Chapter 1 Section 3 Exercise Solutions

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Contents

1 Exercise 5

p	q	r	$q \vee r$	$p \wedge (q \vee r)$	$p \wedge q$	$p \wedge r$	$(p \wedge q) \vee (p \wedge r)$
T	T	T	T	T	T	T	T
T	T	F	T	T	T	F	T
T	F	T	T	T	F	T	T
T	F	F	F	F	F	F	F
F	T	T	T	F	F	F	F
F	T	F	T	F	F	F	F
F	F	T	T	F	F	F	F
F	F	F	F	F	F	F	F

Since the truth values of the compound propositions $p \wedge (q \vee r)$ and $(p \wedge q) \vee (p \wedge r)$ agree for all possible combinations of the truth values of p, q, and r, said compound propositions are logically equivalent.

2 Exercise 6

p	q	$p \wedge q$	$\neg(p \wedge q)$	$\neg p$	$\neg q$	$\neg p \vee \neg q$
T	T	T	F	F	F	F
T	F	F	T	F	T	T
F	T	F	T	T	F	T
F	F	F	T	T	T	T

Since the truth values of the compound propositions $\neg(p \wedge q)$ and $\neg p \vee \neg q$ agree for all possible combinations of the truth values of p and q, said compound propositions are logically equivalent.

3 Exercise 9

3.1 (a)

$$
\begin{aligned}
& p \implies \neg q && \equiv \\
& \neg p \vee \neg q
\end{aligned}
$$

3.2 (b)

$$
\begin{aligned}
& (p \implies q) \implies r && \equiv \\
& \neg(p \implies q) \vee r && \equiv \\
& \neg(\neg p \vee q) \vee r && \equiv \\
& (p \wedge \neg q) \vee r
\end{aligned}
$$

3.3 (c)

$$
\begin{aligned}
& (\neg q \implies p) \implies (p \implies \neg q) && \equiv \\
& \neg(\neg q \implies p) \vee (p \implies \neg q) && \equiv \\
& \neg(q \vee p) \vee (\neg p \vee \neg q) && \equiv \\
& (\neg q \wedge \neg p) \vee (\neg p \vee \neg q) && \equiv \\
& (\neg p \wedge \neg q) \vee (\neg p \vee \neg q) && \equiv \\
& ((\neg p \wedge \neg q) \vee \neg p) \vee \neg q && \equiv \\
& (\neg p \vee (\neg p \wedge \neg q)) \vee \neg q && \equiv \\
& \neg p \vee \neg q
\end{aligned}
$$

4 Exercise 10

4.1 (a)

$$\begin{aligned} \neg p \implies \neg q & \equiv \\ \neg(\neg p) \vee \neg q & \equiv \\ p \vee \neg q & \end{aligned}$$

4.2 (b)

$$\begin{aligned} p \vee q \implies \neg p & \equiv \\ \neg(p \vee q) \vee \neg p & \equiv \\ (\neg p \wedge \neg q) \vee \neg p & \equiv \\ \neg p \vee (\neg p \wedge \neg q) & \equiv \\ \neg p & \end{aligned}$$

4.3 (c)

$$\begin{aligned} (p \implies \neg q) \implies (\neg p \implies q) & \equiv \\ \neg(p \implies \neg q) \vee (\neg p \implies q) & \equiv \\ \neg(\neg p \vee \neg q) \vee (p \vee q) & \equiv \\ (p \wedge q) \vee (p \vee q) & \equiv \\ ((p \wedge q) \vee p) \vee q & \equiv \\ (p \vee (p \wedge q)) \vee q & \equiv \\ p \vee q & \end{aligned}$$