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CS332 Homework 1

1.

Base case: (n = 4)

2^4 = 16 < 4 ! = 24 (base case check)

Induction step:

Assume 2^n < n! , We need to show that 2^(n+1) <(n+1)! (n>4)

(n+1) ! = (n+1) * n !

We have: 2^n < n!

As a result : 2^n *(n+1) < (n+1)!   (From inductive hypothesis)

2^n*2 < (n+1)!

2^(n+1) <(n+1)! (proved)

2.

Rank growth (from smallest to the largest):

5n < n*logn < n*sqr(n) < 4n^3 < n^3*log 5n < n^3 *log n *log n < 3^n

3.

The first step of the procedure states that the turing machine has to try every possible assignments of natural numbers to a,b,c. We know that there are infinite numbers of possible assignments of natural numbers to a,b,c which implies that the turing machine will have to run in a infinite time in this procedure to try all these assignments. However, in the description of a turing machine, we have to have a finite number of steps in each stage.As a result, it is not a legitimate algorithm.

4.

Informal Description:

Let M1 be the TM in this question. M1 will accept if its input is a member of L and to reject otherwise. According to a strategy introduced from the textbook, we zig-zag to the corresponding places on two sides of the “2” and determine whether they match. We also make marks on the tape to keep track of these places.The informal description of this machine is as follows:

M1 = “On input string w:

1.M1 zig-zags across the tape to corresponding positions on either side of “2”. Then it checks if either side of “2” contain the same symbols.To be specific. If the symbols do not match or no “2” is found, M1 rejects. The checked symbols will be cut off in order to keep track where the current symbols are of both sides.

2. When there are nothing left on the left side of “2”, M1 accepts(meaning that both sides of 2 contain same symbols). Otherwise, M1 rejects.

5.
Based on the binary observation, the last three digits of the binary string (from the left) of the number 5+8n is always 101 and as long as the the leftmost three digits are 101, the equation 5+8n is satisfied. One more reason is that the binary representation of 8 is 1000 which will not affect the left three digits.
As a result, the turing machine can be described informally as follows:

M = “On input string w:

1. read the input from right to left.
2. If the first digit is not 1 , reject. If it is the case, moves the tape from right to left
3. If the second digit is not 0. Reject. If it is the case, moves the tape from right to left
4. If the third digit is not 1, Reject. If it is the case, accept.
5. Go back to 1 “

The formal description is as follows: