Chapter 1 Section 2 Exercise Solutions

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5 Exercise 5

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7 Exercise 7

7.1 (a)

$$q \implies p$$

7.2 (b)

$$q \wedge \neg p$$

7.3 (c)

$$q \implies p$$

7.4 (d)

$$\neg q \implies \neg p$$

8 Exercise 8

8.1 (a)

$$r \wedge \neg p$$

8.2 (b)

$$r \wedge p \implies q$$

8.3 (c)

$$\neg r \implies \neg q$$

8.4 (d)

$$\neg p \wedge r \implies q$$

9 Exercise 9

Let

$$
\begin{aligned}
p &::= \text{"The system is in multiuser state."} \\
q &::= \text{"The system is operating normally."} \\
r &::= \text{"The kernel is functioning."} \\
s &::= \text{"The system is in interrupt mode"}
\end{aligned}
$$

Then our system specifications can be expressed as the following system of logical expressions:

$$p \iff q \tag{1}$$
$$q \implies r \tag{2}$$
$$\neg r \vee s \tag{3}$$
$$\neg p \implies s \tag{4}$$
$$\neg s \tag{5}$$

In order for (5) to be true, s must be false. Since s is false, p must be true in order for (4) to be true. Since p is true, q must be true in order for (1) to be true. Since q is true, r must be true in order in order for (2) to be true. However, we must conclude that (3) is false since r is true and s is false.

Therefore, there is no assignment of truth values such that all of our logical expressions are true. Hence, our system specifications are inconsistent.

10 Exercise 10

Let

$$
\begin{aligned}
p &::= \text{"The system software is being upgraded."} \\
q &::= \text{"Users can access the file system."} \\
r &::= \text{"Users can save new files"}
\end{aligned}
$$

Then our system specifications can be expressed as the following system of logical expressions:

$$p \implies \neg q \tag{6}$$

$$q \implies r \tag{7}$$

$$\neg r \implies \neg p \tag{8}$$

All of our logical expressions are true if we take $p = true$, $q = false$ and $r = true$. Hence, our system specifications are consistent.

11 Exercise 11

Let

$$p ::= \text{"The router can send packets to the edge system."}$$
$$q ::= \text{"The router supports the new address space."}$$
$$r ::= \text{"The latest software release is installed."}$$

Then our system specifications can be expressed as the following system of logical expressions:

$$p \implies q \tag{9}$$
$$q \implies r \tag{10}$$
$$r \implies p \tag{11}$$
$$\neg q \tag{12}$$

All of our logical expressions are true if we take $p = false$, $q = false$, and $r = false$. Hence, our system specifications are consistent.

12 Exercise 12

Let

$$p ::= \text{"The file system is locked."}$$
$$q ::= \text{"New messages will be queued."}$$
$$r ::= \text{"The system is functioning normally."}$$
$$s ::= \text{"New messages will be sent to the message buffer."}$$

Then our system specifications can be expressed as the following system of logical expressions:

$$\neg p \implies q \tag{13}$$
$$\neg p \iff r \tag{14}$$
$$\neg q \implies s \tag{15}$$
$$\neg p \implies s \tag{16}$$
$$\neg s \tag{17}$$

In order for (17) to be true, s must be false. Since s is false, p must be true in order for (16) to be true. Since s is false, q must be true in order for (15) to be true. However, since p is false and q is false, we must conclude that (13) is false.

All of our logical expressions are true if we take $p = true$, $q = true$, $r = false$, and $s = false$. Hence, our system specifications are consistent.

13 Exercise 13

13.1 (a)

beaches AND New AND Jersey

13.2 (b)

(beaches AND Jersey) NOT New

14 Exercise 14

14.1 (a)

hiking AND West AND Virginia

14.2 (b)

(hiking AND Virginia) NOT West

15 Exercise 15

Ethiopian AND restaurant AND New AND (York OR Jersey)

16 Exercise 16

(men AND (shoes or boots)) NOT work

17 Exercise 17

17.1 (a)

The statement that "All of the inscriptions are false" is equivalent to the propositional expression:

$$\neg p_3 \wedge \neg p_1 \wedge \neg(\neg p_3) \equiv \neg p_1 \wedge \neg p_3 \wedge p_3 \equiv \neg p_1 \wedge F \equiv F$$

Therefore, the Queen who never lies cannot make this statement.

17.2 (b)

The statement that "Exactly one of the inscriptions is true" is equivalent to the propositional expression:

$$(p_3 \wedge \neg p_1 \wedge \neg(\neg p_3)) \vee (\neg p_3 \wedge p_1 \wedge \neg(\neg p_3)) \vee (\neg p_3 \wedge \neg p_1 \wedge \neg p_3) \equiv (p_3 \wedge \neg p_1) \vee (\neg p_3 \wedge \neg p_1)$$

The Queen who never lies could make this statement if the treasure is in either Trunk 3 or Trunk 2.

17.3 (c)

The statement that "Exactly two of the inscriptions are true" is equivalent to the propositional expression:

$$(p_3 \wedge p_1 \wedge \neg(\neg p_3)) \vee (p_3 \wedge \neg p_1 \wedge \neg p_3) \vee (\neg p_3 \wedge p_1 \wedge \neg p3) \equiv (p_3 \wedge p_1) \vee (\neg p_3 \wedge p_1)$$

The Queen who never lies could make this statement. If we assume that the treasure cannot be in multiple trunks, we can conclude that the treasure is in Trunk 1.

17.4 (d)

The statement that "All three inscriptions are true" is equivalent to the propositional expression:

$$p_3 \wedge p_1 \wedge \neg p_3 \equiv p_1 \wedge p_3 \wedge \neg p_3 \equiv p_1 \wedge F \equiv F$$

The Queen who never lies cannot make this statement.

18 Exercise 18

18.1 (a)

The statement that "All of the inscriptions are false" is equivalent to the propositional expression:

$$\neg(\neg p_1) \land \neg p_1 \land \neg p_2 \equiv p_1 \land \neg p_1 \land \neg p_2 \equiv F \land \neg p_2 \equiv F$$

Therefore, the Queen who never lies cannot make this statement.

18.2 (b)

The statement that "Exactly one of the inscriptions is true" is equivalent to the propositional expression:

$$(\neg p_1 \land \neg p_1 \land \neg p_2) \lor (\neg(\neg p_1) \land p_1 \land \neg p_2) \lor (\neg(\neg p_1) \land \neg p_1 \land p_2) \equiv (\neg p_1 \land p_2) \lor (p_1 \land \neg p_2)$$

The Queen who never lies could make this statement if the treasure is in either Trunk 2 or Trunk 1.

18.3 (c)

The statement that "Exactly two of the inscriptions are true" is equivalent to the propositional expression:

$$(\neg p_1 \qquad p_1 \neg p_2) \lor (\neg p1 \land \neg p_1 \land p_2) \lor (\neg(\neg p1) \land p_1 \land p_2) \equiv (\neg p_1 \land p_2) \lor (p_1 \land p_2)$$

The Queen who never lies could make this statement. If we assume that the treasure cannot be in multiple trunks, we can conclude that the treasure is in Trunk 2.

18.4 (d)

The statement that "All three inscriptions are true" is equivalent to the propositional expression:

$$\neg p_1 \land p_1 \land p_2 \equiv F \land p_2 \equiv F$$

Therefore, the Queen who never lies cannot make this statement.

19 Exercise 23

Let

$$p ::= \text{"A is a knight."}$$
$$q ::= \text{"B is a knight."}$$

Then A's statement is equivalent to the propositional expression:

$$R ::= \neg p \vee \neg q$$

First, let us consider the case where A is a knave. Then $\neg p$ is true and A's statement is false. We could informally reason that the negation of A's statement is "Neither of us is knave" or $p \wedge q$. However, let us try a more formal approach by applying De Morgan's Law for OR:

$$\begin{aligned} &\neg R && \equiv \\ &\neg(\neg p \vee \neg q) && \equiv \\ &p \wedge q \end{aligned}$$

Therefore, we've contradicted our assumption that $\neg p$ is true.

Next, let us consider the case where A is a knight. Then p is true and A's statement is true. In order for R to evaluate to true, $\neg q$ must be true. I.e. B is a knave.

Hence, we conclude that A is a knight and B is a knave.

20 Exercise 24

The case where A is a knight is equivalent to the following system specifications:

$$p$$
$$p \wedge q$$
$$(\neg p \wedge q) \vee (p \wedge \neg q)$$

These specifications are inconsistent. In order for the first two expressions to be true, p must be true and q must be true. However, these truth value assignments result in the third expression evaluating to false.

The case where A is a knave is equivalent to the following system specifications:

$$\neg p$$
$$\neg(p \wedge q) \equiv \neg p \vee \neg q$$
$$(\neg p \wedge q) \vee (p \wedge \neg q)$$

These specifications are all satisfied for $p = F$ and $q = T$.

Hence, we conclude that A is a knave and B is a knight.

21 Exercise 25

The case where A is a knight is equivalent to the following system specifications:

$$p$$
$$\neg p \lor q$$

These specifications are all satisfied for $p = T$ and $q = T$.

The case where A is a knave is equivalent to the following system specifications:

$$\neg p$$
$$\neg(\neg p \lor q) \equiv p \land \neg q$$

These specifications are inconsistent. In order for the first expression to be true, p must be false. However, this truth value assignment results in the second expression evaluating to false regardless of the value for q.

Hence, we conclude that A is a knight and B is a knight.

22 Exercise 26

Let R be the logical expression equivalent to A's statement:

$$R ::= p$$

Let S be the logical expression equivalent to B's statement:

$$S ::= q$$

This scenario is equivalent to the system specifications:

$$(R \wedge p) \vee (\neg R \wedge \neg p) \equiv p \vee \neg p \equiv T$$
$$(S \wedge q) \vee (\neg S \wedge \neg q) \equiv q \vee \neg q \equiv T$$

Any truth value assignment will satisfy all of the specifications.

Hence, we conclude that we cannot draw any conclusions in this scenario. A can be either a knight or a knave independent of the status of B. B can be either a knight or a knave independent of the status of A.

23 Exercise 27

Let R be the logical expression equivalent to A's statement:

$$\neg p \wedge \neg q$$

This scenario is equivalent to the logical expression:

$$\begin{aligned}
&(R \wedge p) \vee (\neg R \wedge \neg p) &\equiv\\
&(\neg p \wedge \neg q \wedge p) \vee (\neg(\neg p \wedge \neg q) \wedge \neg p) &\equiv\\
&(p \wedge \neg p \wedge \neg q) \vee ((p \vee q) \wedge \neg p) &\equiv\\
&(F \wedge \neg q) \vee ((\neg p \wedge p) \vee (\neg p \wedge q)) &\equiv\\
&F \vee (\neg p \wedge q) &\equiv\\
&\neg p \wedge q
\end{aligned}$$

Hence, we conclude that A is a knave and B is a knight.

24 Exercise 28

Brute forcing knight, knave, and spy problems using logical formalism can get quite complicated as we need at least 4 propositional variables to uniquely express who is the knight, who is the knave, and who is the spy. Therefore, for exercises 28-35, we'll try to simplify the problem by applying the constraints before possibly resorting to logical expressions.

B cannot be the knight since then we would have two knights. If C is the knight, then C's statement would result in a contradiction. Therefore, C isn't the knight either. A must the knight.

Hence, we conclude that A is the knight, B is the spy, and C is the knave.

25 Exercise 29

C cannot be the knight since then we would have two knights. If B is the knight, then B's statement would result in a contradiction. Therefore, B isn't the knight either. A must be the knight. Now suppose that B is the knave. Then B's statement would be false, resulting in a contradiction. Therefore, B must be the spy.

Hence, we conclude that A is the knight, B is the spy, and C is the knave.

26 Exercise 30

This scenario is a paradox since it requires the knight to lie. I.e. there is no solution.

27 Exercise 31

If either B or C is the knight, then their respective statements results in a contradiction. A must be the knight. Since this means that B's statement is also true, B must be the spy.

Hence, we conclude that A is the knight, B is the spy, and C is the knave.

28 Exercise 32

Suppose that A is the knight. Then B's statement is also true so B is the spy. However, this also makes C's statement true and we reach a contradiction since there is no knave. Therefore, A isn't the knight.

Suppose that A is spy. Then B's statement is true so B is the knight. However, this also makes C's statement true and we reach a contradiction since there is no knave. Therefore, A isn't the spy.

Suppose that A is the knave. Then B's statement is false so B is the spy. C's statement is true so C is the knight.

Hence, we conclude that A is the knave, B is the spy, and C is the knight.

29 Exercise 33

These statements don't allow us to eliminate any solutions. I.e. all 6 permutations of knight, knave, and spy are possible in this scenario.

30 Exercise 34

Suppose that C is the knight. Then A is the spy and B is the knave.

Suppose that C is the knave. Then C's statement is true, resulting in a contradiction.

Suppose that C is the spy. Then either A or B must be the knave. This results in a contradiction since the knave's statement is true.

Hence, we conclude that A is the spy, B is the knave, and C is the knight.

31 Exercise 35

This scenario is a paradox since it requires the knave to tell the truth. I.e. there is no solution.