1. Prove that if all edge weights are distinct in an undirected graph G = (V, E), then it has a unique minimum spanning tree.

This question can be proved by contradiction.

Suppose there are two minimum spanning trees t1 and t2. As a result, there exists at least one edge that belongs to t1 and does not belong to t2.

Let an edge in that set with lowest weight be edge1. Due to the fact that all edge weights are distinct in the undirected graph, edge1 is unique. As a result, if we add edge1 to t2, a cycle is formed because t2 is also a minimum spanning tree.

If there is a cycle in t2, there must be an edge in the cycle that is not contained in t1 due to the fact that t1 does not contain the cycle. Let us call this edge edge2.

Now we have two edges: edge1 and edge2. Edge1 has the lowest weight which implies edge 2 must be greater than edge1. As a result, if we replace edge2 with edge1 in t2, we can obtain a smaller cost of spanning tree.

The conclusion contradicts our original assumption that t2 is a minimum spanning tree.