

Learning Companion Systems*

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INTRODUCTION

Why There was One Who Studied with the Prince

In the past in China, there existed a child who studied with the Prince under the instruction of a royal teacher, as mentioned by the Chinese proverb "Studying with the Prince." Why? Perhaps the Queen recognized the importance of *learning companionship*. For whatever reason it evolved, the Prince was clearly expected to learn more effectively with the companion than by learning alone.

Learning Companion Systems (LCS) are a form of intelligent tutoring systems (ITS) in which a computer companion is added to the traditional ITS environment. In developing the LCS model, we have been guided by properties of learning companionship which naturally occur in human learning (Chan & Baskin, 1988).

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Learning Facilitation through Peer Interaction in Different Areas

Influence of social interactions on individual cognitive development has also been observed by a number of researchers in different areas. In SOPHIE's game environment (Brown, Burton, & DeKleer, 1982), researchers noticed that "team players are far less self-conscious than a single informant . . . in collecting a protocol of a subject who is working alone, it is extremely difficult to get insights into why he rejects certain moves; subjects usually feel no need to justify why they don't do something. . . . In the two-person team environment, the arguments that naturally arise involve attempts to justify or defeat a proposed move. The record of these justifications provides a rare opportunity to see strategic reasoning unfold and be defended."

In the context of "control" in problem solving, which deals with the resource allocation during problem-solving performance, it seems to Schoenfeld (1985) that Vygotsky's hypothesis about the role of social interaction is plausible: "looking at situations from multiple perspectives, planning, evaluating new ideas, monitoring and assessing solutions in the midst of working problems, and so forth. Where do such behaviors arise, and how does one learn to argue with oneself while solving problems? . . . It seems reasonable that involvement in cooperative problem solving—where one is forced to examine one's ideas when challenged by others, and in turn to keep an eye out for possible mistakes that are made by one's collaborators . . ."

In the studies of cognitive psychology of reading, Goodman (1973) describes receptive language processes in general as hypothesis-based, defining them as "cycles of sampling, predicting, testing and confirming." Spiro (1980) points out that in the dynamic process of reading, "a reader's working hypothesis may be wrong and that at various points during the reading process it may be in a state of limbo, only partially specified, needing more evidence." At some of these intermediate stages, "the reader must back up and rehypothesize about the meaning of a text." But how can such active behavior of the reader be best motivated? It seems to be his need to *convey* his newly learned knowledge, while it is still at a hazy stage, to another agent—his learning companion (Aizenstein, Chan, & Baskin, 1988).

Psychological Studies of Social Interaction on Cognitive Development

In his work, *Mind in Society*, Vygotsky (1978) hypothesizes that social interactions play a fundamental role in shaping internal cognitive structures. Vygotsky's *zone of proximal development* is the distance (as illustrated in Figure 1.1) between the actual developmental level as determined by independent problem solving and the level of potential development as determined through problem solving under adult guidance or in collaboration with more capable peers. He also points out that the role of imitation is particularly important in this concept

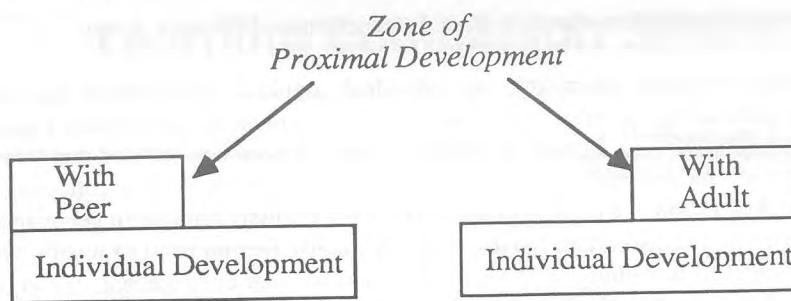


Figure 1.1. Vygotsky's hypothesis on the zone of proximal development.

of zone of proximal development. Experiments by Doise and his colleagues (Doise, Mugny, & Perret-Clermont, 1975) have shown that two children working together can successfully perform a task which cannot be performed by children of the same age working alone. Their subsequent experiment (Mugny & Doise, 1978) further indicates that "more progress takes place when children with different cognitive strategies work together than when children with the same strategies do so, and that not only the less advanced but also the more advanced child make progress when they interact with each other." More recent work by Petitto (1985) also illustrates that the approach taken by a pair of students to an estimation task can often be qualitatively different from the approach taken by either student alone. Once the new approach emerges, it then becomes part of their repertoire.

What accounts for the improved cognitive development through interaction with a peer? Doise and his colleagues (Doise, Mugny, & Perret-Clermont, 1975) suggest that such improvement is caused by *social-cognitive conflict*. In the pair situation, the child finds himself confronted with alternative and conflicting solutions which, while not necessarily offering the correct response, may suggest to him "some relevant dimensions for a progressive elaboration of a mechanism new to him" (Mugny, Perret-Clermont, & Doise, 1981). Therefore, it is the active resolution of the cognitive conflict on the learner's part that accounts for the improved learning under the influence of social interaction.

Doise and his colleagues do not further illustrate what constitutes a cognitive conflict. In a broad sense, we can view it as some idea or point of view that arises whenever one agent's response does not completely match with the other agent's knowledge. Unlike a teacher's response, which intends to focus the student back to the supposed correct path, cognitive conflict represents the conflictual dilemma that needs to be reconciled by both agents. Cognitive conflict is not a rare phenomenon, but occurs very often, for it is seldom that two agents' knowledge overlaps completely. When cognitive conflict occurs, the learner is forced to examine his thinking, looking for alternative perspectives hinted at by the conflict, and at the same time keep an eye out for possible relevancies. In a way,

both agents need to diagnose and evaluate problems indicated by the cognitive conflict and to justify their own perspectives (Figure 1.2).

Learning Companion vs. Collaborative Partner

The idea of building a *collaborative partner* to interact with a human student is being explored by Gilmore and Self (1988). This collaborative partner is similar to a learning companion in that

1. Their learning has to be "psychologically credible," which means that the student must consider the learning performance, by either, whichever's possible for him to emulate.
2. The role of collaborator or companion can de-emphasize the role of transmitting certified knowledge to students in accordance with the general trend of ITS cited by Gilmore and Self.
3. The collaborator and the companion are able to make comments about learning skills in order to improve the student's learning skill acquisition.

Learning Companion Systems and the notion of a collaborative partner are different in several important aspects. The collaborative partner, in the form of a machine learning-based partner, emerged from the effort of trying to apply machine learning techniques to dynamically model a student (Self, 1985, 1986). Unlike Gilmore and Self, who do not suggest that collaboration is necessarily more effective than tutorial, our work has been inspired by insights from peer learning. Specifically, we have incorporated cognitive benefits from collaboration as well as competition which occur in peer learning into our LCS design. As part of our effort to explore the cognitive benefits of the learner of LCS, we have identified both simulation and machine learning as two different ways of implementing the companion part of an LCS system.

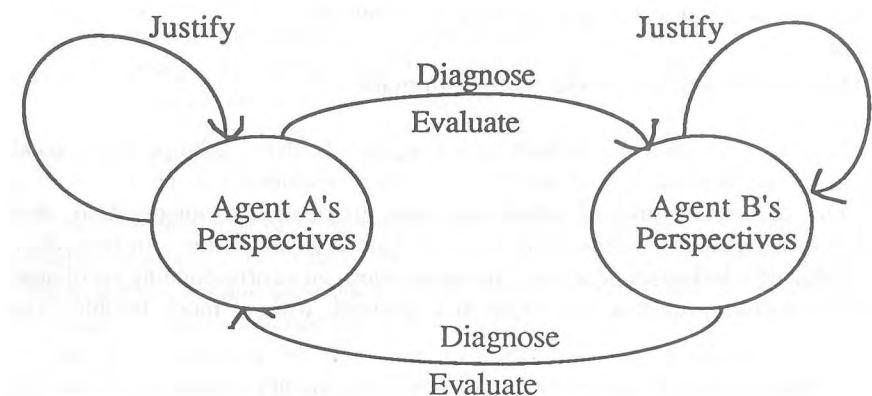


Figure 1.2. Cognitive interaction between two learners.

Our research focuses primarily on exploring an *alternative architecture* in which the usual *two agent model* of ITS, a computer teacher and a human student, is replaced by a *three agent model* consisting of a computer teacher, a human student, and a computer companion. The distinguished notions of computer teacher and computer companion enable the system to interact differently with the student. Integration-Kid (Chan, 1989), a prototype of LCS, adopts a simulation approach to companion learning rather than by first perfecting a machine learning-based companion. In this way, some of the questions raised by Gilmore and Self can be answered. Through the use of a *curriculum hierarchy* to encode concept knowledge about the domain and sets of *rules of behavior* of different agents (Chan, 1989), different learning protocols can be designed in an LCS system, and from which we are able to define what is precisely meant by collaboration and competition. Furthermore, through experience with our prototype of LCS, we are able to test whether the student will accept a computer companion and "learn how to learn" as a result of interaction with the companion.

DESIGN OF AN LCS FOR INTEGRATION

The design of a Learning Companion System (LCS) involves three agents, namely, the human student, the computer learning companion, and the computer teacher. The role of the computer teacher is to offer examples, guidance, and comments to both the student and the learning companion. The goal of the learning companion is to stimulate the student's learning through the process of collaboration and competition. In order to better illustrate the idea of LCS and discuss the two main approaches to the design of the learning companion, we will describe the application of LCS to the domain of indefinite integration. Since our primary goal is to explore the idea of LCS, we confine ourselves to a subset of indefinite integrations at the level of a first year undergraduate, for example, $\int \cos^5 x dx$, $\int e^x \sin x dx$, and so on. The prerequisites are some competence with differentiation and algebraic manipulation.

Indefinite Integration as the Sample Domain

Indefinite integration is a domain which requires heuristic solution and a good deal of resourcefulness and intelligence. Many problems can be solved in a variety of ways—some of which are more efficient than others. Also, this domain is not heavily dependent on other mathematical abilities. In fact, after introducing a technique, students can usually work on a corresponding set of near miss¹ practice problems like those in a textbook without much trouble. Yet

¹ Winston's notion of a *near miss* is a sample which does not qualify as an instance of a class for a small number of reasons. Here we refer to a problem set that each selected problem differs slightly from previously selected problems by a small number of features.

Table 1.1. Basic Rules of Integrations.

(1) $\int x^a dx = \frac{x^{a+1}}{a+1}$ where $a \neq -1$	(2) $\int c dx = cx$ where c is a constant
(3) $\int \sin x dx = -\cos x$	(4) $\int \cos x dx = \sin x$
(5) $\int \frac{1}{x} dx = \ln x $	(6) $\int e^x dx = e^x$
(7) $\int c f(x) dx = c \int f(x) dx$ where c is a constant	(8) $\int (f(x) + g(x)) dx = \int f(x) dx + \int g(x) dx$

students consistently have more difficulty in taking examinations and in doing miscellaneous exercises than they should despite many hours of working problems, as noted by Schoenfeld (1978). Part of the difficulty lies in their lack of adequate integration of separate concepts and techniques learned and therefore better judgment of the form of integrands.

INTEGRATION-KID is our first LCS system currently in development. The learning companion incorporated in INTEGRATION-KID will be a vehicle for our attempt to examine how a student's learning can be affected through the interaction with a computer companion. For example, when the companion acts as a competitor to the student for those less demanding drill-and-practice exercises (discussed below), we hope that the student can be better motivated than when working alone. For this chosen domain, the difficulty addressed by Schoenfeld should be directly addressed by the learning companion.

Outline of the Design of Instructional Material

Similar to the organization of the material in a standard textbook of calculus, the learning activities are divided into five sessions:

- (Session a) Introduce the concept of integration.
- (Session b) Familiarize with basic rules² (Table 1.1).
- (Session c) Introduce the substitution method.
- (Session d) Introduce integration by parts.
- (Session e) Practice miscellaneous exercises..

Learning activities of Session a are rather like a traditional CAI format where teaching material is presented with simple question-and-answer interactions between the teacher and the student. Then, starting from Session b, we introduce the learning companion into the system. In Session b, the student learns to solve problems by applying the basic rules of integration in a rather straightforward

² For simplicity, we omit the arbitrary constant of indefinite integration.

way. After a while, the student will realize that there are many problems that cannot be solved just by applying those basic rules right away. Then, in Session c, he will be introduced to a technique, the *substitution method*, which, incorporated with basic rules, allows the student to solve a wider class of problems. Later, in Session d, he will learn *integration by parts*, another powerful technique, to solve another class of integration problems. In the last session, the student by then is already equipped with all the techniques he has learned. However, since the problems are no longer near-miss problems, the student cannot get any hint straight from previous problems.

THE ROLE OF THE TEACHER AND THE COMPANION

When difficulty or doubt arises, the student may naturally look for the teacher rather than the companion simply because the teacher is the authority on the subject. On the other hand, the student may turn to the companion for assistance in order not to face the teacher with a problem. An important step in the design of an LCS is to know what will actually happen to all the agents, that is, the trilateral relationship involved in the learning activities (Figure 1.3).

Results from Human Protocol Analysis

We have monitored the activity of a small number of human subjects involved in learning behavior similar to that intended for the Integration-Kid. A protocol analysis of these representative sessions has been used to help design the LCS paradigm. Based on our analysis and Schoenfeld's study (1978), we identified the following general concerns when designing an LCS for integration:

Educational goal management. Neither the student nor the companion can be relied upon to know enough about this domain to schedule the major learning objectives in the most effective way. In fact, the tutor may be the only one that could determine the order and the emphasis on a given topic or skill. For example, the human tutor foresees that proficiency of handling basic rules in Session b is critical in the later learning of new techniques or handling more complex problems. In order to achieve the proficiency, the student needs to *overlearn*, that is, practice until he almost *compiles* the knowledge.

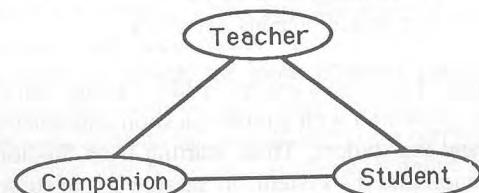


Figure 1.3. Trilateral interpersonal relation of an LCS environment.

Stages of learning. We observe that a student evolves through different stages of learning, for example, in learning a new technique, say, substitution method, first the student takes the teacher's demonstrated example as a template, and in solving his own problem, for each step, he only changes those parts that are needed to be changed—largely maintaining the *syntactic* (form of steps) nature of the problem solution (Figure 1.4a and 1.4b). We call this the *imitation stage*. It seems that the student at that stage tries to acquire all unfamiliar information as much as possible from the demonstrated example. Any mistake he made will be regarded as his misinterpretation of the demonstrated example if it is not the teacher's mistake. After a few problems, the student seems to enter another stage which we call the *developmental stage*. In that stage, the student is getting beyond the burden of syntactic detail. He conceives more about the essential parts of the new technique. For example, his first focus at that stage is “what will be a possible substitution” rather than “remembering that the first step is to find a substitution.” Also, later at that stage, the emphasis shifts from accuracy more towards effectiveness. For example, what is a better substitution than another. When the problems are getting more complex, the student needs to integrate the newly learned technique with previously learned knowledge in order to solve those problems. We call this the *integration stage*. In short, the student's learning of this domain seems to evolve from *recognizing* (through the process of imitation) the new knowledge to *gradual increase of control* (applying the new knowledge with older knowledge to solve different problems) over that knowledge.

Difficulty level of problems. Managing the difficulty level of problems is an important part of any learning situation. Early failure or prolonged failure can discourage a student unduly. Similarly, problems which do not provide a challenge can reduce motivation. We have also noted opportunities to match the behavior of the companion to the level of difficulty of the problems. For example, problems in Session b are easy; the companion may act as an adversary to arouse stimulation for the student. Problems in Session d are more difficult ones; therefore, the problem solving task is subdivided into decision making and execution, and we adopt the responsibility sharing protocol.

Defining the Role of the Teacher

The job of the teacher is to generate problems, demonstrate examples, explain the format of learning activities, negotiate with the student (e.g., how many more problems the student will do), and make final justifications of the solution and/or retrospective insightful comments. It is only under rare situations that the teacher would interrupt the problem-solving process—for example, when the problem is expected to be hard and both the companion and the student cannot solve the problem.

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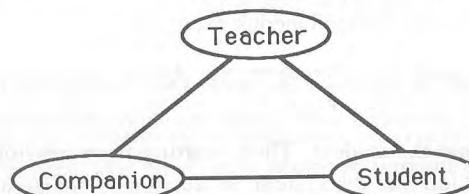


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$$\int (3x - 2)^{234} dx$$

Set $u = 3x - 2$

$$\frac{du}{dx} = \frac{d(3x - 2)}{dx} = \frac{d3x}{dx} - \frac{d2}{dx} = 3$$

$$\therefore \frac{du}{dx} = 3$$

$$du = 3 \cdot dx$$

$$dx = \frac{1}{3} du$$

Substitute u into the integration

$$\begin{aligned}\int (3x - 2)^{234} dx &= \int u^{234} \cdot \frac{1}{3} du \\ &= \frac{1}{3} \cdot \frac{1}{235} u^{235} \\ &= \frac{1}{705} (3x - 2)^{235}\end{aligned}$$

Figure 1.4a. Teacher's demonstrated example.

$$\int (t+1)^2 dt$$

Set $u = t + 1$

$$\int u^2 dt$$

$$\frac{du}{dt} = \frac{d(t+1)}{dt} = \frac{dt}{dt} + \frac{d1}{dt} = 1$$

$$dt = du$$

$$\begin{aligned}\int (t+1)^2 dt &= \int u^2 du \\ &= \frac{1}{2+1} u^3 \\ &= \cancel{\frac{1}{3} t^3} = \frac{1}{3} (t+1)^3\end{aligned}$$

Figure 1.4b. Student's imitation.

Companion as a Competitor

Before a student is able to learn any technique to solve more complex integration problems, proficiency in using the set of basic rules is required. By observing student behavior, we noticed that a student works reliably with problems which

are straightforward applications of the basic rules of integration. At the beginning, a student is very careful to choose and apply the proper rule; later on, he can naturally adopt some mental operations, for example, combining two operators into one macro operator, but still he refers to the table of rules frequently.

In Session b, both the student and the companion work on a set of problems offered by the teacher simultaneously but independently. They are requested to work on the problems slowly and accurately. After they have finished, they compare solutions, discover mistakes, and self-correct the mistakes in their own solution. If there remain mistakes not discovered, the teacher points them out. Those correct solutions will receive credits. Then another set of problems is generated by the teacher. Later on, both the student and the companion are required to solve the problems (e.g., $\int (e^x - x^{-3}) dx$) in one step without consulting the table of basic rules. This additional requirement will encourage the peers to use more macro operations and to memorize the basic rules.

Working Collaboratively with One Working While the Other Watching

In Sessions c and d, a student learns new techniques in order to solve more complex new integrations. The solution plan in employing these techniques by students can be divided into few phases. For example, in learning substitution method in Session c, the first phase is to choose the right substitution, the second phase is to differentiate the substitution. For more complex problems, this phase also verifies whether the substitution chosen is appropriate. In the last phase the original integration is transformed to a simpler one and then solved. To master these techniques, it is important for the student to have the firsthand experience of solving the whole problem with some external help if needed.

In these two sessions, while one is working on a problem, the other is watching—ready to give suggestions if asked or to critique. If they both run out of ideas, then the teacher may interrupt. The learning activities in this session can be represented by the following network of interaction (Figure 1.5).

Working Collaboratively via Responsibility Sharing

The problems in Session e are of various levels of difficulty and may require different kinds or combinations of heuristic strategies and techniques. Working on this type of problem, where all goes together, the student has to constantly make a judgment, proceed, then another judgment and so on. At some point in the process, if the solution path looks to be improving, then proceed or seek a heuristic in order to continue; otherwise, back up. About the judgment at that stage, Schoenfeld (personal communication) recommends what he calls the *three phases model*: “Try simple things before you use more complicated techniques, and only when you’ve exhausted to the possibilities of these do you try some of the *shot in the dark* techniques.”

In this session, the protocol of activities for the student and companion (Fig-

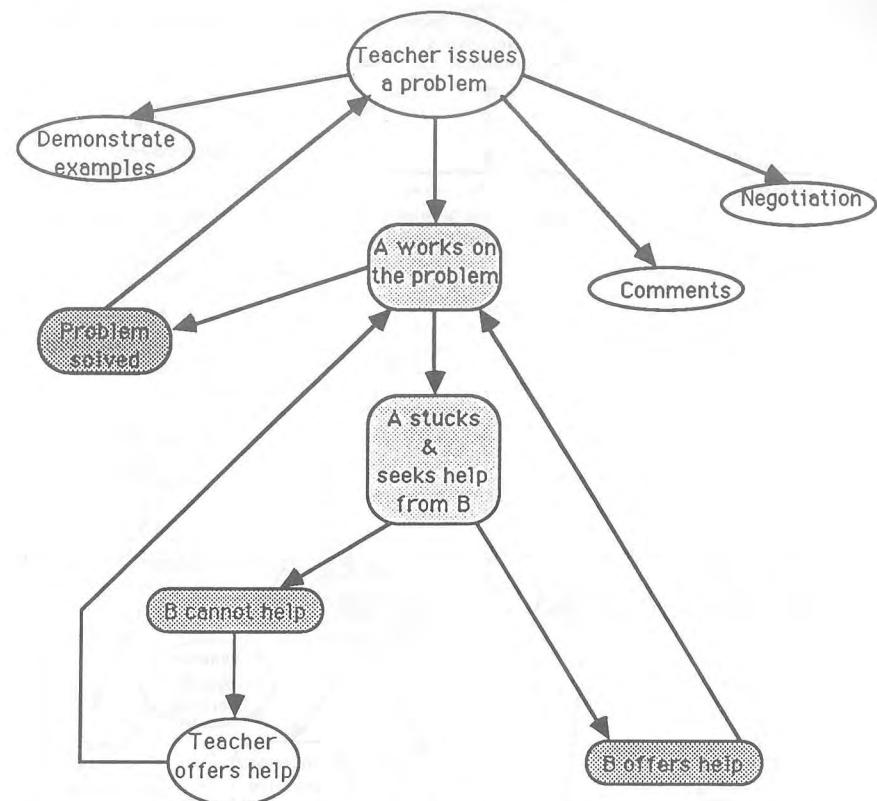


Figure 1.5. Protocol of activity: one working while the other watching.

ure 1.6) is negotiation, decision, and working. One is responsible for decision making and the other is for execution, that is, working the problem according to the decision. The negotiation occurs in a rather simple form. The one who will make decisions first suggests a plausible strategy and explains; for example, he suggests using integration by parts and specifies what is u and what is dv ; then he explains how the problem is similar to the previous one. Then the one who will work on the problem makes a different but plausible suggestion and explains why, if he finds one. Next, the one who is responsible for making decisions decides which suggested strategy to use and the one who is responsible for execution works on the problem according to the decision. This procedure repeats until the next decision point. The roles alternate.

It should be noticed that given a fixed instructional material, it is possible to design other protocols of activities in an LCS. Our choice of these protocols depends on a number of factors described above. Similarly, different domains may have very different design of protocols. Table 1.2 summarizes the protocols of activities.

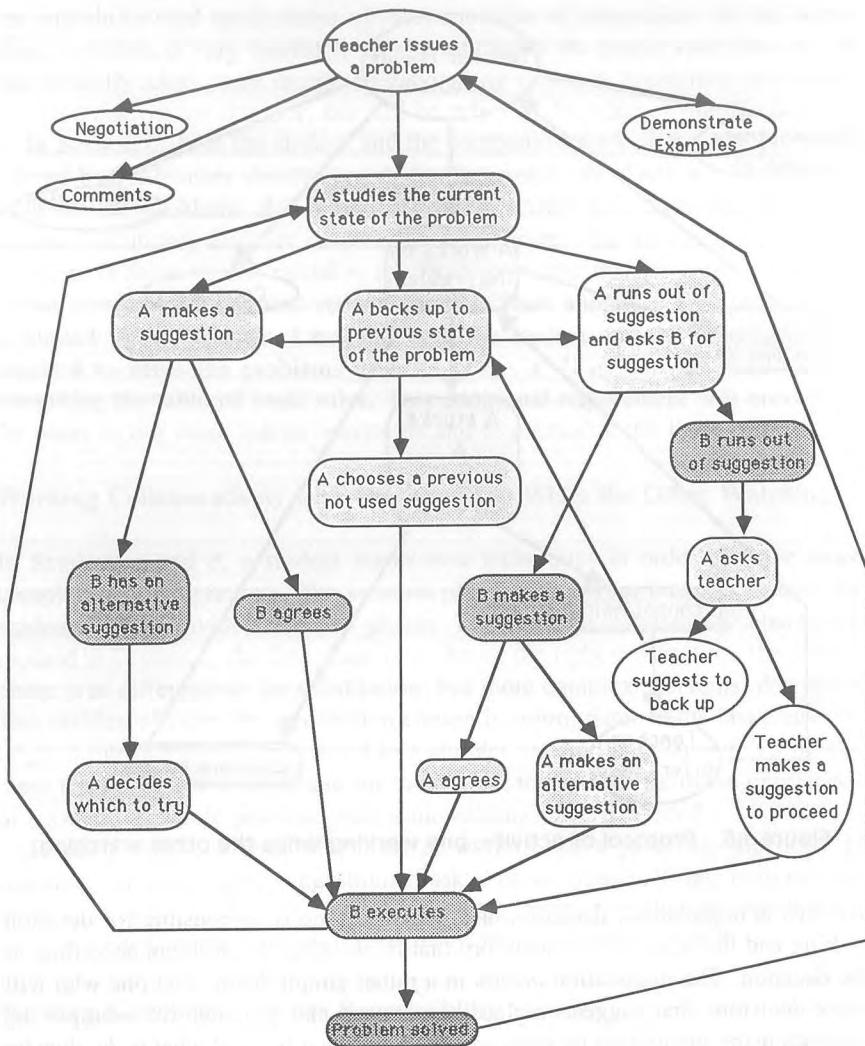


Figure 1.6. Protocol of activity: responsibility sharing.

EXAMPLES OF LCS INTERACTIONS

The following stylized scenes give an illustration of how the student's learning can benefit from an LCS learning environment with respect to the protocols designed above. More subtleties of the advantages of LCS can be revealed in such scenes.

Table 1.2. Summary of Protocols of Activities.

	Same Problems (simple)	Different Problems (near miss)	Different Problems (various)
Working Independently then Comparing (competition)	Session b)		
One Working & One Watching (suggestion)		Sessions c) & d)	
Responsibility Sharing (collaboration)			Session e)

Scene 1 (Companion Working with Student Watching)

Teacher: "Here is a problem, $\int \sqrt{2 - \sin 3t} \cos 3t dt$, would the Companion try it?"

Companion: " $u = 3t$ seems to be a good substitution. Now

$$\frac{du}{dt} = 3,$$

$$\text{so } \frac{1}{3} du = dt.$$

Then the original integration

$$\int \sqrt{2 - \sin 3t} \cos 3t dt$$

$$= \int \sqrt{2 - \sin 3t} \cos u \frac{1}{3} du.$$

Oh ! I don't know how to continue. Do you have any suggestion, Student ?"

Student : "How about trying the substitution $u = 2 - \sin 3t$?"

Companion : "Okay !

$$u = 2 - \sin 3t$$

$$\text{so } \frac{du}{dt} = -3 \cos 3t$$

....

I've got the answer !"

Teacher: "Before going further, it is worthwhile to look back over some of the problems we have just solved:

$$\int e^x \cos(e^x + 1) dx \text{ with substitution } u = e^x + 1,$$

$$\int \frac{x^2 dx}{4 - x^3} = \int (4 - x^3)^{-1} x^2 dx$$

with substitution $u = 4 - x^3$,

$$\int x^2 e^{(1+x^3)} dx = \int e^{(1+x^3)} x^2 dx$$

with substitution $u = (1 + x^3)$, and

$$\int \sqrt{2 - \sin 3t} \cos 3t dt \text{ with substitution } u = 2 - \sin 3t.$$

There are two interesting observations here:

1) All the integrands (e.g. $e^x \cos(e^x + 1)$ in $\int e^x \cos(e^x + 1) dx$) are products of two functions where one is simple and the other is more complex, and for every substitution we chose the inside function of the more complex function; for example, $(1 + x^3)$ is the inside function of $e^{(1+x^3)}$.

(2) If the inside function is a polynomial, the polynomial is one degree higher than the simpler function which is also a polynomial. Since differentiation of a polynomial will decrease its degree by one, that may explain why this choice can work; for example, $d(1 + x^3)$ will lead to $3x^2 dx$ and $x^2 dx$ is what we need. Also, it is quite easy to see that when we differentiate $2 - \sin 3t$, $\cos 3t dt$ will drop out.

Here the student helps the companion by offering an important suggestion. The student is partially involved in the process of solving the problem by watching the companion work and being ready to help. Furthermore, the student can see possibly different strategies for the problem used by the companion. In addition, at this particular scene, the student learns that picking a correct substitution is important in solving the problem. Notice that if the companion does not make any suboptimal substitutions (taking $u = 3t$), then a negative example, and thus a piece of useful information, might not be noticed by the student. Moreover, at the end of a set of practice problems, the teacher looks back and offers insightful comments.

Scene 2 (Student Working with Companion Watching)

Teacher: "Let's go on another problem, $\int \frac{y+1}{\sqrt{y}} dy$."

$$\begin{aligned} \text{Student : } & \int \frac{y+1}{\sqrt{y}} dy \\ &= \int (y+1)(y^{-1/2}) dy \\ &= \int (y+1) \frac{1}{\sqrt{y}} dy \end{aligned}$$

$$u = y + 1$$

$$\frac{du}{dy} = 1$$

Seems no good! . . .

Companion, how about $u = \sqrt{y}$?"

Companion: "Sounds good . . . and I cannot see why this is not a plausible choice!"

Student: "Thanks! Now,

$$u = \sqrt{y} = y^{-1/2}$$

$$\frac{du}{dy} = \frac{1}{2} y^{-1/2} = \frac{1}{2} \frac{1}{\sqrt{y}}$$

$$\frac{1}{2} dy \frac{1}{\sqrt{y}} = du$$

$$dy \frac{1}{\sqrt{y}} = 2 du$$

Putting back to the integration, we have,

$$\int (u^2 + 1) 2du$$

$$= 2 \int (u^2 + 1) du$$

$$= 2 \left[\int (u^2) du + \int du \right]$$

$$= 2 \left[\frac{u^3}{3} + u \right]$$

$$= 2 \left(\frac{1}{3} y^{3/2} + y^{1/2} \right)$$

$$= \frac{2}{3} y^{3/2} + 2y^{1/2}.$$

Teacher : "Good effort! However, there exists an alternative way that you both may overlook :

$$\begin{aligned} & \int \frac{y+1}{\sqrt{y}} dy \\ &= \int \frac{y}{\sqrt{y}} + \frac{1}{\sqrt{y}} dy \\ &= \int y^{1/2} + y^{-1/2} dy \\ &= \frac{2}{3} y^{3/2} + 2y^{1/2}. \end{aligned}$$

Nevertheless, it is a good practice to look for easy alternatives before committing to a particular way too early. In this case, the rule:

$\int (f(x) + g(x)) dx = \int f(x) dx + \int g(x) dx$, that is, integration of sum equals to sum of integrations, tells us that whenever it is possible, split the integrand into sum to obtain simpler integration."

Here the student was floundering at the beginning in searching for an appropriate substitution. At one point, he found a substitution $u = \sqrt{y}$, but hesitated. Then he looked for assurance from the companion. The companion did not seem to have a better alternative. Now the student knew that this was not a stupid try as another student would also do that, so he was more secure to carry on. Finally, both the student and the companion missed an easy method. We can see that the companion's feedback is not totally reliable, in contrast to a teacher's. This student appreciates the other easy alternative only when it takes a big effort.

Scene 3 (Responsibility Sharing)

Teacher: "This, $\int x^2 e^x dx$, is another problem for you both."

Companion: "Try integration by parts with $u = x^2$ and $dv = e^x dx$. This is because we have tried $\int x \sin x dx$ with $u = x$ and $dv = \sin x dx$, and $\int x^2 \cos x dx$ with $u = x^2$ and $dv = \cos x dx$. The current problem looks to be the same kind of problems."

Student: "No objection! So let's take

$$u = x^2, dv = e^x dx$$

$$\text{so, } du = 2xdx, v = e^x.$$

$$\text{Now, } \int x^2 e^x dx$$

$$= x^2 e^x - \int e^x 2x dx$$

$$= x^2 e^x - 2 \int x e^x dx.$$

Companion: "As before, we can continue with $u = x$ and $dv = e^x dx$."

Student: "I agree, so
 $du = dx$ and $v = e^x$, the original integration becomes

$$= x^2 e^x - 2 [x e^x - \int e^x dx]$$

$$= x^2 e^x - 2 [x e^x - e^x]$$

$$= x^2 e^x - 2x e^x + 2e^x.$$

Teacher: "Well done. It is a very good strategy to think of similar problems which have been previously solved in order to tackle the current problem."

It is easy for one to make decisions hurriedly without careful consideration. However, if one has to defend or to unfold his reasoning process, he is bound to give extra attention to the decision process. Here, the companion explained his reasoning process explicitly to the student. Through looking for other possibilities and justifying the companion's suggestion and his reasoning, the student's view of how to approach the problem is broadened. Furthermore, the correct reasoning was further justified and positively reinforced by the teacher.

Scene 4 (Responsibility Sharing)

Teacher: "Here is a problem $\int e^x \sin x dx$ for you both."

Student: "I suggest trying integration by parts with $u = e^x$ and $dv = \sin x dx$ since the integrand is a product of two different functions; furthermore, there is no obvious choice."

Companion: "With the same reason, it is also plausible to try integration by parts with $u = \sin x$ and $dv = e^x dx$."

Student: "I prefer my choice."

Companion: "Okay! $du = e^x dx$ and $v = -\cos x$, so
 $\int e^x \sin x dx$
= $-e^x \cos x - \int (-\cos x) e^x dx$
= $-e^x \cos x + \int \cos x e^x dx.$ "

Student: "No progress. Come back and try your previous suggestion."

Companion: "Okay! $du = \cos x dx$ and $v = e^x$, so
 $\int e^x \sin x dx$
= $e^x \sin x - \int e^x \cos x dx.$ "

Student: "Again, no progress. Any suggestion, Companion?"

Companion: "Perhaps, we can continue with $u = e^x$ and $dv = \cos x dx$ and see."

Student: "Okay!"

Companion: "Now $du = e^x dx$ and $v = \sin x dx$, from the last step, we have
 $e^x \sin x - [\int e^x \sin x dx - \int \sin x e^x dx]$
= $\int e^x \sin x dx.$ "

Student: "Strange! This is the original integration. We've been going nowhere! Any suggestion, Companion?"

Companion: "No!"

Teacher: "You were on the right track! You may back up to the previous step. Instead of $u = e^x$ and $dv = \cos x dx$, try $u = \cos x$ and $dv = e^x dx$."

Companion: "Okay! $du = -\sin x dx$ and $v = e^x$. We now have,
 $e^x \sin x - \int e^x \cos x dx$

$$= e^x \sin x - [e^x \cos x - \int e^x (-\sin x) dx]$$

$$= e^x \sin x - e^x \cos x - \int e^x \sin x dx$$

Moving $-\int e^x \sin x dx$ to the left hand side, we get

$$2 \int e^x \sin x dx$$

$$= e^x \sin x - e^x \cos x.$$

$$\text{Therefore, } \int e^x \sin x dx = (e^x \sin x - e^x \cos x)/2.$$

Teacher: "This is a rather difficult integration. It works because . . ."

This problem is particularly difficult because no similar problems have been solved before. In addition, the structure of the integrand gives no hint concerning which is u and which is dv . As we can see, both the student and the companion are "shooting in the dark." The teacher had to interrupt and offer help or they might have given up.

THE KNOWLEDGE AND LEARNING ABILITY OF THE COMPANION

In the LCS environment, both the student and the companion are presented with the same material. As learning goes along, their problem-solving performance improves. That is, with the interaction between them and the demonstrated examples, advices, and comments from the teacher, they acquire new heuristics, refine learned heuristics, associate better related mathematical knowledge, and know better when to apply the heuristics.

If the student is to learn from the companion, the companion's performance must advance along with and be approximately matched to that of the student. In the same way that most students learn a single task with a limited effort, the learning companion's efforts should be limited to a scale similar to that of the student. For example, if a student cannot solve a problem within 10 minutes or after three attempts, he may regard the problem as unsolvable and give up. Likewise, the companion may only be allowed a few attempts at a single problem.

Moreover, the companion has to be at least as advanced as the student. This means that when the problem has become a trivial problem to the student, it must also be a trivial problem to the companion. When the problem is difficult to the student, the companion may either solve the problem or try meaningful effort. For example, in the case of shooting in the dark, the companion uses what he has learned in a disciplined way to approach the problem. Rather like "wild goose chase" which only results in confusion, the effort at least reveals more about the nature of the problem. That is, before any attempt, the companion is able to reason and explain his actions from what he knows and what he has learned.

Even if the companion is limited to a few attempts and is required to have good performance, the companion clearly has an advantage over the student, that is, a good memory. The companion will not forget what he knows and what he has learned. Furthermore, whatever the prior knowledge of the student that a teacher may assume, the companion may also possess such knowledge. For example, it is hard to integrate $\int (\sin x + \cos x)^2 dx$, but after expanding the integrand $(\sin x + \cos x)^2$ into $\sin^2 x + 2 \sin x \cos x + \cos^2 x$, the companion may further use the identity $\sin^2 x + \cos^2 x = 1$ to simplify the integrand while the student might not discover that.

Furthermore, the companion should be able to benefit from a teacher's advice who knows the domain well. The companion can observe the student's work; he can have all the useful background knowledge and related common sense knowledge (e.g., the notion of complexity of an expression) that the student is assumed to have.

SOME LEARNING TASKS OF THE COMPANION

Since the companion only learns those skills that would be important for the student to learn, the critical problem of building a learning companion is to identify the most important concepts and heuristics to be learned by the human student.

In Session b, if a student is able to write the following integration in one step without hesitation,

$$\int 5 \cos x dx = 5 \sin x$$

$$\int 4 e^x dx = 4 e^x$$

then he has been acquiring a new macro rule (collapse of operator sequences),

$$\int r f(x) dx = r \text{ eval}(\int f(x) dx)$$

where r is a number, $f(x)$ matches an integrand of the integrations listed in Table 1.1 and $\text{eval}(\int f(x) dx)$ means the evaluated integration of $f(x)$, for example,

$\text{eval}(\int \cos x dx) = \sin x$. According to the basic rules listed in Table 1.1, the sequence of operations would have been

$$(7) \quad \int 5 \cos x dx = 5 \int \cos x dx = 5 \sin x \quad (4)$$

$$(7) \quad \int 4 e^x dx = 4 \int e^x dx = 4 e^x \quad (6)$$

Therefore, we can see that two elementary steps have collapsed into one. Furthermore, with more practice, a student may write down in one step,

$$\int (5 \cos x + 4 e^x) dx = 5 \sin x + 4 e^x$$

Thus an even more powerful macro-rule,

$$\int (r f(x) + s g(x)) dx = r \text{ eval}(\int f(x) dx) + s \text{ eval}(\int g(x) dx)$$

the student may have learned. With this rule, three elementary steps have collapsed into one. This also implies that any problem state of the form $r f(x)$ and $r f(x) + s g(x)$ has become a solvable problem state, possibly trivial too, to the student. In the literature of machine learning, such learning techniques (acquiring macro-operators or schemata) have been addressed by several researchers (DeJong & Mooney, 1986; Laird, Rosenbloom, & Newell, 1984).

In Session c and d, sets of near-miss problems corresponding to the intended heuristics are given to the student. For example, as mentioned at the end of Scene 1,

$\int e^x \cos(e^x + 1) dx$ with suitable substitution $u = e^x + 1$,

$$\int \frac{x^2 dx}{4 - x^3} (= \int (4 - x^3)^{-1} x^2 dx) \text{ with substitution } u = 4 - x^3,$$

$\int x^2 e^{(1+x^3)} dx$ with suitable substitution $u = (1 + x^3)$, and

$$\int \sqrt{2 - \sin 3t} \cos 3t dt \text{ with suitable substitution } u = 2 - \sin 3t.$$

A heuristic to learn is:

if the integrand is a product of two functions, then pick the "inside" function of the more complex function as the substitution.

After the student notices such a pattern and adapts the heuristic, the teacher may reveal the essential heuristic:

if the integration is of the form $\int f(g(x)) h(x) dx$ where the $h(x)$ is different from $\frac{dg(x)}{dx}$ by at most a constant factor, then choose the substitution $u = g(x)$.

This is typically viewed as similarity-based learning with a sequence of positive examples (Michalski, 1983). In Session e, through working the various kinds of practice problems, the student learns metaheuristics (Lenat, 1983), for example, the Schoenfeld's three phases model. Of course, both the student and the companion are taking advice (learning by taking advice or being told; Mostow, 1983; Haas & Gary, 1983) from the teacher—an important source of knowledge acquisition.

MACHINE LEARNING APPROACH

In this approach, the growing domain knowledge of the companion, which results in improved performance, is acquired from machine learning techniques. We describe some perspectives and constraints in this approach.

What can a Student Learn from a Machine Learning Based Companion?

The book, *How To Solve It*, by mathematician Polya (1957), which can be imagined largely to be the author's introspective analysis of his own problem-solving process, turns out to be a classic work in teaching problem solving in education. In fact, in providing instructions for how to go about particular tasks we are also providing models for the student for how to go about tasks in general. Papert (1972) has already pointed out that, in teaching children subject material, we are also "teaching children thinking." Being aware of such "meta-teaching," we may provide a more effective educational environment.

Research in machine learning has identified various machine learning techniques. Many of these techniques are inspired by human learning and implemented into computer programs. It is certainly a fruitful effort to explore application perspectives of these researches to education, in particular, in facilitation of students' "learning how to learn." In a machine learning-based LCS environment, the companion explains to the student *how he discovers something and why he speculates something*. By being conscious about the learning behavior in the companion's learning process, the student may acquire "meta-learning" in addition to the domain knowledge. If a student is perceptive of such cognitive benefits in his own learning process, he is in fact exploiting and extending the space of possible approaches to a learning task rather than relying on the mysterious "inspiration" or "cleverness." Also, if a student knows explicitly what learning techniques are effective in a given domain, he may transfer those techniques to other domains when appropriate in a self-conscious way rather than by

spontaneity. However, we should notice that there may be a paradox in this approach if we control the learning performance of the companion according to the student. Should we ask the student to learn those learning strategies that are not worth learning, for these strategies are ineffective and they lead the student to make mistakes?

Some Considerations of Machine Learning Techniques for the Machine Learning Companion

Indefinite integration has been the domain of the learning program LEX (Mitchell, Utgoff, & Banejeri, 1983; Mitchell, Keller, & Kedar-Cabelli, 1986; Utgott, 1986). In LEX, the initial knowledge is a set of basic rules of integrations, important techniques (e.g., integration by parts), algebraic term rewriting rules ($\sin^2x + \cos^2x \rightarrow 1$), and so on—which all are simply listed as a set of operators. The concept to learn is a set of heuristics that recommend in which problem states the various operators should be applied. Each of these learned or partially learned heuristics is represented as a *version space* (Mitchell, 1977) which is specified by a set of its maximally-general members and a set of its minimally-specific members. While the use of version space is elegant to represent partially learned heuristics, it is not sufficient. For example, it cannot represent advice, such as "modify the integrand," provided by the teacher. Furthermore, a version space in LEX cannot capture exceptional cases. For example, a sequence of training examples $\int x^2 \sin x dx$, $\int x^2 \cos x dx$, $\int x e^x dx$, and $\int x^2 \ln x dx$, may lead to a learned heuristic of using u to be the monomial and dv to be any transcendental functions. But, for $\int x^2 \ln x dx$, it is better take u to be $\ln x$ and dv to be the monomial x^2 . The other problem of using a version space to represent a heuristic is lacking the intended purpose of the heuristic, which, however, is addressed by explanation-based generalization (Mitchell, Keller, & Kedar-Cabelli, 1986) (discussed below).

To learn a heuristic, a language for describing the generalization, or applicability condition, of the heuristic is important. LEX adopts a grammar for algebraic expressions but does not include composite functions (e.g., $e^x + x^3$). There are two disadvantages of using grammar as description: (a) The description of an integrand is always down to every detail. For example, at some stage of learning, it may be easier to learn the obvious heuristic if the companion has an appropriate abstract language to describe $\sqrt{2 - \sin 3t} \cos 3t$ as (product (power sin) cos) rather than (product (($-2 \sin (* 3 t)$) 1/2)(cos (* 3 t))); (b) It cannot incorporate prior mathematical knowledge. For example, $\sin x$ and $\cos x$ have very similar properties, not only because they are both trigonometric functions. They have similar properties in differentiation, thus this similarity may extrapolate to indefinite integration too. In fact, it is. Thus, a student may regard $\int x \sin x dx$ and $\int x \cos x dx$ as similar (so that a solution for one would imply similar solution for another) but will not compare them with $\int x \tan x dx$. Another

aspect that LEX has not addressed is the metalevel knowledge, for example, being aware of the usefulness of the Schoenfeld's three phases model.

Mitchell, Keller, and Kedar-Cabelli (1986) describe how the Explanation-Based Generalization technique can be employed to generate an operational heuristic (a macro discussed before) of solving integral of the form $\int k x^r dx$ where k and r are any number and $r \neq 1$ by analyzing the solution of the training example, $\int x^2 dx$, and produce an explanation in terms of knowledge about the domain and goal concept. The goal concept is then formulated by generalizing the explanation structure. The main problem of adapting this technique for the companion in this domain is that the companion looks more like a *mathematician* than an *ordinary student*, since most students acquire such macro without realizing they have to prove it. In fact, as Van Harmelen and Bundy (1988) point out, such technique might not even need the example—just use rule 1 and rule 2 from Table 1.1.

As mentioned before, the companion only employs a limited effort of learning a task. Thus any learning techniques which exploit extensive searching or demand a large number of training examples are not appropriate. Furthermore, the learning strategies should reflect human natural preferences in learning. Also the design of knowledge representation should facilitate multiple learning strategies together with shifting of concepts to be learned in different stages of learning.

SIMULATION APPROACH

In the simulation approach, the increasing skill of the companion is directly coded as part of the companion rather than being produced as a result of machine learning. The simulation companion essentially makes selective use of the complete domain knowledge. The increasing performance of the companion can be derived from a succession of discrete simulation programs, say, a set of rules of behavior, which each simulate a different level of performance. Selection among these programs is made based on the student's performance. Greater responsiveness to the student's increasing skill can be obtained using a knowledge base-driven simulation. In this approach to simulation a single problem-solving engine is given access to an increasingly complete knowledge base. In either approach, any information provided to the student about how the learning takes place must be explicitly included in the simulation.

In order to explore the impact on a human student of a learning companion system, the companion need not actually learn. The image presented to the student must be that of a companion whose skill advances in roughly the same way as that of the student. While machine learning techniques provide a natural explanation ability for the student and an ability to expand the LCS paradigm to a wider domain than hand-crafted simulations of problem-solving performance,

the sophistication of the learning companion seems beyond the scope of current learning systems.

THE SPECTRUM OF LCS

In principle, it is possible to introduce a learning companion to a tutoring system on any domain. In particular, LCS is not restricted to the problem-solving context. A student may be accompanied by a learning companion in learning concepts. For example, in learning the concept of variables in beginning algebra using discovery mode, suppose the student successfully generalizes the pattern $11 + 3 = 3 + 11$, $2 + 1 = 1 + 2$, and $100 + 20 = 20 + 100$ to a rule $a + b = b + a$. However, the rule induced by the companion may be a literally different one, $x + y = y + x$. Now the student has to justify this alternative answer.

The wider view of LCS should not be limited to the one described above. In fact, the paradigm of LCS represents a broad spectrum of ITS design due to the possible varieties on the number and the identities of the agents in a LCS. Each of these varieties gives rise to particular cognitive issues in the student's learning.

First, it is possible to have no teacher involved. For example, in learning simple linear equations, the student may provide rules (e.g., distributive rule) and some examples to the learning companion. Then the student may observe how the companion solves the problems and improves performance. In this way, the student *learns how to learn by teaching the learning companion*. In fact, Neves (1978) has developed a system to learn solving linear equations.

At the other extreme, it is possible to have multiple teachers with different persona. For example, there may be a patient teacher and a demanding teacher. The student may choose one of them to respond adapting to his own learning style.

LCS may also be a simulation of peer group learning, which means more than one learning companion with different knowledge level or persona involved in the learning environment. For example, with companions at different levels of performance, the student can compare both suboptimal and optimal performance in learning. Another example is learning with one simulated and one machine learning companion. An interesting case would be that a human student is learning or solving problems collaboratively under competition with another pair of computer-learning agents.

Imagine in the near future as the price of computers falls and the technology of computer networks becomes more accessible, students can learn together through geographically distributed networks of computers (perhaps without a human teacher). We believe that current LCS research is preparing for such an *intelligent futuristic computer classroom*. In particular, LCS research for such a

learning environment will probably focus more on the design of dynamically structuring the learning activities which may be monitored by a rather passive computer teacher.

RELATED LCS RESEARCH UNDERWAY

The spectrum of LCS described above are essentially different LCS environments which vary over the following parameters: domain, number of agents involved, and role of agents (teaching, collaborating, competing, etc.). Apart from the development of INTEGRATION-KID (Chan, 1989), other LCS research underway if the following:

Perspectives and Implications of LCS Study (Chan, 1989)

Theory of learning companionship. We have noticed that LCS environment stimulates more dimensions for student's learning than the traditional *single-teacher oriented* ITS environment (e.g., collaboration and competition). LCS research, we believe, will spawn a lot of studies of cognition and learning. In particular, a *cognitive model of learning companionship* is under development which will form a theoretical basis for LCS research, in particular, for the implementation of the learning companion.

Counseling. Apart from learning, the idea of LCS can be applied for counseling. Imagine an AIDS patient who is concerned about the general effects of such a disease and the social impact it brings. He probably prefers to discuss such matters with a knowledgeable peer, a patient with the same disease rather than with a human medical counselor.

Implication to general knowledge-based systems. An extrapolation of LCS research will perhaps be the indication that most current expert systems which also care about the user's cognitive benefits apart from offering a solution should have a *separate* component which helps the user justify the recommended solution or motivates the user to obtain a better alternative solution by his own. Critiquing systems (Miller, 1984) can be viewed as an instance from this perspective. Also the role of such a separate component in an expert system seems to be a *natural* environment for an expert system to incorporate a learning agent for knowledge acquisition. Comparison of this perspective with critiquing and apprenticeship systems (Mitchell, Mahadevan, & Steinberg, 1985) is underway.

Reading Companion System (Aizenstein, Chan, & Baskin, 1989)

Apart from INTEGRATION-KID, another ongoing LCS project is Reading Companion System (RCS) for the domain of medical text. Well-written natural

language text is everywhere, in the form of textbooks, magazines, journals, and so on—accumulated over time and written by different authors. In a Reading Companion System, a learning companion is added to a hypertext-like environment. The role of this companion is to engage the student in a peer-style dialogue about what he has read. Aside from the cognitive benefits to the student, this peer-style dialogue also supplements the system's natural language understanding. RCS undergoes three phases of knowledge acquisition in adding new text to the environment. In the first two phases, the system captures a *rough understanding* of a text base using a subprogram called the Knowledge Based Categorization (Aizenstein, 1988) which employs simple natural language processing techniques. In the last phase, the RCS improves its understanding of the text base through the peer-style dialogue with the student. An interesting aspect of RCS is that it takes into account the different strengths and weaknesses of the computer and the human. Most people would agree that a computer is a weak natural language processor but a good data retriever while a human student is a strong natural language understander. Through an interactive, cooperative, and complementary knowledge acquisition system like RCS, both agents—the human student and the learning companion—are benefited by the peer-style dialogue.

CONCLUSION

In this chapter, we have discussed the preliminary idea of a Learning Companion System. In the learning environment of an LCS, aside from a computer teacher, there is a learning companion which learns along with the student, as a peer. The learning behavior of the companion is either simulated or actual machine learning. Moreover, we have discussed different considerations in the design of an LCS in the domain of indefinite integrations. Finally, we have outlined some perspectives of LCS research. Whether LCS is educational effective remains to be tested. Nevertheless, we believe that LCS, being a new subclass of ITS, will expand and emerge many interesting studies of ITS.

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