Conclusion

This thesis explores the potential of physical Human-Robot Interaction (pHRI) in the modern industry. In collaborative settings involving humans and robots, robots are increasingly capable of adapting to dynamic scenarios and offer a unique opportunity to assist human operators by accommodating their physical needs, preferences, and skill levels. However, this flexibility in collaborative workspaces raises safety concerns. To ensure that robotic assistance is not a burden on the human operator, the robot must possess a clear and precise understanding of human capabilities. In particular, knowledge of an operator's physical abilities, such as their force capacities, is crucial.

This thesis focuses on a robotic-based formulation of these force capacities at the hand, employing a set-theoretic framework. Accurately capturing human force capabilities requires experimental measurement. For a fixed upper-limb posture, this involves repeated exertion of maximal isometric forces in various directions. This leads to a first challenge addressed in this thesis:

Challenge 1:
How can the measurement of maximal isometric forces be performed efficiently within a reasonably long experiment?

Collaborative tasks demand varying levels of human physical involvement, necessitating adaptable robot assistance. Given that an individual's force capabilities are influenced by posture, physiology, and anthropometry, the robot should account for these factors. To address this inter- and intra-individual variability, this thesis employs a numerical representation of the human, known as a musculoskeletal model. However, the computational cost of such models increases with their complexity and level of detail. Thus, this thesis addresses this second challenge:

Challenge 2:
How detailed and personalized must a musculoskeletal model of the human upper-limb be to accurately represent the exertable maximal isometric forces at the hand of an individual?

Musculoskeletal modeling offers valuable insights into human biomechanics. A deeper understanding of exerted forces can enhance the design and control of collaborative robots, enabling safer and more effective human-robot interaction. This thesis explores the potential of a set-theoretic formulation of exertable forces to understand how muscle interactions contribute to maximal force production. This leads to the third challenge addressed in this thesis:

Challenge 3:
How can a set-theoretic approach to maximal isometric forces be used to quantitatively characterize and predict muscle activation patterns?

Thesis contributions

In Chapter 1, we describe how maximal isometric forces can be formulated within a set-theoretic approach using musculoskeletal models. These sets, termed *force feasible sets*, have been studied in the literature within both robotic and biomechanical contexts. We first describe the experimental protocol for collecting maximal isometric forces at the hand and review the factors that influence the quality of an exerted maximal force. By comparing experimental measurements from robotics and biomechanics, we highlight a discrepancy between musculoskeletal-based force feasible sets and experimental data. This discrepancy arises from differing biomechanical assumptions regarding muscle interactions, leading to distinct characterizations of force feasible sets, notably as (convex) polytopes or ellipsoids.

Chapter 2 focuses on improving the computational aspects of force polytopes to better understand the combinatorial geometric processes involved in their formulation. We present a new, efficient algorithm for computing the vertices of a zonotope — the projection of a hyperrectangle — which can represent feasible torques. The efficiency of this algorithm is theoretically proven using algorithmic complexity analysis, positioning our approach within the context of recent advances in the field. This chapter elucidates the computational challenges associated with describing torque feasible sets, which are inherently linked to polytopic representations of force feasible sets.

While force feasible sets modeled as polytopes assume independent muscle tensions, Chapter 3 explores alternative representations of muscle tension interactions. Adopting a more theoretical perspective, this chapter integrates mathematical results and their biomechanical implications into the framework of *in silico* force feasible sets. We argue that a large number of muscles in a musculoskeletal model permits the representation of a broad class of force feasible sets as ellipsoids, with a scaling factor indicative of the level of muscle tension interaction. While these shape-related results partially address Challenge 1 and 2, we further investigate this interaction by explicitly computing the scaling factor, termed the *projection constant*. This computation enables a numerical transition from a polytopic to an ellipsoidal representation, potentially mitigating the computational complexity associated with polytopes.

Furthermore, this chapter presents two additional results derived from this new class of representations. The first result provides a deeper understanding of how geometric assumptions about force feasible sets are reflected in muscle tension interaction models. It involves an explicit computation of force feasible sets in the muscle tension space, revealing how muscle tensions are linearly constrained when producing a maximal force. This analysis also demonstrates that, in polytopic representations of force feasible sets, most muscle tensions are either fully activated or inactive, and we specify the maximum number of muscles that deviate from this pattern. The second result introduces a novel index to characterize *in silico* torque and force feasible sets, incorporating the geometric processes involved in our set formulation into its calculation.

Chapter 4 delves into the practical implications of the results established in Chapter

3, quantifying the challenges associated with personalizing a musculoskeletal model using force feasible sets represented as ellipsoids or polytopes. Through an *in silico* study, we attempt to personalize various parameters of a musculoskeletal model, assuming a limited number of upper-limb postures and known *in silico* force feasible sets modeled as polytopes or ellipsoids. We introduce a new index, termed the *enlargement complexity*, to evaluate the difficulty of this personalization process. This index is computed through an analysis of the solutions found during an optimization-based personalization process, considering different search spaces. The computed index values suggest that the challenges associated with personalization are not primarily related to the geometric definition of muscle paths, for both force feasible set representations. Thus, this chapter addresses Challenges 2 and 3.

Chapter 5 directly addresses Challenge 3 by introducing an experimental protocol to gather maximal isometric forces in four different upper-limb postures. The experimental setup is designed to accommodate the anthropometric variability of individuals. This chapter confronts the theoretical ellipsoid representation assumption from Chapter 3 with *in vivo* data. The central objective is to evaluate whether a 50-muscle upper-limb musculoskeletal model is sufficient to apply theoretical muscle tension interaction assumptions induced by an ellipsoid representation of force feasible sets, which necessitate a large number of muscles to be assumed. Thus, we formulate a hypothesis about the behavior of muscle tension interactions across different postures and conduct an optimization-based personalization process using the participants' experimental measurements. The results indicate that while an ellipsoidal representation of force feasible sets may capture the shape and orientation of *in vivo* maximal isometric force measurements, the theoretical results of Chapter 3 do not necessarily lead to improved modeling of muscle tension interactions.

In conclusion, this thesis evaluates the potential of a set-theoretic approach to represent maximal isometric forces at the hand. It examines how this approach addresses experimental challenges and assesses the extent to which it can reveal the biomechanical properties of human upper-limb muscles and their interactions.

Perspectives

This section outlines potential avenues for future research that could enhance and extend the findings of this thesis.

Incorporating biomechanical knowledge for improved force feasible set prediction. While this thesis explored force feasible set prediction through a personalization processes both *in silico* and *in vivo*, these processes were challenged by the set-theoretic nature of force feasible sets. Although primarily theoretical hypotheses were employed, incorporating more specific knowledge of individual biomechanics could improve these personalization processes. For instance, future work could consider integrating maximal isometric torques alongside maximal isometric forces, as explored in Rezzoug et al., 2021.

Standardizing the measurement of multiple isometric force exertions in a posture. Reconstructing an isometric force feasible set relies heavily on the maximal voluntary isometric contraction (MVIC) protocol. We proposed an adaptive setup to accommodate anthropometric variability, but further refinements may be necessary to ensure precise maintenance of upper-limb postures. Our experiments suggest that the cognitive demands of simultaneously maintaining a posture, exerting maximal force, and controlling direction may compromise posture stability. Future studies should investigate how this cognitive overload affects posture stability, force amplitude, and directional accuracy.

Extending to dynamic force feasible sets. This thesis focused on isometric maximal force exertions due to their relative ease of measurement. However, human-robot collaboration often involves dynamic movements and postures. While a dynamical description of force feasible sets exists (as discussed in Chapter 1), its formulation is more complex, requiring detailed knowledge of muscle properties (such as force-velocity relationships) and individual mass and inertial parameters. Future research should investigate the relationship between isometric force feasible sets (and their ellipsoidal representation) and their dynamic counterparts, assessing the extent to which our theoretical assumptions in a static context generalize in dynamic contexts.

Exploring moment feasible sets. This thesis primarily focused on maximal isometric forces, assuming negligible isometric moments. However, moments are inherent to any exertion, even if small. Therefore, generalizing findings such as the ellipsoidal approximation of force feasible sets requires careful consideration. While the theoretical framework developed for maximal isometric forces (with no moment) could potentially apply to maximal isometric moments (with no force), a geometric analysis is needed to characterize the set formed by the combination of both maximal isometric forces and moments. As distinct mathematical entities, they cannot be simply represented as two ellipsoids within the same space. However, given the geometric relationship between forces and moments (as Plücker coordinates of a screw line, Dorst et al., 2007), a unified geometric 3D characterization of maximal isometric force and moment may be possible.