

PROGRAM EVALUATION AND REVIEW TECHNIQUE (PERT)

schedule risk on project

Risk planning:

Risk Planning consists of drawing up **contingency** (Uncertainty) Plans & where appropriate, adding these to the Project's task structure.

With small Projects, Risk Planning is likely to be the Responsibility of the Project Manager, but Medium or Larger Projects will benefit from the appointment of a full-time Risk Manager

Risks can be dealt with by:

- Risk acceptance
- Risk avoidance
- Risk reduction
- Risk transfer
- Risk mitigation/contingency measures

INTRODUCTION

CPM and PERT are:

- Graphically display the precedence relationships & sequence of activities
- Estimate the project's duration
- Identify critical activities that cannot be delayed without delaying the project
- Estimate the amount of slack associated with noncritical activities

DEFINITION

(PERT) is an **event-oriented NETWORK ANALYSIS TECHNIQUE** used to estimate project duration

WHEN individual ACTIVITY DURATION estimates are **highly UNCERTAIN**.

PERT" developed by the United States Department of Defense as a management tool with an acronym "Program Evaluation and Review Technique".

Concept of Pert

- PERT uses probabilistic time estimates
- PERT requires the user to SET THREE DURATIONS that constitute the practical range of the duration for each activity

Three Point Estimating

Using a three point estimate helps find an approximate range for an activity duration

Most Likely Time: M_t

This estimate is based on duration of activity, given the resources likely to be assigned, their productivity realistic expectations of availability for activity dependencies participants & interruptions

Optimistic Time: O_t The activity duration based on analysis of best case scenario for the activity

Pessimistic Time P_t : The duration based in analysis of worst case scenario for the activity

Formula

A Three-Point Estimating Technique: PERT

Most likely time (m) : Estimate for both **favorable** and **unfavorable** conditions, with some risks occurring.

Optimistic time (o) : Estimate for all favorable conditions with no risks or changes

Pessimistic (p) : Estimate for all unfavorable conditions with all negative risks occurring and no mitigation of negative risks

$$\text{EXPECTED TIME (ET)} = \frac{\text{PERT weighted average} = \text{optimistic time} + 4 \times \text{most likely time} + \text{pessimistic time}}{6}$$

$$\text{Expected Time} = (T_o + 4T_m + T_p) / 6$$

CONCEPT OF PERT

Example:

Assembling and Erecting the False work for an Elevated Slab will most likely require 8 days. If all goes well, without interruption, the duration may be cut to 10 days. However, in the practically worst-case scenario, this activity may take 24 days.

The preceding values are estimated by the scheduler or project manager, who uses his or her **EXPERIENCE AND GOOD JUDGMENT** to do so

CONCEPT OF PERT

Example:

where optimistic time= 8 days,
most likely time = **10 days**, and
pessimistic time = 24 days

EXPECTED TIME (ET) or PERT weighted average =

$$\frac{8 \text{ workdays} + 4 \times 10 \text{ workdays} + 24 \text{ workdays}}{6} = \mathbf{12 \text{ days}}$$

Therefore, you'd use **12 days** on the network diagram instead of 10
when using PERT for the above example

Three estimates are produced for each activity

- **Most likely time (T_m)** : the time we would expect the task to take normally
- **Optimistic time (T_o or T_a)** : the shortest time that could be realistically be expected
- **Pessimistic (T_p or T_b)** : worst possible time
- **'expected time' $t_e = (T_o + 4T_m + T_p) / 6$** This is mean or
- **'activity standard deviation' $S = (b-a)/6$**
- **SD is the average deviation from the estimated time. Standard deviation indicates degree of uncertainty for each activity OR Risk for each activity .**
- **Higher the SD is the greater amount of uncertainty exists.**

Variance

['ver-ē-ən(t)s]

A measurement of how far each number in a data set is from the mean (average), and thus from every other number in the set.

Varaince $\sigma^2 = (Tb-Ta)/6$ or $(Tp-To)/6$

Activity	Predecessor	t_a	t_m	t_b	Mean (μ)	Variance (σ^2)	Standard Deviation (σ)
A	-	2	4	9			
B	A	5	8	14			
C	B	4	10	13			
D	B	4	7	10			
E	C	11	14	20			
F	D	9	13	16			
G	E,F	2	4	6			

Mean or Expected Duration = $(T_o + 4T_m + T_p) / 6$

Variance $\sigma^2 = (T_b - T_a) / 6$ or $(T_p - T_o) / 6$ (has larger unit than SD)

Standard Deviation (SD) $\sigma = \sqrt{\sigma^2}$

$$\text{expected duration} = \text{mean} = \frac{(t_a + 4t_m + t_b)}{6}$$

$$\text{Variance} = V = \sigma^2 = \left[\frac{t_b - t_a}{6} \right]^2$$

$$\text{standard deviation} = \sigma = \sqrt{\sigma^2}$$

Most optimistic \swarrow Most Probable \downarrow Most Pessimistic \searrow
 t_a t_m t_b

Activity	Predecessor	t_a	t_m	t_b	Mean (μ)	Variance (σ^2)	Standard Deviation (σ)
A	-	2	4	9			
B	A	5	8	14			
C	B	4	10	13			
D	B	4	7	10			
E	C	11	14	20			
F	D	9	13	16			
G	E,F	2	4	6			

$$\text{expected duration} = \text{mean} = \frac{(t_a + 4t_m + t_b)}{6}$$

$$\text{Variance} = V = \sigma^2 = \left[\frac{t_b - t_a}{6} \right]^2$$

$$\text{Standard deviation} = \sigma = \sqrt{\sigma^2}$$

for Activity A:

$$\mu = \frac{(2 + (4)(4) + 9)}{6} = 4.$$

$$\sigma^2 = \left[\frac{9-2}{6} \right]^2 = 1.36$$

$$\sigma = 1.17$$

Most optimistic Most Probable Most Pessimistic

Activity	Predecessor	t_a	t_m	t_b	Mean (μ)	Variance (σ^2)	Standard Deviation (σ)
A	-	2	4	9	4.9	1.36	1.17
B	A	5	8	14			
C	B	4	10	13			
D	B	4	7	10			
E	C	11	14	20			
F	D	9	13	16			
G	E,F	2	4	6			

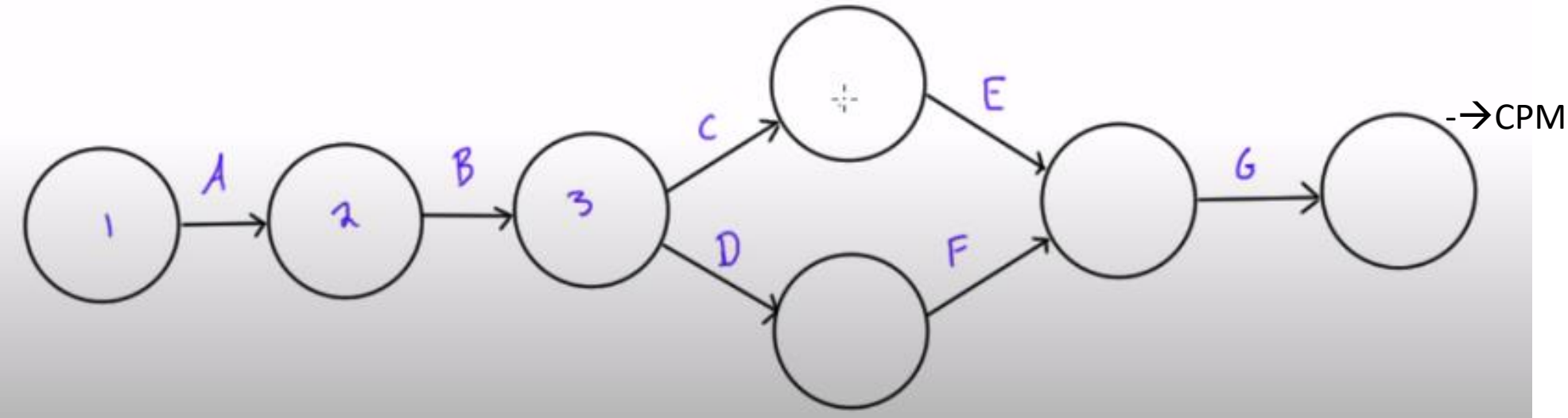
Most optimistic Most Probable Most Pessimistic

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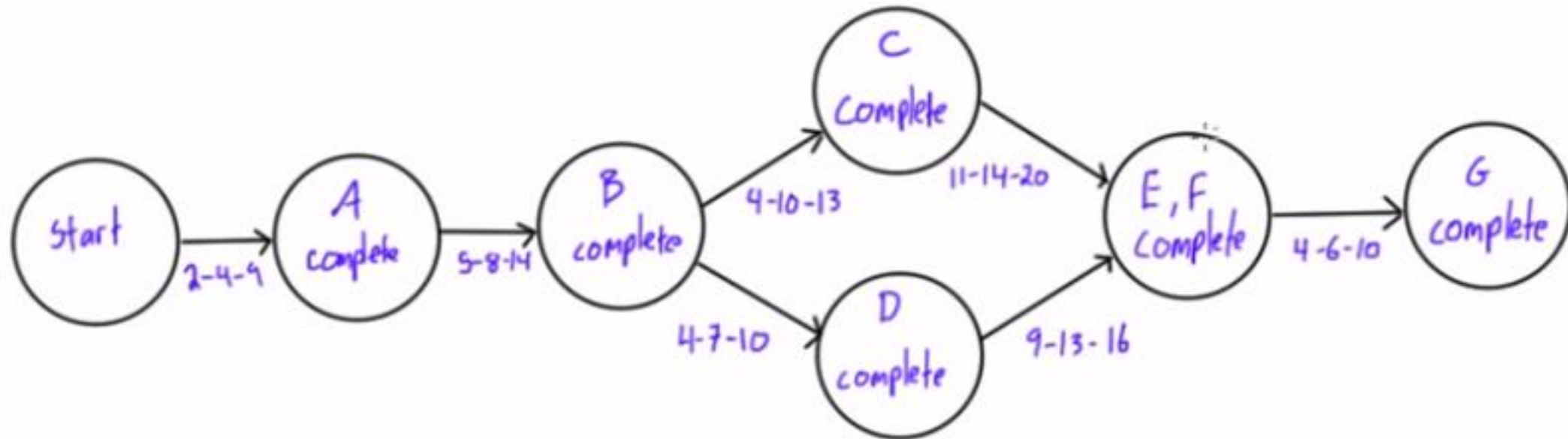
Activity	Predecessor	t_a	t_m	t_b	Mean (μ)	Variance (σ^2)	Standard Deviation (σ)
A	-	2	4	9	4.9	1.36	1.17
B	A	5	8	14	8.5	2.25	1.50
C	B	4	10	13	9.5	2.25	1.50
D	B	4	7	10	7	1.00	1.00
E	C	11	14	20	14.5	2.25	1.50
F	D	9	13	16	12.8	1.36	1.17
G	E,F	2	4	6	4	0.44	0.67

ADM Network diagram for PERT Problems

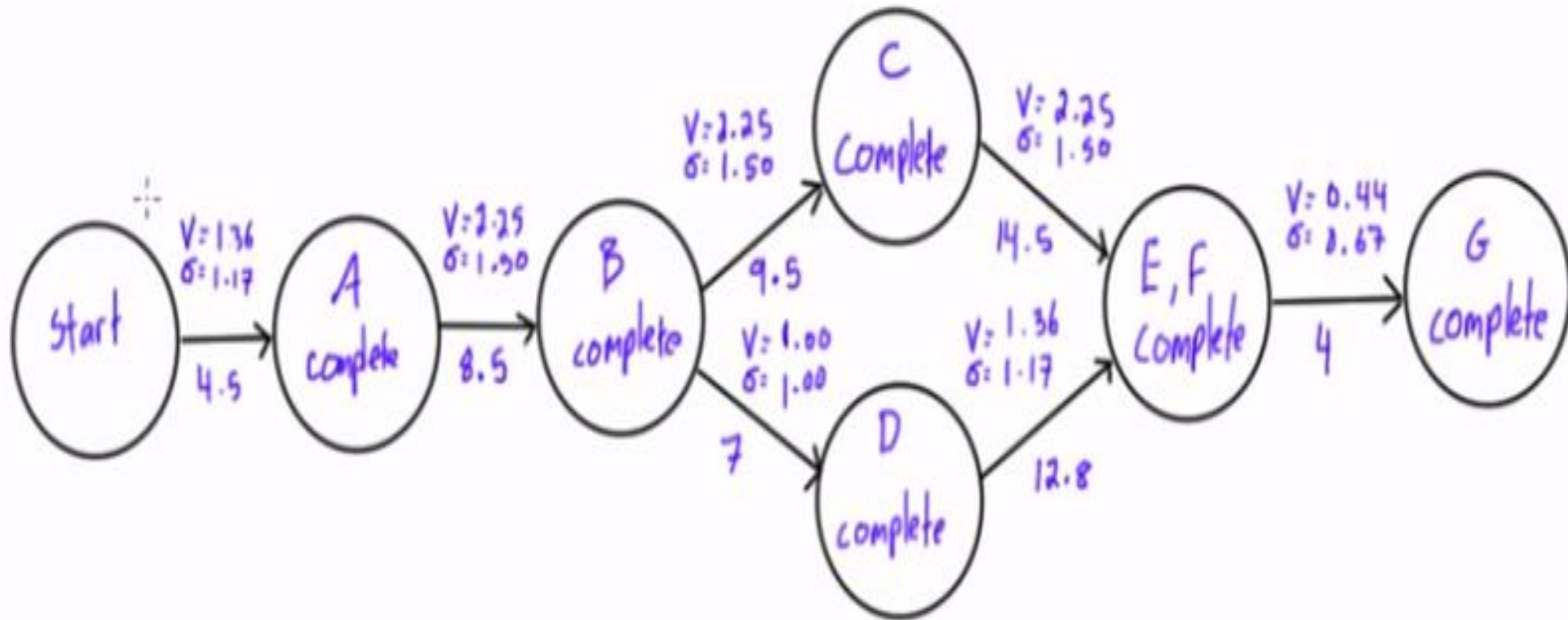
Activity	Predecessor	t_a	t_m	t_o	Mean (μ)	Variance (σ^2)	Standard Deviation (σ)
A	-	2	4	9	4.9	1.36	1.17
B	A	5	8	14	8.5	2.25	1.50
C	B	4	10	13	9.5	2.25	1.50
D	B	4	7	10	7	1.00	1.00
E	C	11	14	20	14.5	2.25	1.50
F	D	9	13	16	12.8	1.36	1.17
G	E,F	2	4	6	4	0.44	0.67



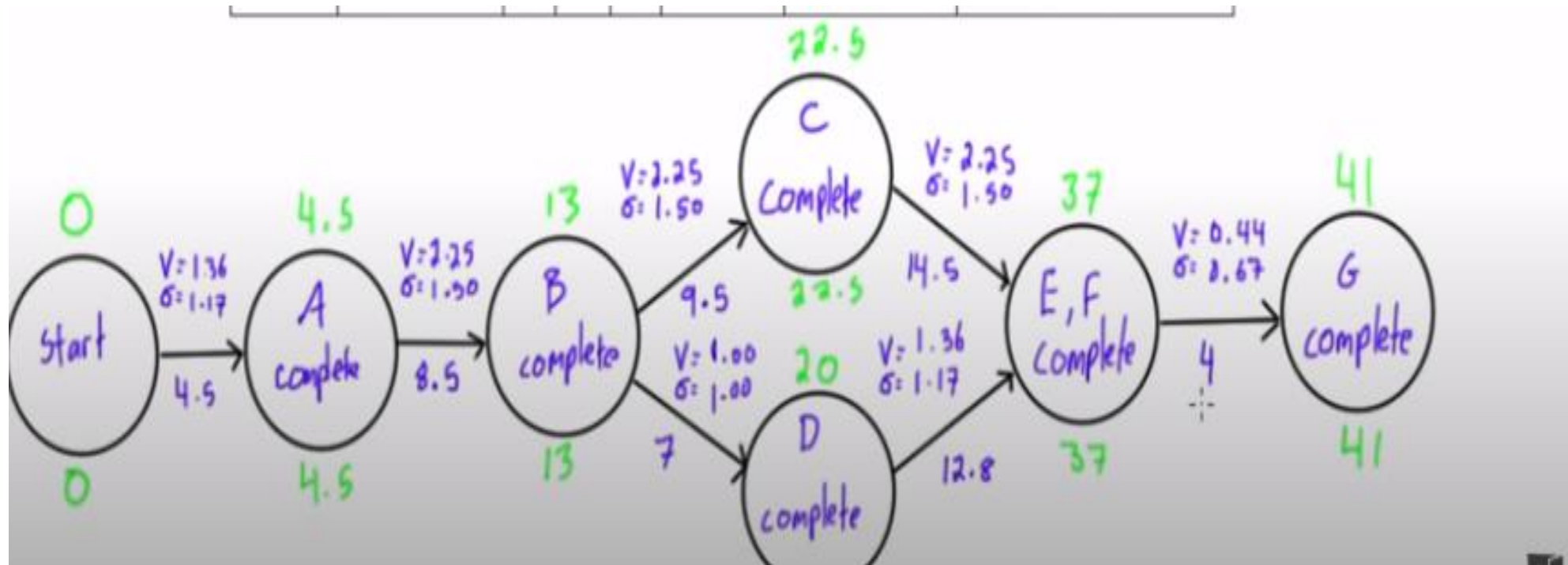
Activity	Predecessor	t_a	t_m	t_b	Mean (μ)	Variance (σ^2)	Standard Deviation (σ)
A	-	2	4	9	4.9	1.36	1.17
B	A	5	8	14	8.5	2.25	1.50
C	B	4	10	13	9.5	2.25	1.50
D	B	4	7	10	7	1.00	1.00
E	C	11	14	20	14.5	2.25	1.50
F	D	9	13	16	12.8	1.36	1.17
G	E, F	2	4	6	4	0.44	0.67



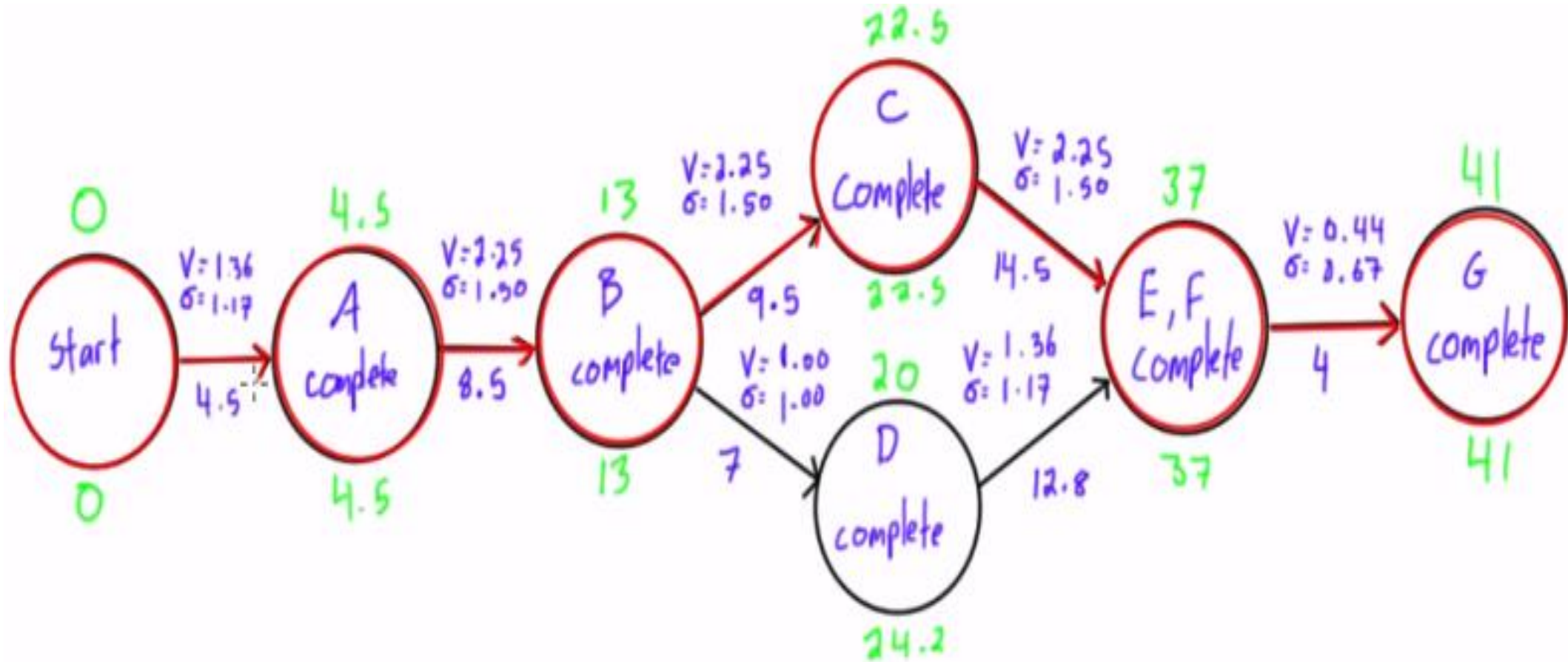
Place expected duration below the arrow and variance and SD above the arrow

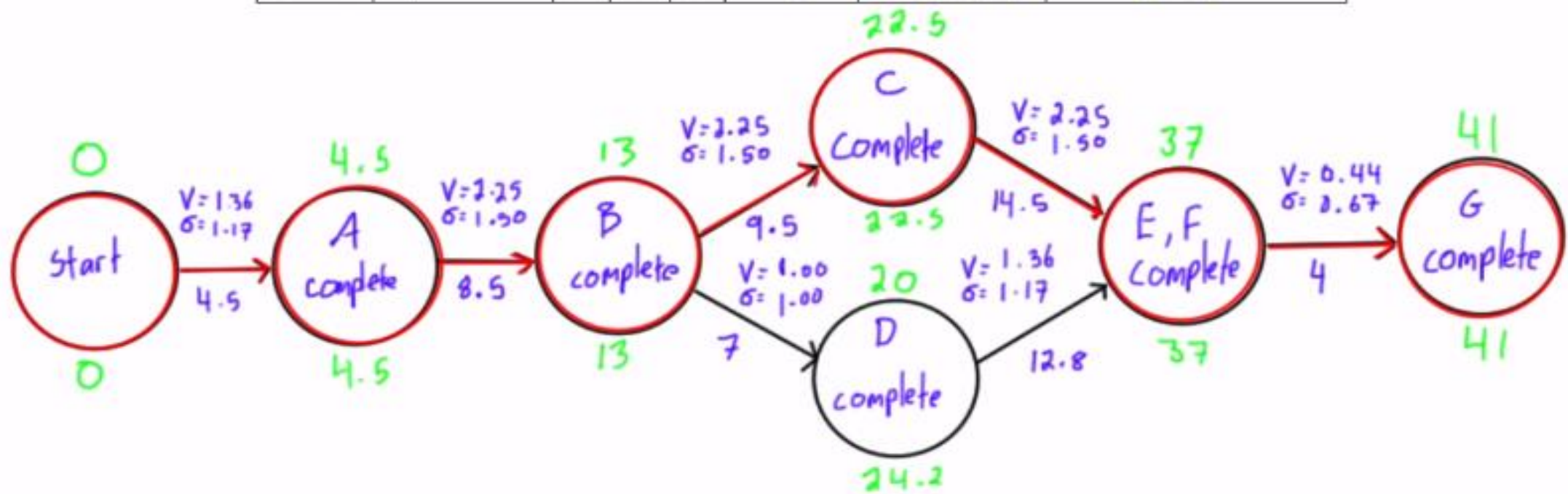


Forward pass and Backward Pass for the PERT Network Diagram



Critical Path





$$\text{Expected Duration} = \sum \mu_{cp} = 4.5 + 8.5 + 9.5 + 14.5 + 4 = 41$$

$$\text{Variance} = \sum V_{cp} = 1.36 + 2.25 + 2.25 + 2.25 + 0.44 = 8.55$$

$$\text{Standard Deviation} = \sqrt{\sum V_{cp}} = \sqrt{8.55} = 2.92$$

Note:

In formulas only the activities in the critical path are included and it gives the info about the whole project

Z-score

The Z-score is a value in statistics that describes the number of standard deviations a data point deviates from the mean.

$$Z = \frac{X - \mu}{\sigma}$$

Z = Z score

x = # days in question

μ = expected duration

σ = standard deviation

What is the probability that the project finishes in less than 38 days?

$$Z = \frac{38 - 41}{2.92} = -1.03$$

P(z) Probability is 0.1515

Assessing the likelihood of meeting a target

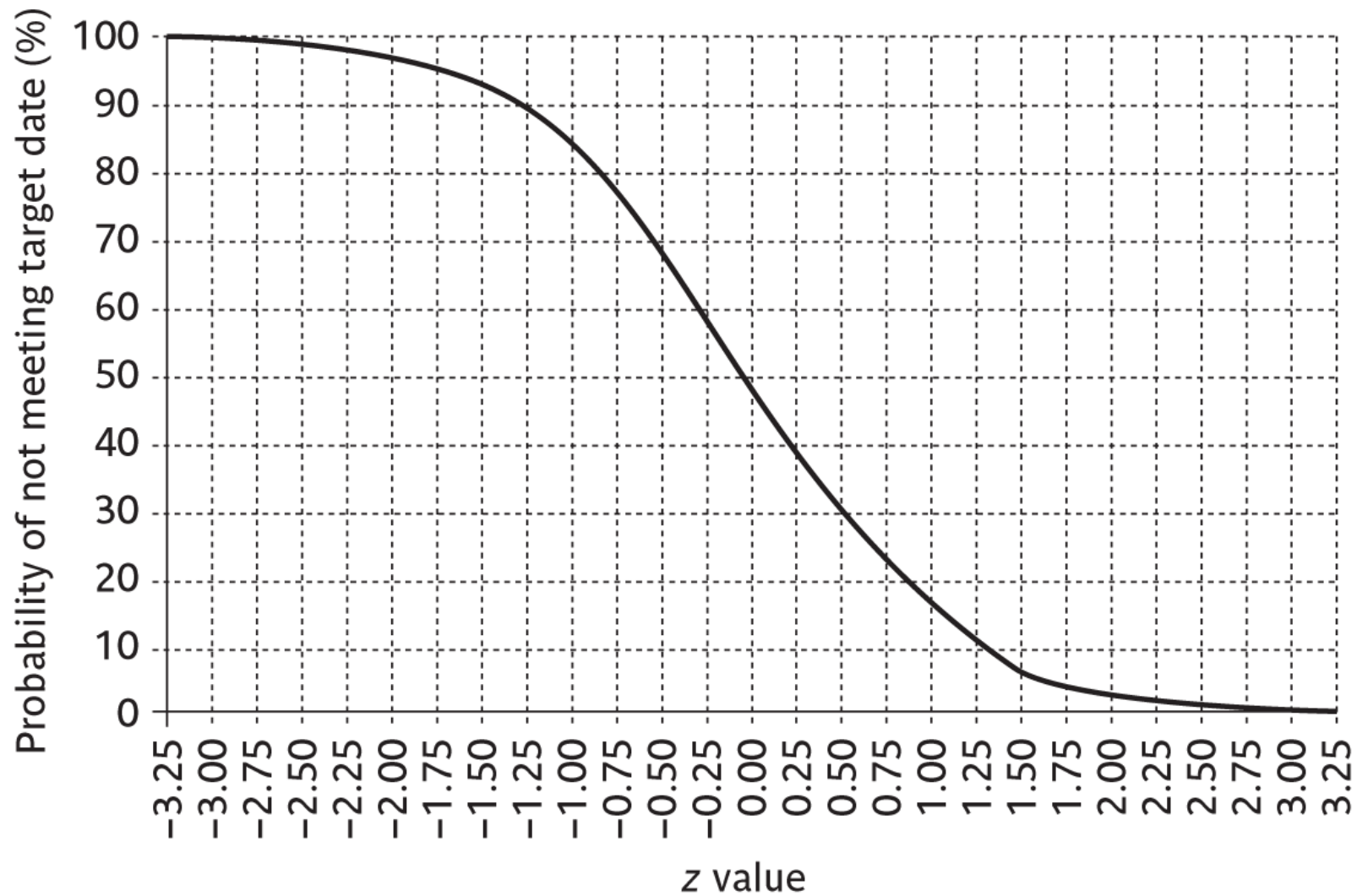
- Z values are calculated for each node that has a target date.
- Standard deviation in this example is 3.32
- Say the target for completing A+B+C was 52 days (T)
- Calculate the z value thus
 $z = (T - t_e)/s$, where T is target date and t_e is expected date .
- In this example $z = (52-48.33)/3.32$ i.e. 1.01

$$0.50 = (T - 30) / 5.4$$

$$0.50 * 5.4 = T - 30$$

$$0.50 * 5.4 + 30 = T$$

Graph of z values



Standard Normal Cumulative Probability Table



Cumulative probabilities for **NEGATIVE** z-values are shown in the following table:

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
-3.4	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0002
-3.3	0.0005	0.0005	0.0005	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0003
-3.2	0.0007	0.0007	0.0006	0.0006	0.0006	0.0006	0.0006	0.0005	0.0005	0.0005
-3.1	0.0010	0.0009	0.0009	0.0009	0.0008	0.0008	0.0008	0.0008	0.0007	0.0007
-3.0	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
-2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
-2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
-2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
-2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
-2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
-2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
-2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
-2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
-2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
-2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
-1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
-1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
-1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
-1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
-1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
-1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
-1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
-1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
-1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
-1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
-0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
-0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
-0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
-0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
-0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
-0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
-0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
-0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859

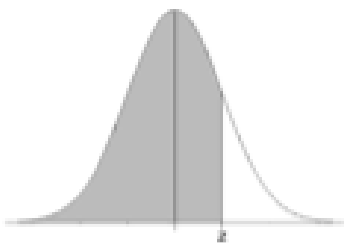
more than 43 days?

$$z = \frac{43 - 41}{2.92} = 0.68 \rightarrow P(z) = 0.7517$$

0.6 + 0.08 means first value from x axis & Second from Y-axis will give P(z)

Normal Distribution Table

Standard Normal Cumulative Probability Table



Cumulative probabilities for POSITIVE z-values are shown in the following table:

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974

0.6 from x axis

0.08 from Y axis

- Finding p value in excel
[\(5\) How to Determine P Value Using Excel - Dr. Rebecca Kreider - YouTube](#)
- Learn z statistics and p value
- [\(5\) P-Value Method For Hypothesis Testing - YouTube](#)

**Knowledge is
knowing a tomato
is a fruit. Wisdom
is not putting it in a
fruit salad.**

