$$y'' = \frac{x+c_1(0) - 1\frac{d}{dx}(x+c_1)}{(x+c_1)^2}$$

$$y'' = \frac{-1(1+0)}{(x+c_1)^2} \Rightarrow \frac{-1}{(x+c_1)^2}$$

equ ① $\Rightarrow$

$$\frac{-1}{(x+c_1)^2} + \left(\frac{1}{x+c_1}\right)^2 = 0$$

$$\Rightarrow \frac{-1}{(x+c_1)^2} + \frac{1}{(x+c_1)^2} = 0$$

$$0 = 0$$

So, equ ② is a sol of equ ①

34. $y'' + y = \tan x \rightarrow ①$

$$y = -\cos x \ln(\sec x + \tan x) \rightarrow ②$$

$$y' = -\cos x \frac{d}{dx}\ln(\sec x + \tan x) + \ln(\sec x + \tan x)\frac{d}{dx}(-\cos x)$$

$$y' = -\cos x \frac{1}{\sec x + \tan x}\frac{d}{dx}(\sec x + \tan x) + \ln(\sec x + \tan x)(+\sin x)$$

$$y' = \frac{-\cos x}{\sec x + \tan x}(\sec x\tan x + \sec^2 x) + \ln(\sec x + \tan x)\sin x$$

$y' = \frac{-\cos x}{\sec x + \tan x}(\tan x + \sec x)\sec x + \ln|\sec x + \tan x| \cdot \sin x$

$y' = -\cancel{\cos x} \cdot \frac{1}{\cancel{\cos x}} + \ln|\sec x + \tan x| \cdot \sin x$

$y'' = -\cos x \frac{d}{dx}\sec x + \sec x \frac{d}{dx}(-\cos x) + \frac{1}{\sec x + \tan x} \frac{d}{dx}(\sec x + \tan x)\sin x + \ln|\sec x + \tan x| \cos x$

$e^x = u$ $e^x = \frac{du}{dx}$ $dx\, e^x = du$

$e^x/ \frac{dy}{dx} = \frac{dy}{dx}$

$y'' = -\cos x \tan x \cdot \sec x + \sec x \sin x + \frac{1}{\sec x + \tan x}(\tan x \sec x + \sec^2 x) + \ln|\sec x + \tan x| \cos x$

$y'' = -\cos x \tan x \cdot \sec x + \sec x \sin x + \frac{1}{\sec x + \cancel{\tan x}}(\tan x + \cancel{\sec x})\sec x + \cos x \ln|\sec x + \tan x|$

$y'' = \sec x(-\cos x \tan x + \sin x + 1) + \cos x \ln|\sec x + \tan x|$

$\frac{-1+2}{2}$

equ (1) $\Rightarrow \sec x(-\cos x \tan x + \sin x + 1) + \cos x \ln|\cancel{\sec x + \tan x}|$ $+ \cos x \ln|\cancel{\sec x + \tan x}| = \tan x$

$\frac{y}{(4+y^2)^{\frac{1}{2}}}$

$\frac{1\sqrt{u-4}}{2(u)^{\frac{1}{2}}\sqrt{\cancel{u-4}}}$ $\frac{1}{2u^{\frac{1}{2}}}$

$u - \sqrt{4}$ $\frac{1}{2}u^{-\frac{1}{2}+1}$ $\frac{1}{2} \frac{u^{1/2}}{1/2}$

$4 + y^2 = u$

$y^2 = u - 4$

$y = \sqrt{u-4}$

$dy = \frac{1}{2\sqrt{u-4}}du$

$$y' = \frac{-\cos x}{\cancel{\sec x + \tan x}} \sec x(\cancel{\tan x + \sec x}) + \ln|\sec x + \tan x| \sin x$$

$$= -\cancel{\cos x} \cdot \frac{1}{\cos x} + \ln|\sec x + \tan x| \sin x$$

$$= -1 + \ln|\sec x + \tan x| \sin x$$

$$y'' = 0 + \frac{d}{dx} \sin x \cdot \ln|\sec x + \tan x|$$

$$= \sin x \frac{d}{dx} \ln|\sec x + \tan x| + \ln|\sec x + \tan x| \frac{d}{dx} \sin x$$

$$= \sin x \cdot \frac{1}{|\sec x + \tan x|} \frac{d}{dx} |\sec x + \tan x| + \cos x \ln|\sec x + \tan x|$$

$$= \frac{\sin x}{\sec x + \tan x} (\sec x \tan x + \sec^2 x) + \cos x \ln|\sec x + \tan x|$$

$$= \frac{\sin x}{\cancel{\sec x + \tan x}} \cdot \sec x(\cancel{\tan x + \sec x}) + \cos x \ln|\sec x + \tan x|$$

$$= \sin x \cdot \sec x + \cos x \ln|\sec x + \tan x|$$

$$y'' = \frac{\sin x}{\cos x} - y$$

equ 1 $\Rightarrow \frac{\sin x}{\cos x} - \cancel{y} + \cancel{y} = \tan x$

$$\frac{\sin x}{\cos x} = \tan x$$

$$\tan x = \tan x$$

So,

equ (2) is a sol of equ (1)

35. $$x\frac{d^2y}{dx^2} + 2\frac{dy}{dx} = 0 \to (1)$$

$$y = C_1 + C_2 x^{-1} \to (2)$$

equ (2) $\Rightarrow$ $$y' = \frac{d}{dx}C_1 + \frac{d}{dx}C_2 x^{-1}$$

$$y' = 0 + C_2(-1x^{-2})$$

$$= -C_2 x^{-2}.$$

$$=$$

$$y'' = -(-2)C_2 x^{-3}$$

$$y'' = 2C_2 x^{-3}$$

equ (1) $\Rightarrow$

$$\Rightarrow x\left(\frac{2C_2}{x^{3}}\right) + 2\left(-\frac{C_2}{x^2}\right) = 0$$

$$\frac{2\cancel{c_2}}{x^2} - \frac{2c_2}{x^2} = 0$$

$$0 = 0$$

So, equ (2) is a sol^n of equ (1)

36. $x^2y'' - xy' + 2y = 0 \rightarrow (1)$

$y = x\cos(\ln x),\ x>0 \rightarrow (2)$

equ (2) $\Rightarrow$ $y' = x\frac{d}{dx}\cos(\ln x) + \cos(\ln x)\frac{dx}{dx}$

$y' = x - \sin(\ln x) \cdot \frac{d}{dx}\ln x + \cos(\ln x)$

$y' = -\cancel{x}\sin(\ln x) \cdot \frac{1}{\cancel{x}}$

$y' = -\sin(\ln x) + \cos(\ln x)$

$y'' = -\cos\ln x \cdot \frac{1}{x} - \sin(\ln x) \cdot \frac{1}{x}$

equ (1) $\Rightarrow$

$-\frac{x^2\cos\ln x}{\cancel{x}} - \frac{x^2\sin(\ln x)}{\cancel{x}} + x\sin\ln x - x\cos\ln x + 2(x\cos(\ln x))$

$\Rightarrow -x\cos\ln x - \cancel{x\sin\ln x} + \cancel{x\sin\ln x} - x\cos\ln x + 2x\cos\ln x$

$\Rightarrow -2x\cos\ln x + 2\cos\ln x = 0$

$0 = 0$

37 $x^2y'' - 3xy' + 4y = 0$

$y = x^2 + x^2 \ln x$

$y' = 2x + x^2 \cdot \frac{1}{x} + \ln x \, 2x$

$= 2x + x + \ln x \, 2x$

$= 3x + \ln x \cdot 2x$

$y'' = 3 + 2\left[x \cdot \frac{1}{x} + \ln x (1)\right]$

$= 3 + 2 + 2\ln x$

$y'' = 5 + 2\ln x$

equ (1) $\Rightarrow x^2(5 + 2\ln x) - 3x(3x + \ln x \, 2x) + 4(x^2 + x^2 \ln x) = 0$

$\Rightarrow 5x^2 + 2x^2\ln x - 9x^2 - 6x^2 \ln x + 4x^2 + 4x^2 \ln x = 0$

$\Rightarrow -\cancel{4x^2} + \cancel{2x^2} \ln x - 2x^2 \ln x + \cancel{4x^2} = 0$

$0 = 0$

So, equ (3) is a sol of equ (1)

38. $y''' - y'' + 9y' - 9y = 0 \rightarrow (1)$

$y = c_1 \sin 3x + c_2 \cos 3x + 4e^x \rightarrow (2)$

equ (2) $\Rightarrow$ $y' = c_1 3 \cos 3x - c_2 3 \sin 3x + 4e^x$

$y'' = -3 \cdot 3 c_1 \sin 3x - 3 \cdot 3 c_2 \cos 3x + 4e^x$

$= -9c_1 \sin 3x - 9c_2 \cos 3x + 4e^x$

$y''' = -27 c_1 \cos 3x + 27 c_2 \sin 3x + 4e^x$

equ (1) $\Rightarrow$ $-27c_1 \cancel{\cos 3x} + 27c_2 \cancel{\sin 3x} + \cancel{4}e^x + 9c_1 \sin 3x + 9c_2 \cancel{\cos} 3x - \cancel{4}e^x + 27c_1 \cos 3x - 27c_2 \sin 3x + 3\cancel{6}e^x - 9c_1 \cancel{\sin} 3x - 9c_2 \cancel{\cos} 3x - \cancel{36e^x} = 0$

So, $0 = 0$

So, equ (2) is a sol of equ (1)

39. $y''' - 3y'' + 3y' - y = 0 \rightarrow (1)$ $\quad y = x^2 e^x \rightarrow (2)$

equ (2) $\Rightarrow$ $y' = x^2 e^x + e^x 2x$

$y'' = x^2 e^x + e^x 2x + 2[e^x(1) + xe^x]$

$= x^2 e^x + 2e^x x + 2e^x + 2xe^x$

$= x^2 e^x + 2e^x + 4xe^x$

$$y''' = x^2e^x + e^x 2x + 2e^x + 4[xe^x + e^x(1)]$$
$$= x^2e^x + 2xe^x + 2e^x + 4xe^x + 4e^x$$
$$= x^2e^x + 6xe^x + 6e^x$$

equ ① $\Rightarrow$ $\cancel{x^2e^x} + \cancel{6xe^x} + \cancel{6e^x} - \cancel{3x^2e^x} - \cancel{6e^x}$
$-12xe^x + \cancel{3x^2e^x} + \cancel{6e^x}x - \cancel{x^2e^x} = 0$

$$0 = 0$$

So, equ ② is a sol of equ ①

40. $x^3\dfrac{d^3y}{dx^3} + 2x^2\dfrac{d^2y}{dx^2} - x\dfrac{dy}{dx} + y = 12x^2 \rightarrow$ ①

$y = c_1x + c_2x\ln x + 4x^2$, $x > 0$, ②

equ ② $\Rightarrow$ $y' = c_1 + c_2\left[x \cdot \dfrac{1}{x} + \ln x(1)\right] + 4(2x)$

$\dfrac{dy}{dx} = y' = c_1 + c_2 + c_2\ln x + 8x$

$y'' = 0 + 0 + c_2 \cdot \dfrac{1}{x} + 8(1)$

$= \dfrac{c_2}{x} + 8$

$y''' = -\dfrac{c_2}{x^2}$

equ ① $\Rightarrow$ $-\frac{c_2}{x^2}\cdot x^3 + 2x^2\left[\frac{c_2}{x} + 8\right] - x\left[c_1 + c_2 + c_2 \ln x + 8x\right] + c_1 x + c_2 x \ln x + 4x^2 = 12x^2$

$\Rightarrow -c_2 x + 2xc_2 + 16x^2 - xc_1 - xc_2 - xc_2 \ln x - 8x^2 + c_1 x + c_2 x \ln x + 4x^2 = 12x^2$

$\Rightarrow -2c_2 x + 2xc_2 + 16x^2 - 4x^2 = 12x^2$

$12x^2 = 12x^2$

So, equ ② is a sol of equ ①

41. $xy' - 2y = 0$; $y = \begin{cases} -x^2, & x < 0 \\ x^2, & x \geq 0 \end{cases}$

For $x < 0$

$y = -x^2$

$y' = -2x$

equ ① $\Rightarrow$ $x(-2x) - 2(-x^2) = 0$

$-2x^2 + 2x^2 = 0$

$0 = 0$

So, $y = -x^2$ is a sol of equ ①

For $x \geq 0$

$y = x^2$

$y' = 2x$