Lai Wei
lw255@duke.edu

CS 570 Homework 0

Question 1

a)

If we continue all the way to the end for this game and ignore the wins and ties. This problem reduces to if we have 5 Xs and in how many ways can we put them on the 3 x 3 board.

This is a simple combination problem and we can apply the formula for combination:

9C5 = **126**

As a result, there are 126 possible final states of the game.

b)

If we care about the order of the move , it's just the number of combinations of the final board times the permutations of Xs and Os.

126 x 5! x 4! = **362880**

Another way to check this answer is that the first move will have 9 options and the second move will have 8 options...etc. As a result:

9 x 8 x 7… x 1 = 9 ! = **362880**

c)

When there is a third player, each player can only have 3 move. The first player can place 3 moves on any of the 9 squares on the board while the second player can place 3 moves on any of the remaining squares of the move etc.

Aa result, the number of final states are:

9C3 x 6C3 = 84 x 20 = **1680**

d)

The first player have 9 squares to place the move, the second player have 8 squares to place the move …..etc. As a result, the total number of possible sequences are:

9 x 8 x 7… x 1 = 9 ! = **362880**

Question 2:

a)

If we place the first rock in a row which have 8 squares, the second rock will only have 7 rows to choose. If the second rock is placed in one of the 7 rows, the third rock will only have 6 rows to choose. As a result, the possible number of final states will be:

8 x 7 x 6 x 5 x 4 x 3 x2 x 1 = 8! = **40320**

Question 2
b)

I will first introduce a 4 x 4 chessboard to talk about my concept:
No matter where the first rock is placed, it will eliminate 4+4 -1= 7 playable squares for the second rock to be placed. The same thing happens to the second move. The second move can only be placed on the rest of 16-7 = 9 squares. Wherever the second move is placed among these nine squares, it will eliminate 7-2 =5 playable squares because it must have 2 squares overlap with the first move(which will be subtracted). As a result, there is only 16- 7 - 5 = 4 playable squares. The same thing applies again and again until the last rock is placed.
By the observation above, let n be the width of the board.
The total possible playable squares for the ith move is:
(n+1-i) ^2 where 1 <= i <= n
Apply this formula to a 8 x 8 chessboard with 8 rocks.
The first rock will have 64 choices to be placed.
The second rock will have 64 - 15 = 49 choices to be placed.
The third rock will gave 49 - 15 + 2 = 36
4th: 36-15 +4 = 25
5th 25- 15 +6 =16
6th 16 -15 + 8 = 9
7th 9 -15 + 10 = 4
8th 4 -15 + 12 = 1.

As a result, one way of count the total possible sequences for a 8 x 8 chessboard with 8 rocks are:

64 x 49 x 36 x 25 x 16 x 9 x 4 x 1 = **1625702400**

Another way to count this is to use :
the total number of final solutions x the permutations of moves
= 40320 x 8 ! = **1625702400**