$$S_0 e^{\left(r-b-\frac{\sigma^2}{2}\right)T+\sigma\sqrt{T}z} > K \quad \leftrightarrow \quad z > \frac{\ln\frac{K}{S_0} - \left(r-b-\frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}} = -\frac{\ln\frac{S_0}{K} + \left(r-b-\frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}}$$

$$\rightarrow \quad x > -\frac{\ln\frac{S_2}{K_2} + \left(r-b_2-\frac{\sigma_2^2}{2}\right)T}{\sigma_2\sqrt{T}} = -y_2 \quad \& \quad y > -\frac{\ln\frac{S_1}{K_1} + \left(r-b_1-\frac{\sigma_1^2}{2}\right)T}{\sigma_1\sqrt{T}} = -y_1$$

$$\therefore e^{-rT}E^Q\left[1_{\{S_1>K_1\}}(S_2-K_2)^+\right]$$

$$= e^{-rT}\int_{-y_1}^{\infty}\int_{-y_2}^{\infty}\left(S_2 e^{\left(r-b_2-\frac{\sigma_2^2}{2}\right)T+\sigma_2\sqrt{T}x} - K_2\right)\frac{1}{2\pi\sqrt{1-\rho^2}}exp\left(-\frac{1}{2(1-\rho^2)}[x^2-2\rho xy+y^2]\right)dxdy$$

$$= e^{-rT}\int_{-y_1}^{\infty}\int_{-y_2}^{\infty} S_2 e^{\left(r-b_2-\frac{\sigma_2^2}{2}\right)T+\sigma_2\sqrt{T}x}\frac{1}{2\pi\sqrt{1-\rho^2}}exp\left(-\frac{1}{2(1-\rho^2)}[x^2-2\rho xy+y^2]\right)dxdy$$

$$- e^{-rT}\int_{-y_1}^{\infty}\int_{-y_2}^{\infty} K_2\frac{1}{2\pi\sqrt{1-\rho^2}}exp\left(-\frac{1}{2(1-\rho^2)}[x^2-2\rho xy+y^2]\right)dxdy$$

For first term:

$$e^{-rT}\int_{-y_1}^{\infty}\int_{-y_2}^{\infty} S_2 e^{\left(r-b_2-\frac{\sigma_2^2}{2}\right)T+\sigma_2\sqrt{T}x}\frac{1}{2\pi\sqrt{1-\rho^2}}exp\left(-\frac{1}{2(1-\rho^2)}[x^2-2\rho xy+y^2]\right)dxdy$$

$$\xRightarrow[z_2=x-\sigma_2\sqrt{T}]{z_1=y-\rho\sigma_2\sqrt{T}} S_2 e^{-b_2T}\int_{-y_1-\rho\sigma_2\sqrt{T}}^{\infty}\int_{-y_2-\sigma_2\sqrt{T}}^{\infty}\frac{1}{2\pi\sqrt{1-\rho^2}}\exp\left(\frac{\sigma_2^2}{2}T+\sigma_2\sqrt{T}z_2\right.$$

$$\left.-\frac{1}{2(1-\rho^2)}\left[\left(z_2+\sigma_2\sqrt{T}\right)^2-2\rho\left(z_2+\sigma_2\sqrt{T}\right)\left(z_1+\rho\sigma_2\sqrt{T}\right)+\left(z_1+\rho\sigma_2\sqrt{T}\right)^2\right]\right)dz_2dz_1$$

$$= S_2e^{-b_2T}\int_{-y_1-\rho\sigma_2\sqrt{T}}^{\infty}\int_{-y_2-\sigma_2\sqrt{T}}^{\infty}\frac{1}{2\pi\sqrt{1-\rho^2}}exp\left(-\frac{1}{2(1-\rho^2)}\left[-\sigma_2^2T+\rho^2\sigma_2^2T-2\sigma_2\sqrt{T}z_2+2\rho^2\sigma_2\sqrt{T}z_2\right.\right.$$

$$+z_2^2+2z_2\sigma_2\sqrt{T}+\sigma_2^2T-2\rho z_1z_2-2\rho^2\sigma_2\sqrt{T}z_2-2\rho\sigma_2\sqrt{T}z_1-2\rho^2\sigma_2^2T+z_1^2+2z_1\rho\sigma_2\sqrt{T}$$

$$\left.\left.+\rho^2\sigma_2^2T\right]\right)dz_2dz_1$$

$$= S_2e^{-b_2T}\int_{-y_1-\rho\sigma_2\sqrt{T}}^{\infty}\int_{-y_2-\sigma_2\sqrt{T}}^{\infty}\frac{1}{2\pi\sqrt{1-\rho^2}}exp\left(-\frac{1}{2(1-\rho^2)}[z_1^2-2\rho z_1z_2+z_2^2]\right)dz_2dz_1$$

$$= S_2e^{-b_2T}\int_{-\infty}^{y_1+\rho\sigma_2\sqrt{T}}\int_{-\infty}^{y_2+\sigma_2\sqrt{T}}\frac{1}{2\pi\sqrt{1-\rho^2}}exp\left(-\frac{1}{2(1-\rho^2)}[z_1^2-2\rho z_1z_2+z_2^2]\right)dz_2dz_1$$

$$= S_2e^{-b_2T}M\left(y_2+\sigma_2\sqrt{T}, y_1+\rho\sigma_2\sqrt{T}, \rho\right)$$

For second term:

$$-e^{-rT}\int_{-y_1}^{\infty}\int_{-y_2}^{\infty} K_2\frac{1}{2\pi\sqrt{1-\rho^2}}exp\left(-\frac{1}{2(1-\rho^2)}[x^2-2\rho xy+y^2]\right)dxdy$$

$$= -K_2e^{-rT}\int_{-\infty}^{y_1}\int_{-\infty}^{y_2}\frac{1}{2\pi\sqrt{1-\rho^2}}exp\left(-\frac{1}{2(1-\rho^2)}[x^2-2\rho xy+y^2]\right)dxdy = -K_2e^{-rT}M(y_2,y_1,\rho)$$

$\therefore$ **call** $= \boldsymbol{S_2e^{-b_2T}M\left(y_2+\sigma_2\sqrt{T}, y_1+\rho\sigma_2\sqrt{T}, \rho\right) - K_2e^{-rT}M(y_2,y_1,\rho)}$ where $y_1 = \frac{\ln\frac{S_1}{K_1}+\left(r-b_1-\frac{\sigma_1^2}{2}\right)T}{\sigma_1\sqrt{T}}$ & $y_2 = \frac{\ln\frac{S_2}{K_2}+\left(r-b_2-\frac{\sigma_2^2}{2}\right)T}{\sigma_2\sqrt{T}}$

Similarly, **put** $= \boldsymbol{K_2e^{-rT}M(-y_2,-y_1,\rho) - S_2e^{-b_2T}M\left(-y_2-\sigma_2\sqrt{T}, -y_1-\rho\sigma_2\sqrt{T}, \rho\right)}$