LABORATORY ACTIVITY

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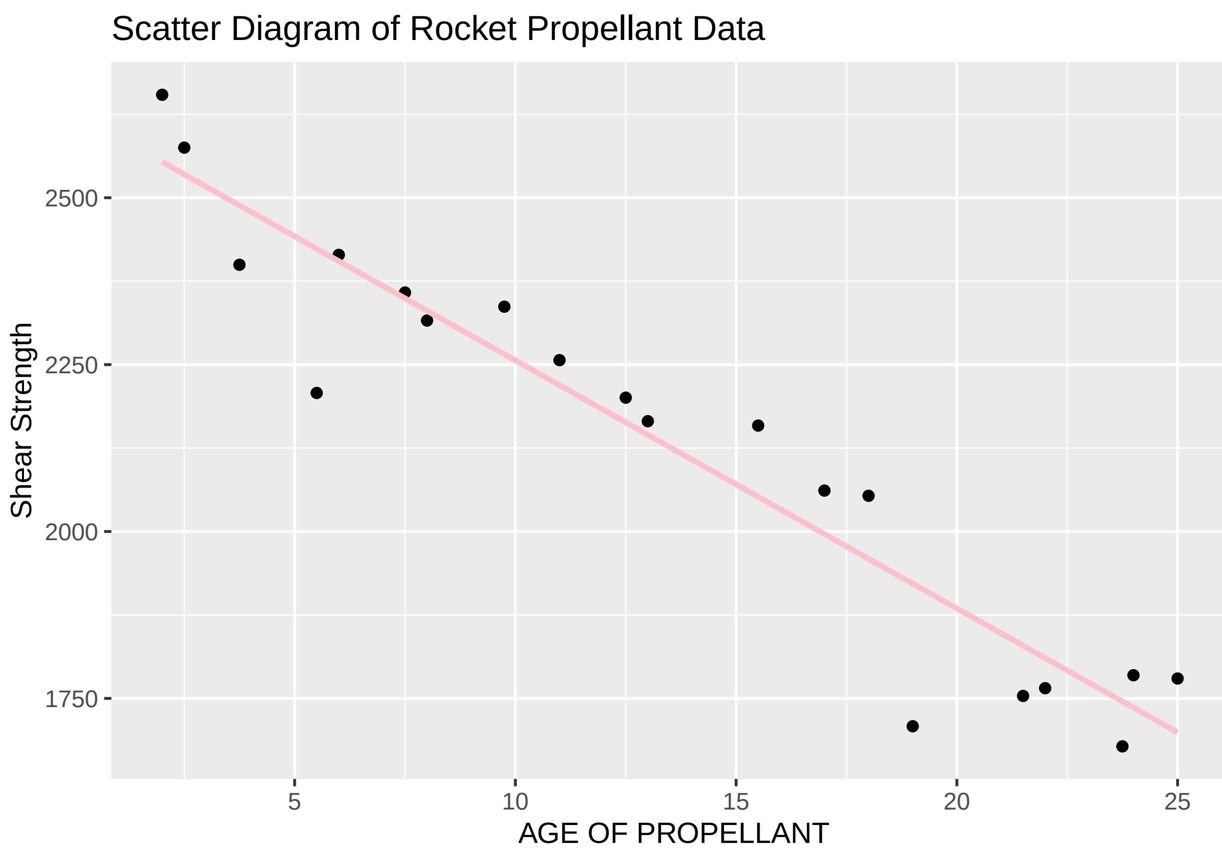
1. The Rocket Propellant Data

A rocket motor is manufactured by bonding an igniter propellant and a sustainer propellant together inside a metal housing. The shear strength of the bond between the two types of propellant is an important quality characteristic. It is suspected that shear strength is related to the age in weeks of the batch of sustainer propellant. Twenty observations on shear strength and the age of the corresponding batch of propellant have been collected.

observations	ShearStrength	AgeofPropellant
1	2158.70	15.50
2	1678.15	23.75
3	2316.00	8.00
4	2061.30	17.00
5	2207.50	5.50
6	1708.30	19.00
7	1784.70	24.00
8	2575.00	2.50
9	2357.90	7.50
10	2256.70	11.00
11	2165.20	13.00
12	2399.55	3.75
13	1779.80	25.00
14	2336.75	9.75
15	1765.30	22.00
16	2053.50	18.00
17	2414.40	6.00
18	2200.50	12.50
19	2654.20	2.00
20	1753.70	21.50

a. Scatter diagram for the data.

```
## `geom_smooth()` using formula = 'y ~ x'
```



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2. Least-Squares Estimation of the Parameters

```
#Use the lm() function to calculate the linear model based on the data set.
# calculate model
model <- lm(data = rocketpropell,
formula = ShearStrength ~ AgeofPropellant)
```

The model object is a list of a number of different pieces of information, which can be seen by looking at the names of the objects in the list.

```
# view the names of the objects in the model
names(model)
```

```
##  [1] "coefficients"  "residuals"     "effects"       "rank"
##  [5] "fitted.values" "assign"        "qr"            "df.residual"
##  [9] "xlevels"       "call"          "terms"         "model"
```

a. The least-squares fit is

The summary() function is a useful way to gather critical information in your model.

```r
model$coefficients
```

```
##     (Intercept) AgeofPropellant
##      2627.82236       -37.15359
```

```r
model_summary <- summary(model)
model_summary$sigma
```

```
## [1] 96.10609
```

```r
# Therefore, the least square fit is y_hat = 2627.822 + (-37 15)x or y_hat= 2627.822 - 37.15x
# view the fitted values
fitted.values <- fitted(model)
fitted.values
```

```
##        1        2        3        4        5        6        7        8
## 2051.942 1745.425 2330.594 1996.211 2423.478 1921.904 1736.136 2534.938
##        9       10       11       12       13       14       15       16
## 2349.170 2219.133 2144.826 2488.496 1698.983 2265.575 1810.443 1959.058
##       17       18       19       20
## 2404.901 2163.402 2553.515 1829.020
```

b. The estimate of σ^2

```r
sigma_hat_squared <- ((model_summary$sigma)^2)
print(paste("Therefore the estimate of sigma squared is: ", sigma_hat_squared))
```

```
## [1] "Therefore the estimate of sigma squared is:  9236.38100372114"
```

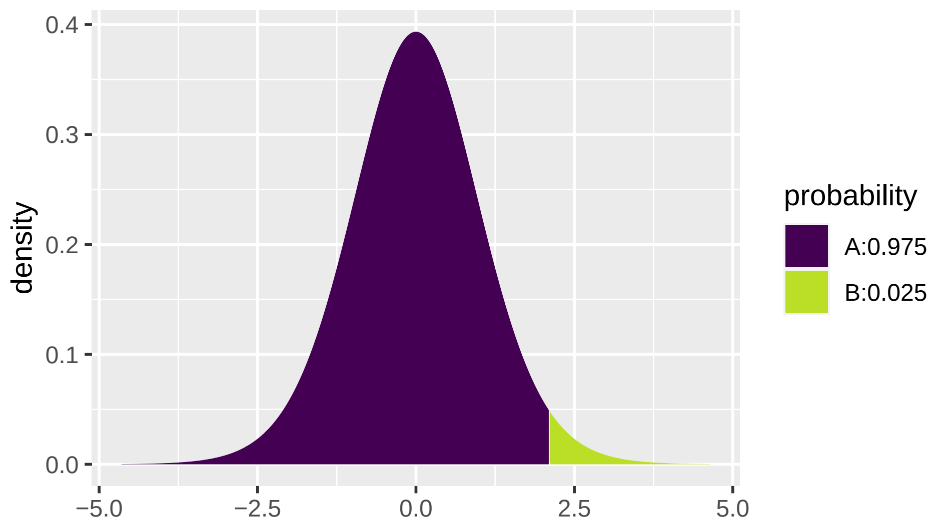
---

3. Hypothesis Testing on the Slope and Intercept

```r
library(mosaic)
model_summary$coefficients["AgeofPropellant",]
```

```
##      Estimate    Std. Error       t value      Pr(>|t|)
## -3.715359e+01  2.889107e+00 -1.285989e+01  1.643344e-10
```

```r
mosaic::xqt(0.975, 18)
```




```
## [1] 2.100922
```

Null Hypothesis: H_0: $\beta_1 = 0$ Alternative Hypothesis: H_1: $\beta_1 ! = 0$

a. Test for significance of regression in the rocket propellant regression model

```
t_value <- model_summary$coefficients["AgeofPropellant","t value"]
print(t_value)
```

```
## [1] -12.85989
```

```
# Find the critical value
df <- 20
significance_level <- 0.05
critical_value <- qt(1-significance_level/2, df-2)
print(paste("The critical value is:", critical_value))
```

```
## [1] "The critical value is: 2.10092204024104"
```

When $\alpha = 0.05$, the critical value of t is $t_{0.025,18} = 2.101$.

Since |t value| > critical value

Thus, we would reject $H_0 : beta_1 = 0$.

Hence, there is a linear relationship between shear strength and the age of the propellant.

```
model_summary
```

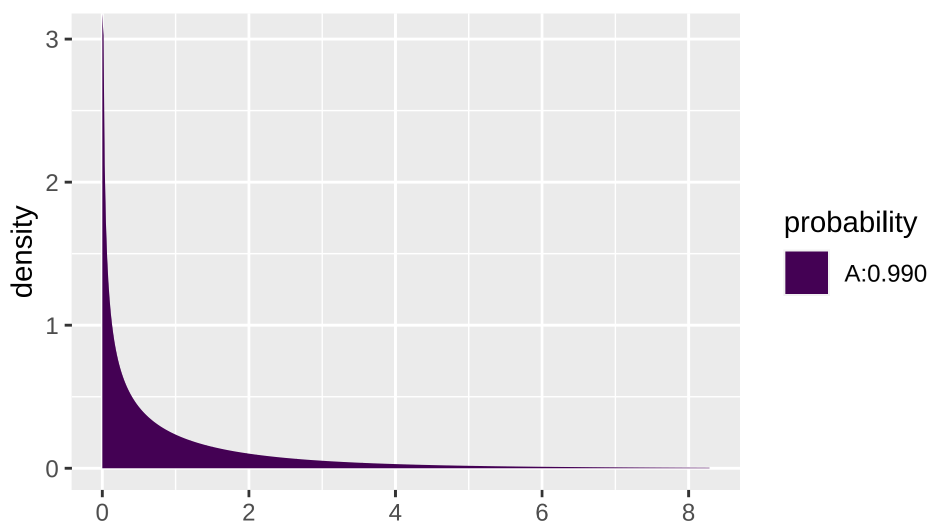
```
##
## Call:
## lm(formula = ShearStrength ~ AgeofPropellant, data = rocketpropell)
```

```
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -215.98  -50.68   28.74   66.61  106.76
##
## Coefficients:
##                 Estimate Std. Error t value Pr(>|t|)
## (Intercept)     2627.822     44.184   59.48  < 2e-16 ***
## AgeofPropellant  -37.154      2.889  -12.86 1.64e-10 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 96.11 on 18 degrees of freedom
## Multiple R-squared:  0.9018, Adjusted R-squared:  0.8964
## F-statistic: 165.4 on 1 and 18 DF,  p-value: 1.643e-10
```

```r
model_summary$fstatistic
```

```
##    value    numdf    dendf
## 165.3768   1.0000  18.0000
```

```r
mosaic::xqf(0.99,1,18)
```




```
## [1] 8.28542
```

b. Use an analysis-of-variance approach to test significance of regression

```r
anova_result <- anova(model)
print(anova_result)
```

```
## Analysis of Variance Table
##
## Response: ShearStrength
##                  Df  Sum Sq Mean Sq F value    Pr(>F)
## AgeofPropellant  1 1527483 1527483  165.38 1.643e-10 ***
## Residuals       18  166255    9236
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Extracting the f-value

```r
f_value <- anova_result$'F value'[1]
print(f_value)
```

```
## [1] 165.3768
```

Finding the critical values

```r
# Find the critical value
df1 <- 1
df2 <- 20
significance_level <- 0.05
critical_fvalue <- qf(1-significance_level, df1, df2-2)
print(paste("The Critical F value is:", critical_fvalue))
```

```
## [1] "The Critical F value is: 4.41387341917057"
```

```r
result <- t.test(AgeofPropellant, ShearStrength)
p_value <- result$p.value
print(p_value)
```

```
## [1] 6.183189e-18
```

The P value for this test is 1.64e-10.

Using F test in testing the analysis of variance and choosing the significance level $\alpha = 0.05$, the critical value of f is computed as $f_{0.05,1,18} = 4.41$

The F value is 163.3768 and the critical F value is 4.41. Since the F value is greater than the critical value, Hence, we reject the null hypothesis.

Therefore, there is a linear relationship between shear strength and the age of the propellant.