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Homework 4

1.
Assume agent 1's valuation is 2.
Suppose agent 2 uses this strategy, due to the valuation is drawn uniform:
there is 50% of chance that agent 2 has valuation either above or below 2.

Case 1: agent 2's valuation is below 2(hence not pay 1):

Agent 1's expected utility if he pays 1:

$0.5 * (\frac{1}{3} * 2 - 1) = - \frac{1}{6}$

Agent 1's expected utility if he does not pay 1:

$0 - 0 = 0$

Case 2: agent 2's valuation is above 2(hence pay 1):

Agent 1's expected utility if he pays 1:
$0.5 * (1* 2 - 1) = \frac{1}{2}$

Agent 1's expected utility if he does not pay 1:
$0.5 * (\frac{1}{3} * 2 - 0) = \frac{1}{3}$

We sum two cases up:
Agent 1's expected utility if he pays 1 with valuation 2:
$- \frac{1}{6} + \frac{1}{2} = \frac{1}{3}$
Agent 1's expected utility if he pays 0 with valuation 2:
$0 + \frac{1}{3} = \frac{1}{3}$

As a result, if agent 1's valuation is 2, given agent 1 uses the same strategy, she is indifferent between the two actions.

b)
Repeat the above process:

Case 1: agent 2' s valuation is below x (with probability F(x)) and hence not pay.

Agent 1's expected utility if he pays 1:
$F(x) * (\frac{1}{3} x - 1)$

Agent 1's expected utility if he does not pay:
0

Case 2: agent 2' s valuation is above x (with probability 1- F(x)) and hence will pay

Agent 1's expected utility if he pays 1:
$(1- F(x))(x -1) = x - 1 - x F(x) + F(x)$

Agent 1's expected utility if he does not pay :

$(1 -F(x)) * \frac{1}{3} * x = \frac{1}{3} x - \frac{1}{3} F(x) x$

Agent 1 should be indifferent between the two actions:
$F(x) * (\frac{1}{3} x - 1) + x - 1 - x F(x) + F(x) = \frac{1}{3} x - \frac{1}{3} F(x) x$
$x - 1 - F(x)x + F(x) + \frac{1}{3} F(x) x - F(x) = \frac{1}{3} x - \frac{1}{3} F(x) x$
$x - 1 - \frac{2}{3} F(x) x = \frac{1}{3} x - \frac{1}{3} F(x) x$
$\frac{2}{3} x - \frac{1}{3} F(x) x - 1 = 0$
Multiply both sides by -3:
$F(x)x - 2x + 3 = 0$

2.
With the free disposal assumption, the optimal solution is:
Take ({b, c}, 9) from the second bid and take ({a}, 3) from the third bid.
The total value is 9 + 3 = 12

Calculating the GVA payments: (the second and third are the winning bidders)

Without the second bidder, the winner would be the third bidder ({a, b, c}, 11)
With the second Bidder, the total value is 3.
As a result, the payment for the second bidder is 11 - 3 = 8

Without the third bidder, the winner would be the first bidder with ({a, b}, 10) or both first and second bidder with ({c}, 4) from the first bidder and ({a, b}, 6) from the second bidder with also a total value of 10.
With the third Bidder, the total value is 9.
As a result, the payment for the third bidder is 10 - 9 = 1