Rosen, Discrete Mathematics and Its Applications, 7th edition

Extra Examples

Section 9.1—Relations and Their Properties

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Examples	

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#1. Let R be the following relation defined on the set $\{a, b, c, d\}$:

$$R = \{(a,a), (a,c), (a,d), (b,a), (b,b), (b,c), (b,d), (c,b), (c,c), (d,b), (d,d)\}.$$

Determine whether R is:

(a) reflexive.

(b) symmetric.

(c) antisymmetric.

Solution:

- (a) R is reflexive because R contains (a, a), (b, b), (c, c), and (d, d).
- (b) R is not symmetric because $(a, c) \in R$, but $(c, a) \notin R$.
- (c) R is not antisymmetric because both $(b, c) \in R$ and $(c, b) \in R$, but $b \neq c$.

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#2. Let R be the following relation on the set of real numbers:

$$aRb \leftrightarrow |a| = |b|$$
, where |x| is the floor of x.

Determine whether R is:

(a) reflexive.

(b) symmetric.

(c) antisymmetric.

Solution:

- (a) R is reflexive: |a| = |a| is true for all real numbers.
- (b) R is symmetric: suppose |a| = |b|; then |b| = |a|.
- (c) R is not antisymmetric: we can have aRb and bRa for distinct a and b. For example, |1.1| = |1.2|.

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#3. Let A be the set of all points in the plane with the origin removed. That is,

$$A = \{(x, y) \mid x, y \in \mathbf{R}\} - \{(0, 0)\}.$$

Define a relation R on A by the rule:

 $(a,b)R(c,d) \leftrightarrow (a,b)$ and (c,d) lie on the same line through the origin.

Determine whether R is:

(a) reflexive.

(b) symmetric.

(c) antisymmetric.

Solution:

- (a) R is reflexive: (a, b) and (a, b) lie on the same line through the origin, namely on the line y = bx/a (if $a \neq 0$), or else on the line x = 0 (if a = 0).
- (b) R is symmetric: if (a, b) and (c, d) lie on the same line through the origin, then (c, d) and (a, b) lie on the same line through the origin.
- (c) R is not antisymmetric: (1,1) and (2,2) lie on the same line through the origin. Therefore, (1,1)R(2,2) and (2,2)R(1,1).

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#4. Let $A = \{(x, y) \mid x, y \text{ integers}\}$. Define a relation R on A by the rule

$$(a,b)R(c,d) \leftrightarrow a < c \text{ and } b < d.$$

Determine whether R is:

(a) reflexive.

(b) symmetric.

(c) antisymmetric.

Solution:

- (a) R is reflexive: (a,b)R(a,b) for all elements (a,b) because $a \le a$ and $b \le b$ is always true.
- (b) R is not symmetric: For example, (1,2)R(3,7) (because $1 \le 3$ and $2 \le 7$), but (3,7)R(1,2).
- (c) R is antisymmetric: Suppose (a,b)R(c,d) and (c,d)R(a,b). Therefore $a \leq c, c \leq a, b \leq d, d \leq b$. Therefore a = c and b = d, or (a,b) = (c,d).

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#5. Let $A = \{(x, y) \mid x, y \text{ integers}\}$. Define a relation R on A by the rule

$$(a,b)R(c,d) \leftrightarrow a = c \text{ or } b = d.$$

Determine whether R is:

(a) reflexive.

(b) symmetric.

(c) antisymmetric.

Solution:

- (a) R is reflexive: (a, b)R(a, b) for all elements (a, b) because a = a and b = b are always true.
- (b) R is symmetric: Suppose (a,b)R(c,d). Therefore a=c or b=d. Therefore c=a or d=b. Therefore (c,d)R(a.b).
- (c) R is not antisymmetric: For example, (1,2)R(1,3) and (1,3)R(1,2) because 1=1, but $(1,2)\neq (1,3)$.

p.578, icon at Example 13

#1. Let R be the following relation defined on the set $\{a, b, c, d\}$:

$$R = \{(a, a), (a, c), (a, d), (b, a), (b, b), (b, c), (b, d), (c, b), (c, c), (d, b), (d, d)\}.$$

Determine whether R is transitive.

Solution:

The relation R is not transitive because, for example, $(a, c) \in R$ and $(c, b) \in R$, but $(a, b) \notin R$.

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#2. Let R be the following relation on the set of real numbers:

$$aRb \leftrightarrow \lfloor a \rfloor = \lfloor b \rfloor$$
, where $\lfloor x \rfloor$ is the floor of x .

Determine whether R is transitive.

Solution:

R is transitive: suppose $\lfloor a \rfloor = \lfloor b \rfloor$ and $\lfloor b \rfloor = \lfloor c \rfloor$; from transitivity of equality of real numbers, it follows that $\lfloor a \rfloor = \lfloor c \rfloor$.

p.578, icon at Example 13

#3. Let A be the set of all points in the plane with the origin removed. That is,

$$A = \{(x, y) \mid x, y \in \mathbf{R}\} - \{(0, 0)\}.$$

Define a relation on A by the rule:

 $(a,b)R(c,d) \leftrightarrow (a,b)$ and (c,d) lie on the same line through the origin.

Determine if R is transitive.

Solution:

R is transitive: suppose (a,b) and (c,d) lie on the same line L through the origin and (c,d) and (e,f) lie on the same line M through the origin. Then L and M both contain the two distinct points (0,0) and (c,d). Therefore L and M are the same line, and this line contains (a,b) and (e,f). Therefore (a,b) and (e,f) lie on the same line through the origin.

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#4. Let $A = \{(x, y) \mid x, y \text{ integers}\}$. Define a relation R on A by the rule

$$(a,b)R(c,d) \ \leftrightarrow \ a \leq c \ \text{ and } \ b \leq d.$$

Determine whether R is transitive.

Solution:

R is transitive: Suppose (a,b)R(c,d) and (c,d)R(e,f). Therefore $a \le c$ and $c \le e$, and $b \le d$ and $d \le f$. Therefore, $a \le e$ and $b \le f$, or (a,b)R(e,f).

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#5. Let $A = \{(x, y) \mid x, y \text{ integers}\}$. Define a relation R on A by the rule $(a, b)R(c, d) \leftrightarrow a = c \text{ or } b = d$.

Determine whether R is transitive.

Solution:

R is not transitive: For example, (1,2)R(1,3) because 1=1, and (1,3)R(4,3) because 3=3. But $(1,2) \neq (4,3)$ because $1 \neq 4$ and $2 \neq 3$.