# Assignment4

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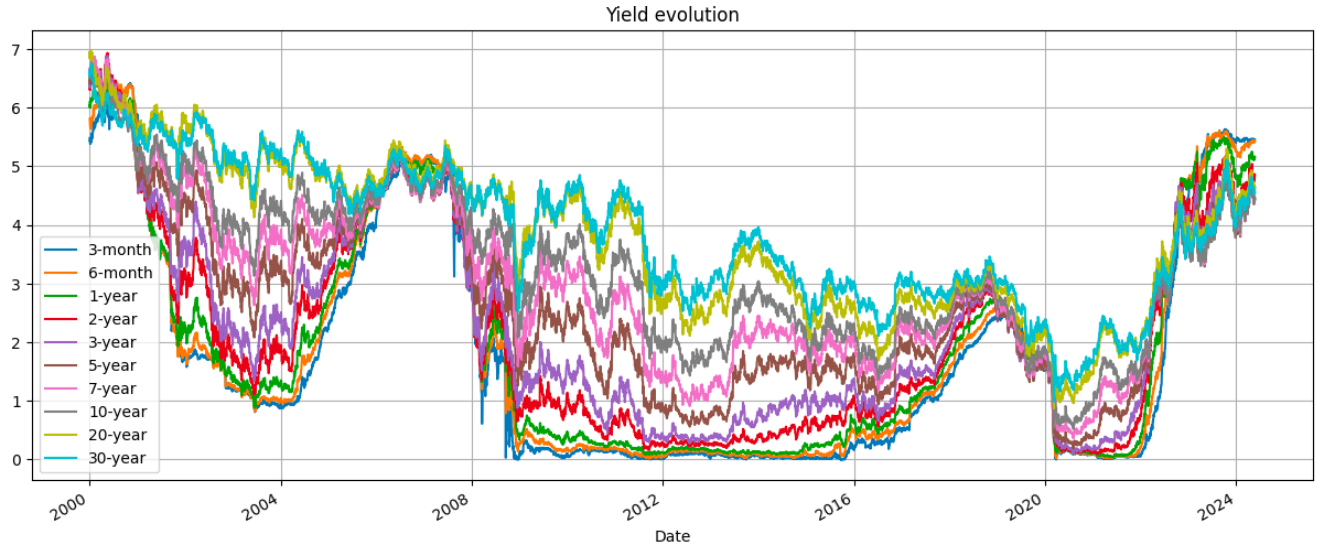
1. **Provide a formal analysis of interest rate behavior over time by applying the econometric concept of cointegration. Summarize and interpret the results of your analysis from a statistical/econometric perspective.**

   i. We begin by visualizing the evolution of yields with different maturities to observe patterns over time.




   ii. The ADF test is implemented for each yield to determine stationarity. The results indicate that all yields are non-stationary

| | ADF Statistic | p-value |
|---|---|---|
| **3-month** | -1.707357 | 0.427326 |
| **6-month** | -1.804738 | 0.378112 |
| **1-year** | -1.702301 | 0.429927 |
| **2-year** | -2.233559 | 0.194292 |
| **3-year** | -2.260166 | 0.185137 |
| **5-year** | -2.339511 | 0.159541 |
| **7-year** | -2.504235 | 0.114447 |
| **10-year** | -2.519285 | 0.110852 |
| **20-year** | -2.424748 | 0.134873 |
| **30-year** | -2.429034 | 0.133709 |

   iii. We split data into training set (80%) and validation set (20%) and do the Johansen test, with a constant term and lag one, is conducted for pairs of yields to check for cointegration. We display the trace statistic as follows.

| | 3-month | 6-month | 1-year | 2-year | 3-year | 5-year | 7-year | 10-year | 20-year | 30-year |
|---|---|---|---|---|---|---|---|---|---|---|
| **3-month** | NaN | 105.672901 | 72.020209 | 50.477974 | 32.399735 | 17.437226 | 13.567883 | 11.043822 | 8.855981 | 9.038284 |
| **6-month** | NaN | NaN | 60.3571 | 45.099833 | 28.527976 | 14.915785 | 11.990479 | 10.510184 | 9.922652 | 10.071221 |
| **1-year** | NaN | NaN | NaN | 39.90001 | 23.348664 | 13.518149 | 11.773586 | 11.077011 | 10.573078 | 10.690947 |
| **2-year** | NaN | NaN | NaN | NaN | 15.182834 | 11.981184 | 11.391304 | 11.075805 | 9.787915 | 10.019489 |
| **3-year** | NaN | NaN | NaN | NaN | NaN | 12.701071 | 11.332583 | 10.691193 | 8.978149 | 9.453938 |
| **5-year** | NaN | NaN | NaN | NaN | NaN | NaN | 12.035779 | 10.193462 | 8.24576 | 9.015199 |
| **7-year** | NaN | NaN | NaN | NaN | NaN | NaN | NaN | 11.833767 | 8.567326 | 9.610683 |
| **10-year** | NaN | NaN | NaN | NaN | NaN | NaN | NaN | NaN | 9.471421 | 10.338029 |
| **20-year** | NaN | NaN | NaN | NaN | NaN | NaN | NaN | NaN | NaN | 17.798997 |
| **30-year** | NaN | NaN | NaN | NaN | NaN | NaN | NaN | NaN | NaN | NaN |

Here is the critical value of 5% significant level.

| | 3-month | 6-month | 1-year | 2-year | 3-year | 5-year | 7-year | 10-year | 20-year | 30-year |
|---|---|---|---|---|---|---|---|---|---|---|
| **3-month** | NaN | 15.4943 | 15.4943 | 15.4943 | 15.4943 | 15.4943 | 15.4943 | 15.4943 | 15.4943 | 15.4943 |
| **6-month** | NaN | NaN | 15.4943 | 15.4943 | 15.4943 | 15.4943 | 15.4943 | 15.4943 | 15.4943 | 15.4943 |
| **1-year** | NaN | NaN | NaN | 15.4943 | 15.4943 | 15.4943 | 15.4943 | 15.4943 | 15.4943 | 15.4943 |
| **2-year** | NaN | NaN | NaN | NaN | 15.4943 | 15.4943 | 15.4943 | 15.4943 | 15.4943 | 15.4943 |
| **3-year** | NaN | NaN | NaN | NaN | NaN | 15.4943 | 15.4943 | 15.4943 | 15.4943 | 15.4943 |
| **5-year** | NaN | NaN | NaN | NaN | NaN | NaN | 15.4943 | 15.4943 | 15.4943 | 15.4943 |
| **7-year** | NaN | NaN | NaN | NaN | NaN | NaN | NaN | 15.4943 | 15.4943 | 15.4943 |
| **10-year** | NaN | NaN | NaN | NaN | NaN | NaN | NaN | NaN | 15.4943 | 15.4943 |
| **20-year** | NaN | NaN | NaN | NaN | NaN | NaN | NaN | NaN | NaN | 15.4943 |
| **30-year** | NaN | NaN | NaN | NaN | NaN | NaN | NaN | NaN | NaN | NaN |

Results show which pairs have at least one cointegration relationship (1) and which do not (0).

| | 3-month | 6-month | 1-year | 2-year | 3-year | 5-year | 7-year | 10-year | 20-year | 30-year |
|---|---|---|---|---|---|---|---|---|---|---|
| **3-month** | NaN | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 |
| **6-month** | NaN | NaN | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 |
| **1-year** | NaN | NaN | NaN | 1 | 1 | 0 | 0 | 0 | 0 | 0 |
| **2-year** | NaN | NaN | NaN | NaN | 0 | 0 | 0 | 0 | 0 | 0 |
| **3-year** | NaN | NaN | NaN | NaN | NaN | 0 | 0 | 0 | 0 | 0 |
| **5-year** | NaN | NaN | NaN | NaN | NaN | NaN | 0 | 0 | 0 | 0 |
| **7-year** | NaN | NaN | NaN | NaN | NaN | NaN | NaN | 0 | 0 | 0 |
| **10-year** | NaN | NaN | NaN | NaN | NaN | NaN | NaN | NaN | 0 | 0 |
| **20-year** | NaN | NaN | NaN | NaN | NaN | NaN | NaN | NaN | NaN | 1 |
| **30-year** | NaN | NaN | NaN | NaN | NaN | NaN | NaN | NaN | NaN | NaN |

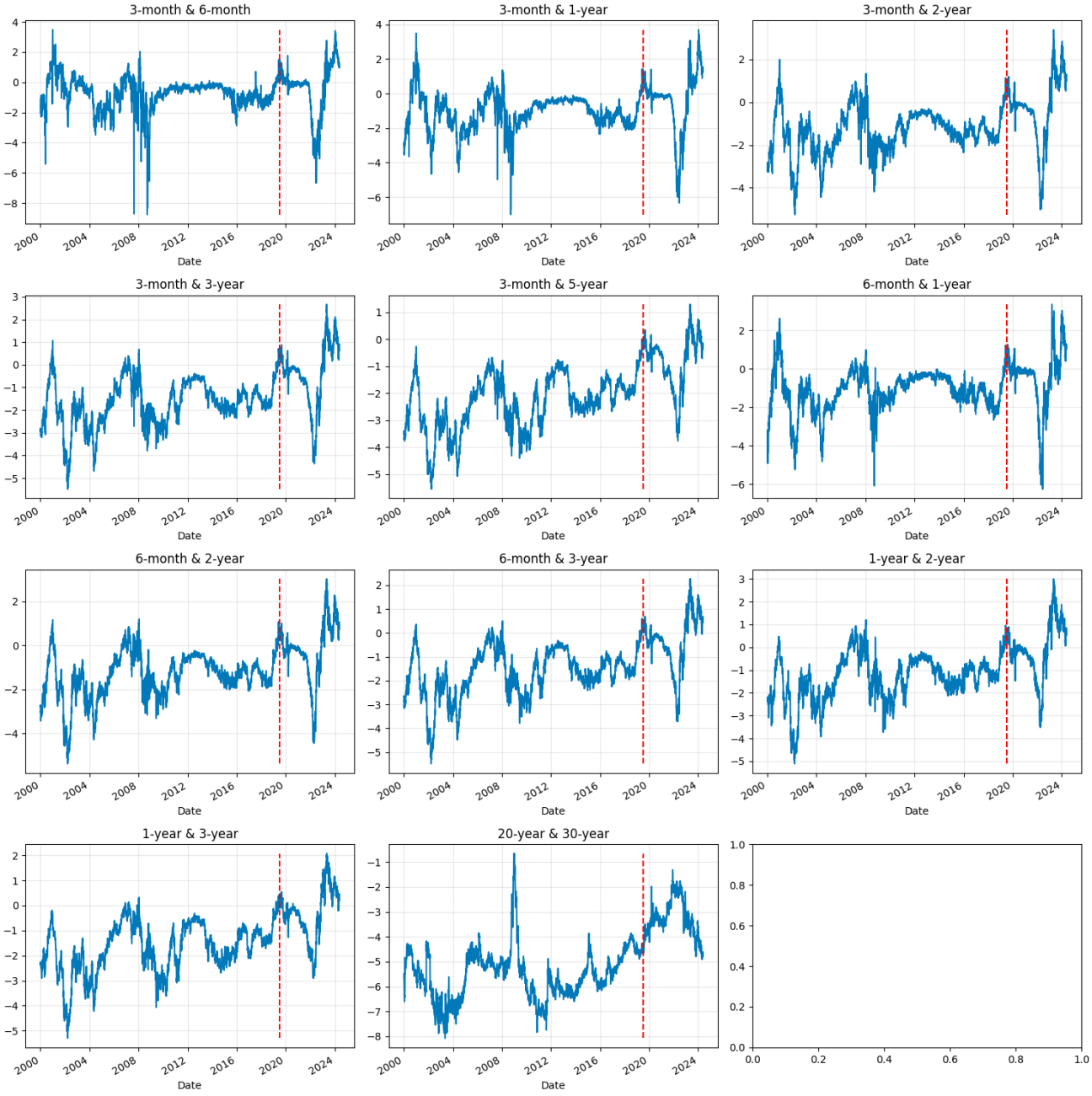
2. **Provide an interpretation of your results from part 1, in terms of what general insights they provide for predicting and managing the interest rate risk associated with the banking business model.**

i. Short-term vs. Long-term Yields
   1. The cointegration analysis indicates that short-term yields (less than 5 years) are more likely to exhibit cointegration relationships compared to long-term yields (more than 5 years).
   2. Short-term yields tend to react more sensitively and quickly to changes in monetary policy. This is because monetary policy interventions, such as changes in the central bank's interest rate, directly affect short-term borrowing and lending rates.
   3. Long-term yields are influenced by broader economic factors, including inflation expectations, economic growth forecasts, and risk premiums. These factors introduce greater variability and less predictable patterns, reducing the likelihood of cointegration with short-term yields.

ii. Predictive Power of Cointegrated Pairs
   1. Identifying cointegrated pairs of interest rates allows for more reliable predictions of their relative movements. When two rates are cointegrated, deviations from their long-term equilibrium relationship are temporary and will correct over time.
   2. For instance, if the 3-month and 5-year yields are cointegrated, a divergence from their historical spread can be expected to revert to the mean, providing a basis for predicting future movements.

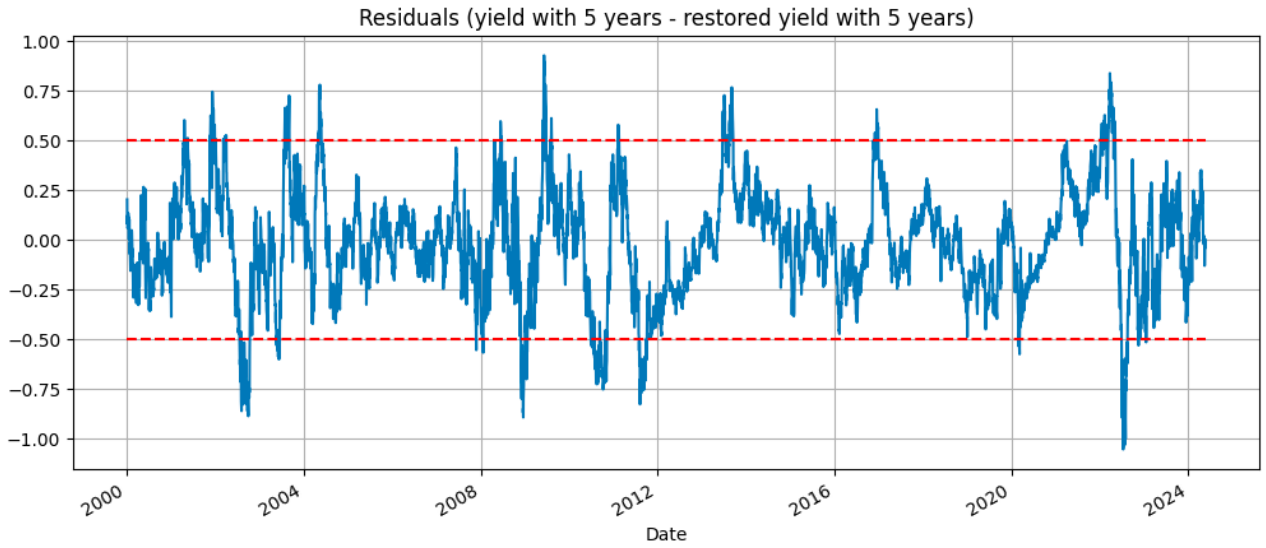
iii. Implications for Risk Management
   1. Interest Rate Risk Hedging: Understanding cointegration helps in designing effective hedging strategies. For instance, a bank can hedge its exposure to short-term interest rate movements by taking positions in instruments tied to the cointegrated long-term rates, thereby reducing the overall risk.
   2. Stress Testing and Scenario Analysis: By incorporating cointegration relationships into stress testing frameworks, banks can better assess the potential impact of extreme market movements on their portfolios. This enhances the robustness of their risk management practices.
   3. Yield Curve Strategies: Cointegration insights can be leveraged to develop trading strategies based on the yield curve. For example, a pairs trading strategy that exploits deviations in the cointegrated spread between short-term and long-term rates can generate profits while managing risk.

iv. Ten pairs with cointegration relationships are identified. Using the maximum eigenvalue, we find the corresponding eigenvector to construct a stationary time series. Out-of-sample periods are marked beyond the vertical red dash line in the visualization.

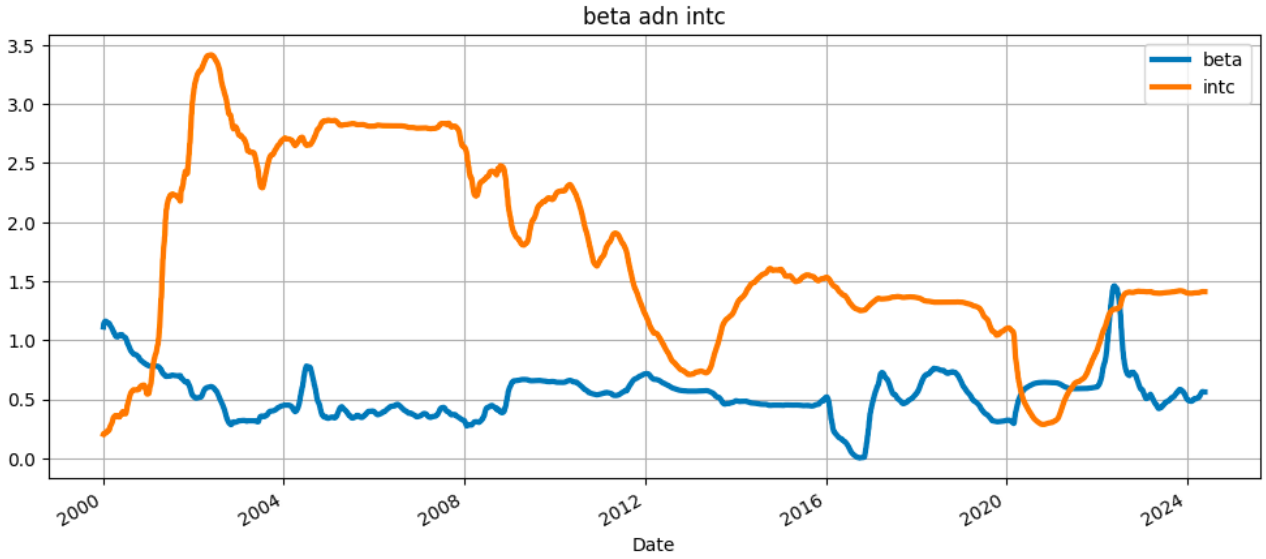


3. **Using the data provided design and test (including back-testing) a pairs trading strategy for the 3-month and 5-year pair of constant maturity rates. For this part you can assume that the constant maturity rates are the closing prices for each day. Provide supporting reasons for any assumptions and choices you make.**

i. We apply the Kalman Filter with dynamic beta hedge to generate residuals.




The time series for beta and intercept are tracked.



ii. Trading strategy:

1. When residuals exceed 0.5, short the pair until residuals fall below -0.5.
2. When residuals drop below -0.5, long the pair until residuals rise above 0.5.
3. The strategy's performance is measured by cumulative profit, winning rate (47%), profit/loss ratio (1.01), and Sharpe ratio (0.33).