Question 3.

STEP 1: Construct the recurrence equation:

Let $M_n$ be the minimum moves required to move $n$ disks to another rod. Now, suppose we want to move the entire stack from Rod 1 to Rod 3. We simply do the following steps:

① Move the top $n-1$ disks (except the largest one) from Rod 1 to Rod 2. $\longrightarrow$ (This will take $\underline{M_{n-1}}$ moves)

② Put the largest disk in Rod 1 to Rod 3. (1 step)

③ Move the all disks on Rod 2 to Rod 3. This will take again $\underline{M_{n-1}}$ steps.

As a result, since all disks are moved from Rod 1 to 3, the objective is finished. The recurrence equation is as followed:

$$M_n = 2M_{n-1} + 1$$

STEP 2: Solve the recurrence equation:

Based on observation: $\underbrace{M_1 = 1}_{\text{Base case}}$, $M_2 = 2M_1 + 1 = 3$, $M_3 = 2M_2 + 1 = 7$

$M_4 = 2M_3 + 1 = 15$ ... etc. The sequence satisfies the formula: $\underline{M_n = 2^n - 1}$. I will prove this observation by induction.

Base case: $M_1 = 1$. IH: $M_n = 2^n - 1$, prove $M_{n+1} = 2^{n+1} - 1$. $(n \geq 1)$

$$M_{n+1} = 2M_n + 1 = 2(2^n - 1) + 1 = 2^{n+1} - 2 + 1 = 2^{n+1} - 1$$

$\therefore$ Proved ∎

which gives $M_n = \Theta(2^n)$