1 Transformers

1.1 Introduction

The Transformer is a deep learning architecture introduced in [Vaswani et al., 2017]. It is a sequence-to-sequence model that uses attention to learn the relationships between the input and output sequences.

In this report we will follow the notation that is used in [Vaswani et al., 2017] where embeddings are represented as rows $\in \mathbb{R}^{1\times d}$ and all matrix multiplications are right multiplications.

1.2 Embedding

A machine is not capable of understand wordings, hence we need to transform it into a vector representation called an *embedding*. Let us denote the embedding of the i-th word in the input sequence as $\boldsymbol{x}_i \in \mathbb{R}^{1\times d}$, where d is the feature dimension. The transformer is able to process these embeddings in **parallel** so we need a way to encode the position of the word in the sequence. We do this by adding a positional encoding $\boldsymbol{p}^{(i)} \in \mathbb{R}^{1\times d}$ to the embedding $\boldsymbol{x}_i$. The positional encoding is a vector that is unique to the position of the word in the sequence. The positional encoding that is used in [Vaswani et al., 2017] is given by:

$$\boldsymbol{p}_j^{(i)} = \begin{cases} \sin\left(\frac{j}{10000^{2i/d}}\right) & \text{if } i \text{ is even} \\ \cos\left(\frac{j}{10000^{2i/d}}\right) & \text{if } i \text{ is odd} \end{cases} \tag{1}$$

where j is the dimension of the positional encoding. The positional encoding is added to the embedding as follows:

$$\boldsymbol{x}_i = \boldsymbol{x}_i + \boldsymbol{p}^{(i)} \in \mathbb{R}^{1\times d} \tag{2}$$

These are all stacked together to form the input matrix $\boldsymbol{X} \in \mathbb{R}^{N\times d}$, where $\boldsymbol{X}_{i:} = \boldsymbol{x_i}$ where N is the number of words in the input sequence.

Alternative Positional Encoding

There are many ways to go about positional encoding. Another way is to use a learned positional encoding. This is done by adding a learnable vector $\boldsymbol{p}^{(i)} \in \mathbb{R}^{1\times d}$ to the embedding. Alternatively to achieve translation equivariance, we can use *Relative Positional Encoding* [Shaw et al., 2018, Wu et al., 2021]. These positional encodings schemes will be useful when trying to build in equivariance into the Transformer model.

1.3 Attention

The attention mechanism is a way to learn the relationships between the input and output sequences.

References

[Shaw et al., 2018] Shaw, P., Uszkoreit, J., and Vaswani, A. (2018). Self-attention with relative position representations.

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[Wu et al., 2021] Wu, K., Peng, H., Chen, M., Fu, J., and Chao, H. (2021). Rethinking and improving relative position encoding for vision transformer.