Lai Wei
lw255@duke.edu

CS 223 Homework 3

Problem 1:

a) A gamble [pays of 5000\$ with probability $\frac{1}{5}$] and gives out [-1250 with probability $\frac{4}{5}$].

Check Fair : $5000 \times \frac{1}{5} + (-1250 \times \frac{4}{5}) = 0$ [indeed Fair]

Bob would end up 6100 with probability $\frac{1}{5}$, -150 (in debt) with probability $\frac{4}{5}$

Therefore, his expected untility is: $\frac{1}{5} \cdot 2 + \frac{4}{5} \cdot 0 = \frac{2}{5} = 0.4 < 1$.

b) A gamble [pays off 2000 with probability $\frac{1}{80}$, $-\frac{2000}{79}$ with probability $\frac{79}{80}$.]

Check fair: $2000 \times \frac{1}{80} + (-\frac{2000}{79} \cdot \frac{79}{80}) = 25 - 25 = 0$

Bob would end up 3100 with probability $\frac{1}{80}$, $1100 - \frac{2000}{79} \approx 1074.68$ with probability $\frac{79}{80}$.

Therefore, his expected utility is: $\frac{1}{80} \cdot 2 + \frac{79}{80} \cdot 1 = \frac{81}{80} > 1$

2. ①

	q	$1-q$
P	4,4	8,2
$1-P$	2,8	7,7

Pure Nash equilibrum:

(4,4)

Mixed strategy Nash equibrum:

player 1's utility = $4q + 8(1-q) = -4q + 8$ (playing up)

$\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad ||$

player's utility = $2q + 7(1-q) = -5q + 7$ (playing down)

$\Rightarrow q = -1$ which is not vaid, as a result, there is no mixed strategy Nash equilibrum.

(No ther equilibium because all cases are considered)

②

	A q	B $1-q$
P	0,8	4,0
$1-P$	2,0	0,1

There is no pure strategy nash equibrium. since both player want to deviate.

Compute Mixed strategy Nash equilibrium:

① $0 \cdot q + 4(1-q) = 2q + 0 \cdot (1-q) \Rightarrow q = \frac{2}{3}$

② $P \cdot 8 + (1-P) \cdot 0 = 0 + 1(1-P) \Rightarrow P = \frac{1}{9}$

As a result, suppose player 1 has move C, D + player 2 has move A, B.
The mixed strategy for this game is $\left[\left(\frac{1}{9}C, \frac{8}{9}D\right), \left(\frac{2}{3}A, \frac{1}{3}B\right)\right]$
(A, B, C, D shown in graph)

(No other Nash equibrium beause all cases are considered)

③

	A q	B $1-q$
P	7,7	6,8
$1-P$	9,2	0,1

Pure strategy: (9,2), (6,8)

Mixed strategy:

① $7 \cdot q + 6(1-q) = 9q \cdot 0 \cdot (1-q) \Rightarrow q = \frac{3}{4}$

② $7P + 2(1-P) = 8P + (1-P) \Rightarrow P = \frac{1}{2}$

As a result, the mixed strategy for this game is:

$\left[\left(\frac{1}{2}C, \frac{1}{2}D\right), \left(\frac{3}{4}A, \frac{1}{4}B\right)\right]$

(No other Nash equibrium because all cases are considered)

④

	P_2 A	B	C
D	4,0	4,0	1,2
E	~~3,5~~	~~3,4~~	~~2,4~~
F	4,0	1,1	5,0

⇓

	P_2 ~~A~~	B	C
D	~~4,0~~	4,0	1,2
F	~~4,0~~	1,1	5,0

⇓

		q: B	$1-q$: C
p	D	4,0	1,2
$1-p$	F	1,1	5,0

① Based on the observation.

Consider player 1 play $(\frac{67}{100} D, \frac{33}{100} F)$, which dominates E.

$$\left.\begin{aligned} &\tfrac{67}{100}\cdot 4 + \tfrac{33}{100}\cdot 4 = 4 > 3 \\ &\tfrac{67}{100}\cdot 4 + \tfrac{33}{100}\cdot 1 = 3.01 > 3 \\ &\tfrac{67}{100}\cdot 1 + \tfrac{33}{100}\cdot 5 = 2.32 > 2 \end{aligned}\right\}$$

The we cross out E.

② It's obvious that player 2 play $(\frac{1}{2}B, \frac{1}{2}, C)$ dominates A.

Then A can be crossed out.

③ Compute mixed strategy nash equibrium:

$$4q + (1-q) = q + (5-5q) \Rightarrow q = \frac{4}{7}$$

$$0 + 1 - p = 2p \Rightarrow p = \frac{1}{3}$$

As a result, $\left[\left(\frac{4}{7}B, \frac{3}{7}C\right), \left(\frac{1}{3}D, \frac{2}{3}F\right)\right]$ is the mixed strategy nash equilibrium.

(There is no pure strategy equilibrium in this game)

This is the only nash equilibrum because:

P_1: ① No mixed strategy between D E that dominates F.

② No mixed strategy EF that dominates D

P_2: ① No mixed strategy AB that dominates C.

② No mixed strategy between AC that dominates B.

And finally, No pure strategy Nash equibrium as well.

3. a): the normal form game is as follows:

Player 2

	LL	LR	RL	RR
L	3,3	3,3	2,4	2,4
layer 1 M	2,4	2,4	3,3	3,3
R	4,2	1,1	4,2	1,1

Since there is a dotted line between L, M. which means a player cannot distinguish between 2 states.

As a result, LR of player 2 stands for:

$$\begin{cases} L \text{ if player 1 choose } \underline{L \text{ or } M}. \\ R \text{ if player 1 choose } R. \end{cases}$$

b): Let player 2 play R when player 1 plays R. As a result, player 1 actually is forced to play (L, M). Hence. R for player 1 can be eliminated.

Now we have the game with imperfect information:

(p)	L	R (q)
L	3,3	2,4
M	2,4	3,3

calculating mixed strategy equilibrium:

$$3q + 2(1-q) = 2q + 3(1-q) \Rightarrow q = \frac{1}{2}$$

$$3p + 4(1-p) = 4p + 3(1-p) \Rightarrow p = \frac{1}{2}$$

As a result, the mixed strategy would be:

$$\Big[\underbrace{\left(\tfrac{1}{2}L, \tfrac{1}{2}M\right)}_{\text{player 1}}, \underbrace{\left(\tfrac{1}{2}L, \tfrac{1}{2}R\right)}_{\text{player 2}}\Big]$$

c) ① In the subgame after player 1 chooses R, subgame is

L	R
4,2	1,1

,

player 2 will choose L because it will give you higher payoff.

In the whole game tree, (R, LL) is also a Nash equilibrium. As a result $\underline{(R, LL)}$ is subgame perfect equilibrium, $\underline{(R, RL)}$ is also a subgame perfect equilibrium.

Due to the same reason

② In the subgame on the left with imperfect information, there is no pure strategy Nash equilibrium.