Chapter 1 Section 3 Exercise Solutions

Samuel Lair

September 2022

Contents

1 Exercise 5 **2**

2 Exercise 6 **2**

3 Exercise 9 **3**
3.1 (a) . . . 3
3.2 (b) . . . 3
3.3 (c) . . . 3

4 Exercise 10 **4**
4.1 (a) . . . 4
4.2 (b) . . . 4
4.3 (c) . . . 4

5 Exercise 15 **5**
5.1 (a) . . . 5
5.2 (b) . . . 5
5.3 (c) . . . 5
5.4 (d) . . . 6
5.5 (e) . . . 6
5.6 (f) . . . 6

6 Exercise 16 **7**
6.1 (a) . . . 7
6.2 (b) . . . 7
6.3 (c) . . . 8
6.4 (d) . . . 8

1 Exercise 5

p	q	r	$q \vee r$	$p \wedge (q \vee r)$	$p \wedge q$	$p \wedge r$	$(p \wedge q) \vee (p \wedge r)$
T	T	T	T	T	T	T	T
T	T	F	T	T	T	F	T
T	F	T	T	T	F	T	T
T	F	F	F	F	F	F	F
F	T	T	T	F	F	F	F
F	T	F	T	F	F	F	F
F	F	T	T	F	F	F	F
F	F	F	F	F	F	F	F

Since the truth values of the compound propositions $p \wedge (q \vee r)$ and $(p \wedge q) \vee (p \wedge r)$ agree for all possible combinations of the truth values of p, q, and r, said compound propositions are logically equivalent.

2 Exercise 6

p	q	$p \wedge q$	$\neg(p \wedge q)$	$\neg p$	$\neg q$	$\neg p \vee \neg q$
T	T	T	F	F	F	F
T	F	F	T	F	T	T
F	T	F	T	T	F	T
F	F	F	T	T	T	T

Since the truth values of the compound propositions $\neg(p \wedge q)$ and $\neg p \vee \neg q$ agree for all possible combinations of the truth values of p and q, said compound propositions are logically equivalent.

3 Exercise 9

3.1 (a)

$$
\begin{aligned}
& p \implies \neg q && \equiv \\
& \neg p \vee \neg q
\end{aligned}
$$

3.2 (b)

$$
\begin{aligned}
& (p \implies q) \implies r && \equiv \\
& \neg(p \implies q) \vee r && \equiv \\
& \neg(\neg p \vee q) \vee r && \equiv \\
& (p \wedge \neg q) \vee r
\end{aligned}
$$

3.3 (c)

$$
\begin{aligned}
& (\neg q \implies p) \implies (p \implies \neg q) && \equiv \\
& \neg(\neg q \implies p) \vee (p \implies \neg q) && \equiv \\
& \neg(q \vee p) \vee (\neg p \vee \neg q) && \equiv \\
& (\neg q \wedge \neg p) \vee (\neg p \vee \neg q) && \equiv \\
& (\neg p \wedge \neg q) \vee (\neg p \vee \neg q) && \equiv \\
& ((\neg p \wedge \neg q) \vee \neg p) \vee \neg q && \equiv \\
& (\neg p \vee (\neg p \wedge \neg q)) \vee \neg q && \equiv \\
& \neg p \vee \neg q
\end{aligned}
$$

4 Exercise 10

4.1 (a)

$$\begin{aligned} \neg p \implies \neg q & \equiv \\ \neg(\neg p) \vee \neg q & \equiv \\ p \vee \neg q & \end{aligned}$$

4.2 (b)

$$\begin{aligned} p \vee q \implies \neg p & \equiv \\ \neg(p \vee q) \vee \neg p & \equiv \\ (\neg p \wedge \neg q) \vee \neg p & \equiv \\ \neg p \vee (\neg p \wedge \neg q) & \equiv \\ \neg p & \end{aligned}$$

4.3 (c)

$$\begin{aligned} (p \implies \neg q) \implies (\neg p \implies q) & \equiv \\ \neg(p \implies \neg q) \vee (\neg p \implies q) & \equiv \\ \neg(\neg p \vee \neg q) \vee (p \vee q) & \equiv \\ (p \wedge q) \vee (p \vee q) & \equiv \\ ((p \wedge q) \vee p) \vee q & \equiv \\ (p \vee (p \wedge q)) \vee q & \equiv \\ p \vee q & \end{aligned}$$

5 Exercise 15

5.1 (a)

$$
\begin{aligned}
(p \wedge q) &\implies p && \equiv \\
\neg(p \wedge q) &\vee p && \equiv \\
(\neg p \vee \neg q) &\vee p && \equiv \\
\neg q &\vee (p \vee \neg p) && \equiv \\
\neg q &\vee T && \equiv \\
T &
\end{aligned}
$$

5.2 (b)

$$
\begin{aligned}
p &\implies (p \vee q) && \equiv \\
\neg p &\vee (p \vee q) && \equiv \\
q &\vee (p \vee \neg p) && \equiv \\
q &\vee T && \equiv \\
T &
\end{aligned}
$$

5.3 (c)

$$
\begin{aligned}
\neg p &\implies (p \implies q) && \equiv \\
\neg p &\implies (\neg p \vee q) && \equiv \\
p &\vee (\neg p \vee q) && \equiv \\
q &\vee (p \vee \neg p) && \equiv \\
q &\vee T && \equiv \\
T &
\end{aligned}
$$

5.4 (d)

$$
\begin{aligned}
&(p \wedge q) \implies (p \implies q) && \equiv \\
&(p \wedge q) \implies (\neg p \vee q) && \equiv \\
&\neg(p \wedge q) \vee (\neg p \vee q) && \equiv \\
&(\neg p \vee \neg q) \vee (\neg p \vee q) && \equiv \\
&(\neg p \vee \neg p) \vee (q \vee \neg q) && \equiv \\
&\neg p \vee T && \equiv \\
&T
\end{aligned}
$$

5.5 (e)

$$
\begin{aligned}
&\neg(p \implies q) \implies p && \equiv \\
&\neg(\neg p \vee q) \implies p && \equiv \\
&(\neg p \vee q) \vee p && \equiv \\
&q \vee (p \vee \neg p) && \equiv \\
&q \vee T && \equiv \\
&T
\end{aligned}
$$

5.6 (f)

$$
\begin{aligned}
&\neg(p \implies q) \implies \neg q && \equiv \\
&\neg(\neg p \vee q) \implies \neg q && \equiv \\
&(\neg p \vee q) \vee \neg q && \equiv \\
&\neg p \vee (q \vee \neg q) && \equiv \\
&\neg p \vee T && \equiv \\
&T
\end{aligned}
$$

6 Exercise 16

6.1 (a)

$$
\begin{aligned}
[\neg p \wedge (p \vee q)] \implies q & \equiv \\
\neg[\neg p \wedge (p \vee q)] \vee q & \equiv \\
p \vee \neg(p \vee q) \vee q & \equiv \\
p \vee (\neg p \wedge \neg q) \vee q & \equiv \\
(p \vee q) \vee (\neg p \wedge \neg q) & \equiv \\
((p \vee q) \vee \neg p) \wedge ((p \vee q) \vee \neg q) & \equiv \\
(q \vee (p \vee \neg p)) \wedge (p \vee (q \vee \neg q)) & \equiv \\
(q \vee T) \wedge (p \vee T) & \equiv \\
T \wedge T & \equiv \\
T &
\end{aligned}
$$

6.2 (b)

$$
\begin{aligned}
[(p \implies q) \wedge (q \implies r)] \implies (p \implies r) & \equiv \\
\neg[(p \implies q) \wedge (q \implies r)] \vee (p \implies r) & \equiv \\
\neg[(\neg p \vee q) \wedge (\neg q \vee r)] \vee (\neg p \vee r) & \equiv \\
[\neg(\neg p \vee q) \vee \neg(\neg q \vee r)] \vee (\neg p \vee r) & \equiv \\
[(p \wedge \neg q) \vee (q \wedge \neg r)] \vee (\neg p \vee r) & \equiv \\
[((p \wedge \neg q) \wedge q) \vee ((p \vee \neg q) \wedge \neg r)] \vee (\neg p \vee r) & \equiv \\
[(p \wedge (q \wedge \neg q)) \vee ((p \vee \neg q) \wedge \neg r)] \vee (\neg p \vee r) & \equiv \\
[(p \wedge F) \vee (\neg r \wedge (p \vee \neg q))] \vee (\neg p \vee r) & \equiv \\
[F \vee ((\neg r \wedge p) \vee (\neg r \wedge \neg q))] \vee (\neg p \vee r) & \equiv \\
((\neg r \wedge p) \vee (\neg r \wedge \neg q)) \vee (\neg p \vee r) & \equiv \\
((\neg p \vee r) \vee (\neg r \wedge p)) \vee (\neg r \wedge \neg q) & \equiv \\
(((\neg p \vee r) \vee \neg r) \wedge ((\neg p \vee r) \vee p)) \vee (\neg r \wedge \neg q) & \equiv \\
((\neg p \vee T) \wedge (r \vee T)) \vee (\neg r \wedge \neg q) & \equiv \\
(T \wedge T) \vee (\neg r \wedge \neg q) & \equiv \\
(\neg r \wedge \neg q) \vee T & \equiv \\
T &
\end{aligned}
$$

6.3 (c)

$$
\begin{aligned}
[p \wedge (p \implies q)] \implies q \quad &\equiv \\
\neg[p \wedge (\neg p \vee q)] \vee q \quad &\equiv \\
[\neg p \vee \neg(\neg p \vee q)] \vee q \quad &\equiv \\
[\neg p \vee (p \wedge \neg q)] \vee q \quad &\equiv \\
(\neg p \vee q) \vee (p \wedge \neg q) \quad &\equiv \\
[((\neg p \vee q) \vee p) \wedge ((\neg p \vee q) \vee \neg q)] \quad &\equiv \\
[(q \vee T) \wedge (\neg p \vee T)] \quad &\equiv \\
T \wedge T \equiv & \\
T &
\end{aligned}
$$

6.4 (d)

$$
\begin{aligned}
[(p \vee q) \wedge (p \implies r) \wedge (q \implies r)] \implies r \quad &\equiv \\
\neg[(p \vee q) \wedge (\neg p \vee r) \wedge (\neg q \vee r)] \vee r \quad &\equiv \\
[\neg(p \vee q) \vee \neg(\neg p \vee r) \vee \neg(\neg q \vee r)] \vee r \quad &\equiv \\
[(\neg p \wedge \neg q) \vee (p \wedge \neg r) \vee (q \wedge \neg r)] \vee r \quad &\equiv \\
[(\neg p \wedge \neg q) \vee (p \wedge \neg r)] \vee [r \vee (q \wedge \neg r)] \quad &\equiv \\
[(\neg p \wedge \neg q) \vee (p \wedge \neg r)] \vee [(r \vee q) \wedge (r \vee \neg r)] \quad &\equiv \\
[(\neg p \wedge \neg q) \vee (p \wedge \neg r)] \vee (r \vee q) \quad &\equiv \\
[(r \vee q) \vee (\neg p \wedge \neg q)] \vee (p \wedge \neg r) \quad &\equiv \\
[((r \vee q) \vee \neg p) \wedge ((r \vee q) \vee \neg q)] \vee (p \wedge \neg r) \quad &\equiv \\
((r \vee q) \vee \neg p) \vee (p \wedge \neg r) \quad &\equiv \\
(((r \vee q) \vee \neg p) \vee p) \wedge (((r \vee q) \vee \neg p) \vee \neg r) \quad &\equiv \\
((r \vee q) \vee (p \vee \neg p)) \wedge ((\neg p \vee q) \vee (r \vee \neg r)) \quad &\equiv \\
((r \vee q) \vee T) \wedge ((\neg p \vee q) \vee T) \quad &\equiv \\
T \wedge T \quad &\equiv \\
T &
\end{aligned}
$$