MEMO Number: UMBC-CMPE320-5
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SUBJECT: Project 5

1. Introduction

In project 5 I graph sets of random functions and the means and correlations of their values. Along with this I will simulate a filter with a sliding window average output with different L values and discuss how this affected the correlation between points of the outputs.

2. Simulation and Discussion

2.1) Create a MATLAB Model

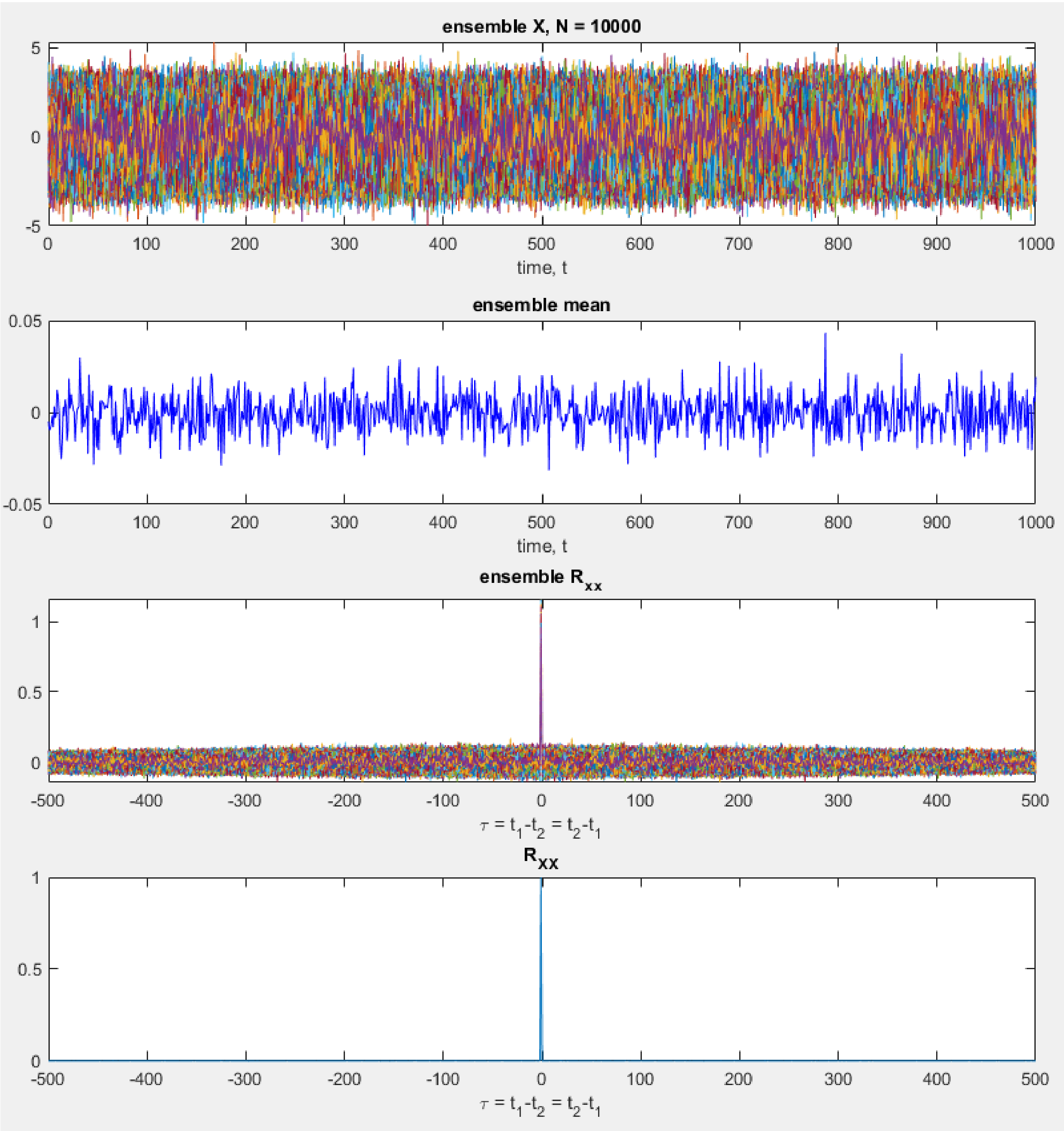

Figure 1: Random Functions

Figure 1 above shows multiple representations of random functions. Plot 1 at the top of Figure 1 is a direct graph of 10000 different functions in different colors. When attempting to pick one out of the ensemble, it can be seen that the function accurately shows a random function, with outputs being completely random in time within the bounds given to the random function. Plot 2 further supports this by graphing the function of the mean of the 10000 random functions. It can be seen that this function is essentially random with no correlation seen between each point in time. This graph is centered at 0 as expected based on the bounds of the input for the initial random functions being centered on 0. Plot 3 is a graph of the statistical autocorrelation of each function shown in Plot 1. This plot is able to accurately show the correlation of the function within time, showing the correlation between two points as a factor of a difference in their time. The graph shows what could be assumed based on the input, there is no correlation beyond a small random amount at any time distance other than 0. This is shown by the impulse at 0 where there should be perfect correlation with itself. Plot 4 is the mean of all of the autocorrelation functions. All random noise is canceled out more cleanly showing the perfect impulse at 0 shown in Plot 3.

2.2) Simulate Filtered Random Gaussian Data

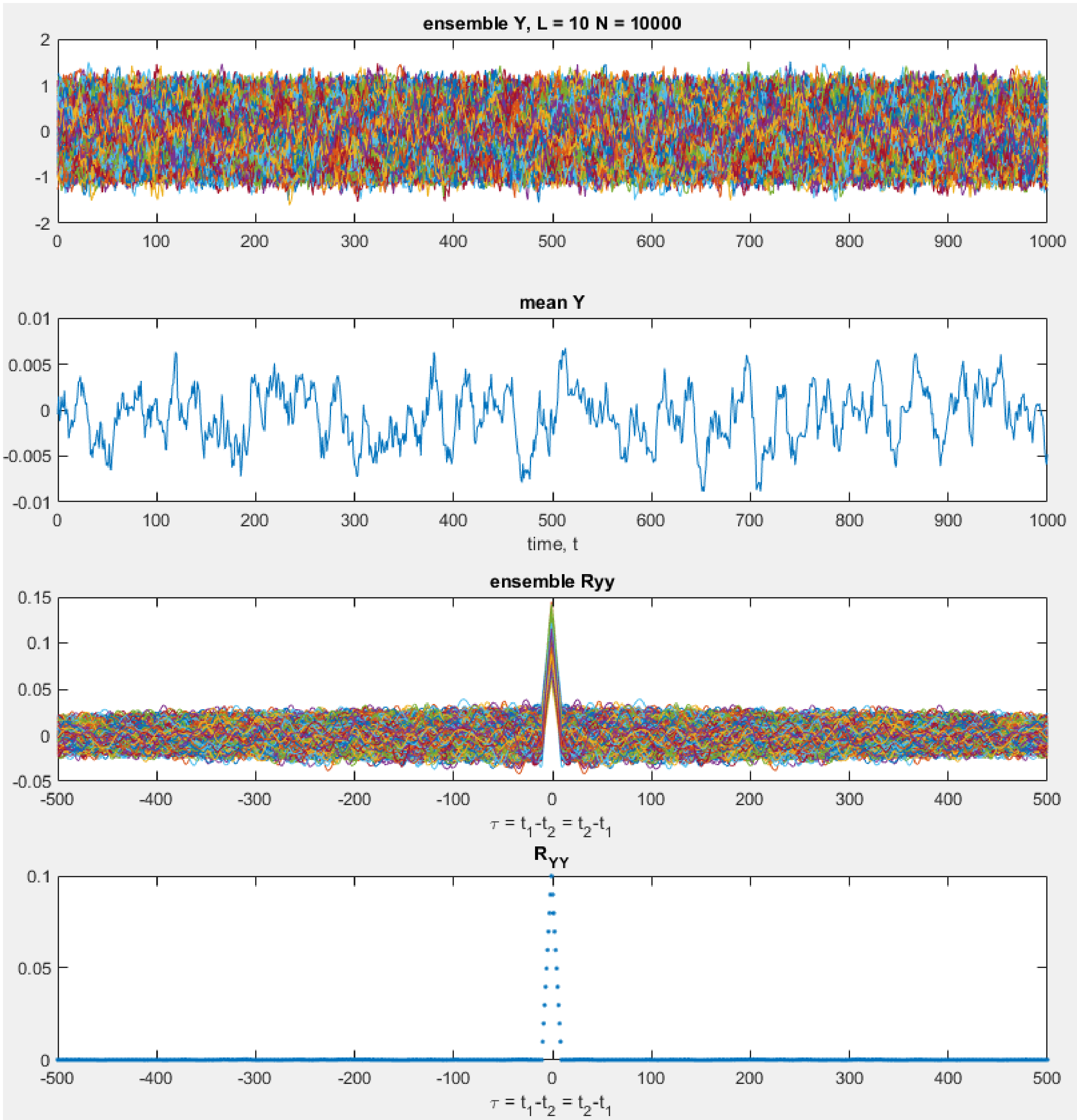

Figure 2: Random Functions L = 10

Figure 2 shows again a collection of random functions only this time they have been evaluated with a sliding window average of L. Due to the addition of the sliding window average these plots are noticeably different from Figure 1. The first difference can be seen in Plot 1, when picking out a function, it can be seen to be noticeably less random than in Figure 1. This is further shown in Plot 2, with the mean function being much smoother when compared to Figure 1, showing the decrease in variance due to the sliding window average. This is also shown by the smaller bounds of the evaluated mean function. The most noticeable difference is in Plot 3. The impulse at 0 has now turned into a spike, showing how at time differences larger than 0 there is noticeable correlation between evaluated function values. The graphs are supported with a calculated variance of ${\sigma^2}_Y =.659x10^{-6}$ and a variance reduction factor of $g = 15.15$.

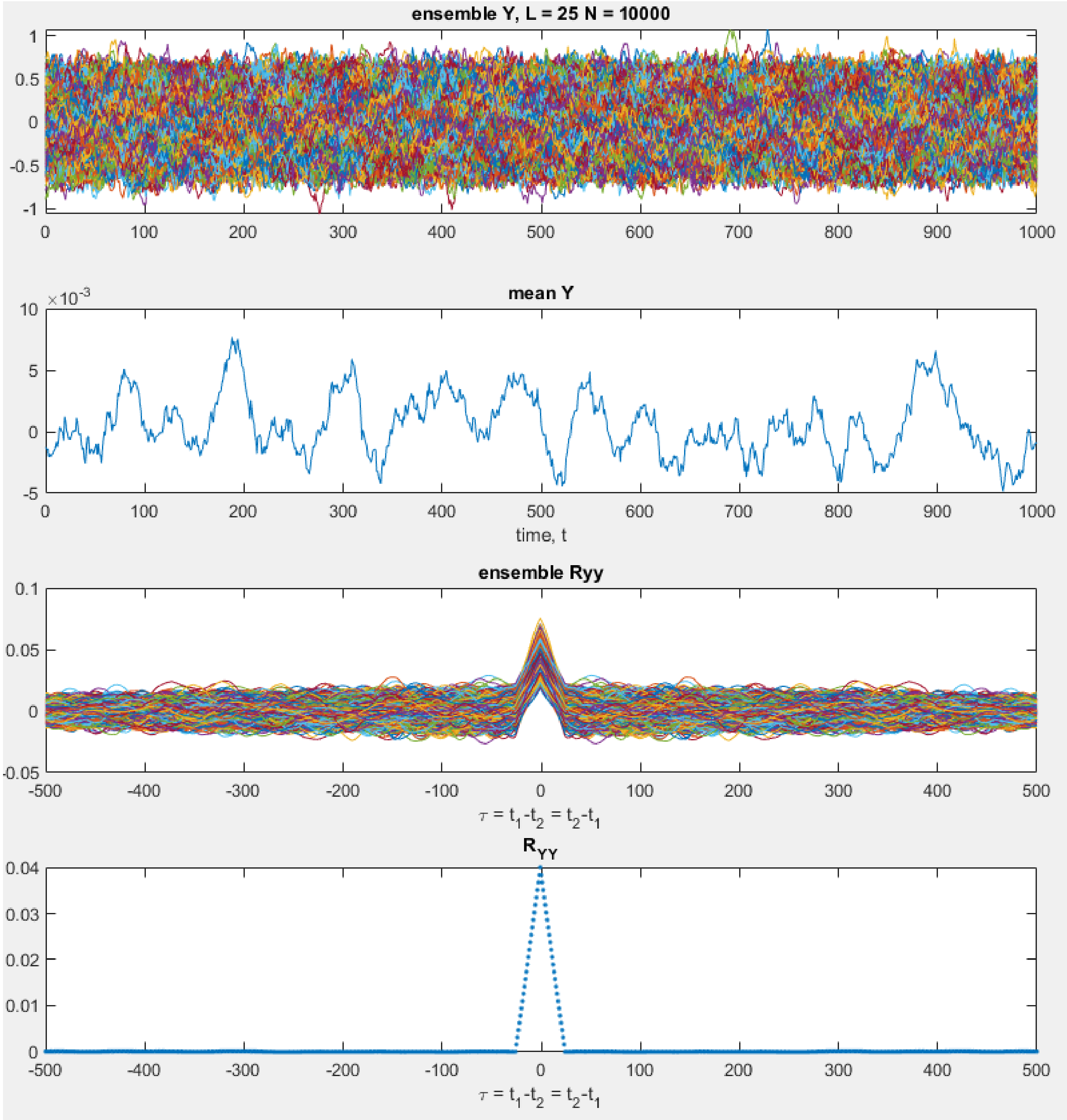

Figure 3: Random Functions L = 25

Figure 3 shows the same plots as Figure 2 but with L = 25. This change in the sliding window average has resulted in noticeable changes in the functions plotted. Firstly, Plot 1 has again smoothed out along with the mean function in Plot 2. The variance of the mean function has further decreased when compared to Figure 2. The largest difference is in Plot 3, with the nonrandom output of the correlation function widening, showing a correlation between two values even farther apart than in Figure 2. The peak of this function has noticeably decreased though from .15 to .1 showing less correlation between two points close together when compared to Figure 2. This increase in average correlation is supported by the calculated variance of $\sigma_Y^2 = .258x10^{-6}$ and a variance reduction factor of $g = 39.34$.

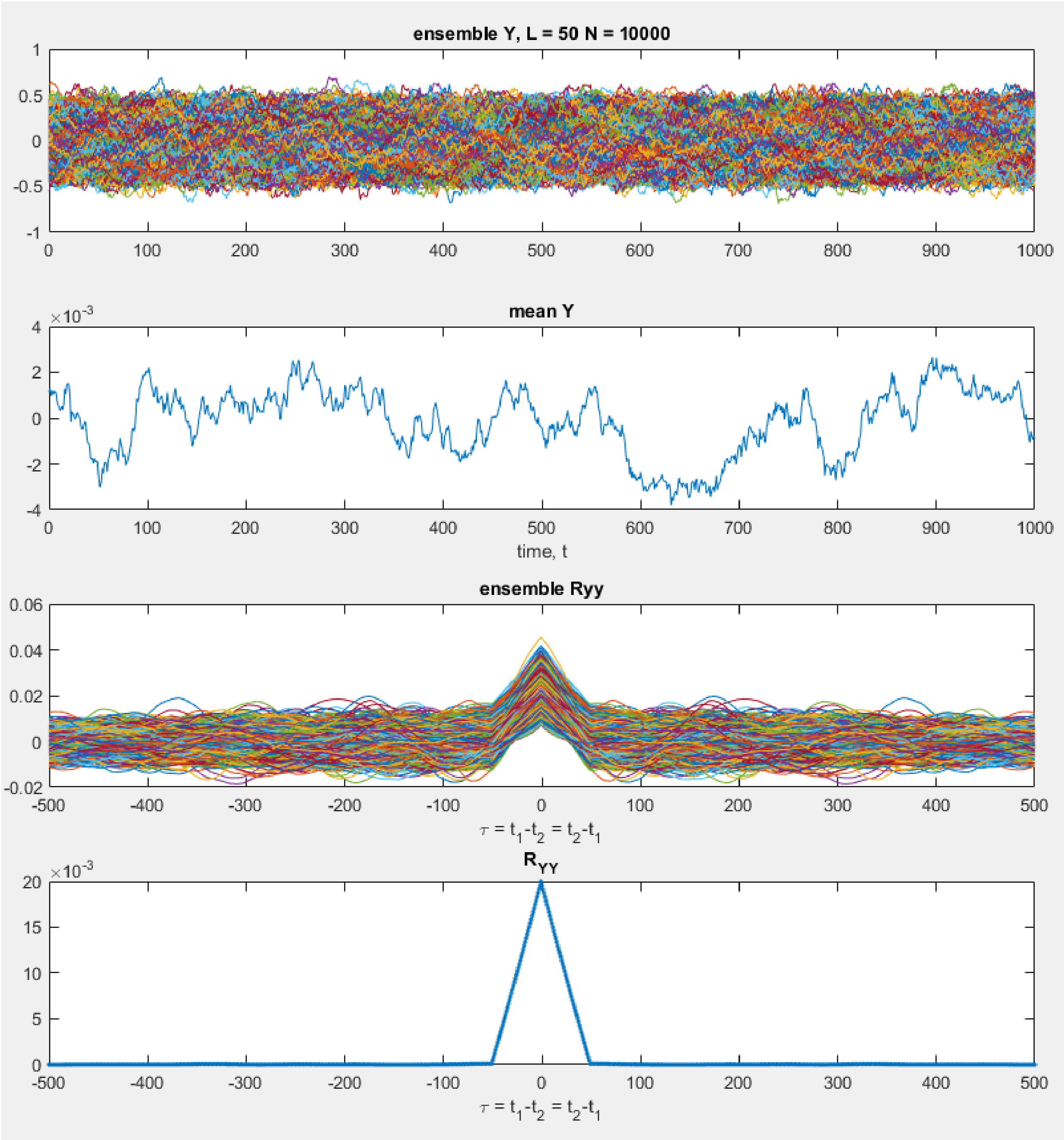

Figure 4: Random Functions L = 50

Figure 4 demonstrated the most noticeable effects of the sliding window average with an L = 50. Plot 1 shows the functions now noticeably far from their bounds with much smoother slopes. The variance of the mean function is even smaller now than in Figure 3. In Plot 3, the function now has a much smaller peak variance with an even wider center demonstrating the larger effect of the increased L value. Even the random variances beyond the center are more linear. This is also shown with a calculated variance of $\sigma^2_Y = .12x10^{-6}$ and a variance reduction factor of $g = 82.79$.

3. What I learned

Project 5 has allowed me to clearly understand how a random function is created. It has also allowed me to further understand the effect a filter can have on an input function, specifically the correlation effects of a sliding window average.