Rosen, Discrete Mathematics and Its Applications, 7th edition

Extra Examples

Section 12.1—Boolean Functions



Page references correspond to locations of Extra Examples icons in the textbook.

p.816, icon at Example 10

#1. Prove the idempotent law $x = x \cdot x$ using the other identities of Boolean algebra listed in Table 5 of Section 11.1 the textbook.

Solution:

$$\begin{array}{ll} x = x \cdot 1 & \text{identity law} \\ = x \cdot (x + \overline{x}) & \text{unit property} \\ = x \cdot x + x \cdot \overline{x} & \text{distributive law} \\ = x \cdot x + 0 & \text{zero property} \\ = x \cdot x. & \text{identity law} \end{array}$$

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#2. Prove the domination law $x \cdot 0 = 0$ using the other identities of Boolean algebra listed in Table 5 in Section 11.1 of the textbook.

Solution:

$$x \cdot 0 = x \cdot (x \cdot \overline{x})$$
 zero property
= $(x \cdot x) \cdot \overline{x}$ associative law
= $x \cdot \overline{x}$ idempotent law
= 0. zero property

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#3. Using the properties of Boolean algebra, prove that

$$yz + x\overline{(xz)} + y(\overline{z} + 1) + \overline{z}x$$

can be simplified to give $y + \overline{z}x$.

Solution:

$$yz + x\overline{(xz)} + y(\overline{z} + 1) + \overline{z}x = yz + x(\overline{x} + \overline{z}) + y(\overline{z} + 1) + \overline{z}x \qquad \text{De Morgan's law}$$

$$= yz + x\overline{x} + x\overline{z} + y\overline{z} + y + \overline{z}x \qquad \text{distributive law; identity law}$$

$$= yz + 0 + x\overline{z} + y\overline{z} + y + \overline{z}x \qquad \text{zero property}$$

$$= yz + x\overline{z} + y\overline{z} + y + \overline{z}x \qquad \text{identity law}$$

$$= y + yz + y\overline{z} + x\overline{z} + \overline{z}x \qquad \text{commutative law}$$

$$= y + y(z + \overline{z}) + x\overline{z} + \overline{z}x \qquad \text{unit property}$$

$$= y + y + x\overline{z} + \overline{z}x \qquad \text{identity law}$$

$=y+x\overline{z}+\overline{z}x$	
$=y+\overline{z}x+\overline{z}x$	
$= y + \overline{z}x.$	

idempotent law
commutative law
idempotent law