Thus, we conclude that, in this example, storing the four highest-order bit planes would allow us to reconstruct the original image in acceptable detail. Storing these four planes instead of the original image requires 50% less storage.

HISTOGRAM EQUALIZATION

to improve the contrast of an image by redistributing the intensity values.

- Assuming initially continuous intensity values, let the variable r denote the intensities of an image to be processed. As usual, we assume that r is in the range $[0, L - 1]$. with $r = 0$ representing black and $r = L - 1$ representing white.

$$s = T(r) \qquad 0 \le r \le L - 1$$

- that produce an output intensity value, s, for a given intensity value r in the input image. We assume that..
- **(a)** $T(r)$ is a monotonic increasing function in the interval $0 \le r \le L - 1$; and
- **(b)** $0 \le T(r) \le L - 1$ for $0 \le r \le L - 1$.

HISTOGRAM EQUALIZATION

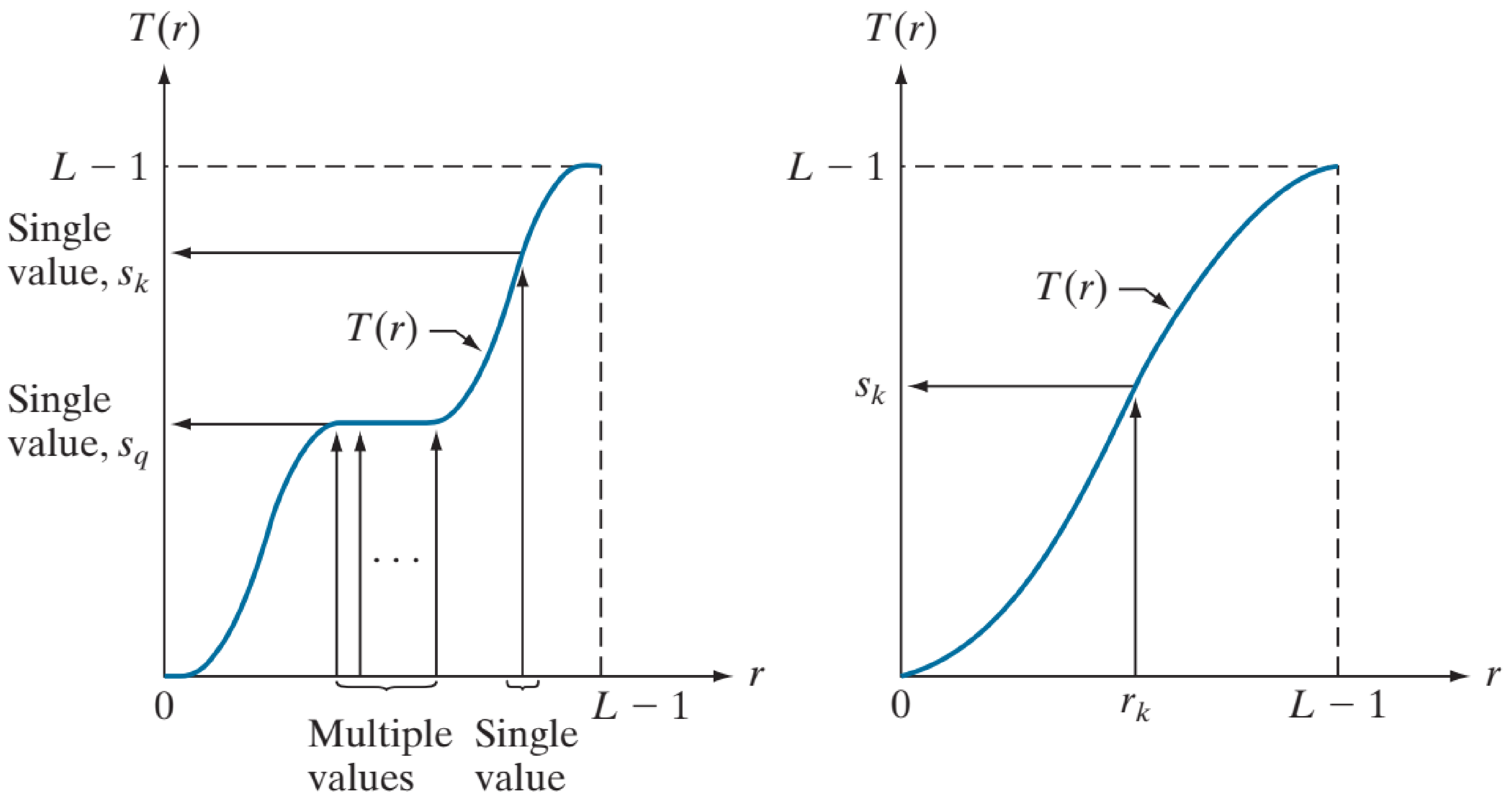


FIGURE 3.17 (a) Monotonic increasing function, showing how multiple values can map to a single value. (b) Strictly monotonic increasing function. This is a one-to-one mapping, both ways.

HISTOGRAM EQUALIZATION

- In some formulations to be discussed shortly, we use the inverse transformation $r = T^{-1}(s) \qquad 0 \le s \le L-1$
- in which case we change condition (a) to:
  (a') *T* (*r*) is a *strictly* monotonic increasing function in the interval 0 ≤ *r* ≤ *L* – 1.

HISTOGRAM EQUALIZATION

The intensity of an image may be viewed as a random variable in the interval $[0, L-1]$. Let $p_r(r)$ and $p_s(s)$ denote the PDFs of intensity values r and s in two different images. The subscripts on p indicate that p_r and p_s are different functions. A fundamental result from probability theory is that if $p_r(r)$ and $T(r)$ are known, and $T(r)$ is continuous and differentiable over the range of values of interest, then the PDF of the transformed (mapped) variable s can be obtained as

$$p_s(s) = p_r(r)\left|\frac{dr}{ds}\right| \tag{3-10}$$

Probability Density Fun

HISTOGRAM EQUALIZATION

$$s = T(r) = (L-1)\int_0^r p_r(w)\,dw \qquad (3\text{-}11)$$

- where w is a dummy variable of integration. The integral on the right side is the *cumulative distribution function* (CDF) of random variable *r*. Because PDFs always are positive, and the integral of a function is the area under the function, it follows that the transformation function of Eq. (3-11) satisfies condition (a). This is because the area under the function cannot decrease as *r* increases.

HISTOGRAM EQUALIZATION

We use Eq. (3-10) to find the $p_s(s)$ corresponding to the transformation just discussed. We know from Leibniz's rule in calculus that the derivative of a definite integral with respect to its upper limit is the integrand evaluated at the limit. That is,

$s = T(r)$

$$\begin{aligned}\frac{ds}{dr} &= \frac{dT(r)}{dr} \\ &= (L-1)\frac{d}{dr}\left[\int_0^r p_r(w)\,dw\right] \\ &= (L-1)p_r(r)\end{aligned} \tag{3-12}$$

HISTOGRAM EQUALIZATION

Substituting this result for dr/ds in Eq. (3-10), and noting that all probability values are positive, gives the result

$$
\begin{aligned}
p_s(s) &= p_r(r)\left|\frac{dr}{ds}\right| \\
&= \cancel{p_r}(r)\left|\frac{1}{(L-1)\cancel{p_r}(r)}\right| \qquad (3\text{-}13) \\
&= \frac{1}{L-1} \qquad 0 \le s \le L-1
\end{aligned}
$$

HISTOGRAM EQUALIZATION

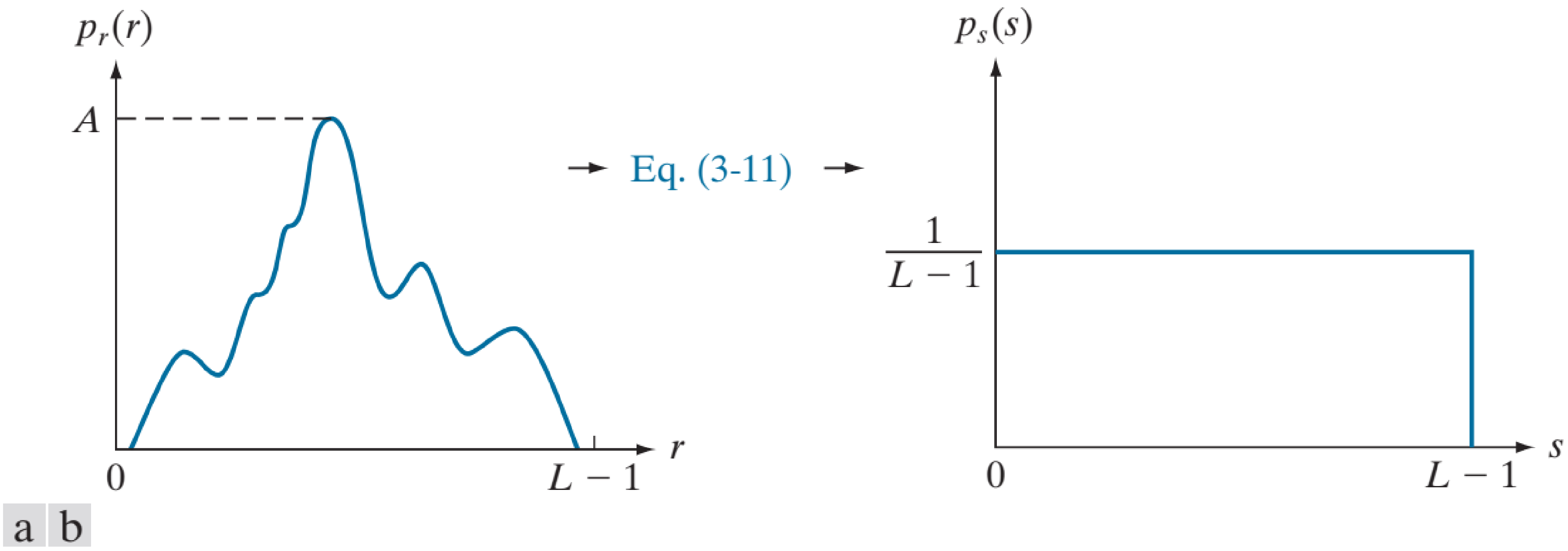


FIGURE 3.18 (a) An arbitrary PDF. (b) Result of applying Eq. (3-11) to the input PDF. The resulting PDF is always uniform, independently of the shape of the input.

HISTOGRAM EQUALIZATION

For discrete values, we work with probabilities and summations instead of probability density functions and integrals (but the requirement of monotonicity stated earlier still applies). Recall that the probability of occurrence of intensity level r_k in a digital image is approximated by

$$p_r(r_k) = \frac{n_k}{MN} \tag{3-14}$$

where MN is the total number of pixels in the image, and n_k denotes the number of pixels that have intensity r_k. As noted in the beginning of this section, $p_r(r_k)$, with $r_k \in [0, L-1]$, is commonly referred to as a normalized image histogram.

The discrete form of the transformation in Eq. (3-11) is

$$s_k = T(r_k) = (L-1)\sum_{j=0}^{k} p_r(r_j) \qquad k = 0, 1, 2, \ldots, L-1 \tag{3-15}$$

EXAMPLE 3.5: Illustration of the mechanics of histogram equalization.

It will be helpful to work through a simple example. Suppose that a 3-bit image ($L = 8$) of size 64×64 pixels ($MN = 4096$) has the intensity distribution in Table 3.1, where the intensity levels are integers in the range $[0, L-1] = [0, 7]$. The histogram of this image is sketched in Fig. 3.19(a). Values of the histogram equalization transformation function are obtained using Eq. (3-15). For instance,

$$s_0 = T(r_0) = 7\sum_{j=0}^{0} p_r(r_j) = 7p_r(r_0) = 1.33$$

Equalize the given histogram

Grey Level	Number of Pixels(nk)	PDF P(rk) = nk/n	CDF Sk = $\sum_{0}^{r} p_r(r)$	(L-1)*Sk	Round off
0	790	0.19	0.19	1.33	1
1	1023	0.25	0.44	3.08	3
2	850	0.21	0.65	4.55	5
3	656	0.16	0.81	5.67	6
4	329	0.08	0.89	6.23	6
5	245	0.06	0.95	6.65	7
6	122	0.03	0.98	6.86	7
7	81	0.02	1	7	7
	4096				

HISTOGRAM EQUALIZATION

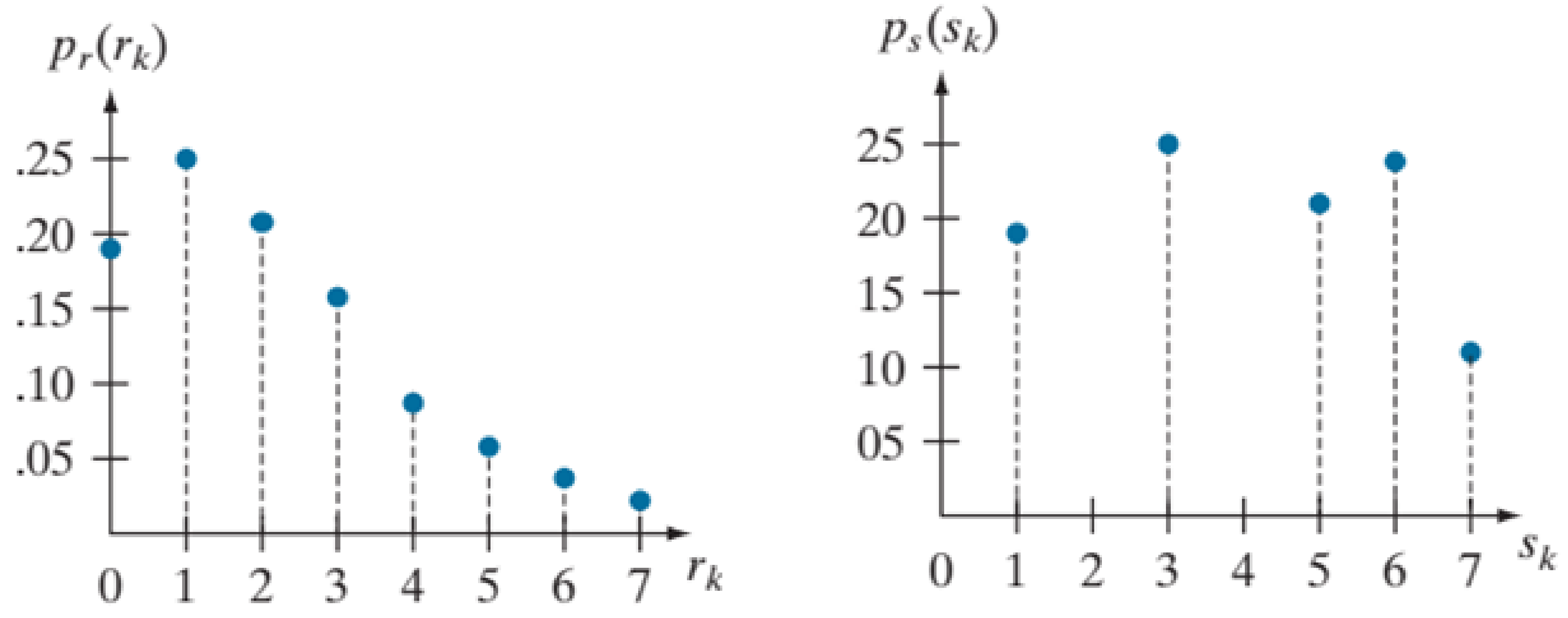
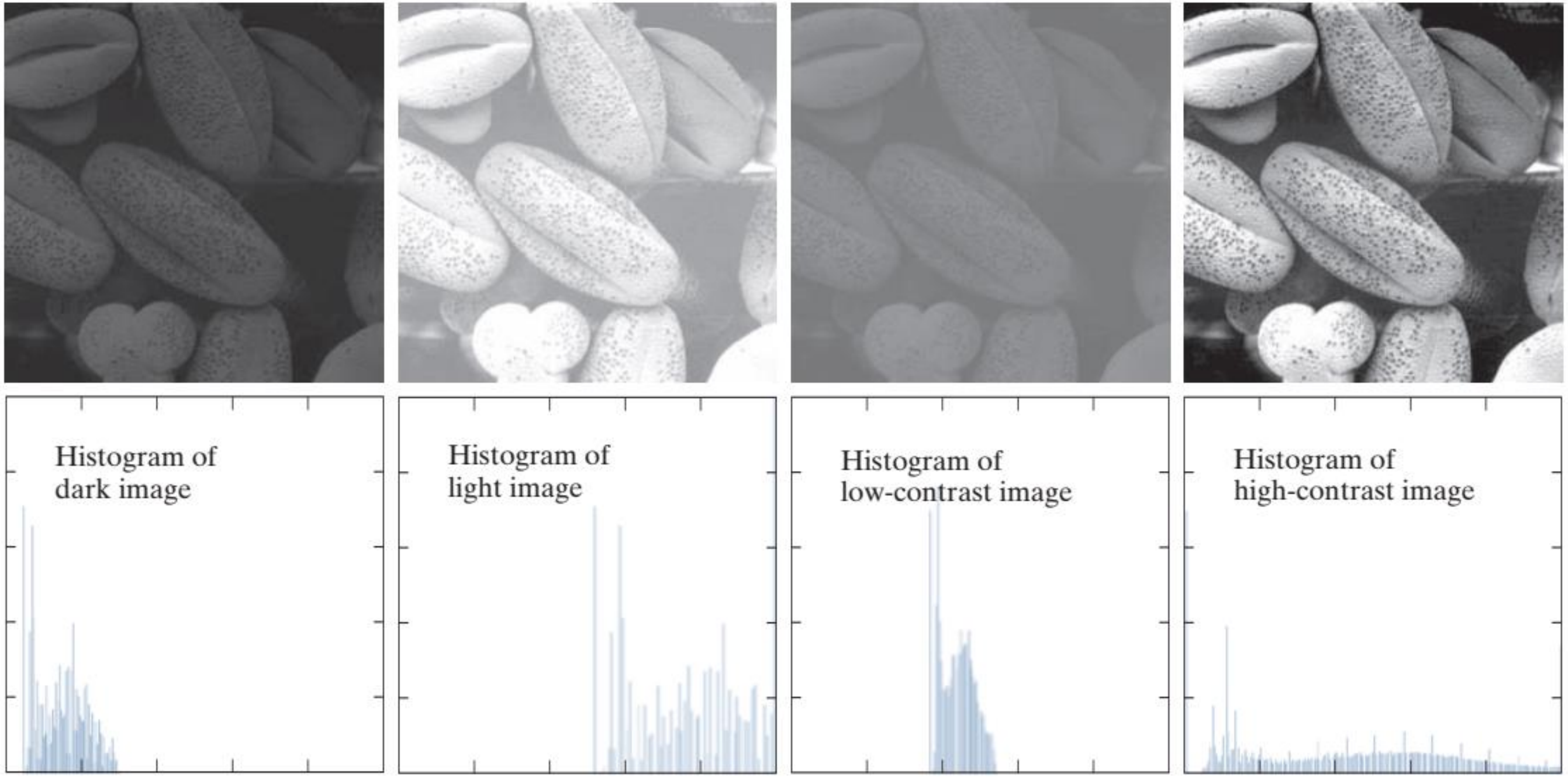


FIGURE 3.19 Histogram equalization. (a) Original histogram. (b) Equalized histogram.

Solve histogram equalization of 3-Bit Image?

4	4	4	4	4
3	4	5	4	3
3	5	5	5	3
3	4	5	4	3
4	4	4	4	4



a b c d

FIGURE 3.16 Four image types and their corresponding histograms. (a) dark; (b) light; (c) low contrast; (d) high contrast. The horizontal axis of the histograms are values of r_k and the vertical axis are values of $p(r_k)$.

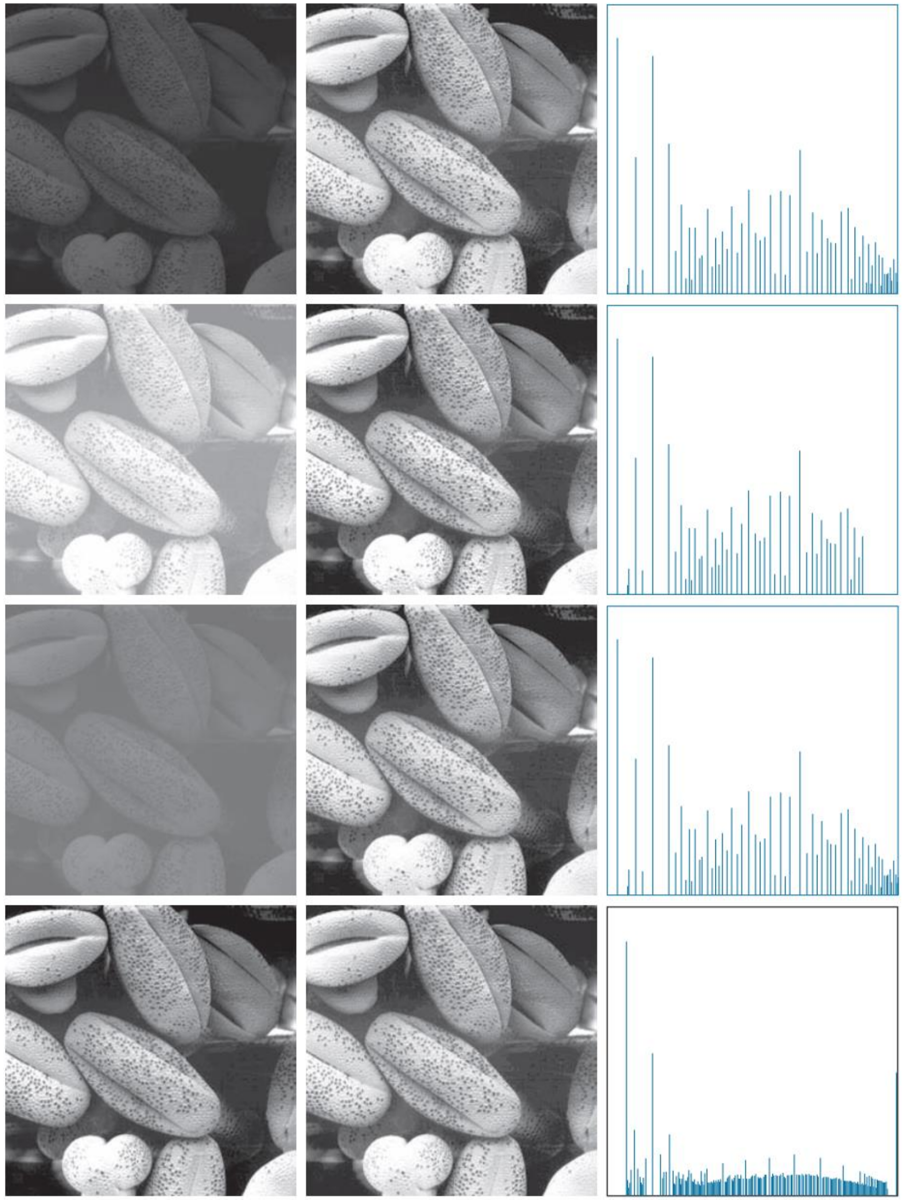


FIGURE 3.20 Left column: Images from Fig. 3.16. Center column: Corresponding histogram-equalized images. Right column: histograms of the images in the center column (compare with the histograms in Fig. 3.16).

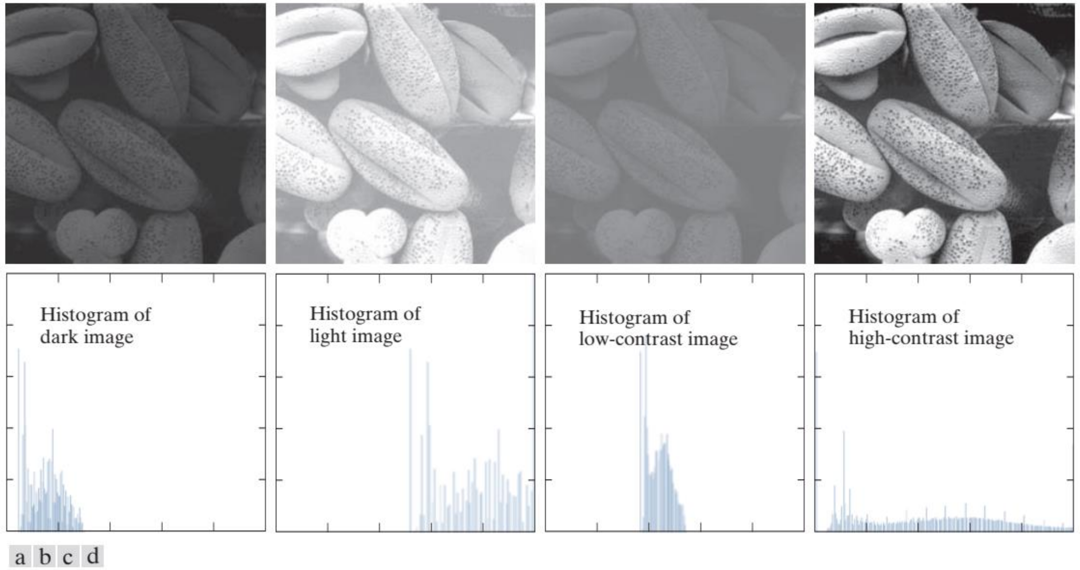


FIGURE 3.16 Four image types and their corresponding histograms. (a) dark; (b) light; (c) low contrast; (d) high contrast. The horizontal axis of the histograms are values of r_k and the vertical axis are values of $p(r_k)$.