Chapter 1 Section 2 Exercise Solutions

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$$\neg a \implies \neg e$$

2 Exercise 2

$$m \implies e \vee p$$

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$$g \implies r \wedge \neg m \wedge \neg b$$

4 Exercise 4

$$\neg s \implies (d \implies w)$$

5 Exercise 5

$$e \implies a \wedge (b \vee p) \wedge r$$

6 Exercise 6

$$u \implies (b_{32} \wedge g_1 \wedge r_1 \wedge h_{16}) \vee (b_{64} \wedge g_2 \wedge r_2 \wedge h_{32})$$

7 Exercise 7

7.1 (a)

$$q \implies p$$

7.2 (b)

$$q \wedge \neg p$$

7.3 (c)

$$q \implies p$$

7.4 (d)

$$\neg q \implies \neg p$$

8 Exercise 8

8.1 (a)

$$r \wedge \neg p$$

8.2 (b)

$$r \wedge p \implies q$$

8.3 (c)

$$\neg r \implies \neg q$$

8.4 (d)

$$\neg p \wedge r \implies q$$

9 Exercise 9

Let

$$p ::= \text{"The system is in multiuser state."}$$
$$q ::= \text{"The system is operating normally."}$$
$$r ::= \text{"The kernel is functioning."}$$
$$s ::= \text{"The system is in interrupt mode"}$$

Then our system specifications can be expressed as the following system of logical expressions:

$$p \iff q \tag{1}$$
$$q \implies r \tag{2}$$
$$\neg r \vee s \tag{3}$$
$$\neg p \implies s \tag{4}$$
$$\neg s \tag{5}$$

In order for (5) to be true, s must be false. Since s is false, p must be true in order for (4) to be true. Since p is true, q must be true in order for (1) to be true. Since q is true, r must be true in order in order for (2) to be true. However, we must conclude that (3) is false since r is true and s is false.

Therefore, there is no assignment of truth values such that all of our logical expressions are true. Hence, our system specifications are inconsistent.

10 Exercise 10

Let

$$p ::= \text{"The system software is being upgraded."}$$
$$q ::= \text{"Users can access the file system."}$$
$$r ::= \text{"Users can save new files"}$$

Then our system specifications can be expressed as the following system of logical expressions:

$$p \implies \neg q \tag{6}$$
$$q \implies r \tag{7}$$
$$\neg r \implies \neg p \tag{8}$$

All of our logical expressions are true if we take $p = true$, $q = false$ and $r = true$. Hence, our system specifications are consistent.

11 Exercise 11

Let

$p ::=$ "The router can send packets to the edge system."

$q ::=$ "The router supports the new address space."

$r ::=$ "The latest software release is installed."

Then our system specifications can be expressed as the following system of logical expressions:

$$p \implies q \tag{9}$$

$$q \implies r \tag{10}$$

$$r \implies p \tag{11}$$

$$\neg q \tag{12}$$

All of our logical expressions are true if we take $p = false$, $q = false$, and $r = false$. Hence, our system specifications are consistent.

12 Exercise 12

Let

$p ::=$ "The file system is locked."
$q ::=$ "New messages will be queued."
$r ::=$ "The system is functioning normally."
$s ::=$ "New messages will be sent to the message buffer."

Then our system specifications can be expressed as the following system of logical expressions:

$$\neg p \implies q \tag{13}$$
$$\neg p \iff r \tag{14}$$
$$\neg q \implies s \tag{15}$$
$$\neg p \implies s \tag{16}$$
$$\neg s \tag{17}$$

In order for (17) to be true, s must be false. Since s is false, p must be true in order for (16) to be true. Since s is false, q must be true in order for (15) to be true. However, since p is false and q is false, we must conclude that (13) is false.

All of our logical expressions are true if we take $p = true$, $q = true$, $r = false$, and $s = false$. Hence, our system specifications are consistent.