1.

$$P(+u|+e) = \frac{P(+u,+e)}{P(+e)}$$

First compute $P(+u,+e)$.

$$P(+u,+e) = \sum_h P(h) \sum_i (+u|i,h) \sum_t P(t|i)\cdot P(i)\cdot P(+e|t,+u)$$

$$= \sum_h P(h) \sum_i (+u|i,h) f_1(i)$$

compute: $f_1(+i) = P(+t|+i)\cdot P(+i)\cdot P(+e|+t,+u)$

$$+ P(-t|+i)\cdot P(+i)\cdot P(+e|-t,+u)$$

$$= 0.8\times 0.7\times 0.9 + 0.2\times 0.7\cdot 0.7$$

$$= 0.602$$

$$f_1(-i) = P(+t|-i)\times P(-i)\times P(+e|+t,+u)$$

$$+ P(-t|-i)\times P(-i)\times P(+e|-t,u)$$

$$= 0.5\times 0.3\times 0.9 + 0.5\times 0.3\times 0.7$$

$$= 0.24$$

original equation $= \sum_h P(h) f_2(h)$

compute $f_2(+h) = (+u|+i,+h)\cdot f(+i) + (+u|-i,+h)\cdot f(-i)$

$$= 0.9\times 0.602 + 0.5\times 0.24$$

$$= 0.6618$$

$$f_2(-h) = (+u|+i,-h)\cdot f(+i) + (+u|-i,-h)\cdot f(-i)$$

$$= 0.3\times 0.602 + 0.1\times 0.24$$

$$= 0.2046$$

Now compute $f_3 = \sum_h P(h) f_2(h)$

$= 0.6 \times 0.6618 + 0.4 \times 0.2046$

$= 0.47892$

As a result, $P(+u, +e) = 0.47892$

To calculate: $P(+e) = P(+u, +e) + P(-u, +e)$

Compute: $P(-u, +e) = \sum_h P(h) \sum_i P(-u|i,h) \sum_t P(t|i) \cdot P(i) \cdot P(+e|t,-u)$

$= \sum_h P(h) \sum_i P(-u|i,h) f_1(i)$

$f_1(+i) = P(+t|+i) \cdot P(+i) \cdot P(+e|+t,-u) + (-t|+i) \cdot P(+i) \cdot P(+e|-t,-u)$

$= 0.8 \cdot 0.7 \cdot 0.5 + 0.2 \times 0.7 \times 0.3 = 0.322$

$f_1(-i) = P(+t|-i) \cdot P(-i) \cdot P(+e|+t,-u) + (-t|-i) \cdot P(-i) \cdot P(+e|-t,-u)$

$= 0.5 \times 0.3 \times 0.5 + 0.5 \times 0.3 \times 0.3 = 0.12$

orginal equation $= \sum_h P(h) f_2(h)$

$f_2(+h) = (-u|+i,+h) \cdot f(+i) + (-u|-i,+h) \cdot f(-i)$

$= 0.1 \cdot 0.322 + 0.5 \cdot 0.12 = 0.0922$

$f_2(-h) = (-u|+i,-h) \cdot f(+i) + (-u|-i,-h) \cdot f(-i)$

$= 0.7 \cdot 0.322 + 0.9 \times 0.12 = 0.3334$

$f_3 = \sum_h P(h) \cdot f_2(h) = 0.6 \cdot 0.0922 + 0.4 \cdot 0.3334 = 0.18868$

$P(+u|+e) = \dfrac{P(+u,+e)}{P(+e)} = \dfrac{P(+u,+e)}{P(+u,+e) + P(-u,+e)}$

$= \dfrac{0.47892}{0.47892 + 0.18868} \approx 0.7174$

1.T and U are independent (FALSE)

There are a total of 2 paths from T to U.

First Path: T -- E -- U
The structure of this path is: T → E ← U

Due to the fact that E is not given and none of E's descendants are given. We can say that this path is blocked

Second Path: T -- I --U
The structure of this path is T ← I → U
Due to the fact that I is not given, we can say that path is not blocked.
As a result, T and U are not independent.

2. T and U are conditionally independent given I, E, and H (FALSE)

There are a total of 2 paths from T to U.

First Path: T -- E -- U
The structure of this path is: T → E ← U
Due to the fact that E are given we can say that this bath is not blocked.

We do not need to check other paths since we have found a path that is not blocked.
As a result, T and U are not conditionally independent given I,E,H

3. T and U are conditionally independent given I and H (TRUE)

There are a total of 2 paths from T to U.

First Path: T -- E -- U
The structure of this path is: T → E ← U
Due to the fact that E is not given and none of E's descendants are given. We can say that this path is blocked

Second Path: T -- I --U
The structure of this path is T ← I → U
Due to the fact that I is given, we can say that this path is blocked
As a result, T and U are conditionally independent given I and H.

4. E and H are conditionally independent given U. (FALSE)
There are two paths from E to H.
First path: E -- U -- H
The structure of this path is: E ← U ← H
Due to the fact that U is given, this path is blocked

Second path: E -- T -- I -- U --H
Check each triple in this path:

1. E ← T ← I (not blocked since G is not given)
2. T ← I → U (not blocked since I is not given)
3. I → U ← H (not blocked since U is given)

As a result, the path E -- T -- I -- U --H is not blocked

E and H are not conditionally independent given U.

5. E and H are conditionally independent given U, I, and T. (TRUE)

There are two paths from E to H.
First path: E -- U -- H
The structure of this path is: E ← U ← H
Due to the fact that U is given, this path is blocked

Second path: E -- T -- I -- U --H
Check each triple in this path:

1. E ← T ← I (blocked since T is given)

Found a blocked triple path, no need to move forward
As a result, the path E -- T -- I -- U --H is blocked.

As a result, E and H are conditionally independent given U, I, and T.

6. I and H are conditionally independent given E. (False)
There are two paths from I to H.
First Path: I -- U -- H
The structure of this path is: I → U ← H
In this case U is not given. However, E, U's descendants , is given.
As a result, this path is not blocked and we do not need to check other paths.

7. I and H are conditionally independent given T (TRUE)

There are two paths from I to H.

First Path: I -- U -- H

The structure of this path is: $I \rightarrow U \leftarrow H$

In this case U is not given and none of U's descendants is given, this path is blocked.

Second path: I --T -- E -- U -- H

Check every triple:

1. $I \rightarrow T \rightarrow E$ (blocked since T is given)

No need to check the rest and the second path is blocked.

As a result, I and H are conditionally independent given T.

8. T and H are independent (TRUE)

There are are two paths between T and H.

First Path: T -- I --U --H

Check every triple in this path:

1. $T \leftarrow I \rightarrow U$ (I is not given, not blocked)
2. $I \rightarrow U \leftarrow H$ (U and its descendants are not given, blocked)

Second path; T -- E -- U -- H

Check every triple in this path:

1. $T \rightarrow E \leftarrow U$ (E is not given and have not descendants, blocked)

No need to check other triple.

As a result, T and H are independent.

9. T and H are conditionally independent given E. (FALSE)

There are are two paths between T and H.

First Path: T -- I --U --H

Check every triple in this path:

1. $T \leftarrow I \rightarrow U$ (I is not given, not blocked)
2. $I \rightarrow U \leftarrow H$ (U's descendants E is given, not blocked)

Since the first path is already not blocked.

T and H are not conditionally independent given E.

10. T and H are conditionally independent given E and U (FALSE)

First Path: T -- I --U --H

Check every triple in this path:

1. $T \leftarrow I \rightarrow U$ (I is not given, not blocked)
2. $I \rightarrow U \leftarrow H$ (U is given, not blocked)

As a result, this path is not blocked, T and H are not conditionally independent given E and U.