APPENDIX A

Angular Momentum

A.1 Allowable angular momentum states for two particles.

The single-particle angular momentum operator J obeys the commutation relationship

$$J \times J = iJ \tag{A.1-1}$$

Ψ_{JM} is an eigenfunction of this operator and has the properties[16]

$$\begin{aligned} J^2\Psi_{JM} &= J(J+1)\Psi_{JM} \\ J_z\Psi_{JM} &= M\Psi_{JM} \end{aligned} \tag{A.1-2}$$

and the shift operators

$$J_{\pm} = J_x \pm iJ_y$$

have the property

$$J_{\pm}\Psi_{JM} = \sqrt{\{(J \mp M)(J \pm M + 1)\}}\Psi_{JM\pm 1} \tag{A.1-3}$$

Using the results and the fact that the total angular momenta J' for the nucleus is [17]

$$J' = \sum_{i=1}^{A} J_i \tag{A.1-4}$$

to deduce the angular momentum states of the many-body system.

We state the following theorems for ease of reference as we progress.

<u>Theorem 1.:</u> A many-particle wavefunction with z-component of angular momentum M which has the property that

$$J_+\Psi_{JM} = 0$$

is an eigenfunction of the operator J^2 and describes a state with angular momentum M.

<u>Theorem 2.:</u> In a given configuration $(j_1, j_2, ..., j_n)$ with n particles; if there are p distinct Slater determinants $\Phi_M(i)(i = 1,..., p)$ with the property $J_z\Phi_M(i) = M\Phi_M(i)$. If there are $(p+q)(q>0)$ linearly independent Slater determinants that have the property $J_z\Phi_{M-1}(i) = (M-1)\Phi_{M-1}(i)$, then q states with angular momentum $I = M-1$ can be constructed.

For the special case $n = 2$, Theorem 2 leads to the following result:

<u>Theorem 3.:</u> For two particles in the states j_1 *and* $j_2 (j_1 \neq j_2)$ the allowable angular momentum values are $I = j_1 + j_2, j_1 + j_2 - 1, j_1 + j_2 - 2, ..., |j_1 - j_2|$

<u>Theorem 4.:</u> Two neutrons or two protons in the same single-particle orbit j (j being a half integer) can only couple their spins to even values of I. That is $I = 0,2,4,...,(2j-1)$

A.2 Allowable states of the configuration j^n

For identical nucleons in the same single-particle orbit, we find that there is only one such state with the maximum value of

$M_{\max} = [n(2j+1-n)]/2$ which is also an eigenstate of J^2 with

$J = M_{\max} = \frac{1}{2} n(2j+1-n)$. Hence,

<u>Theorem 5.:</u> The maximum possible angular momentum that can arise in the configuration j^n is

$$I_M = n\{j-(n-1)/2\}$$

<u>Theorem 6.:</u> There is no state of the configuration j^n with $I = I_M - 1$

<u>Theorem 7.:</u> In the configuration j^n, there is a state with

$$\begin{aligned} I &= I_M - 2 \\ &= n\{j - {}^{(n-1)}\!/_2\} - 2 \end{aligned}$$

Since we will be investigating the c.f.p's of the following configurations of the sd-shell: $(d_{5/2})^4$, $(d_{5/2})^3$, $(d_{5/2})^2$, $(d_{3/2})^4$, $(d_{3/2})^3$, and $(d_{3/2})^2$. Along with the sum method, we have devised a simple and straightforward way to enumerate all the states with the various J values. This is for particles with the same values of n, l, and j in the m-scheme.

TABLE A.1: $(d_{5/2})^4$ configuration's possible states in the m-scheme

M	Possible States
4	$\left(\frac{5}{2}\,\frac{3}{2}\,\frac{1}{2}\,-\frac{1}{2}\right)$
3	$\left(\frac{5}{2}\,\frac{3}{2}\,\frac{1}{2}\,-\frac{3}{2}\right)$
2	$\left(\frac{5}{2}\,\frac{3}{2}\,-\frac{1}{2}\,-\frac{3}{2}\right)\left(\frac{5}{2}\,\frac{3}{2}\,\frac{1}{2}\,-\frac{5}{2}\right)$
1	$\left(\frac{5}{2}\,\frac{1}{2}\,-\frac{1}{2}\,-\frac{3}{2}\right)\left(\frac{5}{2}\,\frac{3}{2}\,-\frac{1}{2}\,-\frac{5}{2}\right)$
0	$\left(\frac{5}{2}\,\frac{3}{2}\,-\frac{3}{2}\,-\frac{5}{2}\right)\left(\frac{5}{2}\,\frac{1}{2}\,-\frac{1}{2}\,-\frac{5}{2}\right)\left(\frac{3}{2}\,\frac{1}{2}\,-\frac{1}{2}\,-\frac{3}{2}\right)$

Possible J values: 4, 2, 0. These are true because the re is no state with J = 3 since the number of independent states in the m-scheme with M = 3 is same as that with M = 4.

TABLE A.2: $(d_{5/2})^3$ configuration's possible states in the m-scheme

M	Possible States
$\frac{9}{2}$	$\left(\frac{5}{2}\,\frac{3}{2}\,\frac{1}{2}\right)$
$\frac{7}{2}$	$\left(\frac{5}{2}\,\frac{3}{2}\,-\frac{1}{2}\right)$
$\frac{5}{2}$	$\left(\frac{5}{2}\,\frac{3}{2}\,-\frac{3}{2}\right)\left(\frac{5}{2}\,\frac{1}{2}\,-\frac{1}{2}\right)$
$\frac{3}{2}$	$\left(\frac{5}{2}\,\frac{3}{2}\,-\frac{5}{2}\right)\left(\frac{3}{2}\,\frac{1}{2}\,-\frac{1}{2}\right)\left(\frac{5}{2}\,\frac{1}{2}\,-\frac{3}{2}\right)$
$\frac{1}{2}$	$\left(\frac{3}{2}\,\frac{1}{2}\,-\frac{3}{2}\right)\left(\frac{5}{2}\,\frac{1}{2}\,-\frac{5}{2}\right)\left(\frac{5}{2}\,-\frac{3}{2}\,-\frac{1}{2}\right)$

Possible J values: $\frac{9}{2}$, $\frac{5}{2}$, and $\frac{3}{2}$

TABLE A.3: $(d_{5/2})^2$ configuration's possible states in the m-scheme

M	Possible States
4	$\left(\frac{5}{2}\,\frac{3}{2}\right)$
3	$\left(\frac{5}{2}\,\frac{1}{2}\right)$
2	$\left(\frac{5}{2}\,-\frac{1}{2}\right)\left(\frac{3}{2}\,\frac{1}{2}\right)$
1	$\left(\frac{5}{2}\,-\frac{3}{2}\right)\left(\frac{3}{2}\,-\frac{1}{2}\right)$
0	$\left(\frac{5}{2}\,-\frac{5}{2}\right)\left(\frac{3}{2}\,-\frac{3}{2}\right)\left(\frac{1}{2}\,-\frac{1}{2}\right)$

TABLE A.4: $(d_{3/2})^4$ configuration's possible states in the m-scheme

M	Possible States
0	$\left(\frac{3}{2}\,\frac{1}{2}\,-\frac{1}{2}\,-\frac{3}{2}\right)$

Possible J value: 0

TABLE A.5: $(d_{3/2})^3$ configuration's possible states in the m-scheme

M	Possible States
$\frac{3}{2}$	$\left(\frac{3}{2}\,\frac{1}{2}\,-\frac{1}{2}\right)$
$\frac{1}{2}$	$\left(\frac{3}{2}\,\frac{1}{2}\,-\frac{3}{2}\right)$

Possible J-values: $\frac{3}{2}$

TABLE A.6: $(d_{3/2})^2$ configuration's possible states in the m-scheme

M	Possible States
2	$\left(\tfrac{3}{2}\ \tfrac{1}{2}\right)$
1	$\left(\tfrac{3}{2}\ -\tfrac{1}{2}\right)$
0	$\left(\tfrac{3}{2}\ -\tfrac{3}{2}\right)\left(\tfrac{1}{2}\ -\tfrac{1}{2}\right)$

Possible States: J = 2 and 0.

APPENDIX B

Construction of many-particle wavefunctions for the $d^4, d^3, and\ d^2$ configurations in the sd-shell

For the d-level in the sd-shell the spin-orbit coupling model of nuclei provides that the d-level be split into the two sub-levels $d_{5/2}\ and\ d_{3/2}$.

In the d^4 configuration, we have both the $d^4_{5/2}\ and\ d^4_{3/2}$ configurations according to the fore-going argument. In 4.1, we describe the wavefunctions of $d^4_{5/2}$ while in 4.2, we enumerate the wavefunctions of the $d^4_{3/2}$ configuration.

A.1 Wavefunctions for the $d^4_{5/2}$ configuration.

From the Theorem 5 in App. A, we find that the maximum J -value for the $d^4_{5/2}$ configuration is

$$\begin{aligned} J_{\max} &= 4\{5/2 - {}^{(4-1)}/2\} \\ &= 4 \end{aligned}$$

From theory 3 in App. A, we can deduce that in the configuration $(d_{5/2})^4$, the possible $J - values$ are 4, 3, 2, 1, 0. Since we have shown in

Table A.1 that the allowable $J-values$ in accordance with the Pauli principle are

$J = 4,2,\ and\ 0$.

We first enumerate the wavefunctions for the J = 4 state in the $(d_{5/2})^4$ state.

A.1.1 J = 4.

1a $\Phi_{4,4} = a^+_{5/2} a^+_{3/2} a^+_{1/2} a^+_{-1/2} |0\rangle$ where we have suppressed the $j = 5/2$ index since we are operating with the same j value all through.

b Suppose we represent the only term in the foregoing representation with x, we find that the normalization integral is 1 as follows:

$$\begin{aligned}\langle \Phi_{4,4} | \Phi_{4,4} \rangle &= \langle x | x \rangle \\ &= 1\end{aligned}$$

Since $\langle \Phi_{4,4} | \Phi_{4,4} \rangle = 1$, we rightly conclude that the wavefunction is normalized and our derived expression for it is correct.

The operator J_- is defined through the following

$$J_{-}\Phi_{JM} = \sqrt{\{(J+M)(J-M+1)\}}\Phi_{JM-1}$$

Appling the operator J_{-} to $\Phi_{4,4}$, we find

2a.

$$J_{-}\Phi_{4,4} = j_{-}^{(1)}\{a^{+}_{5/2}a^{+}_{3/2}a^{+}_{1/2}a^{+}_{-1/2}\left|0\right\rangle\} + j_{-}^{(2)}\{a^{+}_{5/2}a^{+}_{3/2}a^{+}_{1/2}a^{+}_{-1/2}\left|0\right\rangle\} + j_{-}^{(3)}\{a^{+}_{5/2}a^{+}_{3/2}a^{+}_{1/2}a^{+}_{-1/2}\left|0\right\rangle\} + j_{-}^{(4)}\{a^{+}_{5/2}a^{+}_{3/2}a^{+}_{1/2}a^{+}_{-1/2}\left|0\right\rangle\}$$

The first three terms in the above expression vanish because in applying the destruction operator to them, we obtain states with two quantum projection occurring in the same state. Such states are not allowed by the Pauli principle since the J_{-} operator produces a quantum state in which the projection quantum number is reduced by 1[24].

Hence,

$$2\sqrt{2}\Phi_{4,3} = 2\sqrt{2}a^{+}_{5/2}a^{+}_{3/2}a^{+}_{1/2}a^{+}_{-3/2}\left|0\right\rangle$$

from where

$$\underline{\Phi_{4,3} = a^{+}_{5/2}a^{+}_{3/2}a^{+}_{1/2}a^{+}_{-3/2}\left|0\right\rangle}$$

b.

$$\left\langle\Phi_{4,3}\middle|\Phi_{4,3}\right\rangle = \left\langle x\middle|x\right\rangle \qquad (here, x = a^{+}_{5/2}a^{+}_{3/2}a^{+}_{1/2}a^{+}_{-3/2}\left|0\right\rangle)$$
$$= 1$$

We therefore conclude that $\Phi_{4,3}$ is normalized.

Proceeding as before , we can obtain states of lower M but same J until we get to the point where M = - 4.

$$3.a \qquad J_{-}\Phi_{4,3} = j_{-}^{(3)}\{a_{5/2}^{+}a_{3/2}^{+}a_{1/2}^{+}a_{-3/2}^{+}|0\rangle\} + j_{-}^{(4)}\{a_{5/2}^{+}a_{3/2}^{+}a_{1/2}^{+}a_{-3/2}^{+}|0\rangle\}$$

$$or \qquad \sqrt{14}\Phi_{4,2} = \{3a_{5/2}^{+}a_{3/2}^{+}a_{-1/2}^{+}a_{-3/2}^{+}|0\rangle + \sqrt{5}a_{5/2}^{+}a_{3/2}^{+}a_{1/2}^{+}a_{-5/2}^{+}|0\rangle$$

$$or \qquad \underline{\Phi_{4,2} = \{3/\sqrt{14}\, a_{5/2}^{+}a_{3/2}^{+}a_{-1/2}^{+}a_{-3/2}^{+} + \sqrt{5/14}\, a_{5/2}^{+}a_{3/2}^{+}a_{1/2}^{+}a_{-5/2}^{+}\}|0\rangle}$$

If we represent the states on the r.h.s of $\Phi_{4,2}$ by x_1 *and* x_2, we find

$$\begin{aligned}
\langle\Phi_{4,2}|\Phi_{4,2}\rangle &= \left\langle 3/\sqrt{14}\, x_1 + \sqrt{5/14}\, x_2 \,\middle|\, 3/\sqrt{14}\, x_1 + \sqrt{5/14}\, x_2 \right\rangle \\
&= 9/14\, x_1^2 + 3\sqrt{5}/14\, x_1 x_2 + 3\sqrt{5}/14\, x_2 x_1 + 5/14\, x_2^2 \\
&= 9/14\, x_1^2 + 5/14\, x_2^2 \\
&= 9/14 + 5/14 \\
&= 14/14 \\
&= 1
\end{aligned}$$

Hence, $\Phi_{4,2}$ *is normalized*.

$$4.a \qquad J_-\Phi_{4,2} = \left(\tfrac{3}{\sqrt{14}}\{j_-^2(a^+_{5/2}a^+_{3/2}a^+_{-1/2}a^+_{-3/2}) + j_-^4(a^+_{5/2}a^+_{3/2}a^+_{-1/2}a^+_{-3/2})\}\right.$$
$$\left. + \sqrt{5/14}\{j_-^{(3)}(a^+_{5/2}a^+_{3/2}a^+_{1/2}a^+_{-5/2})\}\right)\left|0\right\rangle$$

where we have neglected the terms that vanish)

$$or \qquad 3\sqrt{2}\Phi_{4,1} = \left(\tfrac{3}{\sqrt{14}}\{2\sqrt{2}a^+_{5/2}a^+_{1/2}a^+_{-1/2}a^+_{-3/2} + \sqrt{5}a^+_{5/2}a^+_{3/2}a^+_{-1/2}a^+_{-5/2}\}\right.$$
$$\left. + \sqrt{5/14}\{3a^+_{5/2}a^+_{3/2}a^+_{-1/2}a^+_{-5/2}\}\right)\left|0\right\rangle$$
$$= \{\tfrac{6}{\sqrt{7}}a^+_{5/2}a^+_{1/2}a^+_{-1/2}a^+_{-3/2} + 3\sqrt{5/14}a^+_{5/2}a^+_{3/2}a^+_{-1/2}a^+_{-5/2} + 3\sqrt{5/14}a^+_{5/2}a^+_{3/2}a^+_{-1/2}a^+_{-5/2}\}\left|0\right\rangle$$
$$= \{\tfrac{6}{\sqrt{7}}a^+_{5/2}a^+_{1/2}a^+_{-1/2}a^+_{-3/2} + 6\sqrt{5/14}a^+_{5/2}a^+_{3/2}a^+_{-1/2}a^+_{-5/2}\}\left|0\right\rangle$$

Therefore,

$$\underline{\Phi_{4,1} = \{\tfrac{2}{\sqrt{14}}a^+_{5/2}a^+_{1/2}a^+_{-1/2}a^+_{-3/2} + 2\sqrt{5/28}a^+_{5/2}a^+_{3/2}a^+_{-1/2}a^+_{-5/2}\}\left|0\right\rangle}$$

b.

$$\left\langle \Phi_{4,1} \middle| \Phi_{4,1} \right\rangle = \left\langle \tfrac{2}{\sqrt{14}}x_1 + \sqrt{5/7}x_2 \middle| \tfrac{2}{\sqrt{14}}x_1 + \sqrt{5/7}x_2 \right\rangle$$
$$= \tfrac{4}{14}x_1^2 + \tfrac{5}{7}x_2^2$$
$$= \tfrac{7}{7}$$
$$= 1$$

Since the integral evaluates to 1, *we therefore conclude that the wavefunction is normalized*.

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$$J_-\Phi_{4,1} = (\tfrac{2}{\sqrt{14}}\{ j_-^{(1)}(a^+_{5/2}a^+_{1/2}a^+_{-1/2}a^+_{-3/2}) + j_-^{(4)}(a^+_{5/2}a^+_{1/2}a^+_{-1/2}a^+_{-3/2})$$

$$+\sqrt{5/7}\{ j_-^{(2)}(a^+_{5/2}a^+_{3/2}a^+_{-1/2}a^+_{-5/2}) + j_-^{(3)}(a^+_{5/2}a^+_{3/2}a^+_{-1/2}a^+_{-5/2})\})\left|0\right\rangle$$

$$or \qquad 2\sqrt{5}\Phi_{4,0} = (\tfrac{2}{\sqrt{14}}\{\sqrt{5}a^+_{3/2}a^+_{1/2}a^+_{-1/2}a^+_{-3/2} + \sqrt{5}a^+_{5/2}a^+_{1/2}a^+_{-1/2}a^+_{-5/2}\}$$

$$+\sqrt{5/7}\{2\sqrt{2}a^+_{5/2}a^+_{1/2}a^+_{-1/2}a^+_{-5/2} + 2\sqrt{2}a^+_{5/2}a^+_{3/2}a^+_{-3/2}a^+_{5/2}\})\left|0\right\rangle$$

$$or \qquad \underline{\Phi_{4,0} = \{\tfrac{1}{\sqrt{14}}a^+_{3/2}a^+_{1/2}a^+_{-1/2}a^+_{-3/2} + \tfrac{3}{\sqrt{14}}a^+_{5/2}a^+_{1/2}a^+_{-1/2}a^+_{-5/2} + \tfrac{2}{\sqrt{14}}a^+_{5/2}a^+_{3/2}a^+_{-3/2}a^+_{-5/2}\})\left|0\right\rangle}$$

The normalization integral is given by

$$\left\langle \Phi_{4,0} \middle| \Phi_{4,0} \right\rangle = \left\langle \tfrac{1}{\sqrt{14}}x_1 + \tfrac{3}{\sqrt{14}}x_2 + \tfrac{2}{\sqrt{14}}x_3 \middle| \tfrac{1}{\sqrt{14}}x_1 + \tfrac{3}{\sqrt{14}}x_2 + \tfrac{2}{\sqrt{14}}x_3 \right\rangle$$

$$= \tfrac{1}{14}x_1^2 + \tfrac{9}{14}x_2^2 + \tfrac{4}{14}x_3^2$$

$$= \tfrac{14}{14}$$

$$= 1$$

Since $\left\langle \Phi_{4,0} \middle| \Phi_{4,0} \right\rangle = 1$, *we conclude that* $\Phi_{4,0}$ *is normalized.*

$$\therefore \qquad \Phi_{4,1} = \{\tfrac{2}{\sqrt{14}}a^+_{3/2}a^+_{1/2}a^+_{-1/2}a^+_{-5/2} + \sqrt{5/7}a^+_{5/2}a^+_{1/2}a^+_{-3/2}a^+_{-5/2}\}\left|0\right\rangle$$

6. $J_-\Phi_{4,0}$

$$=2\sqrt{5}\Phi_{4,-1} = \left(\tfrac{1}{\sqrt{14}}\{\sqrt{5}a^+_{3/2}a^+_{1/2}a^+_{-1/2}a^+_{-5/2}\} + \tfrac{3}{\sqrt{14}}\{\sqrt{5}a^+_{3/2}a^+_{1/2}a^+_{-1/2}a^+_{-5/2} + 2\sqrt{2}a^+_{5/2}a^+_{1/2}a^+_{-3/2}a^+_{-5/2}\}\right.$$
$$\left. + \tfrac{2}{\sqrt{14}}\{2\sqrt{2}a^+_{5/2}a^+_{1/2}a^+_{-3/2}a^+_{-5/2}\}\right)|0\rangle$$
$$= \{\sqrt{\tfrac{5}{14}}a^+_{3/2}a^+_{1/2}a^+_{-1/2}a^+_{-5/2} + 3\sqrt{\tfrac{5}{14}}a^+_{3/2}a^+_{1/2}a^+_{-1/2}a^+_{-5/2} + \tfrac{6}{\sqrt{7}}a^+_{5/2}a^+_{1/2}a^+_{-3/2}a^+_{-5/2} + \tfrac{4}{\sqrt{7}}a^+_{5/2}a^+_{1/2}a^+_{-3/2}a^+_{-5/2}\}|0\rangle$$
$$= \{4\sqrt{\tfrac{5}{14}}a^+_{3/2}a^+_{1/2}a^+_{-1/2}a^+_{-5/2} + \tfrac{10}{\sqrt{7}}a^+_{5/2}a^+_{1/2}a^+_{-3/2}a^+_{-5/2}\}|0\rangle$$

$$\therefore \qquad \underline{\Phi_{4,-1} = \{\tfrac{2}{\sqrt{14}}a^+_{3/2}a^+_{1/2}a^+_{-1/2}a^+_{-5/2} + \sqrt{\tfrac{5}{7}}a^+_{5/2}a^+_{1/2}a^+_{-3/2}a^+_{-5/2}\}|0\rangle}$$

Suppose we put $x_1 = a^+_{3/2}a^+_{1/2}a^+_{-1/2}a^+_{-5/2}|0\rangle$ *and* $x_2 = a^+_{5/2}a^+_{1/2}a^+_{-3/2}a^+_{-5/2}\}|0\rangle$

$$\langle\Phi_{4,-1}|\Phi_{4,-1}\rangle = \left\langle \tfrac{2}{\sqrt{14}}x_1 + \sqrt{\tfrac{5}{7}}x_2 \middle| \tfrac{2}{\sqrt{14}}x_1 + \sqrt{\tfrac{5}{7}}x_2 \right\rangle$$
$$= \tfrac{4}{14}x_1^2 + \tfrac{10}{14}x_2^2$$
$$= \tfrac{4}{14} + \tfrac{10}{14}$$
$$= 1$$

$\therefore$ *the* exp*ression for* $\Phi_{4,-1}$ *is correct*.

7. $J_-\Phi_{4,-1}$

$$= 3\sqrt{2}\Phi_{4,-2} = \left(\tfrac{2}{\sqrt{14}}\{2\sqrt{2}a^+_{3/2}a^+_{1/2}a^+_{-3/2}a^+_{-5/2}\} + \sqrt{\tfrac{5}{7}}\{\sqrt{5}a^+_{3/2}a^+_{1/2}a^+_{-3/2}a^+_{-5/2} + 3a^+_{5/2}a^+_{-1/2}a^+_{-3/2}a^+_{-5/2}\}\right)|0\rangle$$
$$= \{\tfrac{4}{\sqrt{7}}a^+_{3/2}a^+_{1/2}a^+_{-3/2}a^+_{-5/2} + \sqrt{\tfrac{5}{7}}a^+_{3/2}a^+_{1/2}a^+_{-3/2}a^+_{-5/2} + \tfrac{15}{\sqrt{35}}a^+_{5/2}a^+_{-1/2}a^+_{-3/2}a^+_{-5/2}\}|0\rangle$$

$$\therefore \qquad \underline{\Phi_{4,-2} = \{\tfrac{3}{\sqrt{14}}a^+_{3/2}a^+_{1/2}a^+_{-3/2}a^+_{-5/2} + \sqrt{\tfrac{5}{14}}a^+_{5/2}a^+_{-1/2}a^+_{-3/2}a^+_{-5/2}\}|0\rangle}$$

$$I_{4,-2} = \left\langle \Phi_{4,-2} \middle| \Phi_{4,-2} \right\rangle = \left\langle {}^{3}\!/\!_{\sqrt{14}}\, x_1 + \sqrt{{}^{5}\!/\!_{14}}\, x_2 \middle| {}^{3}\!/\!_{\sqrt{14}}\, x_1 + \sqrt{{}^{5}\!/\!_{14}}\, x_2 \right\rangle$$

$$= {}^{9}\!/\!_{14}\, x_1^2 + {}^{5}\!/\!_{14}\, x_2^2$$

$$= 1$$

Hence, $\Phi_{4,-2}$ *is normalized since the normalization integral is unity.*

8. $J_{-}\Phi_{4,-2} = \sqrt{14}\,\Phi_{4,-3}$

$$= \left({}^{3}\!/\!_{\sqrt{14}} \{ 3 a^{+}_{3/2} a^{+}_{-1/2} a^{+}_{-3/2} a^{+}_{-5/2} \} + \sqrt{{}^{5}\!/\!_{14}} \{ \sqrt{5}\, a^{+}_{3/2} a^{+}_{-1/2} a^{+}_{-3/2} a^{+}_{-5/2} \} \right) \left| 0 \right\rangle$$

or $\sqrt{14}\,\Phi_{4,-3} = \{ {}^{9}\!/\!_{\sqrt{14}}\, a^{+}_{3/2} a^{+}_{-1/2} a^{+}_{-3/2} a^{+}_{-5/2} + {}^{5}\!/\!_{\sqrt{14}}\, a^{+}_{3/2} a^{+}_{-1/2} a^{+}_{-3/2} a^{+}_{-5/2} \}) \left| 0 \right\rangle$

or $\Phi_{4,-3} = a^{+}_{3/2} a^{+}_{-1/2} a^{+}_{-3/2} a^{+}_{-5/2} \left| 0 \right\rangle$

$\therefore$ $\underline{\Phi_{4,-3} = a^{+}_{3/2} a^{+}_{-1/2} a^{+}_{-3/2} a^{+}_{-5/2} \left| 0 \right\rangle}$

Suppose we put $x_1 = a^{+}_{3/2} a^{+}_{-1/2} a^{+}_{-3/2} a^{+}_{-5/2} \left| 0 \right\rangle$ *in the foregoing, we find that*

$$\left\langle \Phi_{4,-3} \middle| \Phi_{4,-3} \right\rangle = \left\langle x_1 \middle| x_1 \right\rangle$$

$$= x_1^2 = 1$$

whence $\Phi_{4,3}$ *is normalized.*

9. $J_{-}\Phi_{4,-3} = 2\sqrt{2}\,\Phi_{4,-4} = j^{(1)}_{-} \{ a^{+}_{3/2} a^{+}_{-1/2} a^{+}_{-3/2} a^{+}_{-5/2} \} \left| 0 \right\rangle$

$$= 2\sqrt{2}\, a^{+}_{1/2} a^{+}_{-1/2} a^{+}_{-3/2} a^{+}_{-5/2} \left| 0 \right\rangle$$

$\therefore$ $\underline{\Phi_{4,-4} = a^{+}_{1/2} a^{+}_{-1/2} a^{+}_{-3/2} a^{+}_{-5/2} \left| 0 \right\rangle}$

It is easy to see that $\Phi_{4,-4}$ *is normalised as* $\left\langle \Phi_{4,-4} \middle| \Phi_{4,-4} \right\rangle = 1$

A.1.2 J = 3

In order to find the wavefunctions for the J = 3 state of the $(d_{5/2})^4$ configuration. we note that the presupposed wavefunctions are $\Phi_{3,3}, \Phi_{3,2}, \Phi_{3,1}, \Phi_{3,0}, \Phi_{3,-1}, \Phi_{3,-2}, \Phi_{3,-3.}$

Since, $\Phi_{3,3}$ should be orthogonal to $\Phi_{4,3}$ from our earlier arguments. Representing $\Phi_{3,3}$ as

$$\Phi_{3,3} = ca^+_{5/2}a^+_{3/2}a^+_{1/2}a^+_{-3/2}|0\rangle \textit{ and } \sin ce$$

$$\Phi_{4,3} = a^+_{5/2}a^+_{3/2}a^+_{1/2}a^+_{-3/2}|0\rangle \qquad \textit{from } A.1.1-2$$

$$\langle\Phi_{3,3}|\Phi_{4,3}\rangle = \langle cx_1|x_1\rangle \qquad = cx_1^2 = 0$$

$$\textit{whence } c = 0.$$

$$\therefore \quad \underline{\Phi_{3,3} = 0}$$

Applying the operator J_- repeatedly to $\Phi_{3,3}$, we find that

2. $J_-\Phi_{3,3} = 3\Phi_{3,2} = 0$

   or $\underline{\Phi_{3,2} = 0}$

3. $J_-\Phi_{3,2} = \sqrt{5}\,\Phi_{3,1} = 0$

   or $\underline{\Phi_{3,1} = 0}$

4. $J_-\Phi_{3,1} = 2\sqrt{3}\,\Phi_{3,0} = 0$

   or $\underline{\Phi_{3,0} = 0}$

5. $J_-\Phi_{3,0} = 2\sqrt{3}\,\Phi_{3,-1} = 0$

   or $\underline{\Phi_{3,-1} = 0}$

6. $J_-\Phi_{3,-1} = \sqrt{10}\,\Phi_{3,-2} = 0$

or $\underline{\Phi_{3,-2} = 0}$

7. $J_-\Phi_{3,-2} = \sqrt{7}\,\Phi_{3,-3} = 0$

or $\underline{\Phi_{3,-3} = 0}$

Thus we see that all wavefunctions in the J = 3 state of the $(d_{5/2})^4$ *configuration* are all null.

A.1.3 J = 2

Representing $\Phi_{2,2}$ by the linear combination:

$$\Phi_{2,2} = \{x_1 a^+_{5/2} a^+_{3/2} a^+_{-1/2} a^+_{-3/2} + x_2 a^+_{5/2} a^+_{3/2} a^+_{1/2} a^+_{-5/2}\}\left|0\right\rangle \quad .... \quad ... \quad ...(1)$$

Since $\Phi_{4,2} = \{3/\sqrt{14}\, a^+_{5/2} a^+_{3/2} a^+_{-1/2} a^+_{-3/2} + \sqrt{5/14}\, a^+_{5/2} a^+_{3/2} a^+_{1/2} a^+_{-5/2}\}\left|0\right\rangle$

and the orthogonality condition requires that $\Phi_{2,2}$ be orthogonal to states with higher J values, we conclude that (1) $\left\langle \Phi_{2,2} \middle| \Phi_{4,2} \right\rangle$ *must be zero.*

Moreso, the normalization condition tells us $\left\langle \Phi_{2,2} \middle| \Phi_{2,2} \right\rangle$ must equal 1. Therefore,

$x_1^2 + x_2^2 = 1$ ... ... (2)

and $3/\sqrt{14}\, x_1 + \sqrt{5/14}\, x_2 = 0$

or $x_1 = -\sqrt{5}/3\, x_2$ ... ... (3)

Putting (3) into (2), *we find that*

$5/9\, x_2^2 + x_2^2 = 1$

$or \quad {}^{14}\!/_{9}\, x_2^2 = 1$

$$or \quad x_2 = \pm\, {}^{3}\!/_{\sqrt{14}} \qquad \ldots \qquad \ldots \qquad \ldots \qquad (4)$$

$putting\ (4)\ \text{int}\, o\ (3), we\ find\ that$

$$x_1 = -\,{}^{\sqrt{5}}\!/_{3}\,(\pm\, {}^{3}\!/_{\sqrt{14}}) = \mp\sqrt{{}^{5}\!/_{14}} \qquad \ldots \qquad \ldots \qquad \ldots \qquad (5)$$

$$Hence\ x_1 = \mp\sqrt{{}^{5}\!/_{14}}\ and\ x_2 = \pm\, {}^{3}\!/_{\sqrt{14}}$$

$\therefore\ Eq.(1)\ becomes$

$$\Phi_{2,2} = \left\{\mp\sqrt{{}^{5}\!/_{14}}\, a^{+}_{5/2} a^{+}_{3/2} a^{+}_{-1/2} a^{+}_{-3/2} \pm {}^{3}\!/_{\sqrt{14}}\, a^{+}_{5/2} a^{+}_{3/2} a^{+}_{1/2} a^{+}_{-5/2}\right\}|0\rangle$$

In the foregoing derived wavefunction, we are posed with a question of which phase to adopt. The choice of phase will determine the mathematical symmetry of the wavefunction. The mathematical symmetry/asymmetry does not represent any physical symmetry or asymmetry between the two states(pp. 95-105; Nuclear Shell Theory by de-Shalit, A., and Talmi, I.).Whenever the phase of a function is of any importance, it is the relative phase of the two states that really matters . If the phase (relative phase) can be measured, it is unaffected by any phase convention. The purpose of having a definite phase convention is to facilitate the actual computations of different phenomena and has nothing to do with physical properties of the system in consideration.

Adopting the positive sign of the square root in (4), we re-write $\Phi_{2,2}$ as follows:

1. $$\Phi_{2,2} = \left\{ \tfrac{3}{\sqrt{14}} a^{+}_{5/2} a^{+}_{3/2} a^{+}_{1/2} a^{+}_{-5/2} - \sqrt{\tfrac{5}{14}} a^{+}_{5/2} a^{+}_{3/2} a^{+}_{-1/2} a^{+}_{-3/2} \right\} |0\rangle$$

$$\langle \Phi_{2,2} | \Phi_{2,2} \rangle = \left\langle \tfrac{3}{\sqrt{14}} x_1 - \sqrt{\tfrac{5}{14}} x_2 \,\middle|\, \tfrac{3}{\sqrt{14}} x_1 - \sqrt{\tfrac{5}{14}} x_2 \right\rangle$$
$$= \tfrac{9}{14} x_1^2 - \tfrac{3}{14}\sqrt{5} x_1 x_2 - \tfrac{3}{14}\sqrt{5} x_2 x_1 + \tfrac{5}{14} x_2^2$$
$$= \tfrac{9}{14} + \tfrac{5}{14} = 1$$

Hence, $\Phi_{2,2}$ *is normalized.*

Applying the J_- *operator to* $\Phi_{2,2}$*, we find that*

2. $$2\Phi_{2,1} = \left(\tfrac{3}{\sqrt{14}} \{3 a^{+}_{5/2} a^{+}_{3/2} a^{+}_{-1/2} a^{+}_{-5/2}\} - \sqrt{\tfrac{5}{14}} \{2\sqrt{2} a^{+}_{5/2} a^{+}_{1/2} a^{+}_{-1/2} a^{+}_{-3/2} + \sqrt{5} a^{+}_{5/2} a^{+}_{3/2} a^{+}_{-1/2} a^{+}_{-5/2} \} |0\rangle$$
$$= \{ \tfrac{9}{\sqrt{14}} a^{+}_{5/2} a^{+}_{3/2} a^{+}_{-1/2} a^{+}_{-5/2} - 2\sqrt{\tfrac{5}{7}} a^{+}_{5/2} a^{+}_{1/2} a^{+}_{-1/2} a^{+}_{-3/2} - \tfrac{5}{\sqrt{14}} a^{+}_{5/2} a^{+}_{3/2} a^{+}_{-1/2} a^{+}_{-5/2} \} |0\rangle$$
$$= \{ \tfrac{4}{\sqrt{14}} a^{+}_{5/2} a^{+}_{3/2} a^{+}_{-1/2} a^{+}_{-5/2} - 2\sqrt{\tfrac{5}{7}} a^{+}_{5/2} a^{+}_{1/2} a^{+}_{-1/2} a^{+}_{-3/2} \} |0\rangle$$

$$\therefore \quad \underline{\Phi_{2,1} = \{ \tfrac{2}{\sqrt{14}} a^{+}_{5/2} a^{+}_{3/2} a^{+}_{-1/2} a^{+}_{-5/2} - \sqrt{\tfrac{5}{7}} a^{+}_{5/2} a^{+}_{1/2} a^{+}_{-1/2} a^{+}_{-3/2} \} |0\rangle}$$

Check :

$$\langle \Phi_{2,1} | \Phi_{2,1} \rangle = \left\langle \tfrac{2}{\sqrt{14}} x_1 - \sqrt{\tfrac{5}{7}} x_2 \,\middle|\, \tfrac{2}{\sqrt{14}} x_1 - \sqrt{\tfrac{5}{7}} x_2 \right\rangle$$
$$= \tfrac{4}{14} x_1^2 + \tfrac{5}{7} x_2^2 = \tfrac{4}{14} \qquad + \tfrac{5}{7}$$
$$= 1$$

Since the int *egral* $\langle \Phi_{2,1} | \Phi_{2,1} \rangle = 1$, *we conclude that* $\Phi_{2,1}$ *is normalized.*

$J_{-}\Phi_{2,1}$:

3. $$\sqrt{6}\Phi_{2,0} = (\tfrac{2}{\sqrt{14}}\{2\sqrt{2}a^+_{5/2}a^+_{1/2}a^+_{-1/2}a^+_{-5/2} + 2\sqrt{2}a^+_{5/2}a^+_{3/2}a^+_{-3/2}a^+_{-5/2}\}$$
$$-\sqrt{5/7}\{a^+_{3/2}a^+_{1/2}a^+_{-1/2}a^+_{-3/2} + \sqrt{5}a^+_{5/2}a^+_{1/2}a^+_{-1/2}a^+_{-5/2}\})|0\rangle$$
$$= \{\tfrac{4}{\sqrt{7}}a^+_{5/2}a^+_{1/2}a^+_{-1/2}a^+_{-5/2} + \tfrac{4}{\sqrt{7}}a^+_{5/2}a^+_{3/2}a^+_{-3/2}a^+_{-5/2} - \sqrt{5/7}a^+_{3/2}a^+_{1/2}a^+_{-1/2}a^+_{-3/2} - \tfrac{5}{\sqrt{7}}a^+_{5/2}a^+_{1/2}a^+_{-1/2}a^+_{-5/2}\}|0\rangle$$

$\therefore$ $$\underline{\Phi_{2,0} = \{\tfrac{4}{\sqrt{42}}a^+_{5/2}a^+_{3/2}a^+_{-3/2}a^+_{-5/2} - \tfrac{1}{\sqrt{42}}a^+_{5/2}a^+_{1/2}a^+_{-1/2}a^+_{-5/2} - \tfrac{5}{\sqrt{42}}a^+_{3/2}a^+_{1/2}a^+_{-1/2}a^+_{-3/2}\}|0\rangle}$$

$$\langle\Phi_{2,0}|\Phi_{2,0}\rangle = \langle \tfrac{4}{\sqrt{42}}x_1 - \tfrac{1}{\sqrt{42}}x_2 - \tfrac{5}{\sqrt{42}}x_3 | \tfrac{4}{\sqrt{42}}x_1 - \tfrac{1}{\sqrt{42}}x_2 - \tfrac{5}{\sqrt{42}}x_3\rangle$$
$$= \tfrac{16}{42}x_1^2 + \tfrac{1}{42}x_2^2 + \tfrac{25}{42}x_3^2$$
$$= \tfrac{42}{42} = 1$$

Hence, $\Phi_{2,0}$ *is normalized.*

Again applying J_- *to* $\Phi_{2,0}$, *we find*

4. $$\sqrt{6}\Phi_{2,-1} = (\tfrac{4}{\sqrt{42}}\{2\sqrt{2}a^+_{5/2}a^+_{1/2}a^+_{-3/2}a^+_{-5/2}\} - \tfrac{1}{\sqrt{42}}\{\sqrt{5}a^+_{3/2}a^+_{1/2}a^+_{-1/2}a^+_{-5/2} + 2\sqrt{2}a^+_{5/2}a^+_{1/2}a^+_{-3/2}a^+_{-5/2}\}$$
$$-\tfrac{5}{\sqrt{42}}\{\sqrt{5}a^+_{3/2}a^+_{1/2}a^+_{-1/2}a^+_{-3/2}\})|0\rangle$$
$$= \{\tfrac{8}{\sqrt{21}}a^+_{5/2}a^+_{1/2}a^+_{-3/2}a^+_{-5/2} - \sqrt{5/42}a^+_{3/2}a^+_{1/2}a^+_{-1/2}a^+_{-5/2} - \tfrac{2}{\sqrt{21}}a^+_{5/2}a^+_{1/2}a^+_{-3/2}a^+_{-5/2} - 5\sqrt{5/42}a^+_{3/2}a^+_{1/2}a^+_{-1/2}a^+_{-3/2}\})|0\rangle$$
$$= \{\tfrac{6}{\sqrt{21}}a^+_{5/2}a^+_{1/2}a^+_{-3/2}a^+_{-5/2} - 6\sqrt{5/42}a^+_{3/2}a^+_{1/2}a^+_{-1/2}a^+_{-5/2}\}|0\rangle$$

$\therefore$ $$\underline{\Phi_{2,-1} = \{\sqrt{2/7}a^+_{5/2}a^+_{1/2}a^+_{-3/2}a^+_{-5/2} - \sqrt{5/7}a^+_{3/2}a^+_{1/2}a^+_{-1/2}a^+_{-5/2}\}|0\rangle}$$

$$\langle\Phi_{2,-1}|\Phi_{2,-1}\rangle = \tfrac{2}{7} + \tfrac{5}{7}$$
$$= 1$$

We therefore conclude that the integral $\langle\Phi_{2,-1}|\Phi_{2,-1}\rangle$ *is normalized.*

5. $$2\Phi_{2,-2} = (\sqrt{2/7}\{\sqrt{5}a^+_{3/2}a^+_{1/2}a^+_{-1/2}a^+_{-5/2} + 3a^+_{5/2}a^+_{-1/2}a^+_{-3/2}a^+_{-5/2}\}$$
$$-\sqrt{5/7}\{2\sqrt{2}a^+_{3/2}a^+_{1/2}a^+_{-3/2}a^+_{-5/2}\})\left|0\right\rangle$$
$$= \sqrt{10/7}a^+_{3/2}a^+_{1/2}a^+_{-3/2}a^+_{-5/2} + 3\sqrt{2/7}a^+_{5/2}a^+_{-1/2}a^+_{-3/2}a^+_{-5/2}$$
$$-2\sqrt{10/7}a^+_{3/2}a^+_{1/2}a^+_{-3/2}a^+_{-5/2}\}\left|0\right\rangle$$
$$= \{3\sqrt{2/7}a^+_{5/2}a^+_{-1/2}a^+_{-3/2}a^+_{-5/2} - \sqrt{10/7}a^+_{3/2}a^+_{1/2}a^+_{-3/2}a^+_{-5/2}\}\left|0\right\rangle$$

$\therefore$ $$\underline{\Phi_{2,-2} = 3/2\sqrt{2/7}a^+_{5/2}a^+_{-1/2}a^+_{-3/2}a^+_{-5/2} - 1/2\sqrt{10/7}a^+_{3/2}a^+_{1/2}a^+_{-3/2}a^+_{-5/2}\}\left|0\right\rangle}$$

$$\left\langle\Phi_{2,-2}\middle|\Phi_{2,-2}\right\rangle = \left\langle 3/2\sqrt{2/7}\,x_1 - 1/2\sqrt{10/7}x_2\middle| 3/2\sqrt{2/7}\,x_1 - 1/2\sqrt{10/7}x_2\right\rangle$$
$$= 9/14\,x_1^2 + 5/14\,x_2^2$$
$$= 1$$

Hence, $\Phi_{2,-2}$ *is normalized.*

A.1.4 J = 1.

For the configuration in view the state with J = 1 does not exist as reflected in the Table 3.1. We therefore go right on to evaluating the wavefunction in the J = 0 state.

4.1.5 J = 0.

Let $\Phi_{0,0}$ be denoted by the linear combination :

$$\Phi_{0,0} = \{\alpha a^+_{5/2} a^+_{3/2} a^+_{-3/2} a^+_{-5/2} + \beta a^+_{5/2} a^+_{1/2} a^+_{-1/2} a^+_{-5/2} + \gamma a^+_{5/2} a^+_{1/2} a^+_{-1/2} a^+_{-3/2}\}|0\rangle$$

$$\text{since } \Phi_{2,0} = \{4/\sqrt{42}\, a^+_{5/2} a^+_{3/2} a^+_{-3/2} a^+_{5/2} - 1/\sqrt{42}\, a^+_{5/2} a^+_{1/2} a^+_{-1/2} a^+_{5/2} - 5/\sqrt{42}\, a^+_{3/2} a^+_{1/2} a^+_{-1/2} a^+_{-3/2}\}|0\rangle$$

$$\text{and } \Phi_{4,0} = \{2/\sqrt{42}\, a^+_{5/2} a^+_{3/2} a^+_{-3/2} a^+_{5/2} + 3/\sqrt{42}\, a^+_{5/2} a^+_{1/2} a^+_{-1/2} a^+_{5/2} + 1/\sqrt{42}\, a^+_{3/2} a^+_{1/2} a^+_{-1/2} a^+_{-3/2}\}|0\rangle$$

$$\langle \Phi_{0,0} | \Phi_{2,0} \rangle = 4\alpha - \beta - 5\gamma = 0 \quad \ldots \quad \ldots \quad \ldots(1)$$

$$\text{Similarly, } \langle \Phi_{0,0} | \Phi_{4,0} \rangle = 2\alpha + 3\beta + \gamma = 0 \quad \ldots \quad \ldots(2)$$

$$\langle \Phi_{0,0} | \Phi_{0,0} \rangle = \alpha^2 + \beta^2 + \gamma^2 = 1 \quad \ldots \quad \ldots(3)$$

From the equations above, we find that $\alpha = \gamma = -1/\sqrt{3}$ and $\beta = 1/\sqrt{3}$

and thus

$$\underline{\Phi_{0,0} = \{-1/\sqrt{3}\, a^+_{5/2} a^+_{3/2} a^+_{-3/2} a^+_{-5/2} + 1/\sqrt{3}\, a^+_{5/2} a^+_{1/2} a^+_{-1/2} a^+_{-5/2} + \gamma - 1/\sqrt{3}\, a^+_{3/2} a^+_{1/2} a^+_{-1/2} a^+_{-3/2}\}|0\rangle}$$

$$\langle \Phi_{0,0} | \Phi_{0,0} \rangle = 1/3 + 1/3 + 1/3 = 1$$

A.2 Wavefunction(s) in the $(d_{3/2})^4$ configuration.

In the $(d_{3/2})^4$ configuration, we find that

$$\begin{aligned} J_{\max} &= 4\{3/2 - (4-1)/2\} \\ &= 4\{3/2 - 3/2\} \\ \therefore\ J_{\max} &= 0 \end{aligned}$$

and hence, the only possible wavefunction is $\Phi_{0,0}$ and is written thus:

$$\underline{\Phi_{0,0} = a^+_{3/2} a^+_{1/2} a^+_{-1/2} a^+_{-3/2} |0\rangle}$$

APPENDIX C

EVALUATION OF THE MATRIX ELEMENTS

C.1 d^3 CONFIGURATION

C.1.1 $(d_{5/2})^3$ CONFIGURATION.

1. $$\left\langle (d_{5/2})^3_{3/2\,3/2} \middle| a^+_{5/2\,3/2} \middle| (d_{5/2})^2_{00} \right\rangle$$

$$(\mathrm{d}_{5/2})^3_{3/2\,3/2} = \{2\sqrt{2/21}\,a^+_{5/2}a^+_{3/2}a^+_{-5/2} - \sqrt{5/21}\,a^+_{5/2}a^+_{1/2}a^+_{-3/2} + 2\sqrt{2/21}\,a^+_{3/2}a^+_{1/2}a^+_{-1/2}\}\left|0\right\rangle$$

$$(\mathrm{d}_{5/2})^2_{0,0} = \{1/\sqrt{3}\,a^+_{5/2}a^+_{-5/2} - 1/\sqrt{3}\,a^+_{3/2}a^+_{-3/2} + 1/\sqrt{3}\,a^+_{1/2}a^+_{-1/2}\}\left|0\right\rangle$$

$$a^+_{5/2\,3/2}\left|(d_{5/2})^2_{00}\right\rangle = \{1/\sqrt{3}\,a^+_{5/2}a^+_{3/2}a^+_{-5/2} + 1/\sqrt{3}\,a^+_{3/2}a^+_{1/2}a^+_{-1/2}\}\left|0\right\rangle$$

$$\left\langle (d_{5/2})^3_{3/2\,3/2} \middle| a^+_{5/2\,3/2} \middle| (d_{5/2})^2_{00} \right\rangle = 2/3\sqrt{2/7} + 2/3\sqrt{2/7}$$

$$\therefore \left\langle (d_{5/2})^3_{3/2\,3/2} \middle| a^+_{5/2\,3/2} \middle| (d_{5/2})^2_{00} \right\rangle = 4/3\sqrt{2/7}$$

2. $$\left\langle (d_{5/2})^3_{5/2\,5/2} \middle| a^+_{5/2\,5/2} \middle| (d_{5/2})^2_{00} \right\rangle$$

$$(\mathrm{d}_{5/2})^3_{5/2\,5/2} = \{1/\sqrt{2}\,a^+_{5/2}a^+_{3/2}a^+_{-3/2} - 1/\sqrt{2}\,a^+_{5/2}a^+_{1/2}a^+_{-1/2}\}\left|0\right\rangle$$

$$(\mathrm{d}_{5/2})^2_{0,0} = \{1/\sqrt{3}\,a^+_{5/2}a^+_{-5/2} - 1/\sqrt{3}\,a^+_{3/2}a^+_{-3/2} + 1/\sqrt{3}\,a^+_{1/2}a^+_{-1/2}\}\left|0\right\rangle$$

$$a^+_{5/2\,5/2}\left|(d_{5/2})^2_{00}\right\rangle = \{-1/\sqrt{3}\,a^+_{5/2}a^+_{3/2}a^+_{-3/2} + 1/\sqrt{3}\,a^+_{5/2}a^+_{1/2}a^+_{-1/2}\}\left|0\right\rangle$$

$$\left\langle (d_{5/2})^3_{5/2\,5/2}\left|a^+_{5/2\,5/2}\right|(d_{5/2})^2_{00}\right\rangle = -\tfrac{1}{\sqrt{6}} - \tfrac{1}{\sqrt{6}} = -\tfrac{2}{\sqrt{6}}$$

$$\underline{\therefore \left\langle (d_{5/2})^3_{5/2\,5/2}\left|a^+_{5/2\,5/2}\right|(d_{5/2})^2_{00}\right\rangle = -\sqrt{2/3}}$$

3. $$\left\langle (d_{5/2})^3_{9/2\,3/2}\left|a^+_{5/2\,3/2}\right|(d_{5/2})^2_{00}\right\rangle$$

$$(\mathrm{d}_{5/2})^3_{9/2\,3/2} = \{\sqrt{5/42}\,a^+_{5/2}a^+_{3/2}a^+_{-5/2} + \tfrac{4}{\sqrt{21}}\,a^+_{5/2}a^+_{1/2}a^+_{-5/2} + \sqrt{5/42}\,a^+_{3/2}a^+_{1/2}a^+_{-1/2}\}\left|0\right\rangle$$

$$(\mathrm{d}_{5/2})^2_{0,0} = \{-\tfrac{1}{\sqrt{3}}\,a^+_{5/2}a^+_{-5/2} + \tfrac{1}{\sqrt{3}}\,a^+_{3/2}a^+_{-3/2} - \tfrac{1}{\sqrt{3}}\,a^+_{1/2}a^+_{-1/2}\}\left|0\right\rangle$$

$$a^+_{5/2\,3/2}\left|(d_{5/2})^2_{00}\right\rangle = \{-\tfrac{1}{\sqrt{3}}\,a^+_{5/2}a^+_{3/2}a^+_{-5/2} - \tfrac{1}{\sqrt{3}}\,a^+_{3/2}a^+_{1/2}a^+_{-1/2}\}\left|0\right\rangle$$

$$\left\langle (d_{5/2})^3_{9/2\,3/2}\left|a^+_{5/2\,3/2}\right|(d_{5/2})^2_{00}\right\rangle = -\tfrac{1}{3}\sqrt{5/14} - \tfrac{1}{3}\sqrt{5/14} = -\tfrac{2}{3}\sqrt{5/14}$$

$$\underline{\therefore \left\langle (d_{5/2})^3_{9/2\,3/2}\left|a^+_{5/2\,3/2}\right|(d_{5/2})^2_{00}\right\rangle = -\tfrac{2}{3}\sqrt{5/14}}$$

4. $$\left\langle (d_{5/2})^3_{3/2\,3/2}\left|a^+_{5/2\,5/2}\right|(d_{5/2})^2_{2,-1}\right\rangle$$

$$(\mathrm{d}_{5/2})^3_{3/2\,3/2} = \{-2\sqrt{2/21}\,a^+_{5/2}a^+_{3/2}a^+_{-5/2} + \sqrt{5/21}\,a^+_{5/2}a^+_{1/2}a^+_{-3/2} - 2\sqrt{2/21}\,a^+_{3/2}a^+_{1/2}a^+_{-1/2}\}\left|0\right\rangle$$
$$(\mathrm{d}_{5/2})^2_{2,-1} = \{\sqrt{5/7}\,a^+_{3/2}a^+_{-5/2} - \sqrt{2/7}\,a^+_{1/2}a^+_{-3/2}\}\left|0\right\rangle$$

$$\mathit{Hence,}\left\langle (d_{5/2})^3_{3/2\,3/2}\left|a^+_{5/2\,5/2}\right|(d_{5/2})^2_{2,-1}\right\rangle = -\tfrac{2}{7}\sqrt{10/3} - \tfrac{1}{7}\sqrt{10/3} = -\tfrac{3}{7}\sqrt{10/3}$$

$$\underline{\therefore \left\langle (d_{5/2})^3_{3/2\,3/2}\left|a^+_{5/2\,5/2}\right|(d_{5/2})^2_{2,-1}\right\rangle = -\tfrac{3}{7}\sqrt{10/3}}$$

5. $$\left\langle (d_{5/2})^3_{5/2\,5/2}\left|a^+_{5/2\,3/2}\right|(d_{5/2})^2_{2,1}\right\rangle$$

$$(\mathrm{d}_{5/2})^3_{5/2\,5/2} = \{\tfrac{1}{\sqrt{2}}\,a^+_{5/2}a^+_{3/2}a^+_{-3/2} - \tfrac{1}{\sqrt{2}}\,a^+_{5/2}a^+_{1/2}a^+_{-1/2}\}\left|0\right\rangle$$

$$(\mathrm{d}_{5/2})^2_{2,1} = \{-\tfrac{2}{\sqrt{14}}\, a^+_{3/2} a^+_{-1/2} + \sqrt{5/7}\, a^+_{5/2} a^+_{-3/2}\}\left|0\right\rangle$$

$$\therefore a^+_{5/2\,3/2}\left|(d_{5/2})^2_{2,1}\right\rangle = \sqrt{5/7}\, a^+_{5/2} a^+_{3/2} a^+_{-3/2}\left|0\right\rangle$$

and hence,

$$\left\langle (d_{5/2})^3_{5/2\,5/2}\left| a^+_{5/2\,3/2} \right| (d_{5/2})^2_{2,1}\right\rangle = \sqrt{5/14}$$

6. $$\left\langle (\mathrm{d}_{5/2})^3_{9/2\,7/2}\left| a^+_{5/2\,5/2} \right| (\mathrm{d}_{5/2})^2_{2,1}\right\rangle$$

$$(\mathrm{d}_{5/2})^3_{9/2\,7/2} = a^+_{5/2} a^+_{3/2} a^+_{-1/2}\left|0\right\rangle$$

$$(\mathrm{d}_{5/2})^2_{2,1} = \{-\tfrac{2}{\sqrt{14}}\, a^+_{3/2} a^+_{-1/2} + \sqrt{5/7}\, a^+_{5/2} a^+_{-3/2}\}\left|0\right\rangle$$

$$a^+_{5/2\,5/2}\left|(\mathrm{d}_{5/2})^2_{2,1}\right\rangle = -\tfrac{2}{\sqrt{14}}\, a^+_{5/2} a^+_{3/2} a^+_{-1/2}\left|0\right\rangle$$

$$\underline{\left\langle (\mathrm{d}_{5/2})^3_{9/2\,7/2}\left| a^+_{5/2\,5/2} \right| (\mathrm{d}_{5/2})^2_{2,1}\right\rangle = -\tfrac{2}{\sqrt{14}}}$$

7 $$\left\langle (\mathrm{d}_{5/2})^3_{3/2\,3/2}\left| a^+_{5/2\,3/2} \right| (\mathrm{d}_{5/2})^2_{4,0}\right\rangle$$

$$(\mathrm{d}_{5/2})^3_{3/2\,3/2} = \{-2\sqrt{2/21}\, a^+_{5/2} a^+_{3/2} a^+_{-5/2} + \sqrt{5/21}\, a^+_{5/2} a^+_{1/2} a^+_{-3/2} - 2\sqrt{2/21}\, a^+_{3/2} a^+_{1/2} a^+_{-1/2}\}\left|0\right\rangle$$

$$(\mathrm{d}_{5/2})^2_{4,0} = \{\tfrac{2}{\sqrt{14}}\, a^+_{1/2} a^+_{-1/2} + \tfrac{3}{\sqrt{14}}\, a^+_{3/2} a^+_{-3/2} + \tfrac{1}{\sqrt{14}}\, a^+_{5/2} a^+_{-5/2}\left|0\right\rangle$$

$$a^+_{5/2\,3/2}\left|(\mathrm{d}_{5/2})^2_{4,0}\right\rangle = \{\tfrac{2}{\sqrt{14}}\, a^+_{3/2} a^+_{1/2} a^+_{-1/2} + \tfrac{1}{\sqrt{14}}\, a^+_{5/2} a^+_{3/2} a^+_{-5/2}\}\left|0\right\rangle$$

$$\underline{\therefore \left\langle (\mathrm{d}_{5/2})^3_{3/2\,3/2}\left| a^+_{5/2\,3/2} \right| (\mathrm{d}_{5/2})^2_{4,0}\right\rangle = -\tfrac{6}{\sqrt{147}}}$$

8. $$\left\langle (\mathrm{d}_{5/2})^3_{5/2\,5/2}\left| a^+_{5/2\,5/2} \right| (\mathrm{d}_{5/2})^2_{4,0}\right\rangle$$

$$(\mathrm{d}_{5/2})^3_{5/2\,5/2} = \{ 1/\sqrt{2}\, a^+_{5/2} a^+_{3/2} a^+_{-3/2} - 1/\sqrt{2}\, a^+_{5/2} a^+_{1/2} a^+_{-1/2} \} |0\rangle$$

$$(\mathrm{d}_{5/2})^2_{4,0} = \{ 2/\sqrt{14}\, a^+_{1/2} a^+_{-1/2} + 3/\sqrt{14}\, a^+_{3/2} a^+_{-3/2} + 1/\sqrt{14}\, a^+_{5/2} a^+_{-5/2} |0\rangle$$

$$a^+_{5/2\,5/2} \left| (\mathrm{d}_{5/2})^2_{4,0} \right\rangle = \{ 2/\sqrt{14}\, a^+_{5/2} a^+_{1/2} a^+_{-1/2} + 3/\sqrt{14}\, a^+_{5/2} a^+_{3/2} a^+_{-3/2} \} |0\rangle$$

$$\left\langle (\mathrm{d}_{5/2})^3_{5/2\,5/2} \left| a^+_{5/2\,5/2} \right| (\mathrm{d}_{5/2})^2_{4,0} \right\rangle = 3/\sqrt{28} - 2/\sqrt{28} = 1/\sqrt{28}$$

$$\therefore \quad \underline{\left\langle (\mathrm{d}_{5/2})^3_{5/2\,5/2} \left| a^+_{5/2\,5/2} \right| (\mathrm{d}_{5/2})^2_{4,0} \right\rangle = 1/2\sqrt{7}}$$

9. $\left\langle (\mathrm{d}_{5/2})^3_{9/2\,9/2} \left| a^+_{5/2\,5/2} \right| (\mathrm{d}_{5/2})^2_{4,2} \right\rangle$

$$(\mathrm{d}_{5/2})^3_{9/2\,9/2} = a^+_{5/2} a^+_{3/2} a^+_{1/2} |0\rangle$$

$$(\mathrm{d}_{5/2})^2_{4,2} = \{ \sqrt{5/14}\, a^+_{3/2} a^+_{1/2} + 3/\sqrt{14}\, a^+_{5/2} a^+_{-1/2} |0\rangle$$

$$a^+_{5/2\,5/2} \left| (\mathrm{d}_{5/2})^2_{4,2} \right\rangle = \sqrt{5/14}\, a^+_{5/2} a^+_{3/2} a^+_{-1/2} |0\rangle$$

Therefore, $\underline{\left\langle (\mathrm{d}_{5/2})^3_{9/2\,9/2} \left| a^+_{5/2\,5/2} \right| (\mathrm{d}_{5/2})^2_{4,2} \right\rangle = \sqrt{5/14}}$

C.1.2 $(\mathrm{d}_{3/2})^3$ CONFIGURATION

1. $\left\langle (\mathrm{d}_{3/2})^3_{3/2\,3/2} \left| a^+_{3/2\,3/2} \right| (\mathrm{d}_{3/2})^2_{0,0} \right\rangle$

$$(\mathrm{d}_{3/2})^3_{3/2\,3/2} = a^+_{3/2} a^+_{1/2} a^+_{-1/2} |0\rangle$$

$$(\mathrm{d}_{3/2})^2_{0,0} = \{ 1/\sqrt{2}\, a^+_{1/2} a^+_{-1/2} - 1/\sqrt{2}\, a^+_{3/2} a^+_{-3/2} \} |0\rangle$$

$$a^+_{3/2\,3/2} \left| (\mathrm{d}_{3/2})^2_{0,0} \right\rangle = 1/\sqrt{2}\, a^+_{3/2} a^+_{1/2} a^+_{-1/2} |0\rangle$$

$$hence,\ \underline{\left\langle (\mathrm{d}_{3/2})^3_{3/2\,3/2} \middle| a^+_{3/2\,3/2} \middle| (\mathrm{d}_{3/2})^2_{0,0} \right\rangle = 1/\sqrt{2}}$$

2. $$\left\langle (\mathrm{d}_{3/2})^3_{1/2\,1/2} \middle| a^+_{3/2\,1/2} \middle| (\mathrm{d}_{3/2})^2_{2,0} \right\rangle$$

$$(\mathrm{d}_{3/2})^3_{3/2\,1/2} = a^+_{3/2} a^+_{1/2} a^+_{-3/2} |0\rangle$$

$$(\mathrm{d}_{3/2})^2_{0,0} = \{ 1/\sqrt{2}\, a^+_{1/2} a^+_{-1/2} - 1/\sqrt{2}\, a^+_{3/2} a^+_{-3/2} \} |0\rangle$$

$$a^+_{3/2\,1/2} \left| (\mathrm{d}_{3/2})^2_{0,0} \right\rangle = -1/\sqrt{2}\, a^+_{3/2} a^+_{1/2} a^+_{-3/2} |0\rangle$$

$$hence,\ \underline{\left\langle (\mathrm{d}_{3/2})^3_{1/2\,1/2} \middle| a^+_{3/2\,1/2} \middle| (\mathrm{d}_{3/2})^2_{2,0} \right\rangle = -1/\sqrt{2}}$$

3. $$\left\langle (\mathrm{d}_{3/2})^3_{3/2\,3/2} \middle| a^+_{3/2\,-1/2} \middle| (\mathrm{d}_{3/2})^2_{2,2} \right\rangle$$

$$(\mathrm{d}_{3/2})^3_{3/2\,3/2} = a^+_{3/2} a^+_{1/2} a^+_{-1/2} |0\rangle$$

$$(\mathrm{d}_{3/2})^2_{2,2} = a^+_{3/2} a^+_{1/2} |0\rangle$$

$$a^+_{3/2\,-1/2} \left| (\mathrm{d}_{3/2})^2_{2,2} \right\rangle = a^+_{3/2} a^+_{1/2} a^+_{-1/2} |0\rangle$$

$$hence,\ \underline{\left\langle (\mathrm{d}_{3/2})^3_{3/2\,3/2} \middle| a^+_{3/2\,-1/2} \middle| (\mathrm{d}_{5/2})^2_{2,2} \right\rangle = 1}$$

C.2. $(\mathrm{d})^2$ CONFIGURATION

C.2.1 $(\mathrm{d}_{5/2})^2$ CONFIGURATION

1. $$\left\langle (\mathrm{d}_{5/2})^3_{00} \middle| a^+_{5/2\,-5/2} \middle| (\mathrm{d}_{5/2})^2_{5/2\,5/2} \right\rangle$$

$$(\mathrm{d}_{5/2})^2_{0,0} = \{ 1/\sqrt{3}\, a^+_{5/2} a^+_{-5/2} - 1/\sqrt{3}\, a^+_{3/2} a^+_{-3/2} + 1/\sqrt{3}\, a^+_{1/2} a^+_{-1/2} \} |0\rangle$$

$$(\mathrm{d}_{5/2})^{1}_{5/2\,5/2} = a^{+}_{5/2}|0\rangle$$

$$a^{+}_{5/2\,-5/2}\left|(\mathrm{d}_{5/2})^{1}_{5/2\,5/2}\right\rangle = a^{+}_{5/2}a^{+}_{-5/2}|0\rangle$$

hence,

$$\underline{\left\langle(\mathrm{d}_{5/2})^{2}_{00}\left|a^{+}_{5/2\,-5/2}\right|(\mathrm{d}_{5/2})^{1}_{5/2\,5/2}\right\rangle = 1/\sqrt{3}}$$

2. $$\left\langle(\mathrm{d}_{5/2})^{2}_{22}\left|a^{+}_{5/2\,1/2}\right|(\mathrm{d}_{5/2})^{1}_{5/2\,3/2}\right\rangle$$

$$(\mathrm{d}_{5/2})^{2}_{2,2} = \{-3/\sqrt{14}\,a^{+}_{3/2}a^{+}_{1/2} + \sqrt{5/14}\,a^{+}_{5/2}a^{+}_{-1/2}\}|0\rangle$$

$$a^{+}_{5/2\,1/2}\left|(\mathrm{d}_{5/2})^{1}_{5/2\,3/2}\right\rangle = a^{+}_{3/2}a^{+}_{1/2}|0\rangle$$

hence,

$$\left\langle(\mathrm{d}_{5/2})^{2}_{22}\left|a^{+}_{5/2\,1/2}\right|(\mathrm{d}_{5/2})^{1}_{5/2\,3/2}\right\rangle = -3/\sqrt{14}$$

3. $$\left\langle(\mathrm{d}_{5/2})^{2}_{4,4}\left|a^{+}_{5/2\,5/2}\right|(\mathrm{d}_{5/2})^{1}_{5/2\,5/2}\right\rangle$$

$$(\mathrm{d}_{5/2})^{2}_{4,4} = a^{+}_{5/2}a^{+}_{3/2}|0\rangle$$

$$(\mathrm{d}_{5/2})^{1}_{5/2\,5/2} = a^{+}_{5/2}|0\rangle$$

$$a^{+}_{5/2\,3/2}\left|(\mathrm{d}_{5/2})^{1}_{5/2\,5/2}\right\rangle = a^{+}_{5/2}a^{+}_{3/2}|0\rangle$$

hence, $\left\langle(\mathrm{d}_{5/2})^{2}_{4,4}\left|a^{+}_{5/2\,3/2}\right|(\mathrm{d}_{5/2})^{1}_{5/2\,5/2}\right\rangle = 1$

C.2.2 $(d_{3/2})^2$ CONFIGURATION

1. $\left\langle (\mathrm{d}_{3/2})^2_{0,0} \middle| a^+_{3/2\,-3/2} \middle| (\mathrm{d}_{3/2})^1_{3/2\,3/2} \right\rangle$

Since,

$$(\mathrm{d}_{3/2})^2_{0,0} = \{ 1/\sqrt{2}\, a^+_{1/2} a^+_{-1/2} - 1/\sqrt{2}\, a^+_{3/2} a^+_{-3/2} \} |0\rangle$$

$$(\mathrm{d}_{3/2})^1_{3/2\,3/2} = a^+_{3/2} |0\rangle$$

Whence, $a^+_{3/2\,-3/2} \left| (\mathrm{d}_{3/2})^1_{3/2\,3/2} \right\rangle = a^+_{3/2} a^+_{-3/2} |0\rangle$

And, $\left\langle (\mathrm{d}_{3/2})^2_{0,0} \middle| a^+_{3/2\,-3/2} \middle| (\mathrm{d}_{3/2})^1_{3/2\,3/2} \right\rangle = -1/\sqrt{2}$

2. $\left\langle (\mathrm{d}_{3/2})^2_{2,2} \middle| a^+_{3/2\,1/2} \middle| (\mathrm{d}_{3/2})^1_{3/2\,3/2} \right\rangle$

Since,

$$(\mathrm{d}_{3/2})^2_{2,2} = a^+_{3/2} a^+_{1/2} |0\rangle$$

$$(\mathrm{d}_{3/2})^1_{3/2\,3/2} = a^+_{3/2} |0\rangle$$

$$a^+_{3/2\,1/2} \left| (\mathrm{d}_{3/2})^1_{3/2\,3/2} \right\rangle = a^+_{3/2} a^+_{1/2} |0\rangle$$

Hence,

$$\left\langle (\mathrm{d}_{3/2})^2_{2,2} \middle| a^+_{3/2\,1/2} \middle| (\mathrm{d}_{3/2})^1_{3/2\,3/2} \right\rangle = 1$$

APPENDIX D

SUMMARY OF CLEBSCH-GORDAN COEFFICIENTS USED IN THIS WORK.

The following expression evaluates the Clebsch-Gordan coefficients

$$\langle j_1 m_1, j_2 m_2 \mid j_3 m_3 \rangle = \delta_{m_1+m_2, m_3} \left[\frac{(2j_3+1)(s-2j_3)!(s-2j_2)!}{(s+1)!} \right.$$
$$\left. \times (j_1+m_1)!(j_1-m_1)!(j_2+m_2)!(j_2-m_2)!(j_3+m_3)!(j_3-m_3)! \right]^{1/2}$$
$$\times \sum_v (-1)^v \Big/ \left[v!(j_1+j_2-j_3-v)!(j_1-m_1-v)!(j_2+m_2-v)! \right.$$
$$\left. \times (j_3-j_2+m_1+v)!(j_3-j_1-m_2+v)! \right]$$

where $s = j_1 + j_2 + j_3$ *and the index* v *ranges over all*

Integral values for which the factorial arguments are non-negative[26k].The Clebsch-Gordan coefficient(cgc's) vanishes except if

$$|j_1 - j_2| \le j_3 \le j_1 + j_2 \;\&\; m_3 = m_1 + m_2$$

The following are the Clebsch-Gordan coefficients that are used in this work:

1. $\langle \tfrac{5}{2}\tfrac{3}{2}, \tfrac{3}{2}\,{-\tfrac{3}{2}} \mid 0\,0 \rangle = ne$
2. $\langle \tfrac{5}{2}\tfrac{5}{2}, \tfrac{3}{2}\,{-\tfrac{1}{2}} \mid 2\,2 \rangle = 2\sqrt{5/42}$
3. $\langle \tfrac{5}{2}\tfrac{3}{2}, \tfrac{3}{2}\,{-\tfrac{1}{2}} \mid 4,1 \rangle = \sqrt{15/56}$
4. $\langle \tfrac{5}{2}\,{-\tfrac{5}{2}}, \tfrac{5}{2}\tfrac{5}{2} \mid 2,0 \rangle = -5/\sqrt{84}$
5. $\langle \tfrac{5}{2}\tfrac{3}{2}, \tfrac{5}{2}\tfrac{5}{2} \mid 4,4 \rangle = 1/\sqrt{2}$
6. $\langle \tfrac{5}{2}\,{-\tfrac{3}{2}}, \tfrac{9}{2}\tfrac{7}{2} \mid 2,2 \rangle = \tfrac{1}{3}\sqrt{5}$
7. $\langle \tfrac{5}{2}\tfrac{1}{2}, \tfrac{9}{2}\tfrac{7}{2} \mid 4,1 \rangle = \sqrt{21/55}$
8. $\langle \tfrac{3}{2}\,{-\tfrac{3}{2}}, \tfrac{3}{2}\tfrac{3}{2} \mid 0,0 \rangle = -\tfrac{1}{2}$
9. $\langle \tfrac{3}{2}\,{-\tfrac{3}{2}}, 0\,0 \mid \tfrac{3}{2}\tfrac{3}{2} \rangle = ne$

10. $\langle \tfrac{5}{2}\,\tfrac{5}{2}, 0\,0 | \tfrac{5}{2}\,\tfrac{5}{2} \rangle = 1$

11. $\langle \tfrac{5}{2}\,\tfrac{3}{2}, 0\,0 | \tfrac{9}{2}\,\tfrac{3}{2} \rangle = ne$

12. $\langle \tfrac{5}{2}\,\tfrac{5}{2}, 2\ -1 | \tfrac{3}{2}\,\tfrac{3}{2} \rangle = 3\sqrt{\tfrac{2}{63}}$

13. $\langle \tfrac{5}{2}\,\tfrac{3}{2}, 2\,1 | \tfrac{5}{2}\,\tfrac{5}{2} \rangle = -\sqrt{\tfrac{3}{7}}$

14. $\langle \tfrac{5}{2}\,\tfrac{5}{2}, 2\,1 | \tfrac{9}{2}\,\tfrac{7}{2} \rangle = \tfrac{2}{3}$

15. $\langle \tfrac{5}{2}\,\tfrac{5}{2}, 4\ 0 | \tfrac{3}{2}\,\tfrac{3}{2} \rangle = -\sqrt{\tfrac{2}{7}}$

16. $\langle \tfrac{5}{2}\,\tfrac{5}{2}, 4\ 0 | \tfrac{5}{2}\,\tfrac{5}{2} \rangle = -\tfrac{1}{2}\sqrt{\tfrac{2}{21}}$

17. $\langle \tfrac{5}{2}\,\tfrac{5}{2}, 4\ 2 | \tfrac{9}{2}\,\tfrac{9}{2} \rangle = \sqrt{\tfrac{5}{33}}$

18. $\langle \tfrac{3}{2}\,\tfrac{3}{2}, 0\ 0 | \tfrac{3}{2}\,\tfrac{3}{2} \rangle = 1$

19. $\langle \tfrac{3}{2}\,\tfrac{-1}{2}, 2\ 2 | \tfrac{3}{2}\,\tfrac{3}{2} \rangle = \sqrt{\tfrac{2}{5}}$

20. $\langle \tfrac{5}{2}\,\tfrac{-5}{2}, 0\ 0 | \tfrac{5}{2}\,\tfrac{-5}{2} \rangle = 1$

21. $\langle \tfrac{5}{2}\,\tfrac{1}{2}, \tfrac{5}{2}\,\tfrac{3}{2} | 2\,2 \rangle = 3\sqrt{\tfrac{3}{70}}$

22. $\langle \tfrac{5}{2}\,\tfrac{3}{2}, \tfrac{5}{2}\,\tfrac{5}{2} | 4\,4 \rangle = \tfrac{-1}{\sqrt{2}}$

23. $\langle \tfrac{5}{2}\,\tfrac{-3}{2}, \tfrac{3}{2}\,\tfrac{3}{2} | 0\ 0 \rangle = \tfrac{-1}{2}$

24. $\langle \tfrac{3}{2}\,\tfrac{1}{2}, \tfrac{3}{2}\,\tfrac{3}{2} | 2\ 2 \rangle = \tfrac{-1}{\sqrt{2}}$

**where we have represented the expression on the right-hand side of any clebsch-gordan coefficient by ne, we have meant that the Clebsch-Gordan coefficient is non-existent as the triangle conditions are unfulfilled in such cases.

C++ Program for generating CGCs

```
#include <iostream>

#include<cmath>

using namespace std;

int Quit(int quit);

double F(double n);

double Max(double a, double b, double c, double d, double e, double f);

//double Min(double l, double m, double n, double x, double y, double z);

int main()  {

        int q;

        while (q!=0) {

        double j1,j2,j3;

        double m1, m2, m3;

        cout<<"Enter j1: ";cin>>j1;

        cout<<"\nEnter m1: ";cin>> m1;

        cout<<"\nEnter j2: ";cin>>j2;

        cout<<"\nEnter m2: ";cin>>m2;

        cout<<"\nEnter j3: ";cin>>j3;

        cout<<"\nEnter m3: ";cin>>m3;

        /*if (m1 + m2 != m3 && (abs(j1-j2) <= j3 && j3 <= j1+j2))
```

```
                    cout<<"\nThe Clebsch-Gordan coefficient does not exist for
the values you entered for";
          else*/
          //double X = abs(m3);
          //cout<<"X="<<X;
          double s,v;
          s = j1 + j2 + j3;//cout<<"s: "<<s<<endl;
          cout<<"\ns is "<<s<<endl;
          double FirstNum= 2*j3 +1;
          //cout<<"\n2j3+1: "<< FirstNum << endl;
          double SecondNum = F(s - 2*j1);
          cout<<"\ns - 2j1 is "<<s-2*j1<<endl;
          //cout<<" and (s-2j1)! is "<< SecondNum;
          double ThirdNum  = F(s - 2*j2);
          cout<<"\n(s-2j2): "<<s-2*j2<<"      (s - 2j2)! : "<< ThirdNum;
          double FourthNum = F(s - 2*j3) ;
          double FifthNum  = F(j1 + m1);
          double SixthNum        = F(j1-m1);
          double SeventhNum= F(j2+m2);
          double EigthNum  = F(j2-m2);
```

```
double NinthNum  = F(j3+m3);
double TenthNum  = F(j3-m3);
double DenThese  = F(s+1);
double NumOne        =
FirstNum*SecondNum*ThirdNum*FourthNum;
double NumTwo        =
FifthNum*SixthNum*SeventhNum*EigthNum;
double NumThree      = NinthNum*TenthNum;
double NumAll        = NumOne*NumTwo*NumThree;
double FirstpProd= sqrt (NumAll/DenThese);
cout<<"\nFirst partial product is "<<FirstpProd<<endl;
double va=0.0;//cout<<"\nv1: "<<va<<endl;
double vb=j1+j2-j3;//cout<<"\nv2=j1+j2-j3: "<<vb<<endl;
double vc=j1-m1;//cout<<"\nv3=j1-m1: "<<vc<<endl;
double vd=j2+m2;//cout<<"\nv4=j2+m2: "<<vd<<endl;
double ve=j2-j3-m1;//cout<<"\nv5=j2-j3-m1: "<<ve<<endl;
double vf=j1-j3+m2;//cout<<"\nv6=j1-j3+m2: "<<vf<<endl;
double vmax = Max(va, vb, vc, vd, ve, vf);
//double vmin=Min(v1,v2,v3,v4,v5,v6);
    //cout<<"\nThe max is "<<vmax<<endl;
```

```
	//	cout<<"The min is "<<vmin<<endl;
double SumTerm,Sum = 0.0;
for( v =va; v <= vmax ; v++) {
	{cout<<"v: "<<v<<", ";}
	double Num		= pow(-1,v);
	cout<<"Parity: "<<Num<<endl;
	double DenOne   = F(v);
	cout<<"v! is "<< DenOne <<endl;
	double DenTwo   = F(j1+j2-j3-v);
	//cout<<"(j1+j2+j3-v)! is "<< DenTwo <<endl;
	double DenThree = F(j1-m1-v);
	//cout<<"(j1-m1-v)! is "<<DenThree<<endl;
	double DenFour  = F(j2+m2-v);
	//cout<<"(j2+m2-v)! is "<< DenFour <<endl;
	long double DenFive  = F(j3-j2+m1+v);
	//cout<<"(j3-j2+m1+v)! is "<<DenFive<<endl;
	long double DenSix    = F(j3-j1-m2+v);
	//cout<<"(j3-j1-m2+v)! is "<<DenSix<<endl;
	double Den1		= DenOne*DenTwo*DenThree;
	double Den2		= DenFour*DenFive*DenSix;
```

```
		double Den		= Den1 * Den2;
		//cout<<"Den: "<<Den<<endl;
		SumTerm	= Num/Den;
		Sum+=SumTerm;	}
	//cout<<"The summation evaluates to "<< Sum <<endl;
	double CGCoeff	= FirstpProd * Sum;
	if (CGCoeff	> 1 || CGCoeff < 0 )
				cout<<"\nThe Clebsch-Gordan coefficient
vanishes identically"<<endl;
	else
	cout<<"\n("<<j1<<","<<m1<<";"<<j2<<","<<m2<<"|"<<j3<<","<<m
3<<") is "<< CGCoeff <<endl;

	cout<<"\nEnter 0 if U want to quit"<<endl;
	cout<<"\n"<<endl;
	cin>>q;
	}
	return 0;}
//Tests for the factorial function
```

```
double F(double n) {

        double prod=1.0;

                if(n==0.5) return 0.5;

                 if(n < 0) return 1;
                  if(n>=0)
                        for (double i = n;i > 0;i--)
                                        prod=prod*i;
                return prod;}
//Tests for the maximum of v in the summation
double Max(double v1, double v2, double v3, double v4, double v5, double
v6) {
        double max;
        if(v1>v2) max = v1;
                else max = v2;
        if(v3>max) max = v3;
        else if(v4>max) max = v4;
        if(v5>max) max = v5;
```

```
else if(v6>max) max = v6;
return max;
                }
/************************************************************
********/
//returns the Clebsch-Gordan coefficient in root rational form
class Ratio
{       friend istream& operator >> (istream&, Ratio&);
        friend ostream& operator << (ostream&,const Ratio&);
        public:
                Ratio(int n = 0, int d = 1): num(n), den(d){reduce();}
        operator print();
        private:
        int num, den;
                void reduce();
};
int gcd(int,int);//returns the greatest common divisor(fuction prototype)
void Ratio::reduce()
{//enforce invariant(den>o)
        if(num==0 || den==0)
```

```
{ num=0;
den=1;return;}
if(den<0)
{ den*=-1;
 num*=-1;
}
//enforce invariant(gcd(num,den)==1):
if(den==1) return;//already reduced
int sgn = (num < 0 ? -1 : 1);//nonegatives to gcd()
int g= gcd(sgn*num, den);
num/=g;
den/=g;
            }


int gcd(double m,double n)
{//returns the greatest common divisor of m and n:
            if (m<n) {double temp = m; m = n; n = temp ;}
            while(n>0)
            { int r = m%n;
            m=n;
```

```
            n=r;
            }
            return m;
}
 double NUMALL= sqrt(NumAll);
 double DENTHESE=sqrt(DENTHESE);
            Ratio x (NUMALL,DENTHESE);
            Ratio y(Num,Den)
istream& operator>>(istream& istr, Ratio& r)
            {           x.reduce();
                        y.reduce()
                        Ratio::operator print()
{ return cout << NumAll << '/' << DenThese << endl;}
x.print( );
return 0;
```

APPENDIX D.

REDUCED MATRIX ELEMENTS ARISING IN OUR C.F.P. COMPUTATIONS.

The reduced matrix elements which arise in our c.f.p. calculations are obtainable via the Wigner-Eckart relations earlier described in the text of our work. To evaluate the reduced matrix element $\left\langle \Phi_0 \left\| a^+_{5/2} \right\| \varphi_{3/2} \right\rangle$ for example, we proceed thus making use of the Wigner-Eckart relations:

$$\left\langle \Phi_0 \left\| a^+_{5/2} \right\| \varphi_{3/2} \right\rangle = \frac{\left\langle (d_{5/2})^4_{00} \left| a^+_{5/2\,3/2} \right| (d_{5/2})^3_{3/2\,-3/2} \right\rangle}{\left\langle 5/2\,3/2, 3/2\,-3/2 \middle| 00 \right\rangle} \qquad \text{from where}$$

$\left\langle (d_{5/2})^4_{00} \left| a^+_{5/2\,3/2} \right| (d_{5/2})^3_{3/2\,-3/2} \right\rangle = -4/3\,\sqrt{2/7}$ and $\left\langle 5/2\,3/2, 3/2\,-3/2 \middle| 00 \right\rangle$ do not exist(see appendixes C and D).Hence, the reduced matrix element(RME) likewise vanishes.

The RME we just described arises in the $(d_{5/2})^4$ configuration and we henceforward proceed to find the necessary RMEs required for our c.f.p. calculations in this work.

<u>E.1 d^4 CONFIGURATIION</u>

<u>E.1a $(d_{5/2})^4$ CONFIGURATIION</u>

1. $$\left\langle (d_{5/2})^4_0 \left\| a^+_{5/2} \right\| (d_{5/2})^3_{3/2} \right\rangle = ne$$

2. $$\left\langle (d_{5/2})^4_2 \left\| a^+_{5/2} \right\| (d_{5/2})^3_{3/2} \right\rangle = \frac{\left\langle (d_{5/2})^4_{2,2} \left| a^+_{5/2\,5/2} \right| (d_{5/2})^3_{3/2\,-1/2} \right\rangle}{\left\langle 5/2\,5/2, 3/2\,-1/2 \middle| 2\;2 \right\rangle}$$

From the appendices C and D respectively, we find that

$\left\langle (d_{5/2})^4_2 \left\| a^+_{5/2} \right\| (d_{5/2})^3_{3/2} \right\rangle = -4/7\,\sqrt{5/7}$ and $\left\langle 5/2\,5/2, 3/2\,-1/2 \middle| 2\;2 \right\rangle = 2\sqrt{5/42}$.

Hence, $\underline{\left\langle (d_{5/2})^4_2 \left\| a^+_{5/2} \right\| (d_{5/2})^3_{3/2} \right\rangle = -2\sqrt{3/7}}$

3. $$\left\langle (d_{5/2})^4_4 \left\| a^+_{5/2} \right\| (d_{5/2})^3_{3/2} \right\rangle = \frac{\left\langle (d_{5/2})^4_{4,1} \left| a^+_{5/2\,3/2} \right| (d_{5/2})^3_{3/2\,-1/2} \right\rangle}{\left\langle 5/2\,3/2, 3/2\,-1/2 \middle| 4,1 \right\rangle}$$

From the appendices C and D respectively, we find that

$\left\langle (d_{5/2})^4_{4,1} \left| a^+_{5/2\,3/2} \right| (d_{5/2})^3_{3/2\,-1/2} \right\rangle = 1/7\,\sqrt{5}$ and $\left\langle 5/2\,3/2, 3/2\,-1/2 \middle| 4,1 \right\rangle = \sqrt{15/56}$ .Hence,

$$\underline{\left\langle (d_{5/2})^4_4 \left\| a^+_{5/2} \right\| (d_{5/2})^3_{3/2} \right\rangle = 2\sqrt{2/21}}$$

4. $$\left\langle (d_{5/2})^4_0 \left\| a^+_{5/2} \right\| (d_{5/2})^3_{5/2} \right\rangle = \frac{\left\langle (d_{5/2})^4_{0,0} \left| a^+_{5/2\,-5/2} \right| (d_{5/2})^3_{5/2\,5/2} \right\rangle}{\left\langle 5/2\,-5/2, 5/2\,5/2 \middle| 0,0 \right\rangle}$$

From the appendices B and C respectively, we find that

$\left\langle (d_{5/2})^4_{0,0} \left| a^+_{5/2\,-5/2} \right| (d_{5/2})^3_{5/2\,5/2} \right\rangle = -2/\sqrt{6}$ and $\left\langle 5/2\,-5/2, 5/2\,5/2 \middle| 0,0 \right\rangle = -1/\sqrt{6}$. Hence,

$$\left\langle (d_{5/2})^4_0 \left\| a^+_{5/2} \right\| (d_{5/2})^3_{5/2} \right\rangle = -2$$

5. $$\left\langle (d_{5/2})^4_2 \left\| a^+_{5/2} \right\| (d_{5/2})^3_{5/2} \right\rangle = \frac{\left\langle (d_{5/2})^4_{2,0} \left| a^+_{5/2\,-5/2} \right| (d_{5/2})^3_{5/2\,5/2} \right\rangle}{\left\langle 5/2\ {-5/2}, 5/2\ 5/2 \middle| 2,0 \right\rangle}$$

From the appendices C and D respectively, we find that $\left\langle (d_{5/2})^4_{2,0} \left| a^+_{5/2\,-5/2} \right| (d_{5/2})^3_{5/2\,5/2} \right\rangle = 5/2\sqrt{21}$ and $\left\langle 5/2\ {-5/2}, 5/2\ 5/2 \middle| 2,0 \right\rangle = -5/\sqrt{84}$. Hence,

$$\left\langle (d_{5/2})^4_2 \left\| a^+_{5/2} \right\| (d_{5/2})^3_{5/2} \right\rangle = -1$$

6. $$\left\langle (d_{5/2})^4_4 \left\| a^+_{5/2} \right\| (d_{5/2})^3_{5/2} \right\rangle = \frac{\left\langle (d_{5/2})^4_{4,4} \left| a^+_{5/2\,3/2} \right| (d_{5/2})^3_{5/2\,5/2} \right\rangle}{\left\langle 5/2\ 3/2, 5/2\ 5/2 \middle| 4,4 \right\rangle}$$

From the appendices C and D respectively, we find that $\left\langle (d_{5/2})^4_{4,4} \left| a^+_{5/2\,3/2} \right| (d_{5/2})^3_{5/2\,5/2} \right\rangle = -1/\sqrt{2}$ and $\left\langle 5/2\ 3/2, 5/2\ 5/2 \middle| 4,4 \right\rangle = 1/\sqrt{2}$. Hence,

$$\left\langle (d_{5/2})^4_4 \left\| a^+_{5/2} \right\| (d_{5/2})^3_{5/2} \right\rangle = -1/2$$

7. $$\left\langle (d_{5/2})^4_0 \left\| a^+_{5/2} \right\| (d_{5/2})^3_{9/2} \right\rangle = ne$$

8. $$\left\langle (d_{5/2})^4_2 \left\| a^+_{5/2} \right\| (d_{5/2})^3_{9/2} \right\rangle = \frac{\left\langle (d_{5/2})^4_{2,2} \left| a^+_{5/2\,-3/2} \right| (d_{5/2})^3_{9/2\,7/2} \right\rangle}{\left\langle 5/2\ -3/2, 9/2\ 7/2 \middle| 2,2 \right\rangle}$$

From the appendices C and D respectively, we find that $\left\langle (d_{5/2})^4_{2,2} \left| a^+_{5/2\,-3/2} \right| (d_{5/2})^3_{9/2\,7/2} \right\rangle = -\sqrt{5/14}$ and $\left\langle 5/2\ -3/2, 9/2\ 7/2 \middle| 2,2 \right\rangle = 1/3\sqrt{5/2}$. Hence,

$$\underline{\left\langle (d_{5/2})^4_2 \left\| a^+_{5/2} \right\| (d_{5/2})^3_{9/2} \right\rangle = -3/\sqrt{7}}$$

9. $$\left\langle (d_{5/2})^4_4 \left\| a^+_{5/2} \right\| (d_{5/2})^3_{9/2} \right\rangle = \frac{\left\langle (d_{5/2})^4_{4,4} \left| a^+_{5/2\,1/2} \right| (d_{5/2})^3_{9/2\,7/2} \right\rangle}{\left\langle 5/2\ 1/2, 9/2\ 7/2 \middle| 4,4 \right\rangle}$$

From the appendices C and D respectively, we find that $\left\langle (d_{5/2})^4_{4,4} \left| a^+_{5/2\,1/2} \right| (d_{5/2})^3_{9/2\,7/2} \right\rangle = 1$ and $\left\langle 5/2\ 1/2, 9/2\ 7/2 \middle| 4,4 \right\rangle = \sqrt{21/55}$. Hence,

$$\underline{\left\langle (d_{5/2})^4_4 \left\| a^+_{5/2} \right\| (d_{5/2})^3_{9/2} \right\rangle = \sqrt{55/21}}$$

E.1b $(\mathrm{d}_{3/2})^4$ CONFIGURATION

$$\left\langle (d_{3/2})^4_0 \left\| a^+_{3/2} \right\| (d_{3/2})^3_{3/2} \right\rangle = \frac{\left\langle (d_{3/2})^4_{0,0} \left| a^+_{3/2\,-3/2} \right| (d_{3/2})^3_{3/2\,3/2} \right\rangle}{\left\langle 3/2\ -3/2, 3/2\ 3/2 \middle| 0,0 \right\rangle}$$

From the appendices B and C respectively, we find that

$\left\langle (d_{3/2})^4_{0,0} \left| a^+_{3/2\,-3/2} \right| (d_{3/2})^3_{3/2\,3/2} \right\rangle = 1$ and $\left\langle 3/2\ {-3/2}, 3/2\ 3/2 \middle| 0,0 \right\rangle = -1/2$ .Hence,

$$\underline{\left\langle (d_{3/2})^4_0 \left\| a^+_{3/2} \right\| (d_{3/2})^3_{3/2} \right\rangle = -2}$$

E.2 d^3 CONFIGURATION

E.2a $(d_{5/2})^3$ CONFIGURATION

1. $$\left\langle (d_{5/2})^3_{3/2} \left\| a^+_{5/2} \right\| (d_{5/2})^2_0 \right\rangle = \frac{\left\langle (d_{5/2})^3_{3/2,3/2} \left| a^+_{5/2\,3/2} \right| (d_{5/2})^2_{00} \right\rangle}{\left\langle 5/2\ 3/2, 00 \middle| 3/2, 3/2 \right\rangle}$$

From the appendices B and C respectively, we find that $\left\langle (d_{5/2})^3_{3/2,3/2} \left| a^+_{5/2\,3/2} \right| (d_{5/2})^2_{00} \right\rangle = -4/3\sqrt{2/7}$ and we discover that the Clebsch-Gordan coefficient $\left\langle 5/2\ 3/2, 00 \middle| 3/2, 3/2 \right\rangle$ does not exist. In this light, we see that the RME likewise does not exist.

2. $$\left\langle (d_{5/2})^3_{3/2} \left\| a^+_{5/2} \right\| (d_{5/2})^2_0 \right\rangle = \frac{\left\langle (d_{5/2})^3_{5/2,5/2} \left| a^+_{5/2\,5/2} \right| (d_{5/2})^2_{00} \right\rangle}{\left\langle 5/2\ 5/2, 00 \middle| 5/2, 5/2 \right\rangle}$$

From the appendices B and C respectively, we find that $\left\langle (d_{5/2})^3_{5/2,5/2} \left| a^+_{5/2\,5/2} \right| (d_{5/2})^2_{00} \right\rangle = -2/\sqrt{6}$ and $\left\langle 5/2\ 5/2, 00 \middle| 5/2, 5/2 \right\rangle = 1$. Hence,

$$\underline{\left\langle (d_{5/2})^3_{9/2} \left\| a^+_{5/2} \right\| (d_{5/2})^2_0 \right\rangle = -2/\sqrt{6}}$$

3. $$\left\langle (d_{5/2})^3_{9/2} \left\| a^+_{5/2} \right\| (d_{5/2})^2_0 \right\rangle = \frac{\left\langle (d_{5/2})^3_{9/2,3/2} \left| a^+_{5/2\,3/2} \right| (d_{5/2})^2_{00} \right\rangle}{\left\langle 5/2\, 3/2, 00 \middle| 9/2, 3/2 \right\rangle}$$

Since from the appendix C, the value of the cgc $\left\langle 5/2\, 3/2, 00 \middle| 9/2, 3/2 \right\rangle$ is non-existent, we find the RME is likewise not existing in nature.

4. $$\left\langle (d_{5/2})^3_{3/2} \left\| a^+_{5/2} \right\| (d_{5/2})^2_2 \right\rangle = \frac{\left\langle (d_{5/2})^3_{3/2,3/2} \left| a^+_{5/2\,5/2} \right| (d_{5/2})^2_{2,-1} \right\rangle}{\left\langle 5/2\, 5/2, 2-1 \middle| 3/2, 3/2 \right\rangle}$$

But $\left\langle (d_{5/2})^3_{3/2,3/2} \left| a^+_{5/2\,5/2} \right| (d_{5/2})^2_{2,-1} \right\rangle = -3/7 \sqrt{10/3}$ (App. C)

and $\left\langle 5/2\, 5/2, 2-1 \middle| 3/2, 3/2 \right\rangle = 3\sqrt{2/63}$ (App. D)

Hence, $\underline{\left\langle (d_{5/2})^3_{3/2} \left\| a^+_{5/2} \right\| (d_{5/2})^2_2 \right\rangle = -1/7 \sqrt{105}}$

5. $$\left\langle (d_{5/2})^3_{5/2} \left\| a^+_{5/2} \right\| (d_{5/2})^2_2 \right\rangle = \frac{\left\langle (d_{5/2})^3_{5/2,5/2} \left| a^+_{5/2\,3/2} \right| (d_{5/2})^2_{2,1} \right\rangle}{\left\langle 5/2\, 3/2, 2\,1 \middle| 5/2, 5/2 \right\rangle}$$

But $\left\langle (d_{5/2})^3_{5/2,5/2} \left| a^+_{5/2\,3/2} \right| (d_{5/2})^2_{2,1} \right\rangle = \sqrt{5/14}$ (App. C)

And $\left\langle 5/2\, 3/2, 2\,1 \middle| 5/2, 5/2 \right\rangle = -\sqrt{3/7}$ (App. D)

Therefore, $\underline{\left\langle (d_{5/2})^3_{5/2} \left\| a^+_{5/2} \right\| (d_{5/2})^2_2 \right\rangle = -\sqrt{5/6}}$

6. $$\left\langle (d_{5/2})^3_{9/2} \left\| a^+_{5/2} \right\| (d_{5/2})^2_2 \right\rangle = \frac{\left\langle (d_{5/2})^3_{9/2,7/2} \left| a^+_{5/2\,5/2} \right| (d_{5/2})^2_{2,1} \right\rangle}{\left\langle 5/2\; 5/2, 2\; 1 \middle| 9/2, 7/2 \right\rangle}$$

But $\left\langle (d_{5/2})^3_{9/2,7/2} \left| a^+_{5/2\,5/2} \right| (d_{5/2})^2_{2,1} \right\rangle = {}^{-2}/_{\sqrt{14}}$ (App. C)

And $\left\langle 5/2\; 5/2, 2\; 1 \middle| 9/2, 7/2 \right\rangle = 2/3$ (App. D)

Therefore, $\underline{\left\langle (d_{5/2})^3_{9/2} \left\| a^+_{5/2} \right\| (d_{5/2})^2_2 \right\rangle = {}^{-3}/_{\sqrt{14}}}$

7. $$\left\langle (d_{5/2})^3_{3/2} \left\| a^+_{5/2} \right\| (d_{5/2})^2_4 \right\rangle = \frac{\left\langle (d_{5/2})^3_{3/2,3/2} \left| a^+_{5/2\,5/2} \right| (d_{5/2})^2_{4,0} \right\rangle}{\left\langle 5/2\; 5/2, 4\; 0 \middle| 3/2, 3/2 \right\rangle}$$

But $\left\langle (d_{5/2})^3_{3/2,3/2} \left| a^+_{5/2\,5/2} \right| (d_{5/2})^2_{4,0} \right\rangle = {}^{-6}/_{\sqrt{147}}$ (App. C)

And $\left\langle 5/2\; 5/2, 4\; 0 \middle| 3/2, 3/2 \right\rangle = -\sqrt{2/7}$ (App. D)

Therefore, $\underline{\left\langle (d_{5/2})^3_{3/2} \left\| a^+_{5/2} \right\| (d_{5/2})^2_4 \right\rangle = -\sqrt{6/7}}$

8. $$\left\langle (d_{5/2})^3_{5/2} \left\| a^+_{5/2} \right\| (d_{5/2})^2_4 \right\rangle = \frac{\left\langle (d_{5/2})^3_{5/2,5/2} \left| a^+_{5/2\,5/2} \right| (d_{5/2})^2_{4,0} \right\rangle}{\left\langle 5/2\; 5/2, 4\; 0 \middle| 5/2, 5/2 \right\rangle}$$

But $\left\langle (d_{5/2})^3_{5/2,5/2} \left| a^+_{5/2\,5/2} \right| (d_{5/2})^2_{4,0} \right\rangle = 1/_{2\sqrt{7}}$ (App. C)

And $\left\langle 5/2\; 5/2, 4\; 0 \middle| 5/2, 5/2 \right\rangle = 1/2\sqrt{2/21}$ (App. D)

Therefore, $\underline{\left\langle (d_{5/2})^3_{5/2} \left\| a^+_{5/2} \right\| (d_{5/2})^2_4 \right\rangle = \sqrt{3/2}}$

9. $$\left\langle (d_{5/2})^3_{9/2} \left\| a^+_{5/2} \right\| (d_{5/2})^2_4 \right\rangle = \frac{\left\langle (d_{5/2})^3_{9/2,9/2} \left| a^+_{5/2\,5/2} \right| (d_{5/2})^2_{4,2} \right\rangle}{\left\langle 5/2\; 5/2, 4\; 2 \middle| 9/2, 9/2 \right\rangle}$$

But $\left\langle (d_{5/2})^3_{9/2,9/2} \left| a^+_{5/2\,5/2} \right| (d_{5/2})^2_{4,2} \right\rangle = \sqrt{5/14}$ (App. C)

And $\left\langle 5/2\; 5/2, 4\; 2 \middle| 9/2, 9/2 \right\rangle = \sqrt{5/33}$ (App. D)

Therefore, $\underline{\left\langle (d_{5/2})^3_{9/2} \left\| a^+_{5/2} \right\| (d_{5/2})^2_4 \right\rangle = \sqrt{33/14}}$

E.2b $(d_{3/2})^3$ CONFIGURATION

1. $$\left\langle (d_{3/2})^3_{3/2} \left\| a^+_{3/2} \right\| (d_{3/2})^2_0 \right\rangle = \frac{\left\langle (d_{3/2})^3_{3/2,3/2} \left| a^+_{3/2\,3/2} \right| (d_{3/2})^2_{0,0} \right\rangle}{\left\langle 3/2\; 3/2, 0\; 0 \middle| 3/2, 3/2 \right\rangle}$$

But $\left\langle (d_{3/2})^3_{3/2,3/2} \left| a^+_{3/2\,3/2} \right| (d_{3/2})^2_{0,0} \right\rangle = 1/\sqrt{2}$ (App. C)

And $\left\langle 3/2\; 3/2, 0\; 0 \middle| 3/2, 3/2 \right\rangle = 1$ (App. D)

Therefore, $\underline{\left\langle (d_{3/2})^3_{3/2} \left\| a^+_{3/2} \right\| (d_{3/2})^2_0 \right\rangle = 1/\sqrt{2}}$

2. $$\left\langle (d_{3/2})^3_{3/2} \left\| a^+_{3/2} \right\| (d_{3/2})^2_2 \right\rangle = \frac{\left\langle (d_{3/2})^3_{3/2,3/2} \left| a^+_{3/2\,-1/2} \right| (d_{3/2})^2_{2,2} \right\rangle}{\left\langle 3/2\; -1/2, 2\; 2 \middle| 3/2, 3/2 \right\rangle}$$

But $\left\langle (d_{3/2})^3_{3/2,3/2} \left| a^+_{3/2\,-1/2} \right| (d_{3/2})^2_{2,2} \right\rangle = 1$ (App. C)

And $\left\langle 3/2\; -1/2, 2\; 2 \middle| 3/2, 3/2 \right\rangle = \sqrt{2/5}$ (App. D)

Therefore, $\left\langle (d_{3/2})^3_{3/2} \left\| a^+_{3/2} \right\| (d_{3/2})^2_2 \right\rangle = \sqrt{5/2}$

E.3 $\quad$ d^2 CONFIGURATION

E.3a $\quad$ (d$_{5/2}$)^2 CONFIGURATION

1. $$\left\langle (d_{5/2})^2_0 \left\| a^+_{5/2} \right\| (d_{5/2})_{5/2} \right\rangle = \frac{\left\langle (d_{5/2})^2_{0,0} \left| a^+_{5/2\,-5/2} \right| (d_{5/2})_{5/2\,5/2} \right\rangle}{\left\langle 5/2\,-5/2, 5/2\,5/2 \middle| 0,0 \right\rangle}$$

But $\left\langle (d_{5/2})^2_{0,0} \left| a^+_{5/2\,-5/2} \right| (d_{5/2})_{5/2\,5/2} \right\rangle = 1/\sqrt{3}$ (App. D)

And $\left\langle 5/2\,-5/2, 5/2\,5/2 \middle| 0,0 \right\rangle = -1/\sqrt{6}$ (App. D)

Hence, $\left\langle (d_{5/2})^2_0 \left\| a^+_{5/2} \right\| (d_{5/2})_{5/2} \right\rangle = \sqrt{2}$

2. $$\left\langle (d_{5/2})^2_2 \left\| a^+_{5/2} \right\| (d_{5/2})_{5/2} \right\rangle = \frac{\left\langle (d_{5/2})^2_{2,2} \left| a^+_{5/2\,1/2} \right| (d_{5/2})_{5/2\,3/2} \right\rangle}{\left\langle 5/2\,1/2, 5/2\,3/2 \middle| 2,2 \right\rangle}$$

But $\left\langle (d_{5/2})^2_{2,2} \left| a^+_{5/2\,1/2} \right| (d_{5/2})_{5/2\,3/2} \right\rangle = -3/\sqrt{14}$ (App. C)

And $\left\langle 5/2\,1/2, 5/2\,3/2 \middle| 2,2 \right\rangle = 3\sqrt{3/70}$ (App. D)

Therefore, $\left\langle (d_{5/2})^2_2 \left\| a^+_{5/2} \right\| (d_{5/2})_{5/2} \right\rangle = -\sqrt{5/3}$

3. $$\left\langle (d_{5/2})^2_4 \left\| a^+_{5/2} \right\| (d_{5/2})_{5/2} \right\rangle = \frac{\left\langle (d_{5/2})^2_{4,4} \left| a^+_{5/2\,3/2} \right| (d_{5/2})_{5/2\,5/2} \right\rangle}{\left\langle 5/2\,3/2, 5/2\,5/2 \middle| 4,4 \right\rangle}$$

But $\left\langle (d_{5/2})^2_{4,4} \middle| a^+_{5/2\,3/2} \middle| (d_{5/2})_{5/2\,5/2} \right\rangle = 1$ (App. C)

And $\left\langle 5/2\ 3/2, 5/2\ 5/2 \middle| 4,4 \right\rangle = -1/\sqrt{2}$ (App. D)

Therefore, $\underline{\left\langle (d_{5/2})^2_4 \middle\| a^+_{5/2} \middle\| (d_{5/2})_{5/2} \right\rangle = -\sqrt{2}}$

E.3b $(\mathrm{d}_{3/2})^2$ CONFIGURATION

1. $$\left\langle (d_{3/2})^2_0 \middle\| a^+_{3/2} \middle\| (d_{3/2})_{3/2} \right\rangle = \frac{\left\langle (d_{3/2})^2_{0,0} \middle| a^+_{3/2\,-3/2} \middle| (d_{3/2})_{3/2\,3/2} \right\rangle}{\left\langle 3/2\ -3/2, 3/2\ 3/2 \middle| 0,0 \right\rangle}$$

But $\left\langle (d_{3/2})^2_{0,0} \middle| a^+_{3/2\,-3/2} \middle| (d_{3/2})_{3/2\,3/2} \right\rangle = -1/\sqrt{2}$ (App. C)

And $\left\langle 3/2\ -3/2, 3/2\ 3/2 \middle| 0,0 \right\rangle = -1/2$ (App. D)

Therefore, $\underline{\left\langle (d_{3/2})^2_0 \middle\| a^+_{3/2} \middle\| (d_{3/2})_{3/2} \right\rangle = \sqrt{2}}$

2. $$\left\langle (d_{3/2})^2_2 \middle\| a^+_{3/2} \middle\| (d_{3/2})_{3/2} \right\rangle = \frac{\left\langle (d_{3/2})^2_{2,2} \middle| a^+_{3/2\,1/2} \middle| (d_{3/2})_{3/2\,3/2} \right\rangle}{\left\langle 3/2\ 1/2, 3/2\ 3/2 \middle| 2,2 \right\rangle}$$

But $\left\langle (d_{3/2})^2_{2,2} \middle| a^+_{3/2\,1/2} \middle| (d_{3/2})_{3/2\,3/2} \right\rangle = 1$ (App. C)

And $\left\langle 3/2\ 1/2, 3/2\ 3/2 \middle| 2,2 \right\rangle = -1/\sqrt{2}$ (App. D)

Therefore, $\underline{\left\langle (d_{3/2})^2_2 \middle\| a^+_{3/2} \middle\| (d_{3/2})_{3/2} \right\rangle = -\sqrt{2}}$

NB: “ne” implies the RME is non-existent in such cases.

APPENDIX F

Single Fractional Parentage Coefficients' Calculations.

F.1 (d^4) CONFIGURATION

F.1a ($d^4_{5/2}$) CONFIGURATION

a. with $J = 3/2$,

i. $\left\langle (d_{5/2})^3\, 3/2, 5/2 \middle| \right\}(d_{5/2})^4\, 0 \Big\rangle = -1/2 \langle \Phi_0 \| a^+_{5/2} \| \varphi_{3/2} \rangle$

But the RME above vanishes due to the selection rules. Hence, the c.f.p. likewise vanishes!

ii. $\left\langle (d_{5/2})^3\, 3/2, 5/2 \middle| \right\}(d_{5/2})^4\, 2 \Big\rangle = -1/2 \left\langle (d_{5/2})^4_2 \| a^+_{5/2} \| (d_{5/2})^3_{3/2} \right\rangle$

But, $\left\langle (d_{5/2})^4_2 \| a^+_{5/2} \| (d_{5/2})^3_{3/2} \right\rangle = -2\sqrt{3/7}$ (App. E)

Therefore, $\underline{\left\langle (d_{5/2})^3\, 3/2, 5/2 |\}(d_{5/2})^4\, 2 \right\rangle = \sqrt{3/7}}$

iii. $\left\langle (d_{5/2})^3\, 3/2, 5/2 |\}(d_{5/2})^4\, 4 \right\rangle = -1/2 \left\langle (d_{5/2})^4_4 \| a^+_{5/2} \| (d_{5/2})^3_{3/2} \right\rangle$

But, $\left\langle (d_{5/2})^4_4 \| a^+_{5/2} \| (d_{5/2})^3_{3/2} \right\rangle = 2\sqrt{2/21}$ (App. E)

Therefore, $\underline{\left\langle (d_{5/2})^3\, 3/2, 5/2 |\}(d_{5/2})^4\, 4 \right\rangle = -\sqrt{2/21}}$

b. with $J = 5/2$, we find

i. $\left\langle (d_{5/2})^3\, 5/2, 5/2 |\}(d_{5/2})^4\, 0 \right\rangle = -1/2 \left\langle (d_{5/2})^4_0 \| a^+_{5/2} \| (d_{5/2})^3_{5/2} \right\rangle$

But, $\left\langle (d_{5/2})^4_0 \| a^+_{5/2} \| (d_{5/2})^3_{5/2} \right\rangle = -2$ (App. E)

Therefore, $\underline{\left\langle (d_{5/2})^3\, 5/2, 5/2 |\}(d_{5/2})^4\, 0 \right\rangle = 1}$

ii. $\left\langle (d_{5/2})^3\, 5/2, 5/2 |\}(d_{5/2})^4\, 2 \right\rangle = -1/2 \left\langle (d_{5/2})^4_2 \| a^+_{5/2} \| (d_{5/2})^3_{5/2} \right\rangle$

But, $\left\langle (d_{5/2})^4_2 \left\| a^+_{5/2} \right\| (d_{5/2})^3_{5/2} \right\rangle = -1$ (App. E)

Therefore, $\underline{\left\langle (d_{5/2})^3\, {5/2}, {5/2} \,|\}\, (d_{5/2})^4\, 2 \right\rangle = 1/2}$

iii. $\left\langle (d_{5/2})^3\, {5/2}, {5/2} \,|\}\, (d_{5/2})^4\, 4 \right\rangle = {-1/2} \left\langle (d_{5/2})^4_4 \left\| a^+_{5/2} \right\| (d_{5/2})^3_{5/2} \right\rangle$

But, $\left\langle (d_{5/2})^4_4 \left\| a^+_{5/2} \right\| (d_{5/2})^3_{5/2} \right\rangle = 1$ (App. E)

Therefore, $\underline{\left\langle (d_{5/2})^3\, {5/2}, {5/2} \,|\}\, (d_{5/2})^4\, 4 \right\rangle = {-1/2}}$

c. with J = $9/2$ .we find

i. $\left\langle (d_{5/2})^3\, {9/2}, {5/2} \,|\}\, (d_{5/2})^4\, 0 \right\rangle = {-1/2} \left\langle (d_{5/2})^4_0 \left\| a^+_{5/2} \right\| (d_{5/2})^3_{9/2} \right\rangle$

From the appendix E, we see that this particular RME vanishes and so does the c.f.p. vanish.

ii. $\left\langle (d_{5/2})^3\, {9/2}, {5/2} \,|\}\, (d_{5/2})^4\, 2 \right\rangle = {-1/2} \left\langle (d_{5/2})^4_2 \left\| a^+_{5/2} \right\| (d_{5/2})^3_{9/2} \right\rangle$

But, $\left\langle (d_{5/2})^4_2 \left\| a^+_{5/2} \right\| (d_{5/2})^3_{9/2} \right\rangle = -\, 3/\sqrt{7}$ (App. E)

Therefore, $\underline{\left\langle (d_{5/2})^3\, {9/2}, {5/2} \,|\}\, (d_{5/2})^4\, 2 \right\rangle = 3/2\sqrt{7}}$

iii. $\left\langle (d_{5/2})^3\, {9/2}, {5/2} \,|\}\, (d_{5/2})^4\, 4 \right\rangle = {-1/2} \left\langle (d_{5/2})^4_4 \left\| a^+_{5/2} \right\| (d_{5/2})^3_{9/2} \right\rangle$

But, $\left\langle (d_{5/2})^4_4 \left\| a^+_{5/2} \right\| (d_{5/2})^3_{9/2} \right\rangle = \sqrt{55/21}$ (App. E)

Therefore, $\underline{\left\langle (d_{5/2})^3\, {9/2}, {5/2} \,|\}\, (d_{5/2})^4\, 4 \right\rangle = {-1/2} \sqrt{55/21}}$

$\underline{F.1.b \quad (d_{3/2})^4 \text{ CONFIGURATION.}}$

Using Eq. (4.1), we write out the c.f.p. relation for the $(d_{3/2})^4$ configuration:

$$\left\langle (d_{3/2})^3\, J, 3/2 \,|\}(d_{3/2})^4\, I \right\rangle = \tfrac{(-1)^{4-1}}{\sqrt{4}} \left\langle (d_{3/2})^4_I \left\| a^+_{3/2} \right\| (d_{3/2})^3_J \right\rangle$$
$$= -1/2 \left\langle (d_{3/2})^4_I \left\| a^+_{3/2} \right\| (d_{3/2})^3_J \right\rangle$$

where I represents the angular momentum states of the $(d_{3/2})^4$ configuration while J represents the angular momenta of the $(d_{3/2})^3$ configuration.

The only possible c.f.p. here is $\left\langle (d_{3/2})^3\, 3/2, 3/2 \,|\}(d_{3/2})^4 0 \right\rangle$ and it is here evaluated:

$$\left\langle (d_{3/2})^3\, 3/2, 3/2 \,|\}(d_{3/2})^4 0 \right\rangle = -1/2 \left\langle (d_{3/2})^4_0 \left\| a^+_{3/2} \right\| (d_{3/2})^3_{3/2} \right\rangle$$

But, $\left\langle (d_{3/2})^4_0 \left\| a^+_{3/2} \right\| (d_{3/2})^3_{3/2} \right\rangle = -2$ (App. E)

Therefore, $\underline{\left\langle (d_{3/2})^3\, 3/2, 3/2 \,|\}(d_{3/2})^4 0 \right\rangle = 1}$

F.2 (d)^3 CONFIGURATION

F.2a (d$_{5/2}$)^3 CONFIGURATION

a. with J = 0,

i. $\left\langle (d_{5/2})^2\, 0, 5/2 \,|\}(d_{5/2})^3\, 3/2 \right\rangle = 1/\sqrt{3} \left\langle (d_{5/2})^3_{3/2} \left\| a^+_{5/2} \right\| (d_{5/2})^2_0 \right\rangle$

Since the Clebsch-Gordan coefficient $\left\langle 5/2\; 3/2;\, 0\,0 \middle| 3/2\; 3/2 \right\rangle$ vanishes(App. E, No. 9), we clearly see that the RME vanishes accordingly and so does the c.f.p.

ii. $\left\langle (d_{5/2})^2\, 0, 5/2 \,|\}(d_{5/2})^3\, 5/2 \right\rangle = 1/\sqrt{3} \left\langle (d_{5/2})^3_{5/2} \left\| a^+_{5/2} \right\| (d_{5/2})^2_0 \right\rangle$

But, $\left\langle (d_{5/2})^3_{5/2} \left\| a^+_{5/2} \right\| (d_{5/2})^2_0 \right\rangle = -2/\sqrt{6}$ (App. E)

Therefore, $\underline{\left\langle (d_{5/2})^2\, 0, 5/2 \,|\}(d_{5/2})^3\, 5/2 \right\rangle = 1/3\sqrt{2}}$

iii. $\left\langle (d_{5/2})^2\, 0, 5/2 \,|\}(d_{5/2})^3\, 9/2 \right\rangle = 1/\sqrt{3} \left\langle (d_{5/2})^3_{9/2} \left\| a^+_{5/2} \right\| (d_{5/2})^2_0 \right\rangle$

But $\left\langle (d_{5/2})^3_{9/2} \left\| a^+_{5/2} \right\| (d_{5/2})^2_0 \right\rangle$ is non-existent from the triangle conditions and so does the c.f.p.: $\left\langle (d_{5/2})^2\, 0, 5/2 \,|\}(d_{5/2})^3\, 9/2 \right\rangle$

b. with J = 2,

i. $\left\langle (d_{5/2})^2\, 2, 5/2 \,|\}(d_{5/2})^3\, 3/2 \right\rangle = 1/\sqrt{3}\, \left\langle (d_{5/2})^3_{3/2} \left\| a^+_{5/2} \right\| (d_{5/2})^2_2 \right\rangle$

But, $\left\langle (d_{5/2})^3_{3/2} \left\| a^+_{5/2} \right\| (d_{5/2})^2_2 \right\rangle = {}^{-1}/_7 \sqrt{105}$ (App. E)

Therefore, $\underline{\left\langle (d_{5/2})^2\, 2, 5/2 \,|\}(d_{5/2})^3\, 3/2 \right\rangle = -\sqrt{5/7}}$

ii. $\left\langle (d_{5/2})^2\, 2, 5/2 \,|\}(d_{5/2})^3\, 5/2 \right\rangle = 1/\sqrt{3}\, \left\langle (d_{5/2})^3_{5/2} \left\| a^+_{5/2} \right\| (d_{5/2})^2_2 \right\rangle$

But, $\left\langle (d_{5/2})^3_{5/2} \left\| a^+_{5/2} \right\| (d_{5/2})^2_2 \right\rangle = -\sqrt{5/6}$ (App. E)

Therefore, $\underline{\left\langle (d_{5/2})^2\, 2, 5/2 \,|\}(d_{5/2})^3\, 5/2 \right\rangle = {}^{-1}/_3 \sqrt{5/2}}$

iii. $\left\langle (d_{5/2})^2\, 2, 5/2 \,|\}(d_{5/2})^3\, 9/2 \right\rangle = 1/\sqrt{3}\, \left\langle (d_{5/2})^3_{9/2} \left\| a^+_{5/2} \right\| (d_{5/2})^2_2 \right\rangle$

But, $\left\langle (d_{5/2})^3_{9/2} \left\| a^+_{5/2} \right\| (d_{5/2})^2_2 \right\rangle = -\, 3/\sqrt{14}$ (App. E)

Therefore, $\underline{\left\langle (d_{5/2})^2\, 2, 5/2 \,|\}(d_{5/2})^3\, 9/2 \right\rangle = -\sqrt{3/14}}$

c. with J = 4, we find

i. $\left\langle (d_{5/2})^2\, 4, 5/2 \,|\}(d_{5/2})^3\, 3/2 \right\rangle = 1/\sqrt{3}\, \left\langle (d_{5/2})^3_{3/2} \left\| a^+_{5/2} \right\| (d_{5/2})^2_4 \right\rangle$

But, $\left\langle (d_{5/2})^3_{3/2} \left\| a^+_{5/2} \right\| (d_{5/2})^2_4 \right\rangle = -\sqrt{6/7}$ (App. E)

Therefore, $\underline{\left\langle (d_{5/2})^2\, 4, 5/2 \,|\}(d_{5/2})^3\, 3/2 \right\rangle = -\sqrt{2/7}}$

ii. $\left\langle (d_{5/2})^2\, 4, 5/2 \,|\}(d_{5/2})^3\, 5/2 \right\rangle = 1/\sqrt{3}\, \left\langle (d_{5/2})^3_{5/2} \left\| a^+_{5/2} \right\| (d_{5/2})^2_4 \right\rangle$

But, $\left\langle (d_{5/2})^3_{5/2} \left\| a^+_{5/2} \right\| (d_{5/2})^2_4 \right\rangle = \sqrt{3/2}$ (App. E)

Therefore, $\underline{\left\langle (d_{5/2})^2\, 4, 5/2 \,|\}(d_{5/2})^3\, 5/2 \right\rangle = 1/\sqrt{2}}$

iii. $\left\langle (d_{5/2})^2\, 4, 5/2 \,|\}(d_{5/2})^3\, 9/2 \right\rangle = 1/\sqrt{3}\, \left\langle (d_{5/2})^3_{9/2} \left\| a^+_{5/2} \right\| (d_{5/2})^2_4 \right\rangle$

But, $\left\langle (d_{5/2})^3_{9/2} \left\| a^+_{5/2} \right\| (d_{5/2})^2_4 \right\rangle = \sqrt{5/33}$ (App. E)

Therefore, $\underline{\left\langle (d_{5/2})^2\, 4, 5/2 \,|\}(d_{5/2})^3\, 9/2 \right\rangle = \sqrt{11/14}}$

F.2b $(d_{3/2})^3$ CONFIGURATION

a. $\left\langle (d_{3/2})^2 0, 3/2 |\} (d_{3/2})^3\ 3/2 \right\rangle = 1/\sqrt{3} \left\langle (d_{3/2})^3_{3/2} \left\| a^+_{3/2} \right\| (d_{3/2})^2_0 \right\rangle$

But, $\left\langle (d_{3/2})^3_{3/2} \left\| a^+_{3/2} \right\| (d_{3/2})^2_0 \right\rangle = 1/\sqrt{2}$ (App. E)

Therefore, $\underline{\left\langle (d_{3/2})^2 0, 3/2 |\} (d_{3/2})^3\ 3/2 \right\rangle = 1/\sqrt{6}}$

b. $\left\langle (d_{3/2})^2\ 2, 3/2 |\} (d_{3/2})^3\ 3/2 \right\rangle = 1/\sqrt{3} \left\langle (d_{3/2})^3_{3/2} \left\| a^+_{3/2} \right\| (d_{3/2})^2_2 \right\rangle$

But, $\left\langle (d_{3/2})^3_{3/2} \left\| a^+_{3/2} \right\| (d_{3/2})^2_2 \right\rangle = \sqrt{5/2}$ (App. E)

Therefore, $\underline{\left\langle (d_{3/2})^2\ 2, 3/2 |\} (d_{3/2})^3\ 3/2 \right\rangle = \sqrt{5/6}}$

F.3 $(d)^2$ CONFIGURATION

F.3a $(d_{5/2})^2$ CONFIGURATION

1. $\left\langle (d_{5/2})^1\ 5/2, 5/2 |\} (d_{5/2})^2\ 0 \right\rangle = -1/\sqrt{2} \left\langle (d_{5/2})^2_0 \left\| a^+_{5/2} \right\| (d_{5/2})^1_{5/2} \right\rangle$

But, $\left\langle (d_{5/2})^2_0 \left\| a^+_{5/2} \right\| (d_{5/2})^1_{5/2} \right\rangle = \sqrt{2}$ (App. E)

Therefore, $\underline{\left\langle (d_{5/2})^1\ 5/2, 5/2 |\} (d_{5/2})^2\ 0 \right\rangle = -1}$

2. $\left\langle (d_{5/2})^1\ 5/2, 5/2 |\} (d_{5/2})^2\ 2 \right\rangle = -1/\sqrt{2} \left\langle (d_{5/2})^2_2 \left\| a^+_{5/2} \right\| (d_{5/2})^1_{5/2} \right\rangle$

But, $\left\langle (d_{5/2})^2_2 \left\| a^+_{5/2} \right\| (d_{5/2})^1_{5/2} \right\rangle = -\sqrt{5/3}$ (App. E)

Therefore, $\underline{\left\langle (d_{5/2})^1\ 5/2, 5/2 |\} (d_{5/2})^2\ 2 \right\rangle = -\sqrt{5/6}}$

3. $\left\langle (d_{5/2})^1\ 5/2, 5/2 |\} (d_{5/2})^2\ 4 \right\rangle = -1/\sqrt{2} \left\langle (d_{5/2})^2_4 \left\| a^+_{5/2} \right\| (d_{5/2})^1_{5/2} \right\rangle$

But, $\left\langle (d_{5/2})^2_4 \left\| a^+_{5/2} \right\| (d_{5/2})^1_{5/2} \right\rangle = -\sqrt{2}$ (App. E)

Therefore, $\underline{\left\langle (d_{5/2})^1\ 5/2, 5/2 |\} (d_{5/2})^2\ 4 \right\rangle = 1}$

F.3b $(d_{3/2})^2$ CONFIGURATION

1. $\left\langle (d_{3/2})^1\, 3/2, 3/2 \,|\}\, (d_{3/2})^2\, 0 \right\rangle = -1/\sqrt{2} \left\langle (d_{3/2})^2_0 \left\| a^+_{3/2} \right\| (d_{3/2})^1_{3/2} \right\rangle$

   But, $\left\langle (d_{3/2})^2_0 \left\| a^+_{3/2} \right\| (d_{3/2})^1_{3/2} \right\rangle = \sqrt{2}$ (App. E)

   Therefore, $\underline{\left\langle (d_{3/2})^1\, 3/2, 3/2 \,|\}\, (d_{3/2})^2\, 0 \right\rangle = -1}$

2. $\left\langle (d_{3/2})^1\, 3/2, 3/2 \,|\}\, (d_{3/2})^2\, 2 \right\rangle = -1/\sqrt{2} \left\langle (d_{3/2})^2_2 \left\| a^+_{3/2} \right\| (d_{3/2})^1_{3/2} \right\rangle$

   But, $\left\langle (d_{3/2})^2_2 \left\| a^+_{3/2} \right\| (d_{3/2})^1_{3/2} \right\rangle = -\sqrt{2}$ (App. E)

   Therefore, $\underline{\left\langle (d_{3/2})^1\, 3/2, 3/2 \,|\}\, (d_{3/2})^2\, 2 \right\rangle = 1}$

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