Program Structures and Algorithms

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1 Task: RandomWalk

2 Conclusion

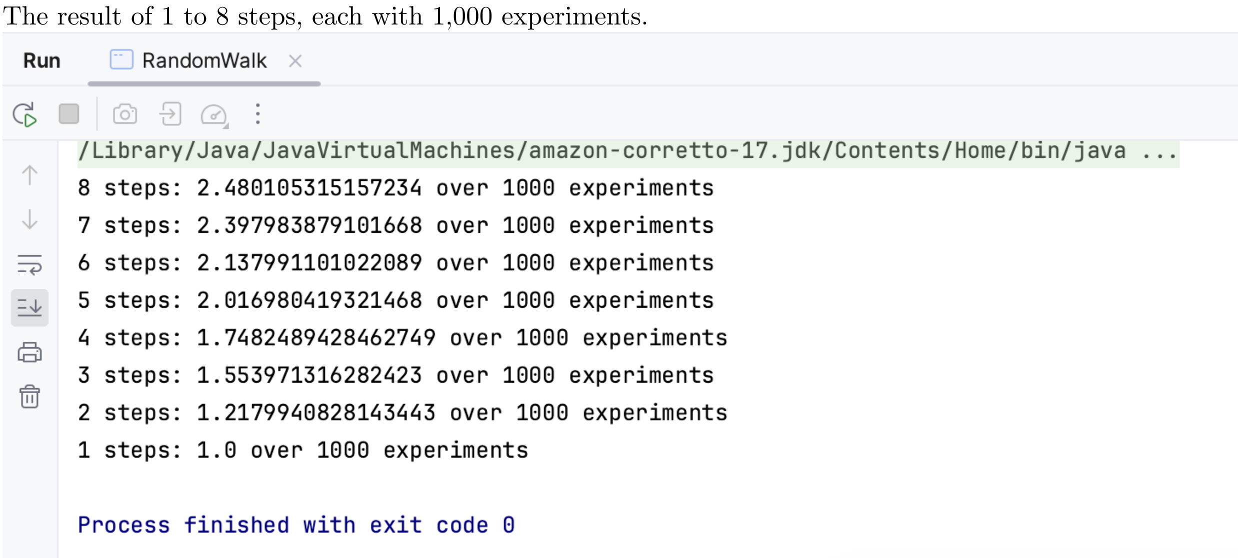
There exists a constant x, so that $d^x \propto m$. After calculating and testing, $x \approx 2.1$, thus, we could say

$$d^{2.1} \propto m$$

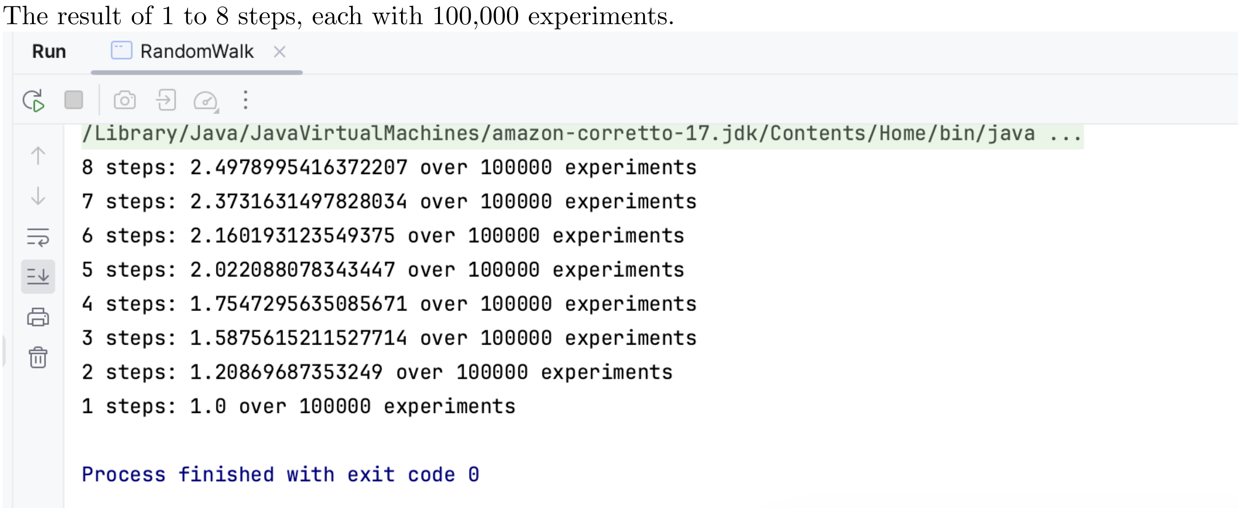
If we simplify further, we see that $d = \sqrt{m}$.

3 Evidence

By running more experiments on different steps, it appears that as the quantity of experiments increases significantly, the average of m steps gravitates towards a specific value. As evident in the following screenshots, using the example of 8 steps, when the number of experiments increases from 1,000 to 10,000,000, the average distance increasingly approximates 2.50.
The result of 1 to 8 steps, each with 1,000 experiments.



The result of 1 to 8 steps, each with 100,000 experiments.



The result of 1 to 8 steps, each with 10,000,000 experiments.

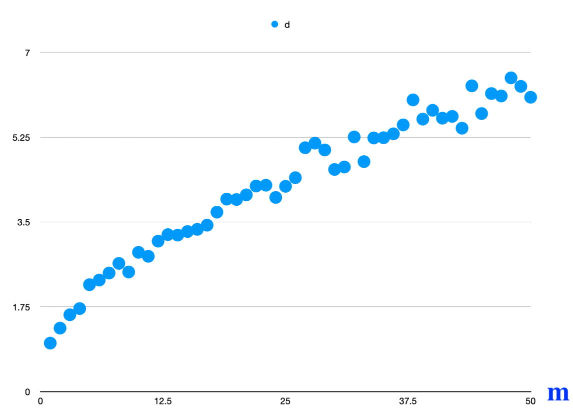
```
Run    RandomWalk
/Library/Java/JavaVirtualMachines/amazon-corretto-17.jdk/Contents/Home/bin/java ...
8 steps: 2.5019684396396307 over 10000000 experiments
7 steps: 2.3743983299462337 over 10000000 experiments
6 steps: 2.161117002598111 over 10000000 experiments
5 steps: 2.0192316682168943 over 10000000 experiments
4 steps: 1.752922793879511 over 10000000 experiments
3 steps: 1.5884876005690869 over 10000000 experiments
2 steps: 1.2068515012571677 over 10000000 experiments
1 steps: 1.0 over 10000000 experiments

Process finished with exit code 0
```

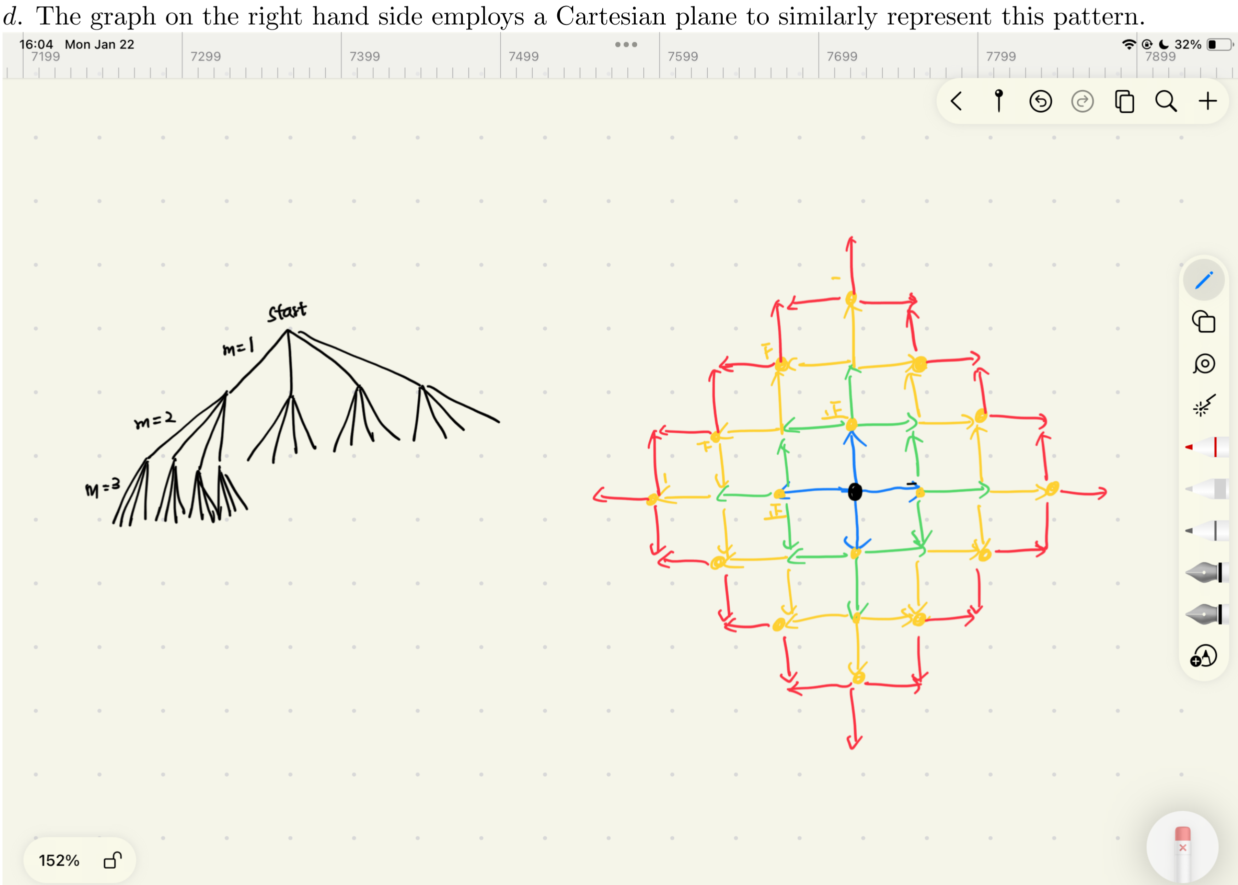
If m falls within the range of $1 - 50$, and we represent each corresponding value of d, the resulting graph appears as shown below.

Table 1

m	d
1	1
2	1.30953318805774
3	1.58498447189992
4	1.7132571111152
5	2.20622677231019
6	2.30244357735359
7	2.44685458334945
8	2.64586320400746
9	2.46782943111834
10	2.87403113671183
11	2.79271320341481
12	3.10500930521652
13	3.23989364058809
14	3.23030838692565
15	3.30381757546736
16	3.34815940504823
17	3.43484615272132
18	3.7040918538462
19	3.97348296597678
20	3.96482423602793
21	4.06099252393569
22	4.2430124212509
23	4.25970879913755
24	4.007497939968
25	4.23716980699841
26	4.41495179260279
27	5.03353584894956
28	5.12891588345788
29	4.9881747896184
30	4.58487106611913
31	4.63592649424437
32	5.25570363824746
33	4.74703061195663
34	5.23593210051403
35	5.2402678007176



I've utilized two graphs to identify the pattern. On the left, a tree diagram illustrates that every time m increases by one, the number of potential d scenarios experiences an exponential growth of four. For example, when m equals 1, there are four instances of d. When m equals 2, there are 4^2 which equals 16 instances of d. The graph on the right hand side employs a Cartesian plane to similarly represent this pattern.



Therefore, an exponential relationship appears to exist between d and m, which I've proposed as $C * d^x = m$, with C and x being constants. Simplifying, this expression can be written as $d^x \propto m$. Then we have:

$$log(d^x) = log(m)$$

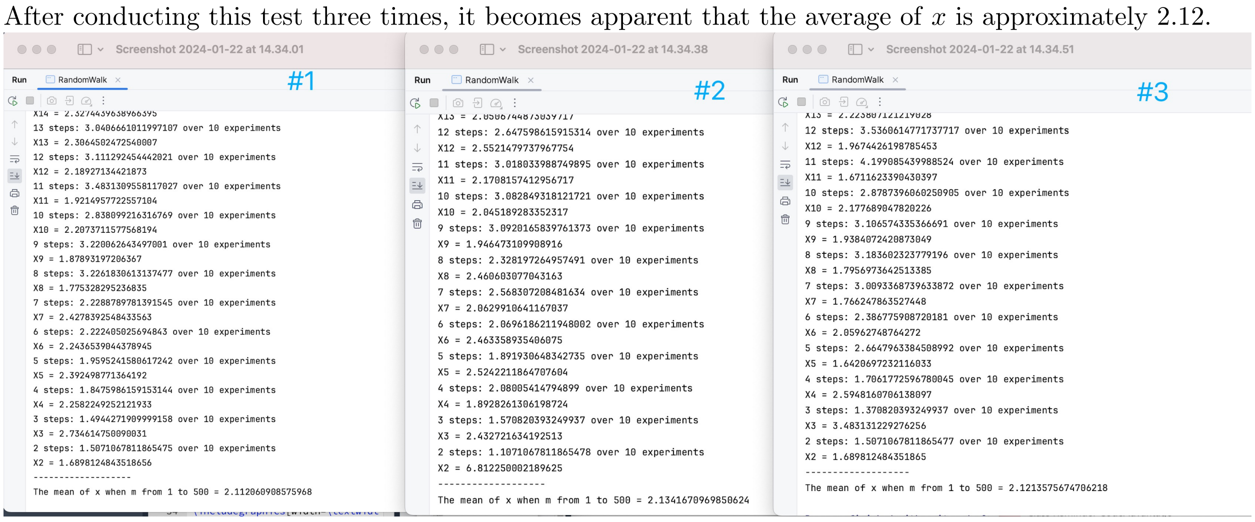
$$x * log(d) = log(m)$$

$$x = \frac{log(m)}{log(d)}$$

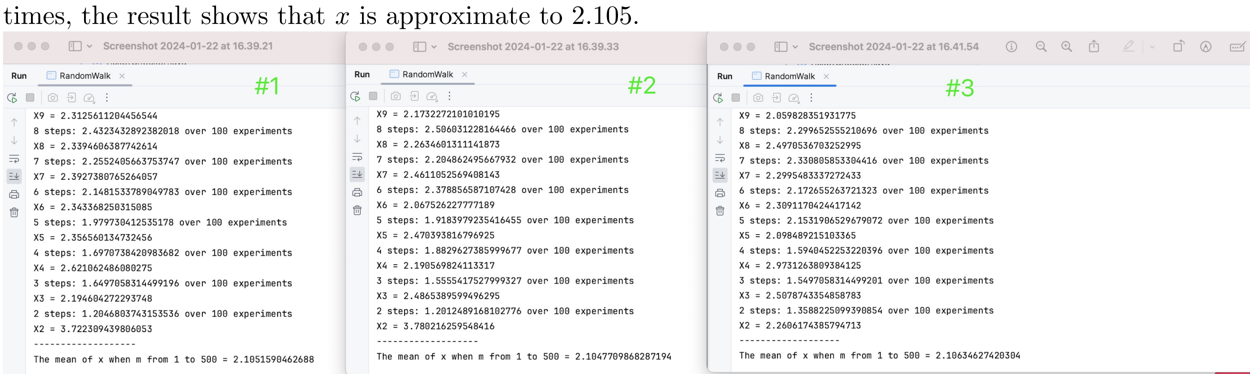
$$x = log(\frac{m}{d})$$

$$x = log_d(m)$$

Then, we run the code using m from $1 - 500$, each step with 10 experiments, to calculate the average of x. After conducting this test three times, it becomes apparent that the average of x is approximately 2.12.



Then we run another test, when m from $1 - 500$ and each m with 100 experiments. After conducting three times, the result shows that x is approximate to 2.105.



Thus, we deduces that $x \approx 2.1$, then $d^{2.1} \propto m$.

4 Code

Please see GitHub repository.

5 Unit Test Screenshots