In [ ]:
```
import pandas as pd
import numpy as np
import matplotlib.pyplot as plt
import yfinance as yf
import scipy.stats as sps
import scipy.optimize as spo
from datetime import timedelta
```

In [ ]:
```
tk = yf.Ticker("QQQ")
# Expiration dates
exps = tk.options
# Get options for each expiration
options = pd.DataFrame()
for e in exps:
    opt = tk.option_chain(e)
    opt.calls['optionType'] = 'Call'
    opt.puts['optionType'] = 'Put'
    opt.calls['expirationDate'] = e
    opt.puts['expirationDate'] = e

    options = pd.concat([options, pd.concat([opt.calls, opt.puts])])
options = options.reset_index(drop=True)
options['midPrice'] = (options['bid'] + options['ask']) / 2
options['expirationDate'] = pd.to_datetime(options['expirationDate'])
```

In [ ]:
```
options
```

Out[ ]:

	contractSymbol	lastTradeDate	strike	lastPrice	bid	ask	change	percentChange	volume	openInterest	impliedVolatility	inTheMor
0	QQQ240412C00300000	2024-04-10 16:52:38+00:00	300.00	137.85	145.51	145.79	0.0	0.0	10.0	11.0	3.371095	Tı
1	QQQ240412C00310000	2024-04-02 14:12:18+00:00	310.00	128.88	135.52	135.79	0.0	0.0	6.0	6.0	3.126955	Tı
2	QQQ240412C00315000	2024-04-10 19:15:07+00:00	315.00	123.31	130.52	130.79	0.0	0.0	11.0	11.0	3.003909	Tı
3	QQQ240412C00320000	2024-04-10 19:10:09+00:00	320.00	117.64	125.52	125.80	0.0	0.0	1.0	1.0	2.890628	Tı
4	QQQ240412C00325000	2024-03-08 14:33:17+00:00	325.00	122.30	115.44	115.89	0.0	0.0	1.0	1.0	0.000010	Tı
...	...	...	...	...	...	...	...	...	...	...	...	
6947	QQQ261218P00589780	2024-01-19 18:47:33+00:00	589.78	171.94	157.50	162.36	0.0	0.0	40.0	0.0	0.199860	Tı
6948	QQQ261218P00594780	2024-03-18 20:12:55+00:00	594.78	157.10	146.50	151.48	0.0	0.0	2.0	0.0	0.110986	Tı
6949	QQQ261218P00620000	2024-03-04 19:46:53+00:00	620.00	174.00	175.00	179.91	0.0	0.0	1.0	1.0	0.150887	Tı
6950	QQQ261218P00640000	2024-02-01 17:09:06+00:00	640.00	219.46	192.42	196.62	0.0	0.0	NaN	0.0	0.131082	Tı
6951	QQQ261218P00650000	2024-03-04 19:41:36+00:00	650.00	203.61	205.00	209.91	0.0	0.0	1.0	0.0	0.165856	Tı

6952 rows × 17 columns

In [ ]:
```
pd.to_datetime(opt.underlying['regularMarketTime'] * 1000, unit='ms').normalize()
```

Out[ ]:
```
Timestamp('2024-04-11 00:00:00')
```

In [ ]:
```
# get trading date
t0 = pd.to_datetime(opt.underlying['regularMarketTime'] * 1000, unit='ms').normalize() - timedelta(0)
options['ttm'] = [(i - t0).days / 365 for i in options['expirationDate']]

# get rid of options that were not traded
idx = np.where(options.lastTradeDate > t0.tz_localize('UTC'))
options = options.loc[idx].reset_index(drop=True)
options = options[options['optionType'] == 'Call'].reset_index(drop=True)

S_0 = opt.underlying['regularMarketPrice']
rf = 0.03
options["Moneyness"] = round(options["strike"]/S_0,2)
```

In [ ]:
```
def bs_call(S, strike, ttm, rf, sigma):
    d1 = (np.log(S / strike) + (rf + sigma**2 / 2.0) * ttm) / (sigma * np.sqrt(ttm))
    d2 = d1 - sigma * np.sqrt(ttm)
    return S * sps.norm.cdf(d1) - strike * np.exp(-rf * ttm) * sps.norm.cdf(d2)

def bs_call_iv(c, S, strike, ttm, rf):
    def _inner(sigma):
        d1 = (np.log(S / strike) + (rf + sigma**2 / 2.0) * ttm) / (sigma * np.sqrt(ttm))
        d2 = d1 - sigma * np.sqrt(ttm)
        return c - (S * sps.norm.cdf(d1) - strike * np.exp(-rf * ttm) * sps.norm.cdf(d2))
    return _inner


def bs_put(S, strike, ttm, rf, sigma):
    d1 = (np.log(S / strike) + (rf + sigma**2 / 2.0) * ttm) / (sigma * np.sqrt(ttm))
    d2 = d1 - sigma * np.sqrt(ttm)
    return sps.norm.cdf(-d2) * strike * np.exp(-rf * ttm) - sps.norm.cdf(-d1) * S

def bs_put_iv(p, S, strike, ttm, rf):
```

```python
    def _inner(sigma):
        d1 = (np.log(S / strike) + (rf + sigma**2 / 2.0) * ttm) / (sigma * np.sqrt(ttm))
        d2 = d1 - sigma * np.sqrt(ttm)
        return p - (sps.norm.cdf(-d2) * strike * np.exp(-rf * ttm) - sps.norm.cdf(-d1) * S)
    return _inner
```

In [ ]:
```python
ivs = []
for i in options.index:
    if options.loc[i, 'optionType'] == 'Call':
        c = options.loc[i, 'midPrice']
        strike = options.loc[i, 'strike']
        ttm = options.loc[i, 'ttm']
        try:
            ivs.append(spo.bisect(bs_call_iv(c ,S_0, strike, ttm, rf), 0.001, 1))
        except:
            ivs.append(np.nan)
    else:
        p = options.loc[i, 'midPrice']
        strike = options.loc[i, 'strike']
        ttm = options.loc[i, 'ttm']
        try:
            ivs.append(spo.bisect(bs_put_iv(p ,S_0, strike, ttm, rf), 0.001, 1))
        except:
            ivs.append(np.nan)

options['impliedVolatility'] = ivs
options = options.dropna().reset_index(drop=True)
```

In [ ]:
```python
options
```

Out[ ]:

	contractSymbol	lastTradeDate	strike	lastPrice	bid	ask	change	percentChange	volume	openInterest	impliedVolatility	inTheMor
0	QQQ240412C00405000	2024-04-11 17:23:33+00:00	405.00	38.54	40.57	40.84	5.160000	15.458358	54.0	118.0	0.998647	Tı
1	QQQ240412C00406000	2024-04-11 17:03:51+00:00	406.00	35.68	39.57	39.84	-0.060001	-0.167883	67.0	46.0	0.977067	Tı
2	QQQ240412C00409000	2024-04-11 16:43:11+00:00	409.00	32.45	36.57	36.96	0.760000	2.398234	8.0	2.0	0.943696	Tı
3	QQQ240412C00410000	2024-04-11 19:45:41+00:00	410.00	36.35	35.57	35.85	8.679998	31.369707	78.0	147.0	0.893316	Tı
4	QQQ240412C00411000	2024-04-11 16:01:22+00:00	411.00	30.70	34.57	34.96	3.800001	14.126399	7.0	21.0	0.899329	Tı
...	...	...	...	...	...	...	...	...	...	...	...	
1102	QQQ261218C00510000	2024-04-11 18:01:40+00:00	510.00	53.50	54.00	58.81	0.820000	1.556567	1.0	17.0	0.229216	Fa
1103	QQQ261218C00540000	2024-04-11 19:51:42+00:00	540.00	44.88	42.00	47.00	2.470001	5.824101	6.0	16.0	0.220665	Fa
1104	QQQ261218C00544780	2024-04-11 19:51:42+00:00	544.78	42.58	40.50	45.21	3.220001	8.180898	6.0	21.0	0.219661	Fa
1105	QQQ261218C00630000	2024-04-11 14:43:31+00:00	630.00	18.53	17.50	22.50	3.520001	23.451036	1.0	6.0	0.202185	Fa
1106	QQQ261218C00670000	2024-04-11 18:41:34+00:00	670.00	14.77	11.50	16.00	2.770001	23.083338	11.0	50.0	0.197686	Fa

1107 rows × 19 columns

$$\sigma(t,y) := \sqrt{\frac{2\frac{\partial C^M}{\partial t}(t,y) + 2ry\frac{\partial C^M}{\partial y}(t,y)}{y^2\frac{\partial^2 C^M}{\partial y^2}(t,y)}}$$

$$, where \quad \frac{\partial C^M}{\partial t}(t,y) = -rK\frac{\partial C^M}{\partial K}(t,x,K) + \frac{\sigma^2 x^2}{2}\frac{K^2}{x^2}\frac{\partial^2 C^M}{\partial K^2}(t,x,K)$$

$$\frac{\partial C^M}{\partial K}(t,y) = \frac{1}{K}C(t,x,K) - \frac{x}{K}\frac{\partial C^M}{\partial x}(t,x,K)$$

$$\frac{\partial^2 C^M}{\partial K^2}(t,y) = \frac{x^2}{K^2}\frac{\partial^2 C^M}{\partial K^2}(t,x,K)$$

In [ ]:
```python
def dcdk_func(c, S, strike, ttm, rf, sigma):
    d1 = (np.log(S / strike) + (rf + sigma**2 / 2.0) * ttm) / (sigma * np.sqrt(ttm))
    delta = sps.norm.cdf(d1)
    return S / strike * (1 / S * c - delta)

def d2cdk2_func(S, strike, ttm, rf, sigma):
    d1 = (np.log(S / strike) + (rf + sigma**2 / 2.0) * ttm) / (sigma * np.sqrt(ttm))
    nprimed1 = sps.norm.pdf(d1)
    gamma =  nprimed1 / (S * sigma * np.sqrt(ttm))
    return (S ** 2 / strike ** 2) * gamma

def dcdt_func(c, S, strike, ttm, rf, sigma):
    dcdk = dcdk_func(c, S, strike, ttm, rf, sigma)
    d2cdk2 = d2cdk2_func(S, strike, ttm, rf, sigma)
    return - rf * strike * dcdk + (sigma ** 2 * S ** 2) / 2 * (strike ** 2 / S ** 2) * d2cdk2
```

In [ ]:
```python
lvs = []
for i in options.index:
    c = options.loc[i, 'midPrice']
    strike = options.loc[i, 'strike']
```

```python
    ttm = (options.loc[i, 'expirationDate'] - t0).days / 365
    sigma = options.loc[i, 'impliedVolatility']

    dcdt = dcdt_func(c, S_0, strike, ttm, rf, sigma)
    dcdk = dcdk_func(c, S_0, strike, ttm, rf, sigma)
    d2cdk2 = d2cdk2_func(S_0, strike, ttm, rf, sigma)

    lv = np.sqrt((2 * dcdt + 2 * rf * strike * dcdk) / (strike ** 2 * d2cdk2))
    lvs.append(lv)

options['localVolatility'] = lvs
```

In [ ]:
```python
options
```

Out[ ]:

	contractSymbol	lastTradeDate	strike	lastPrice	bid	ask	change	percentChange	volume	openInterest	impliedVolatility	inTheMor
0	QQQ240412C00405000	2024-04-11 17:23:33+00:00	405.00	38.54	40.57	40.84	5.160000	15.458358	54.0	118.0	0.998647	Tı
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4	QQQ240412C00411000	2024-04-11 16:01:22+00:00	411.00	30.70	34.57	34.96	3.800001	14.126399	7.0	21.0	0.899329	Tı
...	...	...	...	...	...	...	...	...	...	...	...	
1102	QQQ261218C00510000	2024-04-11 18:01:40+00:00	510.00	53.50	54.00	58.81	0.820000	1.556567	1.0	17.0	0.229216	Fa
1103	QQQ261218C00540000	2024-04-11 19:51:42+00:00	540.00	44.88	42.00	47.00	2.470001	5.824101	6.0	16.0	0.220665	Fa
1104	QQQ261218C00544780	2024-04-11 19:51:42+00:00	544.78	42.58	40.50	45.21	3.220001	8.180898	6.0	21.0	0.219661	Fa
1105	QQQ261218C00630000	2024-04-11 14:43:31+00:00	630.00	18.53	17.50	22.50	3.520001	23.451036	1.0	6.0	0.202185	Fa
1106	QQQ261218C00670000	2024-04-11 18:41:34+00:00	670.00	14.77	11.50	16.00	2.770001	23.083338	11.0	50.0	0.197686	Fa

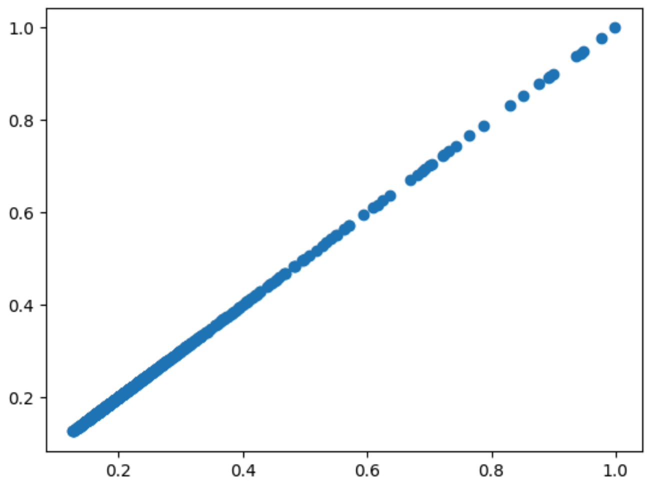
1107 rows × 20 columns

In [ ]:
```python
options['impliedVolatility'] / options['localVolatility']
```

Out[ ]:
```
0       1.0
1       1.0
2       1.0
3       1.0
4       1.0
       ...
1102    1.0
1103    1.0
1104    1.0
1105    1.0
1106    1.0
Length: 1107, dtype: float64
```

In [ ]:
```python
plt.scatter(options['impliedVolatility'], options['localVolatility'])
```

Out[ ]:
```
<matplotlib.collections.PathCollection at 0x1da7a403f50>
```



In [ ]:
```python
lv_surface = pd.DataFrame()
for t in exps:
    ivc = options[(options["expirationDate"] == t)].sort_values("strike").set_index('Moneyness')["localVolatility"]
    adj_ivc = ivc.groupby("Moneyness").mean()
    adj_ivc.name = t
    lv_surface = pd.concat([lv_surface, adj_ivc], axis = 1)
lv_surface.columns = pd.to_datetime(lv_surface.columns)
lv_surface = lv_surface.sort_index()
```

```python
lv_surface = lv_surface.interpolate(limit_area="inside")
lv_surface = lv_surface.loc[(lv_surface.index > 0.9) & (lv_surface.index < 1.1)]
lv_surface = lv_surface.iloc[:,:25]
```

In [ ]:

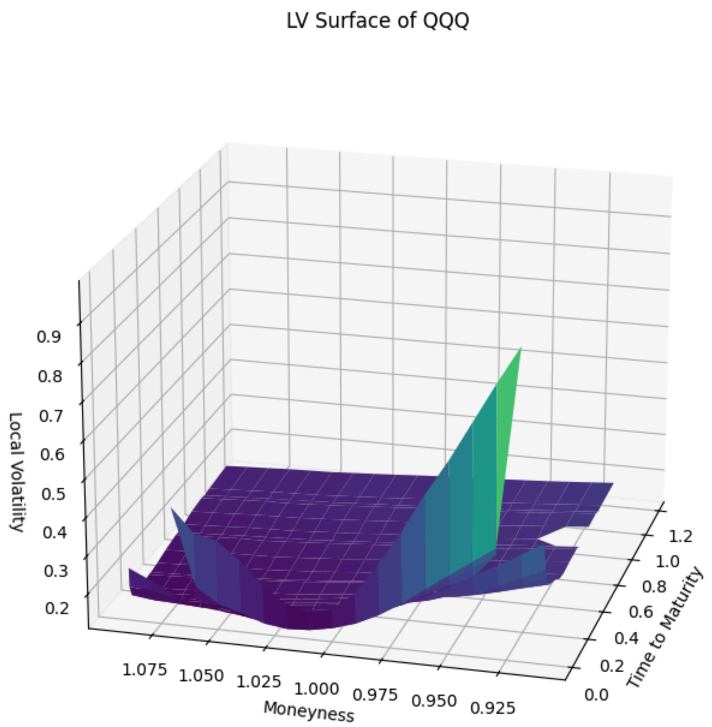
```python
from matplotlib import cm
from mpl_toolkits.mplot3d import Axes3D
from datetime import datetime
#Singapore time is ahead 1 day to the US time
time2maturity = [((i - datetime.today()).days + 1) / 365 for i in lv_surface.columns]

%matplotlib inline
fig = plt.figure(figsize=(8,8))
ax = fig.add_subplot(111, projection='3d')


X, Y = np.meshgrid(time2maturity, lv_surface.index)
ax.plot_surface(X, Y, lv_surface, linewidth=0.2, cmap='viridis')
ax.view_init(20, -165)
ax.set_xlabel("Time to Maturity")
ax.set_ylabel("Moneyness")
ax.set_zlabel("Local Volatility")
ax.set_title('LV Surface of QQQ')
```

Out[ ]:

```
Text(0.5, 0.92, 'LV Surface of QQQ')
```



In [ ]: