FN6905 Exotic Options and Structured Products - Assignment

Due on or before Friday **April 12**, 2024.

Instructions:

- Clarity and quality of presentation will be taken into account in marking.
- All numerical results should be *reproducible* with the codes provided.
- All data/software should be explicitly provided and *accessible*.
- All calculations should be provided and carefully justified.
- The report can be submitted in hardcopy, or uploaded to NTULearn as a single pdf file.
- Codes have to be submitted on NTULearn.
- Graphs are welcome.
- Please adhere to the general coding guidelines available at https://www.tidyverse.org/blog/2017/12/workflow-vs-script/.

Question 1.

The goal of this question is to clarify the issues raised in the attached email.

a) Recover the call and put formulas in § 5.3 of [2] by explicit calculations.

   You may use Chapter 6 of [1] or other sources to be quoted in your report.

b) Price those options by Monte Carlo computations and compare the Monte Carlo outputs to the recovered call and put formulas.

   A discussion of numerical results can be included, for example depending on parameter ranges.

Question 2.

Modify the attached R **code** by replacing finite differences with exact derivatives of the Black-Scholes formula, see for example the solution of Exercise 6.3 of FN6905_Slides.pdf.

This question requires to choose an underlying asset different from SPY, please send your choice of asset by email. The list of reserved assets will be updated on NTULearn (first come, first serve). You may use Bloomberg and/or another language instead of R and quantmod.

Alternative to Question 1 + Question 2.

Redo the numerical simulations in https://arxiv.org/abs/2201.07880.

Subject: Question from a MFE student.
From:
Date:
To: "Nicolas Privault (Prof)" <NPRIVAULT@ntu.edu.sg>

Hi Prof Privault,

Hope this email finds you well.

I am a student of your stochastic calculus class. As a derivative researcher, I am currently working on a project of developing a pricing engine for rainbow options which have analytical solutions.

When I went through **Correlation Digital Option**, I found the analytical solution for the **put option** gives a different result compared with Monte-Carlo method. (I use MC method to verify the analytical solutions). But after deriving the solution myself and searching around for several days, I still cannot get the right analytical solution for the put option. I wonder if you can give some insight about what is wrong with the current analytical. I would be really appreciating it.

Here's the details of the correlation digital option and its pricing formula. which is wrong when compared with MC method.

5.3 TWO-ASSET CORRELATION OPTIONS

This call option pays off $\max(S_2 - X_2; 0)$ if $S_1 > X_1$ and 0 otherwise. The put pays off $\max(X_2 - S_2)$ if $S_1 < X_1$ and 0 otherwise. These options can be priced using the formulas of Zhang (1995a):

$$\begin{aligned} c = {} & S_2 e^{(b_2 - r)T} M(y_2 + \sigma_2\sqrt{T}, y_1 + \rho\sigma_2\sqrt{T}; \rho) \\ & - X_2 e^{-rT} M(y_2, y_1; \rho) \end{aligned} \qquad (5.5)$$

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$$\begin{aligned} p = {} & X_2 e^{-rT} M(-y_2, -y_1; \rho) \\ & - S_2 e^{(b_2 - r)T} M(-y_2 - \sigma_2\sqrt{T}, -y_1 - \rho\sigma_2\sqrt{T}; \rho), \end{aligned} \qquad (5.6)$$

where ρ is the correlation coefficient between the returns on the two assets and

$$y_1 = \frac{\ln(S_1/X_1) + (b_1 - \sigma_1^2/2)T}{\sigma_1\sqrt{T}} \qquad y_2 = \frac{\ln(S_2/X_2) + (b_2 - \sigma_2^2/2)T}{\sigma_2\sqrt{T}}$$

References

[1] P. Buchen. *An introduction to exotic option pricing.* Chapman & Hall/CRC Financial Mathematics Series. CRC Press, Boca Raton, FL, 2012.

[2] E.P. Haug. *The complete guide to option pricing formulas.* McGraw Hill, second edition, 2006.