

Lotfi Aliasker Zadeh<sup>[5]</sup> (/ˈzɑːdeɪ/; Azerbaijani: Lütfi Rəhim oğlu Ələsgərzadə;<sup>[6]</sup> Persian: لطفی; [2] 4 February 1921 – 6 September 2017)<sup>[1][3]</sup> was a mathematician, computer scientist, electrical engineer, artificial intelligence researcher, and professor<sup>[7]</sup> of computer science at the University of California, Berkeley. Zadeh is best known for proposing fuzzy mathematics, consisting of several fuzzy-related concepts: fuzzy sets, [8] fuzzy logic, [9] fuzzy algorithms, [10] fuzzy semantics, [11] fuzzy languages, [12] fuzzy control, [13] fuzzy systems, [14] fuzzy probabilities, [15] fuzzy events, [15] and fuzzy information. [16] Zadeh was a founding member of the Eurasian Academy. [11][17]

#### Early life and career

#### **Azerbaijan**

Zadeh was born in Baku, Azerbaijan SSR,<sup>[18]</sup> as **Lotfi Aliaskerzadeh**.<sup>[19]</sup> His father was Rahim Aleskerzade, an Iranian Muslim Azerbaijani<sup>[20]</sup> journalist from Ardabil on assignment from Iran, and his mother was Fanya (Feyga<sup>[21]</sup>) Korenman, a Jewish pediatrician from Odesa, Ukraine, who was an Iranian citizen.<sup>[22][23][24][25]</sup> The Soviet government at this time courted foreign correspondents, and the family lived well while in Baku.<sup>[26]</sup> Zadeh attended elementary school for three years there, <sup>[26]</sup> which he said "had a significant and long-lasting influence on my thinking and my way of looking at things."<sup>[27]</sup>

#### Iran

In 1931, when Stalin began agricultural collectivization,<sup>[21]</sup> and Zadeh was ten, his father moved his family back to Tehran, Iran. Zadeh was enrolled in Alborz High School, a missionary school,<sup>[21]</sup> where he was educated for the next eight years, and where he met his future wife,<sup>[26]</sup> Fay (Faina<sup>[21]</sup>) Zadeh, who said that he was "deeply influenced" by the "extremely decent, fine, honest and helpful" Presbyterian missionaries from the United States who ran the college. "To me they represented the best that you could find in the United States – people from the Midwest with strong roots. They were really 'Good Samaritans' – willing to give of themselves for the benefit of others. So this kind of attitude influenced me deeply. It also instilled in me a deep desire to live in the United States." During this time, Zadeh was awarded several patents. [26]

#### Lotfi A. Zadeh



(2016)

Born Lotfi Aliaskerzadeh

4 February 1921

Baku, Azerbaijan Soviet

Socialist Republic

**Died** 6 September 2017 (aged 96)<sup>[1]</sup>

[3]

Berkeley, California, US<sup>[4]</sup>

Alma mater University of Tehran

Massachusetts Institute of

Technology

Columbia University

Known for Founder of fuzzy mathematics,

fuzzy set theory, and fuzzy

logic, Z numbers, Z-transform

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### Fuzzy reasoning

- In real world, there exists much fuzzy knowledge; Knowledge that is vague, imprecise, uncertain, ambiguous, inexact, or probabilistic in nature. Human thinking and reasoning frequently involve fuzzy information, originating from inherently inexact human concepts.
- Humans, can give satisfactory answers, which are probably true.
- However, our systems are unable to answer many questions. The reason is, most systems are designed based upon classical set theory and two-valued logic which is unable to cope with unreliable and incomplete information and give expert opinions.
- We want, our systems should also be able to cope with unreliable and incomplete information and give expert opinions. Fuzzy sets have been able provide solutions to many real world problems.
- Fuzzy Set theory is an extension of classical set theory where elements have degrees of membership.

### **Classical Set Theory**

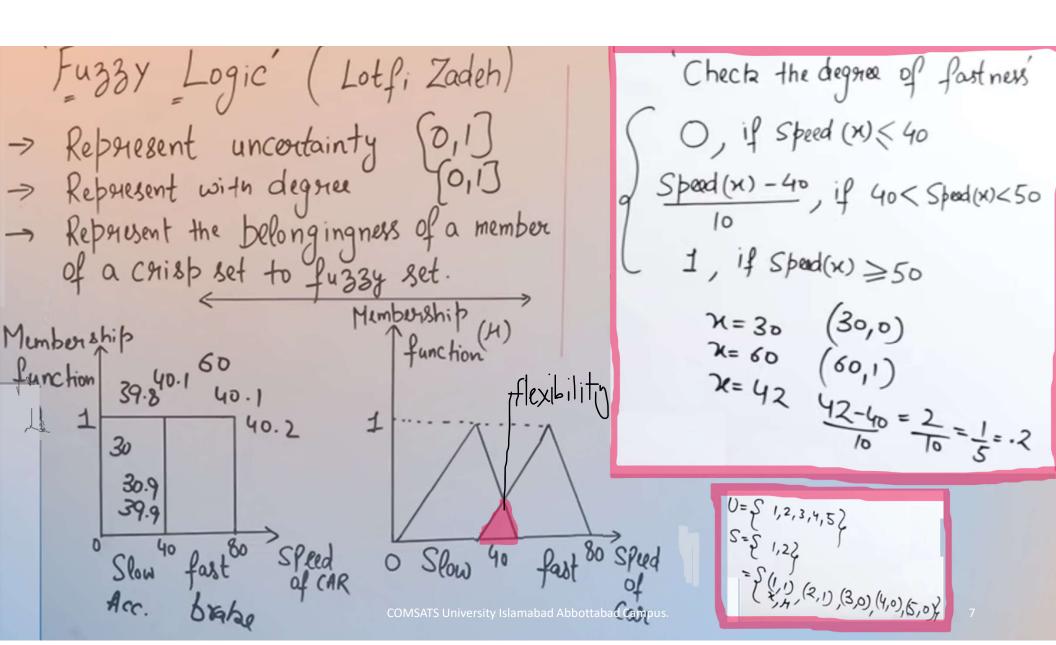
- A Set is any well defined collection of objects. An object in a set is called an element or member of that set. Sets are defined by a simple statement describing whether a particular element having a certain property belongs to that particular set.
- Sets are defined by a simple statement describing whether a particular element having a certain property belongs to that particular set.
- Classical set theory enumerates all its elements using
- A={a1, a2, a3, ....., an}

### Fuzzy Set

• The word "fuzzy" means "vagueness". Fuzziness occurs when the boundary of a piece of information is not clear-cut. Classical set theory allows the membership of the elements in the set in binary terms, a bivalent condition - an element either belongs or does not belong to the set. Fuzzy set theory permits the gradual assessment of the membership of elements in a set, described with the aid of a membership function valued in the real unit interval [0, 1].

### Why Fuzzy Logic?

- Fuzzy logic is useful for commercial and practical purposes.
- It can control machines and consumer products.
- It may not give accurate reasoning, but acceptable reasoning.
- Fuzzy logic helps to deal with the uncertainty in engineering.



# Classical or crisp set Example 1

- Classical or Crisp set is a collection of distinct objects.
- For example,
  - A set of all positive integers.
  - —A set of all the planets in the solar system.
  - —A set of all the lowercase letters of the alphabet.
  - Roster or Tabular Form
    - -Set of vowels in English alphabet, A = {a, e, i, o, u}
    - -Set of odd numbers less than 10, B =  $\{1,3,5,7,9\}$
  - 2. Set Builder Notation
    - -The set {a,e,i,o,u} is written as A = {x | x is a vowel in English alphabet}
    - -The set  $\{1,3,5,7,9\}$  is written as B =  $\{x \mid 1 \le x < 10 \text{ and } (x\%2) \neq 0\}$

#### Finite Set

A set which contains a definite number of elements is called a finite set.

- Example - S = 
$$\{x \mid 1 \le x < 10 \text{ and } (x\%2) \neq 0\}$$

#### Infinite Set

A set which contains infinite number of elements is called an infinite set.

$$-$$
 Example  $-$  S = {x | x ∈ N and x > 10}

#### Empty Set or Null Set

An empty set contains no elements. It is denoted by Φ.

**– Example** – S = 
$$\{x \mid x ∈ N \text{ and } 7 < x < 8\} = Φ$$

#### Subset

- A set X is a subset of set Y (Written as  $X \subseteq Y$ ) if every element of X is an element of set Y.
- **Example** Let, Y =  $\{1,2,3,4,5,6\}$  and X =  $\{1,2\}$ . Hence, we can write X⊆Y.

#### Proper Subset

- The term "proper subset" can be defined as "subset of but not equal to".
- A Set X is a proper subset of set Y (Written as X ⊂ Y) if every element of X is an element of set Y and |X| < |Y|.</li>
- Example Let, Y = {1,2,3,4,5,6} and X = {1,2}. Here set X ⊂ Y, since all elements in X are contained in Y too and Y has at least one element which is more than set X.

#### · Singleton Set or Unit Set

- A Singleton set or Unit set contains only one element.
- A singleton set is denoted by {s}.
- **Example** S =  $\{x \mid x \in \mathbb{N}, 7 < x < 9\} = \{8\}$

#### Equal Set

- If two sets contain the same elements, they are said to be equal.
- **Example** If A =  $\{1,2,6\}$  and B =  $\{6,1,2\}$

#### Equivalent Set

- If the cardinalities of two sets are same, they are called equivalent sets.
- Example If A =  $\{1,2,6\}$  and B =  $\{16,17,22\}$ , they are equivalent as cardinality of A is equal to the cardinality of B. i.e. |A| = |B| = 3

#### Overlapping Set

- Two sets that have at least one common element are called overlapping sets.
- Example Let, A =  $\{1,2,6\}$  and B =  $\{6,12,42\}$ . There is a common element '6'

#### Disjoint Set

- Two sets A and B are called disjoint sets if they do not have even one element in common.
- Example Let,  $A = \{1,2,6\}$  and  $B = \{7,9,14\}$ , there is not a single common element.



Fuzzy set operations Example 2

$$\tilde{A} = \left\{ \frac{1}{2} + \frac{0.3}{4} + \frac{0.5}{6} + \frac{0.2}{8} \right\}$$

$$\tilde{B} = \left\{ \frac{0.5}{2} + \frac{0.4}{4} + \frac{0.1}{6} + \frac{1}{8} \right\}$$

### Union, Intersection

- For the given fuzzy sets we have the following
  - (a) Union

$$\underline{\tilde{A} \cup \tilde{B}} = \max\{\mu_{\underline{A}}(x), \mu_{\underline{B}}(x)\}$$

$$= \left\{\frac{1}{2} + \frac{0.4}{4} + \frac{0.5}{6} + \frac{1}{8}\right\}$$
Degree of membership

$$\tilde{A} = \left\{ \frac{1}{2} + \frac{0.3}{4} + \frac{0.5}{6} + \frac{0.2}{8} \right\}$$

$$\tilde{B} = \left\{ \frac{0.5}{2} + \frac{0.4}{4} + \frac{0.1}{6} + \frac{1}{8} \right\}$$

(b) Intersection

$$\underline{A} \cap \underline{B} = \min\{\mu_{\underline{A}}(x), \ \mu_{\underline{B}}(x)\}$$

$$= \left\{ \frac{0.5}{2} + \frac{0.3}{4} + \frac{0.1}{6} + \frac{0.2}{8} \right\}$$

### Complement

(c) Complement

$$\tilde{A} = 1 - \mu_{\tilde{A}}(x) = \left\{ \frac{0}{2} + \frac{0.7}{4} + \frac{0.5}{6} + \frac{0.8}{8} \right\}$$

$$\underline{B} = 1 - \mu_{\underline{B}}(x) = \left\{ \frac{0.5}{2} + \frac{0.6}{4} + \frac{0.9}{6} + \frac{0}{8} \right\}$$

(d) Difference

$$A \mid B = A \cap \overline{B} = \left\{ \frac{0.5}{2} + \frac{0.3}{4} + \frac{0.5}{6} + \frac{0}{8} \right\}$$

$$\tilde{A} = \left\{ \frac{1}{2} + \frac{0.3}{4} + \frac{0.5}{6} + \frac{0.2}{8} \right\}$$

$$\tilde{B} = \left\{ \frac{0.5}{2} + \frac{0.4}{4} + \frac{0.1}{6} + \frac{1}{8} \right\}$$

### Fuzzy relationship (MAX-MIN Composition) example 3

$$\begin{array}{cccc}
z_1 & z_2 & z_3 \\
T = R \circ S = x_1 \\
x_2 & z_3
\end{array}$$

$$\tilde{R} = \begin{bmatrix} y_1 & y_2 & z_1 & z_2 & z_3 \\ x_1 & 0.6 & 0.3 \\ 0.2 & 0.9 \end{bmatrix} \tilde{S} = \begin{bmatrix} y_1 & 0.5 & 0.3 \\ y_2 & 0.8 & 0.4 & 0.7 \end{bmatrix}$$

$$\tilde{T} = \tilde{R} \circ \tilde{S} = \begin{bmatrix} x_1 & z_2 & z_3 \\ x_2 & 0.6 & 0.5 & 0.3 \\ 0.8 & 0.4 & 0.7 \end{bmatrix}$$

$$\tilde{Z} = \tilde{R} \circ \tilde{S} = \begin{cases}
x_1 & z_2 & z_3 \\
0.6 & 0.5 & 0.3 \\
0.8 & 0.4 & 0.7
\end{cases}$$

$$\mu_{T}(x_{1}, z_{1}) = \max\{\min[\mu_{R}(x_{1}, y_{1}), \mu_{S}(y_{1}, z_{1})], \\ \min[\mu_{R}(x_{1}, y_{2}), \mu_{S}(y_{2}, z_{1})]\}$$

$$= \max[\min(0.6, 1), \min(0.3, 0.8)]$$

$$= \max(0.6, 0.3) = 0.6$$

$$\mu_{T}(x_{1}, z_{2}) = \max[\min(0.6, 0.5), \min(0.3, 0.4)]$$

$$= \max(0.5, 0.3) = 0.5$$

$$\begin{split} \mu_{\underline{T}}(x_1, z_3) &= \max[\min(0.6, 0.3), \min(0.3, 0.7)] \\ &= \max(0.3, 0.3) = 0.3 \\ \mu_{\underline{T}}(x_2, z_1) &= \max[\min(0.2, 1), \min(0.9, 0.8)] \\ &= \max(0.2, 0.8) = 0.8 \\ \mu_{\underline{T}}(x_2, z_2) &= \max[\min(0.2, 0.5), \min(0.9, 0.4)] \\ &= \max(0.2, 0.4) = 0.4 \\ \mu_{\underline{T}}(x_2, z_3) &= \max[\min(0.2, 0.3), \min(0.9, 0.7)] \\ &= \max(0.2, 0.7) = 0.7 \end{split}$$

### Fuzzy relationship (MAX-product Composition) example 3

$$\begin{array}{ccc}
z_1 & z_2 & z_3 \\
T = R \circ S = x_1 \\
x_2
\end{array}$$

$$\tilde{R} = \begin{bmatrix} y_1 & y_2 & z_1 & z_2 & z_3 \\ x_1 & 0.6 & 0.3 \\ x_2 & 0.2 & 0.9 \end{bmatrix} \tilde{S} = \begin{bmatrix} y_1 & 0.5 & 0.3 \\ y_2 & 0.8 & 0.4 & 0.7 \end{bmatrix} \qquad \tilde{T} = \begin{bmatrix} z_1 & z_2 & z_3 \\ 0.6 & 0.3 & 0.21 \\ 0.72 & 0.36 & 0.63 \end{bmatrix}$$

$$\mu_{\underline{T}}(x_1, z_1) = \max\{ [\mu_{\underline{R}}(x_1, y_1) \cdot \mu_{\underline{S}}(y_1, z_1)], \\ [\mu_{\underline{R}}(x_1, y_2) \cdot \mu_{\underline{S}}(y_2, z_1)] \}$$

$$= \max(0.6, 0.24) = 0.6$$

$$\mu_{\underline{T}}(x_1, z_2) = \max[(0.6 \times 0.5), (0.3 \times 0.4)]$$

 $= \max(0.3, 0.12) = 0.3$ 

$$\mu_{T}(x_{1}, z_{3}) = \max[(0.6 \times 0.3), (0.3 \times 0.7)]$$

$$= \max(0.18, 0.21) = 0.21$$

$$\mu_{T}(x_{2}, z_{1}) = \max[(0.2 \times 1), (0.9 \times 0.8)]$$

$$= \max(0.2, 0.72) = 0.72$$

$$\mu_{T}(x_{2}, z_{2}) = \max[(0.2 \times 0.5), (0.9 \times 0.4)]$$

$$= \max(0.1, 0.36) = 0.36$$

$$\mu_{T}(x_{2}, z_{3}) = \max[(0.2 \times 0.3), (0.9 \times 0.7)]$$

$$= \max(0.06, 0.63) = 0.63$$

### Apply fuzzy logic- example 4

based upon their detection levels and gain settings. The table of gain settings and sensor detection levels with a standard item being monitored providing typical membership values to represent the detection levels for each sensor is given in Table.

Gain	Detection Level Sensor 1	Detection Level Sensor 2
0	0	0
10	0.2	0.35
20	0.35	0.25
30	0.65	0.8
40	0.85	0.95
50	1	1

(a) 
$$\mu_{\underline{D}_1 \cup \underline{D}_2}(x)$$
; (b)  $\mu_{\underline{D}_1 \cap \underline{D}_2}(x)$ ; (c)  $\mu_{\overline{\underline{D}_1}}(x)$ ; (d)  $\mu_{\overline{\underline{D}_2}}(x)$ ; (e)  $\mu_{\underline{D}_1 \cup \overline{\underline{D}_1}}(x)$ ; (f)  $\mu_{\underline{D}_1 \cap \overline{\underline{D}_1}}(x)$ ;

### Apply fuzzy logic- example 4

Now given the universe of discourse
 X = {0, 10, 20, 30, 40, 50} and the
 membership functions for the two
 sensors in discrete form as

$$D_1 = \left\{ \frac{0}{0} + \frac{0.2}{10} + \frac{0.35}{20} + \frac{0.65}{30} + \frac{0.85}{40} + \frac{1}{50} \right\}$$

$$D_2 = \left\{ \frac{0}{0} + \frac{0.35}{10} + \frac{0.25}{20} + \frac{0.8}{30} + \frac{0.95}{40} + \frac{1}{50} \right\}$$

Gain	Detection Level Sensor 1	Detection Level Sensor 2
0	0	0
10	0.2	0.35
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40	0.85	0.95
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$$D_1 = \left\{ \frac{0}{0} + \frac{0.2}{10} + \frac{0.35}{20} + \frac{0.65}{30} + \frac{0.85}{40} + \frac{1}{50} \right\}$$

$$D_2 = \left\{ \frac{0}{0} + \frac{0.35}{10} + \frac{0.25}{20} + \frac{0.8}{30} + \frac{0.95}{40} + \frac{1}{50} \right\}$$

(a) 
$$\mu_{\bar{\nu}_1 \cup \bar{\nu}_2}(x)$$
  
=  $\max \{ \mu_{\bar{\nu}_1}(x), \ \mu_{\bar{\nu}_2}(x) \}$   
=  $\left\{ \frac{0}{0} + \frac{0.35}{10} + \frac{0.35}{20} + \frac{0.8}{30} + \frac{0.95}{40} + \frac{1}{50} \right\}$ 

$$D_1 = \left\{ \frac{0}{0} + \frac{0.2}{10} + \frac{0.35}{20} + \frac{0.65}{30} + \frac{0.85}{40} + \frac{1}{50} \right\}$$

$$D_2 = \left\{ \frac{0}{0} + \frac{0.35}{10} + \frac{0.25}{20} + \frac{0.8}{30} + \frac{0.95}{40} + \frac{1}{50} \right\}$$

(b) 
$$\mu_{\underline{p}_1 \cap \underline{p}_2}(x)$$
  

$$= \min \{ \mu_{\underline{p}_1}(x), \mu_{\underline{p}_2}(x) \}$$

$$= \left\{ \frac{0}{0} + \frac{0.2}{10} + \frac{0.25}{20} + \frac{0.65}{30} + \frac{0.85}{40} + \frac{1}{50} \right\}$$

$$D_1 = \left\{ \frac{0}{0} + \frac{0.2}{10} + \frac{0.35}{20} + \frac{0.65}{30} + \frac{0.85}{40} + \frac{1}{50} \right\}$$

$$D_2 = \left\{ \frac{0}{0} + \frac{0.35}{10} + \frac{0.25}{20} + \frac{0.8}{30} + \frac{0.95}{40} + \frac{1}{50} \right\}$$

(c) 
$$\mu_{\overline{Q}_1}(x) = 1 - \mu_{\overline{Q}_1}(x)$$
  
=  $\left\{ \frac{1}{0} + \frac{0.8}{10} + \frac{0.65}{20} + \frac{0.35}{30} + \frac{0.15}{40} + \frac{0}{50} \right\}$ 

$$D_1 = \left\{ \frac{0}{0} + \frac{0.2}{10} + \frac{0.35}{20} + \frac{0.65}{30} + \frac{0.85}{40} + \frac{1}{50} \right\}$$

$$D_2 = \left\{ \frac{0}{0} + \frac{0.35}{10} + \frac{0.25}{20} + \frac{0.8}{30} + \frac{0.95}{40} + \frac{1}{50} \right\}$$

(d) 
$$\mu_{\overline{D}_2}(x) = 1 - \mu_{D_2}(x)$$

$$= \left\{ \frac{1}{0} + \frac{0.65}{10} + \frac{0.75}{20} + \frac{0.2}{30} + \frac{0.05}{40} + \frac{0}{50} \right\}$$

(e) 
$$\mu_{\bar{D}_1 \cup \overline{D}_1}(x) = \max\{\mu_{\bar{D}_1}(x), \mu_{\overline{D}_1}(x)\}$$

$$= \left\{ \frac{1}{0} + \frac{0.8}{10} + \frac{0.65}{20} + \frac{0.65}{30} + \frac{0.85}{40} + \frac{1}{50} \right\}$$

$$D_1 = \left\{ \frac{0}{0} + \frac{0.2}{10} + \frac{0.35}{20} + \frac{0.65}{30} + \frac{0.85}{40} + \frac{1}{50} \right\}$$

$$D_2 = \left\{ \frac{0}{0} + \frac{0.35}{10} + \frac{0.25}{20} + \frac{0.8}{30} + \frac{0.95}{40} + \frac{1}{50} \right\}$$

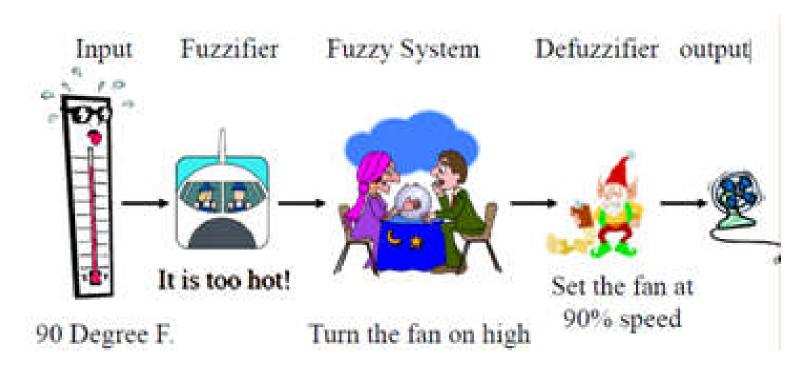
(c) 
$$\mu_{\overline{D_1}}(x) = 1 - \mu_{D_1}(x)$$

(f) 
$$\mu_{\overline{Q_1} \cap \overline{Q_1}}(x)$$

$$= \left\{ \frac{0}{0} + \frac{0.2}{10} + \frac{0.35}{20} + \frac{0.35}{30} + \frac{0.15}{40} + \frac{0}{50} \right\}$$

$$= \left\{ \frac{1}{0} + \frac{0.8}{10} + \frac{0.65}{20} + \frac{0.35}{30} + \frac{0.15}{40} + \frac{0}{50} \right\}$$

### Example 5



#### **Algorithm**

#### BUILD A FUZZY CONTROLLER

5 Steps

#### 1. Pick the linguistic variable

Example: Let temperature (X) be input and motor speed (Y) be output

#### 2. Pick the fuzzy sets

Define fuzzy subsets of the X and Y

#### 3. Pick the fuzzy rules

Associate output to the input

#### 4. Obtain Fuzzy value

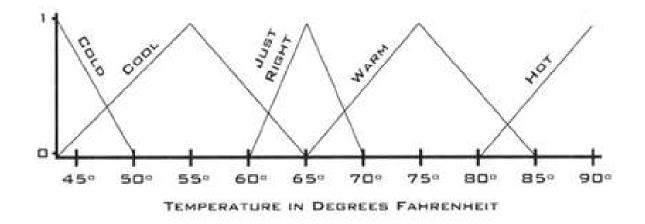
#### 5.Perform Defuzzification

## Goal: Design a motor speed controller for air conditioner

- Step 1: assign input and output variables
- Let X be the temperature in Fahrenheit
- Let Y be the motor speed of the air conditioner
- Step 2: Pick fuzzy sets
- Define linguistic terms of the linguistic variables temperature (X) and motor speed (Y) and associate them with fuzzy sets
- For example, 5 linguistic terms / fuzzy sets on X

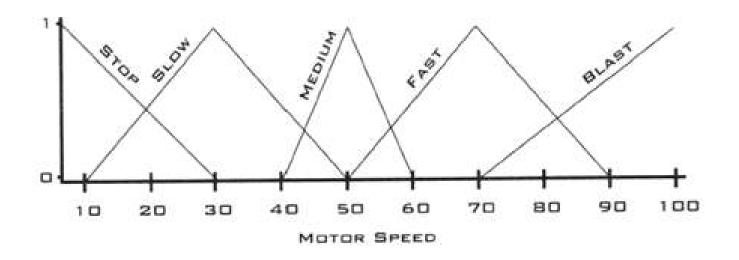
### Cold, Cool, Just Right, Warm, and Hot

#### Input Fuzzy sets



### Stop, Slow, Medium, Fast, and Blast

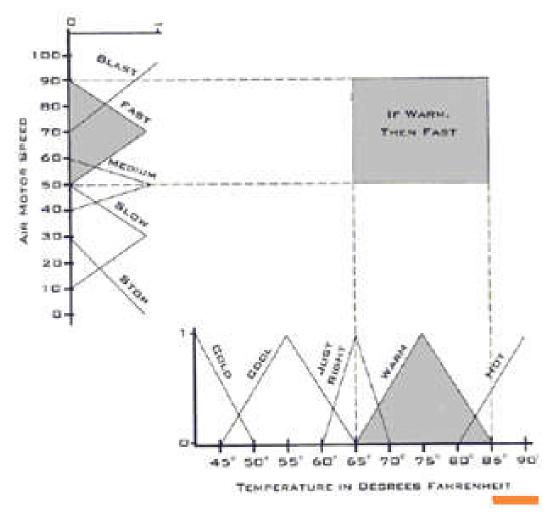
### Output Fuzzy sets



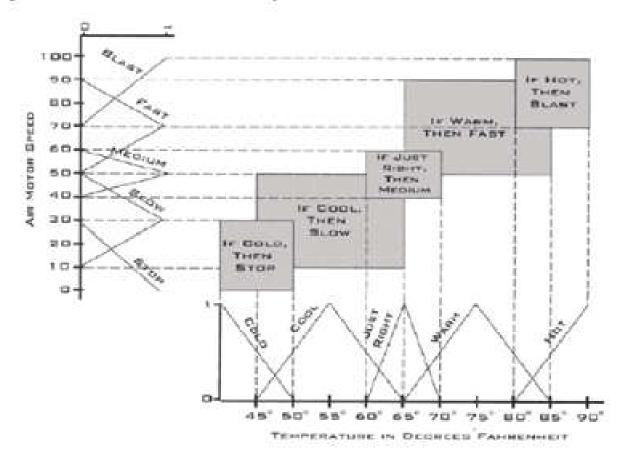
## Step 3: Assign a motor speed set to each temperature set

- If temperature is cold then motor speed is stop
- If temperature is cool then motor speed is slow
- If temperature is just right then motor speed is medium
- If temperature is warm then motor speed is fast
- If temperature is hot then motor speed is blast

 A Fuzzy Relation expressed by a rule



#### A Fuzzy controller with 5 patches



### Step 4: Obtain fuzzy value

• Fuzzy set operations perform evaluation of rules. The operations used for OR and AND are Max and Min respectively. Combine all results of evaluation to form a final result. This result is a fuzzy value.

### Step 5: Perform defuzzification

• Defuzzification is then performed according to membership function for output variable.