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Problem 1.

a)

$\forall x: ((\text{continent}(x) \land (\neg \text{Australia}(x)) \land (\neg \text{Antarctica}(x)))$
$\Longrightarrow \exists y: (\text{continent}(y) \land \text{isconnected}(x, y))$

b) $\forall x \forall y: (\text{isperson}(x) \land \text{isperson}(y) \Longrightarrow (\text{Smart}(x) \land \text{StudyHard}(x) \land$
$(\neg \text{Smart}(y)) \land (\neg \text{StudyHard}(y)) \Longrightarrow \text{GetHigherScore}(x, y)))$

Note: GetHighScore(a, b) means a gets a higher score than b.

c) $\forall x: (\text{WalksLikeDuck}(x) \land \text{TalkslikeDuck}(x))$
$\Longrightarrow (\text{Duck}(x) \lor (\text{isHumman}(x) \land \text{ImitatingDuck}(x)))$

d) $\forall x \forall y: (\text{Gold}(x) \land \text{Silver}(y) \land \text{SameEvent}(x, y)) \Longrightarrow \text{WorthMorethan}(x, y)$

~~e) ∃x ∀y : (Dog(x) ∧ isHuman(y) ∧ (¬Love(x, y)))~~

e) $\forall x \forall y: (\text{isHuman}(x) \land \text{Love}(y, x)) \Longrightarrow \text{isDog}(y)$

Note: Love(a, b) means a loves b.

f) $\exists x \forall y: (\text{isDog}(x) \land \text{isHuman}(y) \land (\neg \text{Love}(x, y)))$

g) $\forall x \forall y : ((\text{isEnemy}(Me, x) \land \text{isEnemy}(x, y)) \Rightarrow \text{isFriend}(Me, y))$

↳ "x is my Enemy"

h) $\exists x \exists y :$ $\text{point}(x) \land \text{point}(y) \land \text{onWorld}(x) \land \text{onworld}(y) \Rightarrow$

~~(1meterS(1meterE(1meterN(x)))) = (1meterS(1meterE(1meterN(y)))) ∧ (¬(x=y))~~

~~(1meterS(1meterE(1meterN(x)))) = (1meterS(1meterE(1meterN(y)~~ ∧ ((1meterS(1meterE(1meterN(y)))) = y) ∧ (¬(x=y)))

(((1meterS(1meteE(1meterN(x)))) = x)

2. First, apply substitution: $\{v/John, x/Rice, y/z, w/z\}$ the orignal statements become:

$\forall z$: LovesTheCombinationOf(John, Rice, z) ∨ MakesSick(Rice, John) ∨ RuinsTasteOf(z, Rice)

$\forall z$: ¬LovesTheCombinationOf(John, Rice, z) ∨ ~~MakeSick(Rice, John)~~ Flavorful(z)

Note that we have the form $\{P, \neg P\}$, we can then apply resolution here:

$\forall z$: Flavorful(z) ∨ MakesSick(Rice, John) ∨ RuinsTasteOf(z, Rice)

In English, a conclusion is:

Everything is Flavorful that ~~w~~ ruins the taste of rice, which (Rice) ~~w~~ makes John sick.

3. knowledge Base :

① $\forall x \forall y \forall z : (IsEnemy(x,y) \wedge IsEnemy(y,z)) \Longrightarrow IsFriend(x,z)$

② $\forall x \exists y \exists z : (IsEnemy(x,y) \wedge IsEnemy(x,z) \wedge (\neg(y=z)))$

③ $\forall x \forall y : IsEnemy(x,y) \Longrightarrow IsEnemy(y,x)$

④ $\exists x \forall y : \quad ((y=x) \vee (\neg IsFriend(y,x)))$ [negation of the goal]

⑤ $\forall x : (IsEnemy(x, f_1(x)) \wedge IsEnemy(x, f_2(x)) \wedge (\neg(f_1(x) = f_2(x))$
[From ② and Skolem function]

⑥ $\forall x : (IsEnemy(f_1(x), f_1(f_1(x))) \wedge IsEnemy(\cancel{x}\, f_1(x), f_1(f_2(x))) \wedge (\neg(f_1(f_1(x)) = f_1(f_2(x)))$
[from ⑤, replace x with $f_1(x)$]

⑦ $(IsEnemy(x, f_1(x)) \wedge IsEnemy(x, f_2(x)) \wedge IsEnemy(f_1(x), f_1(f_1(x)))$
$\wedge IsEnemy(f_1(x) \wedge f_1(f_2(x)))$ [combine ⑤ and ⑥]

⑧ $IsFriend(x, f_1(f_1(x))) \wedge IsFriend(x, f_1(f_2(x)))$ [From ⑦ and ①]

⑨ $x = f_1(f_1(x)) \wedge x = f_1(f_2(x))$ [From ⑧ and ④]

⑩ $f_1(f_1(x)) = f_1(f_2(x))$ [From ⑨]

⑪ $\neg(f_1(f_1(x)) = f_1(f_2(x)))$ [From ⑥]

⑫ EMPTY SET

As a result, a contracdiction is reached, the statement is proved.

In plain English, a thing x's enemy have two enemies. At least one of the 2 enemy is x's friend because it is possible that one of them might be x itself. ~~Also, another x's enemy might be x itself and they share a common enemy which is x's friend~~.