(1) $\underset{x}{\text{minimize}} \quad \frac{1}{2} x^T Q x + p^T z$

subject to $\quad Gx \leq h$

$G_2 =$

$$\begin{bmatrix} -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix} \leq \begin{bmatrix} -0.1 \\ -0.1 \\ -0.1 \\ -0.1 \\ -0.1 \\ -0.1 \end{bmatrix}$$

$\frac{23}{\frac{12}{1}}$

Form 6 x 12 shown on next page

$$G_1 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix} \leq \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$

Lagrangian of (1) is given by

$$L(x, \lambda) = \frac{1}{2} x^T Q x + p^T z + \lambda^T (Gx - h) \quad , \quad \lambda \geq 0$$

KKT conditions for stationarity, primal feasibility and complementary slackness are

$$Qx^* + p + \cancel{A} G^T \lambda^* = 0$$

$$D(\lambda^*)(Gx^* - h) = 0$$

$D(\cdot)$ creates a diagonal matrix from a vector. Then the differentials of the KKT conditions:

$$dQ x^* + Q\,dx + dp + \cancel{dA^T \nu^*} + dG^T \lambda^* + G^T d\lambda = 0$$

$$D(Gx^* - h)\,d\lambda + D(\lambda^*)(dG x^* + G\,dx - dh) = 0$$

In matrix form, we have

$$\begin{bmatrix} Q & G^T \\ D(\lambda^+) G & D(Gz^+ - h) \end{bmatrix} \begin{bmatrix} dz \\ d\lambda \end{bmatrix} = \begin{bmatrix} -dQ z^+ - dp - dG^T \lambda^+ \\ -D(\lambda^+) dG z^+ + D(\lambda^+) dh \end{bmatrix}$$

Form Jacobians of z^+

$$\begin{bmatrix} d_z \\ d_\lambda \end{bmatrix} = \begin{bmatrix} Q & G^T D(\lambda^+) \\ G & D(Gz^+ - h) \end{bmatrix}^{-1} \begin{bmatrix} \left(\frac{\partial \ell}{\partial z^+}\right)^T \\ 0 \end{bmatrix}$$

$$\frac{\partial \ell}{\partial p} = d_z \qquad \frac{\partial \ell}{\partial G} = D(\lambda^+)(d_\lambda x^T + \lambda d_x^T)$$

$$\frac{\partial \ell}{\partial h} = -D(\lambda^*) d_\lambda \qquad \frac{\partial \ell}{\partial Q} = \frac{1}{2}(d_x x^T + x d_x^T)$$

α is the largest step size that maintains dual feasibility and $\sigma > 0$.

&

Symmetricized version of linear system:

$$K_{sym} = \begin{bmatrix} Q & 0 & G^T \\ 0 & D(\tfrac{\lambda}{s}) & I \\ G & I & 0 \end{bmatrix}$$

where $D(\lambda/s)$ is the only version that changes between iterations. Analytically decompose the systems into smaller symmetric systems and pre-factorize portions that do not change between iterations.

Compute affine scaling directions by solving

$$K \begin{bmatrix} \Delta z^{aff} \\ \Delta s^{aff} \\ \Delta \lambda^{aff} \end{bmatrix} = \begin{bmatrix} -(G^T \lambda + Qz + p) \\ -S\lambda \\ -(Gz + s - h) \end{bmatrix}$$

where

$$K = \begin{bmatrix} Q & 0 & G^T \\ 0 & D(\lambda) & D(s) \\ G & I & 0 \end{bmatrix}$$

Centering plus corrector directions:

$$K \begin{bmatrix} \Delta z^{cc} \\ \Delta s^{cc} \\ \Delta \lambda^{cc} \end{bmatrix} = \begin{bmatrix} 0 \\ \sigma \mu 1 - D(\Delta s^{aff}) \Delta \lambda^{aff} \\ 0 \end{bmatrix}$$

$$K_{sym}\begin{bmatrix} d_x \\ d_s \\ d\lambda \end{bmatrix} = \begin{bmatrix} \left(-\frac{\partial L}{\partial x_{t+1}}\right)^T \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix}
-1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix}
\begin{bmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \\ z_5 \\ z_6 \\ z_1 \\ z_2 \\ z_3 \\ z_4 \\ z_5 \\ z_6 \end{bmatrix}
\leq
\begin{bmatrix} -0.1 \\ -0.1 \\ -0.1 \\ -0.1 \\ -0.1 \\ -0.1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$