# Chapter 1 Section 2 Exercise Solutions

Samuel Lair

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# 9 Exercise 9

Let

$$\begin{aligned}
p &::= \text{"The system is in multiuser state."}\\
q &::= \text{"The system is operating normally."}\\
r &::= \text{"The kernel is functioning."}\\
s &::= \text{"The system is in interrupt mode"}
\end{aligned}$$

Then our system specifications can be expressed as the following system of logical expressions:

$$p \iff q \tag{1}$$

$$q \implies r \tag{2}$$

$$\neg r \vee s \tag{3}$$

$$\neg p \implies s \tag{4}$$

$$\neg s \tag{5}$$

In order for (5) to be true, $s$ must be false. Since $s$ is false, $p$ must be true in order for (4) to be true. Since $p$ is true, $q$ must be true in order for (1) to be true. Since $q$ is true, $r$ must be true in order in order for (2) to be true. However, we must conclude that (3) is false since $r$ is true and $s$ is false.

Therefore, there is no assignment of truth values such that all of our logical expressions are true. Hence, our system specifications are inconsistent.

## 10 Exercise 10

Let

$$p ::= \text{"The system software is being upgraded."}$$
$$q ::= \text{"Users can access the file system."}$$
$$r ::= \text{"Users can save new files"}$$

Then our system specifications can be expressed as the following system of logical expressions:

$$p \implies \neg q \tag{6}$$
$$q \implies r \tag{7}$$
$$\neg r \implies \neg p \tag{8}$$

All of our logical expressions are true if we take $p = true$, $q = false$ and $r = true$. Hence, our system specifications are consistent.

## 11 Exercise 11

Let

$$p ::= \text{"The router can send packets to the edge system."}$$
$$q ::= \text{"The router supports the new address space."}$$
$$r ::= \text{"The latest software release is installed."}$$

Then our system specifications can be expressed as the following system of logical expressions:

$$p \implies q \tag{9}$$
$$q \implies r \tag{10}$$
$$r \implies p \tag{11}$$
$$\neg q \tag{12}$$

All of our logical expressions are true if we take $p = false$, $q = false$, and $r = false$. Hence, our system specifications are consistent.

## 12 Exercise 12

Let

$$p ::= \text{"The file system is locked."}$$
$$q ::= \text{"New messages will be queued."}$$
$$r ::= \text{"The system is functioning normally."}$$
$$s ::= \text{"New messages will be sent to the message buffer."}$$

Then our system specifications can be expressed as the following system of logical expressions:

$$\neg p \implies q \tag{13}$$
$$\neg p \iff r \tag{14}$$
$$\neg q \implies s \tag{15}$$
$$\neg p \implies s \tag{16}$$
$$\neg s \tag{17}$$

In order for (17) to be true, s must be false. Since s is false, p must be true in order for (16) to be true. Since s is false, q must be true in order for (15) to be true. However, since p is false and q is false, we must conclude that (13) is false.

All of our logical expressions are true if we take $p = true$, $q = true$, $r = false$, and $s = false$. Hence, our system specifications are consistent.

# 13 Exercise 13

## 13.1 (a)

beaches AND New AND Jersey

## 13.2 (b)

(beaches AND Jersey) NOT New

## 14 Exercise 14

### 14.1 (a)

hiking AND West AND Virginia

### 14.2 (b)

(hiking AND Virginia) NOT West

## 15 Exercise 15

Ethiopian AND restaurant AND New AND (York OR Jersey)

## 16 Exercise 16

(men AND (shoes or boots)) NOT work

# 17 Exercise 17

## 17.1 (a)

The statement that "All of the inscriptions are false" is equivalent to the propositional expression:

$$\neg p_3 \land \neg p_1 \land \neg(\neg p_3) \equiv \neg p_1 \land \neg p_3 \land p_3 \equiv \neg p_1 \land F \equiv F$$

Therefore, the Queen who never lies cannot make this statement.

## 17.2 (b)

The statement that "Exactly one of the inscriptions is true" is equivalent to the propositional expression:

$$(p_3 \land \neg p_1 \land \neg(\neg p_3)) \lor (\neg p_3 \land p_1 \land \neg(\neg p_3)) \lor (\neg p_3 \land \neg p_1 \land \neg p_3) \equiv$$
$$(p_3 \land \neg p_1) \lor (\neg p_3 \land \neg p_1)$$

The Queen who never lies could make this statement if the treasure is in either Trunk 3 or Trunk 2.

## 17.3 (c)

The statement that "Exactly two of the inscriptions are true" is equivalent to the propositional expression:

$$(p_3 \land p_1 \land \neg(\neg p_3)) \lor (p_3 \land \neg p_1 \land \neg p_3) \lor (\neg p_3 \land p_1 \land \neg p3) \equiv$$
$$(p_3 \land p_1) \lor (\neg p_3 \land p_1)$$

The Queen who never lies could make this statement. If we assume that the treasure cannot be in multiple trunks, we can conclude that the treasure is in Trunk 1.

## 17.4 (d)

The statement that "All three inscriptions are true" is equivalent to the propositional expression:

$$p_3 \land p_1 \land \neg p_3 \equiv p_1 \land p_3 \land \neg p_3 \equiv p_1 \land F \equiv F$$

The Queen who never lies cannot make this statement.

# 18 Exercise 18

## 18.1 (a)

The statement that "All of the inscriptions are false" is equivalent to the propositional expression:

$$\neg(\neg p_1) \wedge \neg p_1 \wedge \neg p_2 \equiv p_1 \wedge \neg p_1 \wedge \neg p_2 \equiv F \wedge \neg p_2 \equiv F$$

Therefore, the Queen who never lies cannot make this statement.

## 18.2 (b)

The statement that "Exactly one of the inscriptions is true" is equivalent to the propositional expression:

$$(\neg p_1 \wedge \neg p_1 \wedge \neg p_2) \vee (\neg(\neg p_1) \wedge p_1 \wedge \neg p_2) \vee (\neg(\neg p_1) \wedge \neg p_1 \wedge p_2) \equiv (\neg p_1 \wedge p_2) \vee (p_1 \wedge \neg p_2)$$

The Queen who never lies could make this statement if the treasure is in either Trunk 2 or Trunk 1.

## 18.3 (c)

The statement that "Exactly two of the inscriptions are true" is equivalent to the propositional expression:

$$(\neg p_1 \qquad p_1 \neg p_2) \vee (\neg p1 \wedge \neg p_1 \wedge p_2) \vee (\neg(\neg p1) \wedge p_1 \wedge p_2) \equiv (\neg p_1 \wedge p_2) \vee (p_1 \wedge p_2)$$

The Queen who never lies could make this statement. If we assume that the treasure cannot be in multiple trunks, we can conclude that the treasure is in Trunk 2.

## 18.4 (d)

The statement that "All three inscriptions are true" is equivalent to the propositional expression:

$$\neg p_1 \wedge p_1 \wedge p_2 \equiv F \wedge p_2 \equiv F$$

Therefore, the Queen who never lies cannot make this statement.

## 19 Exercise 19

Let

$$p ::= \text{"The left fork leads to the ruins."}$$
$$q ::= \text{"The villager always tells the truth."}$$

Suppose we ask a straightfoward question like:

$$\text{"Does the left fork lead to the ruins?"} \tag{18}$$

This question is equivalent to the following propositional expression:

$$(p \wedge q) \vee (\neg p \wedge \neg q) \tag{19}$$

In other words, the truth table for (18) is as follows:

| $p$ | $q$ | (19) |
|---|---|---|
| $T$ | $T$ | $T$ |
| $T$ | $F$ | $F$ |
| $F$ | $T$ | $F$ |
| $F$ | $F$ | $T$ |

This is problematic since we want the truth value of our question to match the truth value of p regardless of the truth value of q. We can achieve this by making the liar lie about his answer to (18). Due to the double negation rule, a lie about a lie will be equal to the truth. Therefore, the question we will actually ask is:

$$\text{"If I were to ask you about whether the left fork leads to the ruins, would you say yes?"} \tag{20}$$

The truth table for (20) is as follows:

| $p$ | $q$ | | (20) |
|---|---|---|---|
| $T$ | $T$ | $(18) \equiv$ | $T$ |
| $T$ | $F$ | $\neg(18) \equiv$ | $T$ |
| $F$ | $T$ | $(18) \equiv$ | $F$ |
| $F$ | $F$ | $\neg(18) \equiv$ | $F$ |

The truth value of (20) matches $p$ as desired, allowing us to infer which fork leads to the ruins.

# 20 Exercise 20

## 20.1 (a)

The question "Are you a liar?" does not work because both types of cannibals will answer no. The cannibals who always tell the truth will answer no because that is the truth. The cannibals who always lie will answer no because that is the lie.

## 20.2 (b)

Similar to Exercise 19, the explorer needs to make the liars lie about a lie. Therefore, the explorer should ask: "If I were to ask you whether you are a liar, would you say yes?"

# 21 Exercise 21

From the first professor's response, we can infer that he wants coffee but does not know whether the other two professors want coffee. From the second professor's response, we can infer that he wants coffee but does not know whether the third professor wants coffee. From the third professor's response, we can infer that he does not want coffee.

Therefore, the hostess gives coffee to the first and second professors.

## 22 Exercise 22

Let

$$\begin{aligned}
j &::= \text{"Jasmine attends."}\\
s &::= \text{"Samir attends."}\\
k &::= \text{"Kanti attends."}
\end{aligned}$$

In order for noone to be unhappy, we must satisfy the following system specifications:

$$j \implies \neg s \tag{21}$$
$$s \implies k \tag{22}$$
$$\neg j \implies \neg k \tag{23}$$

Suppose that Jasmine attends. From (21), Samir cannot also attend. Our specifications are satisfied regardless of whether Kanti attends.

Now suppose that Jasmine doesn't attend. From (23), Kanti also doesn't attend. Finally, from (22), Samir also doesn't attend.

Hence, we can invite either Jasmine alone, both Jasmine and Kanti, or none of the three.

## 23 Exercise 23

Let

$$
\begin{aligned}
p &::= \text{"A is a knight."} \\
q &::= \text{"B is a knight."}
\end{aligned}
$$

Then A's statement is equivalent to the propositional expression:

$$R ::= \neg p \vee \neg q$$

First, let us consider the case where A is a knave. Then $\neg p$ is true and A's statement is false. We could informally reason that the negation of A's statement is "Neither of us is knave" or $p \wedge q$. However, let us try a more formal approach by applying De Morgan's Law for OR:

$$
\begin{aligned}
&\neg R && \equiv \\
&\neg(\neg p \vee \neg q) && \equiv \\
&p \wedge q &&
\end{aligned}
$$

Therefore, we've contradicted our assumption that $\neg p$ is true.

Next, let us consider the case where A is a knight. Then $p$ is true and A's statement is true. In order for $R$ to evaluate to true, $\neg q$ must be true. I.e. B is a knave.

Hence, we conclude that A is a knight and B is a knave.

## 24 Exercise 24

The case where A is a knight is equivalent to the following system specifications:

$$
\begin{aligned}
&p \\
&p \wedge q \\
&(\neg p \wedge q) \vee (p \wedge \neg q)
\end{aligned}
$$

These specifications are inconsistent. In order for the first two expressions to be true, $p$ must be true and $q$ must be true. However, these truth value assignments result in the third expression evaluating to false.

The case where A is a knave is equivalent to the following system specifications:

$$
\begin{aligned}
&\neg p \\
&\neg(p \wedge q) \equiv \neg p \vee \neg q \\
&(\neg p \wedge q) \vee (p \wedge \neg q)
\end{aligned}
$$

These specifications are all satisfied for $p = F$ and $q = T$.

Hence, we conclude that A is a knave and B is a knight.

## 25 Exercise 25

The case where A is a knight is equivalent to the following system specifications:

$$
\begin{aligned}
&p \\
&\neg p \vee q
\end{aligned}
$$

These specifications are all satisfied for $p = T$ and $q = T$.

The case where A is a knave is equivalent to the following system specifications:

$$
\begin{aligned}
&\neg p \\
&\neg(\neg p \vee q) \equiv p \wedge \neg q
\end{aligned}
$$

These specifications are inconsistent. In order for the first expression to be true, $p$ must be false. However, this truth value assignment results in the second expression evaluating to false regardless of the value for $q$.

Hence, we conclude that A is a knight and B is a knight.

## 26 Exercise 26

Let $R$ be the logical expression equivalent to A's statement:

$$R ::= p$$

Let $S$ be the logical expression equivalent to B's statement:

$$S ::= q$$

This scenario is equivalent to the system specifications:

$$(R \wedge p) \vee (\neg R \wedge \neg p) \equiv p \vee \neg p \equiv T$$
$$(S \wedge q) \vee (\neg S \wedge \neg q) \equiv q \vee \neg q \equiv T$$

Any truth value assignment will satisfy all of the specifications.

Hence, we conclude that we cannot draw any conclusions in this scenario. $A$ can be either a knight or a knave independent of the status of $B$. $B$ can be either a knight or a knave independent of the status of $A$.

## 27 Exercise 27

Let $R$ be the logical expression equivalent to A's statement:

$$\neg p \wedge \neg q$$

This scenario is equivalent to the logical expression:

$$\begin{aligned}
&(R \wedge p) \vee (\neg R \wedge \neg p) &&\equiv \\
&(\neg p \wedge \neg q \wedge p) \vee (\neg(\neg p \wedge \neg q) \wedge \neg p) &&\equiv \\
&(p \wedge \neg p \wedge \neg q) \vee ((p \vee q) \wedge \neg p) &&\equiv \\
&(F \wedge \neg q) \vee ((\neg p \wedge p) \vee (\neg p \wedge q)) &&\equiv \\
&F \vee (\neg p \wedge q) &&\equiv \\
&\neg p \wedge q
\end{aligned}$$

Hence, we conclude that $A$ is a knave and $B$ is a knight.

# 28 Exercise 28

Brute forcing knight, knave, and spy problems using logical formalism can get quite complicated as we need at least 4 propositional variables to uniquely express who is the knight, who is the knave, and who is the spy. Therefore, for exercises 28-35, we'll try to simplify the problem by applying the constraints before possibly resorting to logical expressions.

$B$ cannot be the knight since then we would have two knights. If $C$ is the knight, then $C$'s statement would result in a contradiction. Therefore, $C$ isn't the knight either. $A$ must the knight.

Hence, we conclude that $A$ is the knight, $B$ is the spy, and $C$ is the knave.

## 29 Exercise 29

$C$ cannot be the knight since then we would have two knights. If $B$ is the knight, then $B$'s statement would result in a contradiction. Therefore, $B$ isn't the knight either. $A$ must be the knight. Now suppose that $B$ is the knave. Then $B$'s statement would be false, resulting in a contradiction. Therefore, $B$ must be the spy.

Hence, we conclude that $A$ is the knight, $B$ is the spy, and $C$ is the knave.

# 30 Exercise 30

This scenario is a paradox since it requires the knight to lie. I.e. there is no solution.

## 31 Exercise 31

If either $B$ or $C$ is the knight, then their respective statements results in a contradiction. $A$ must be the knight. Since this means that $B$'s statement is also true, $B$ must be the spy.

Hence, we conclude that $A$ is the knight, $B$ is the spy, and $C$ is the knave.

# 32 Exercise 32

Suppose that $A$ is the knight. Then $B$'s statement is also true so $B$ is the spy. However, this also makes $C$'s statement true and we reach a contradiction since there is no knave. Therefore, $A$ isn't the knight.

Suppose that $A$ is spy. Then B's statement is true so $B$ is the knight. However, this also makes $C$'s statement true and we reach a contradiction since there is no knave. Therefore, $A$ isn't the spy.

Suppose that $A$ is the knave. Then B's statement is false so $B$ is the spy. $C$'s statement is true so $C$ is the knight.

Hence, we conclude that $A$ is the knave, $B$ is the spy, and $C$ is the knight.

## 33 Exercise 33

These statements don't allow us to eliminate any solutions. I.e. all 6 permutations of knight, knave, and spy are possible in this scenario.

## 34 Exercise 34

Suppose that $C$ is the knight. Then $A$ is the spy and $B$ is the knave.

Suppose that $C$ is the knave. Then C's statement is true, resulting in a contradiction.

Suppose that $C$ is the spy. Then either $A$ or $B$ must be the knave. This results in a contradiction since the knave's statement is true.

Hence, we conclude that $A$ is the spy, $B$ is the knave, and $C$ is the knight.

# 35 Exercise 35

This scenario is a paradox since it requires the knave to tell the truth. I.e. there is no solution.

## 36 Exercise 37

Let

$$\begin{aligned} f &::= \text{"Fred is the highest paid."} \\ j &::= \text{"Janice is the highest paid."} \\ k &::= \text{"Janice is the lowest paid."} \\ m &::= \text{"Maggie is the highest paid."} \end{aligned}$$

Then the known facts are equivalent to the following system specifications:

$$j \implies \neg k \tag{24}$$
$$k \implies \neg j \tag{25}$$
$$f \implies \neg j \tag{26}$$
$$f \implies \neg m \tag{27}$$
$$j \implies \neg f \tag{28}$$
$$j \implies \neg m \tag{29}$$
$$m \implies \neg f \tag{30}$$
$$m \implies \neg j \tag{31}$$
$$\neg f \implies j \tag{32}$$
$$\neg k \implies m \tag{33}$$

Suppose that $f = T$. From (26), $j = F$. From (27), $m = F$. From (33), $k = T$. Therefore, Fred $>$ Maggie $>$ Janice in this case.

Suppose that $f = F$. From (32), $j = T$. From (24), $k = F$. From (33), $m = T$. However, this contradicts both (29) and (31). I.e. Janice and Maggie cannot both be the highest paid.

Hence, we conclude that Fred is the highest paid, Janice is the lowest paid, and Maggie's pay is between that of Fred and Janice.

## 37 Exercise 38

Let

$$
\begin{aligned}
k &::= \text{"Kevin is chatting."}\\
h &::= \text{"Heather is chatting."}\\
r &::= \text{"Randy is chatting."}\\
v &::= \text{"Vijay is chatting."}\\
a &::= \text{"Abby is chatting."}
\end{aligned}
$$

Our scenario is equivalent to the following system specifications:

$$k \vee h \tag{34}$$

$$r \oplus v \tag{35}$$

$$a \implies r \tag{36}$$

$$v \iff k \tag{37}$$

$$h \implies a \wedge k \tag{38}$$

Suppose that $v = T$. From (37), $k = T$. From (35), $r = F$. From (36), $a = F$. From (38), $h = F$. (34) $= T$ since $k = T$.

Suppose that $v = F$. From (37), $k = F$. From (38), $h = F$. However, this a contradiction since (34) $= F$.

Hence, we conclude that Kevin and Vijay are chatting while Heather, Randy and Abby are not chatting.

## 38 Exercise 39

Let

$$\begin{aligned}
b &::= \text{"The butler is telling the truth."} \\
c &::= \text{"The cook is telling the truth."} \\
g &::= \text{"The gardener is telling the truth."} \\
h &::= \text{"The handyman is telling the truth."}
\end{aligned}$$

Our facts are equivalent to the following system specifications:

$$b \implies c \tag{39}$$
$$c \implies \neg g \tag{40}$$
$$g \implies \neg c \tag{41}$$
$$\neg g \implies h \tag{42}$$
$$\neg h \implies g \tag{43}$$
$$h \implies \neg c \tag{44}$$

Suppose that $b = T$. From (39), $c = T$. From (40), $g = F$. From (42), $h = T$. However, (44) leads us to a contradiction where $c = F$.

Suppose that $b = F$. We must also take $c = F$ to avoid the previous contradiction. If we take $g = T$, $h$ can be either true or false without reaching a contradiction. If we take $g = F$, then (42) requires that $h = T$. This doesn't lead to any contradictions either.

Hence, we can conclude that the butler and cook are lying. However, we cannot determine whether the gardener is telling the truth or whether the handyman is telling the truth.

# 39 Exercise 40

## 39.1 (a)

Suppose Alice is telling the truth. Then we reach a contradiction where John is also telling the truth.

Suppose that John is telling the truth. Then we reach a contradiction where either Carlos or Diana is also telling the truth. Since we can conclude that John's statement is false, we now know that he did it. However, let's also see if we can figure out who is telling the truth.

Suppose that Carlos is telling the truth. This contradicts the fact that John's statement is false.

Suppose that Diana is telling the truth. There is no contradiction in this case.

Therefore, Diana is telling the truth while the other three are lying. Hence, we conclude that John accessed the computer system without authorization since his statement is false.

## 39.2 (b)

Alice's and Carlos's statements cannot both be true since they implicate different people. Also, if Diana's statement is true, then Carlos's statement is false.

Suppose that Carlos's statement is true. Then we reach a contradiction where both Alice's and Diana's statements are false. Therefore, Carlos's statement is false while the other three are true.

Hence, we conclude that Carlos accessed the computer system without authorization since Alice's statement is true.

## 40 Exercise 41

If the sign on the first door is true, then the sign on the second door is true as well. Since we know that exactly one sign is true, the sign on the first door must be false and the sign on the second door must be true.

Hence, the lady is in the second room and the tiger is in the first room.

## 41 Exercise 43

If more than one senator is honest, then there exists a pair of senators such that both are honest (i.e. neither is corrupt). This contradicts the requirement that "given any two Freedonian senators, at least one is corrupt."

Hence we conclude that one senator is honest and the other 49 are corrupt.

# 42 Exercise 44

## 42.1 (a)

$$\neg p \vee \neg q$$

## 42.2 (b)

$$\neg(p \vee (\neg p \wedge q))$$

# 43 Exercise 45

## 43.1 (a)

$$\neg(p \wedge (q \vee \neg r))$$

## 43.2 (b)

$$(\neg p \wedge \neg q) \vee (p \wedge r)$$