Simplex method

Simplex method is the method to solve (LPP) models which contain two or more decision variables.

Basic variables:

Are the variables which coefficients One in the equations and Zero in the other equations.

Non-Basic variables:

Are the variables which coefficients are taking any of the values, whether positive or negative or zero.

Slack, surplus & artificial variables:

a) **If the inequality be $\leq$ (less than or equal, then we add a slack variable + S to change $\leq$ to =.**
b) **If the inequality be $\geq$ (greater than or equal, then we subtract a surplus variable - S to change $\geq$ to =.**
c) **If we have = we use artificial variables.**

The steps of the simplex method:

Step 1:

Determine a starting basic feasible solution.

Step 2:

Select an entering variable using the optimality condition. Stop if there is no entering variable.

Step 3:

Select a leaving variable using the feasibility condition.

Optimality condition:

The entering variable in a maximization (minimization) problem is the non-basic variable having the most negative (positive) coefficient in the Z-row.

The optimum is reached at the iteration where all the Z-row coefficient of the non-basic variables are non-negative (non-positive).

Feasibility condition:

For both maximization and minimization problems the leaving variable is the basic associated with the smallest non-negative ratio (with strictly positive denominator).

Pivot row:

a) Replace the leaving variable in the basic column with the entering variable.
b) New pivot row equal to current pivot row divided by pivot element.
c) All other rows:
   New row=current row - (pivot column coefficient) *new pivot row.

Example 1:

Use the simplex method to solve the (LP) model:

$$max\ Z = 5x_1 + 4x_2$$

Subject to

$$6x_1 + 4x_2 \leq 24$$

$$x_1 + 2x_2 \quad \leq 6$$

$$-x_1 + x_2 \quad \leq 1$$

$$x_2 \quad \leq 2$$

$$x_1, x_2 \quad \geq 0$$

Solution:

$$max\ Z - 5x_1 + 4x_2 = 0$$

Subject to

$$6x_1 + 4x_2 + S_1 = 24$$

$$x_1 + 2x_2 + S_2 = 6$$

$$-x_1 + x_2 + S_3 = 1$$

$$x_2 + S_4 = 2$$

Table 1:

Basic	x_1	x_2	S_1	S_2	S_3	S_4	Sol.
S_1	$\boxed{6}$	4	1	0	0	0	24
S_2	1	2	0	1	0	0	6
S_3	-1	1	0	0	1	0	1
S_4	0	1	0	0	0	1	2
Max Z	-5	-4	0	0	0	0	0

$\frac{24}{6} = \boxed{4}$

$\frac{6}{1} = 6$

$\frac{1}{-1} = -1$ (ignore)

$\frac{2}{0} = \infty$ (ignore)

The entering variable is x_1 and S_1 is a leaving variable.

Table 2:

Basic	x_1	x_2	S_1	S_2	S_3	S_4	Sol.
x_1	1	2/3	1/6	0	0	0	4
S_2	0	$\boxed{4/3}$	-1/6	1	0	0	2
S_3	0	5/3	1/6	0	1	0	5
S_4	0	1	0	0	0	1	2
Max Z	0	-2/3	5/6	0	0	0	20

- Pivot row or new x_1-row=$\frac{1}{6}$ [current S_1 –row]

$$= \frac{1}{6}[6 \quad 4 \quad 1 \quad 0 \quad 0 \quad 0 \quad 24]$$

$$= [1 \quad \frac{2}{3} \quad \frac{1}{6} \quad 0 \quad 0 \quad 0 \quad 4]$$

- New S_2-row=[current S_2 –row]-(1)[new x_1 –row]

=[1 2 0 1 0 0 6]- (1)[1 2/3 1/6 0 0 0 0 4]

=[0 4/3 -1/6 1 0 0 2]

- New S_3-row=[current S_3 –row]-(1)[new x_1 –row]

=[-1 1 0 0 1 0 1]- (1)[1 2/3 1/6 0 0 0 0 4]

=[0 5/3 1/6 0 1 0 5]

- New S_4-row=[current S_4 –row]-(0)[new x_1 –row]

=[0 1 0 0 0 1 2]- (0)[1 2/3 1/6 0 0 0 0 4]

=[0 1 0 0 0 1 2]

- New Z-row=[current Z –row]-(-5)[new x_1 –row]

=[-5 -4 0 0 0 0 0]-(-5)[1 2/3 1/6 0 0 0 0 4]

=[0 -2/3 5/6 0 0 0 20]

Now:

$$\frac{4}{\frac{2}{3}} = 6$$

$$\frac{2}{\frac{4}{3}} = \frac{6}{4} = \boxed{\frac{3}{2}}$$

$$\frac{5}{\frac{5}{3}} = 3$$

$$\frac{2}{1} = 2$$

The entering variable is x_2 and S_2 is a leaving variable.

Table 3: (optimal solution):

Basic	x_1	x_2	S_1	S_2	S_3	S_4	Sol.
x_1	1	0	1/4	-1/2	0	0	3
x_2	0	1	-1/8	3/4	0	0	3/2
S_3	0	0	3/8	-5/4	1	0	5/2
S_4	0	0	1/8	-3/4	0	1	1/2
Max Z	0	0	5/6	1/2	0	0	21

- **Pivot row or new x_2-row=$\frac{1}{\frac{4}{3}}$ [current S_2 –row]**

=$\frac{1}{\frac{4}{3}}$[0 4/3 -1/6 1 0 0 2]

=[0 1 -1/8 ¾ 0 0 3/2]

- New x_1-row=[current x_1 –row]-(2/3)[new x_2 –row]

=[1 2/3 1/6 0 0 0 4]- (2/3)[0 1 -1/8 ¾ 0 0 3/2]

=[1 0 ¼ -1/2 0 0 3]

- New S_3-row=[current S_3 –row]-(5/2)[new x_2 –row]

=[0 5/3 1/6 0 1 0 5]-(5/3)[0 1 -1/8 ¾ 0 0 3/2]

=[0 0 3/8 -5/4 1 0 5/3]

- New S_4-row=[current S_4 –row]-(1)[new x_2 –row]

=[0 1 0 0 0 1 2]-(1)[0 1 -1/8 ¾ 0 0 3/2]

=[0 0 1/8 -3/4 0 1 ½]

New Z-row=[current Z –row]-(-2/3)[new x_2 –row]

=[0 -2/3 5/6 0 0 0 20]-(-2/3)[0 1 -1/8 ¾ 0 0 3/2]

=[0 0 ¾ ½ 0 0 21]

Then the solution is:

$$x_1 = 3 \ \& \ x_2 = \frac{3}{2} \ \& S_3 = \frac{5}{2} \ \& S_4 = \frac{1}{2}$$

$$S_1 = 0 \ , \ \ S_2 = 0$$

<u>Example 2:</u>

Use the simplex method to solve the (LP) model:

$$max \ Z = 2x_1 + 3x_2$$

Subject to

$$0.25x_1 + 0.5x_2 \leq 40$$

$$0.4x_1 + 0.2x_2 \quad \leq 40$$

$$0.8x_2 \quad \leq 40$$

$$x_1, x_2 \qquad \geq 0$$

<u>Solution:</u>

$$max \ Z - 2x_1 + 3x_2 = 0$$

Subject to

$$0.25x_1 + 0.5x_2 + S_1 = 40$$

$$0.4x_1 + 0.2x_2 \ + S_2 \ = 40$$

$$0.8x_2 \ \ + S_3 \ \ = 40$$

$$x_1, x_2, +S_1, +S_2, +S_3 \ \geq 0$$

<u>Table 1:</u>

Basic	x_1	x_2 ↓	S_1	S_2	S_3	Sol.
S_1	0.25	0.5	1	0	0	40
S_2	0.4	0.2	0	1	0	40
← S_3	0	0.8	0	0	1	40
Max Z	-2	-3	0	0	0	0

$\frac{40}{0.5} = 80$

$\frac{40}{0.2} = 200$

$\frac{40}{0.8} = \boxed{50}$

- **Pivot row or new S_3-row=$\frac{1}{0.8}$ [0 0.8 0 0 1 40]**
  =[0 1 0 0 1.25 50]

New S_1-row=[current S_1 –row]-(0.5)[new x_2 –row]
=[0.25 0.5 1 0 0 40]-(0.5)[0 1 0 0 1.25 50]
=[0 0.5 0 0 -0.625 15]

New S_2-row=[current S_2 –row]-(0.2)[new x_2 –row]
=[0.4 0.2 0 1 0 40]-(0.2)[0 1 0 0 1.25 50]
[0.4 0 0 1 -0.25 30]

New Z-row=[current Z –row]-(-3)[new x_2 –row]
=[-2 -3 0 0 0 0]-(-3)[0 1 0 0 1.25 50]
=[-2 0 0 0 3.75 150]

Table 2:

Basic	x_1	x_2	S_1	S_2	S_3	Sol.
S_1	$\boxed{0.25}$	0	1	0	-0.625	15
S_2	0.4	0	0	1	-0.25	30
x_2	0	1	0	0	1.25	50
Max Z	-2	0	0	0	3.75	150

$\frac{15}{0.25} = \boxed{60}$

$\frac{30}{0.4} = 75$

$\frac{50}{0} = \infty$ (ignore)

Pivot row or new S_1-row=$\frac{1}{0.25}$[0.25 0 1 0 -0.625 15]

=[1 0 4 0 -2.5 60]

New S_2-row=[current S_2 –row]-(0.4)[new x_1 –row]

=[0.4 0 0 0 -0.25 30]-(0.4)[1 0 4 0 -2. 5 60]

[0 0 -1.6 0 -0.75 6]

New x_2-row=[0 1 0 0 1.25 50]-(0)[1 0 4 0 -2. 5 60]

=[0 1 0 0 1.25 50]

New Z-row=[current Z –row]-(-2)[1 0 4 0 -2.5 60]

=[-2 0 0 0 3.75 150]-(-2)[1 0 4 0 -2. 5 60]

[0 0 8 0 -1.25 270]

<u>Table 3:</u>

Basic	x_1	x_2	S_1	S_2	S_3	Sol.
x_1	1	0	4	0	-2.5	60
S_2	0	0	-1.6	1	0.75	6
x_2	0	1	0	0	1.25	50
Max Z	0	0	8	0	-1.25	270

$\frac{60}{-2.5} = -24$ (ignore)

$\frac{6}{0.75} = \boxed{8}$

$\frac{50}{1.25} = 40$

New S_2-row=$\frac{1}{0.75}$ =[current S_2-row] =$\frac{1}{0.75}$ [0 0 -1.6 0 0.75 6]

=[0 0 -2.133 0 1 8]

New x_1-row= [1 0 4 0 -2.5 60]-(-2.5)[0 0 -2.133 0 1 8]

=[1 0 -1.333 0 0 80]

New x_2-row= [0 1 0 0 1.25 50]-(-1.25)[0 0 -2.133 0 1 8]

=[0 1 -2.76 0 0 40]

New Z-row= [0 0 8 0 -1.25 270]-(-2.5)[0 0 -2.133 0 1 8]

=[0 0 5.33 0 0 280]

Table 3: (optimal solution):

Basic	x_1	x_2	S_1	S_2	S_3	Sol.
x_1	1	0	-1.333	0	0	80
S_3	0	0	-2.133	0	1	8
x_2	0	1	-2.67	0	0	40
Max Z	0	0	5.33	0	0	280

The optimal solution :

x_1=80 , $x_2 = 40$, $S_1 = 0$ & $S_2 = 0$ // Z=280

Example 3:

Use the simplex method to solve the (LP) model:

$$min\ Z = -6x_1 - 10x_2 - 4x_3$$

Subject to

$$x_1 + x_2 + x_3 \le 1000$$

$$x_1 + x_2 \quad \le 500$$

$$x_1 + 2x_2 \quad \le 700$$

$$x_1, x_2, x_3 \quad \ge 0$$

Solution:

$$min\ Z + 6x_1 + 10x_2 + 4x_3 = 0$$

Subject to

$$x_1 + x_2 + x_3 + S_1 = 1000$$

$$x_1 + x_2 \quad + S_2 \qquad = 500$$

$$x_1 + 2x_2 \quad + S_3 \qquad = 700$$

$$x_1, x_2, x_3\ , S_{1,} S_2\ S_3\ \ \geq 0$$

Table 1:

Basic	x_1	x_2	x_3	S_1	S_2	S_3	Sol.
S_1	1	1	1	1	0	0	1000
S_2	1	1	0	0	1	0	500
S_3	1	2	0	0	1	1	700
Max Z	6	10	4	0	0	0	0

$\frac{1000}{1} = 1000$

$\frac{500}{1} = 500$

$\frac{700}{2} = \boxed{350}$

New S_3-row or x_2-row $=\frac{1}{2}$ [1 2 0 0 0 1 700]

$=[\frac{1}{2}$ 1 0 0 0 $\frac{1}{2}$ 350]

New S_1-row = [1 1 1 1 0 0 1000]-(1)[$\frac{1}{2}$ 1 0 0 0 $\frac{1}{2}$ 350]

$=[\frac{1}{2}$ 0 1 1 0 -$\frac{1}{2}$ 650]

New S_2-row = [1 1 0 0 1 0 500]-(1)[$\frac{1}{2}$ 1 0 0 0 $\frac{1}{2}$ 350]

$=[\frac{1}{2}$ 0 0 0 1 -$\frac{1}{2}$ 150]

New Z-row = [6 10 4 0 0 0   0]-(10)[$\frac{1}{2}$ 1 0 0   0 $\frac{1}{2}$ 350]

=[1   0   4   0   0   - 5   -3500]

Table 2:

Basic	x_1	x_2	x_3	S_1	S_2	S_3	Sol.
S_1	1/2	0	1	1	0	-1/2	650
S_2	1/2	0	0	0	1	-1/2	150
x_2	1/2	1	0	0	0	1/2	350
Max Z	1	0	4	0	0	-5	-3500

$\frac{650}{1} = \boxed{650}$

$\frac{150}{0} = \infty$ (ignore)

$\frac{350}{0} = \infty$ (ignore)

New S_1-row or x_3-row =1[$\frac{1}{2}$ 0 1   1 0 -$\frac{1}{2}$ 650]

=[$\frac{1}{2}$ 0 1   1 0 -$\frac{1}{2}$ 650]

New S_2-row = [$\frac{1}{2}$ 0 0 0 1 - $\frac{1}{2}$ 150]-(0)[$\frac{1}{2}$ 0 1   1 0 -$\frac{1}{2}$ 650]

=[$\frac{1}{2}$ 0 0 0 1 - $\frac{1}{2}$ 150]

New x_2-row = [$\frac{1}{2}$ 1 0 0 0 $\frac{1}{2}$ 350]-(0)[$\frac{1}{2}$ 0 1   1 0 -$\frac{1}{2}$ 650]

=[$\frac{1}{2}$ 1 0 0 0 $\frac{1}{2}$ 350]

New Z-row = [1 0 4 0 0 -5   -3500]-(4)[$\frac{1}{2}$ 0 1   1 0 -$\frac{1}{2}$ 650]

=[-1 0 0 -4 0   -3   -6100]

Table 3: (optimal solution):

Basic	x_1	x_2	x_3	S_1	S_2	S_3	Sol.
x_3	1/2	0	1	1	0	-1/2	650
S_2	1/2	0	0	0	1	-1/2	150
x_2	1/2	1	0	0	0	1/2	350
Max Z	-1	0	0	-4	0	-3	-6100

The optimal solution :

x_3=650 , $x_2 = 350$, $S_1 = 0$ S_3 & $S_2 = 150$ $x_1 = 0$ // Z=280

Example 4:

Use the simplex method to solve the (LP) model:

$$max\ Z = 4x_1 - x_2$$

Subject to

$$x_1 + 2x_2 \leq 4$$

$$2x_1 + 3x_2 \leq 12$$

$$x_1 - x_2 \leq 3$$

$$x_1, x_2 \geq 0$$

Solution:

$$max\ Z - 4x_1 + x_2 = 0$$

Subject to

$$x_1 + 2x_2 + S_1 = 4$$

$$2x_1 + 3x_2 + S_2 = 12$$

$$x_1 - x_2 + S_3 = 3$$

$$x_1, x_2, S_1, S_2, S_3 \geq 0$$

Table 1:

Basic	x_1	x_2	S_1	S_2	S_3	Sol.
S_1	1	2	1	0	0	4
S_2	2	3	0	1	0	12
S_3	1	-1	0	0	1	3
Max Z	-4	1	0	0	0	0

$\frac{4}{1} = 4$

$\frac{12}{2} = 6$

$\frac{3}{1} = \boxed{3}$

New S_3-row or x_1-row =1[1 -1 0 0 1 3]

=[1 -1 0 0 1 3]

New S_1-row = [1 2 1 0 0 4]-(1)[1 -1 0 0 1 3]

=[0 3 1 0 -1 1]

New S_2-row = [2 3 0 1 0 12]-(2)[1 -1 0 0 1 3]

=[0 5 0 1 -2 6]

New Z-row = [-4 1 0 0 0 0]-(-4)[1 -1 0 0 1 3]

=[0 -3 0 0 4 12]

Table 2:

Basic	x_1	x_2	S_1	S_2	S_3	Sol.
S_1	0	3	1	0	-1	1
S_2	0	5	0	1	-2	6
x_1	1	-1	0	0	1	3
Max Z	0	-3	0	0	4	12

$\frac{1}{3}=\boxed{\frac{1}{3}}$

$\frac{6}{5}=\frac{6}{5}$

$\frac{3}{-1}=-3$ **(ignore)**

New S_1-row or x_2-row =$(\frac{1}{3})$[0 3 1 0 -1 1]

=[0 1 1/3 0 -1/3 1/3]

New S_2-row = [0 5 0 1 -2 6]-(5)[$0\ \ 1\ \ 1/3\ \ 0\ -1/3\ \ 1/3$]

=[0 0 -2/3 1 11/3 13/3]

New x_1-row = [1 -1 0 0 1 3]-(-1)[$0\ \ 1\ \ 1/3\ \ 0\ -1/3\ \ 1/3$]

=[1 0 1/3 0 2/3 10/3]

New Z-row = [0 -3 0 0 4 12]-(-3)[$0\ \ 1\ \ 1/3\ \ 0\ -1/3\ \ 1/3$]

=[0 0 1 0 3 13]

<u>Table 3: (optimal solution):</u>

Basic	x_1	x_2	S_1	S_2	S_3	Sol.
x_2	0	1	1/3	0	-1/3	1/3
S_2	0	0	-2/3	1	11/3	13/3
x_1	1	0	1/3	0	2/3	10/3
Max Z	0	0	1	0	3	13

The optimal solution :

x_1=10/3 , $x_2=1/3$, $S_2=13/3$ S_1 & $S_3=0$ // Z=13

Example 5:

Use the simplex method to solve the (LP) model:

$$max\ \ Z = 16x_1 + 17x_2 + 10x_3$$

Subject to

$$x_1 + 2x_2 + 4x_3 \ \leq 2000$$

$$2x_1 + x_2 + \ x_3 \quad \leq 3600$$

$$x_1 + 2x_2 + 2x_3 \quad \leq 2400$$

$$x_1 \qquad\qquad\qquad \leq 30$$

$$x_1, x_2,\ x_3 \ \ \geq 0$$

Solution:

$$max\ \ Z - 16x_1 - 17x_2 - 10x_3 = 0$$

Subject to

$$x_1 + 2x_2 + 4x_3 + \ S_1 = 2000$$

$$2x_1 + x_2 + \ x_3 \ + \ S_2 = 3600$$

$$x_1 + 2x_2 + 2x_3 \ + \ S_3 = 2400$$

$$x_1 \ + \ S_4 \qquad\qquad = 30$$

$$x_1, x_2,\ x_3 \ \ \geq 0, \quad S_1, S_2,\ S_3, S_4 \ \geq 0$$

Table 1:

$\downarrow$

Basic	x_1	x_2	x_3	S_1	S_2	S_3	S_4	Sol.
$\leftarrow$ S_1	1	2	4	1	0	0	0	2000
S_2	2	1	1	0	1	0	0	3600
S_3	1	2	2	0	0	1	0	2400
S_4	1	0	0	0	0	0	1	30
Max Z	-16	-17	-10	0	0	0	0	0

$\frac{2000}{2} = \boxed{1000}$

$\frac{3600}{1} = 3600$

$\frac{2400}{2} = 1200$

$\frac{30}{0} = \infty$ (ignore)

New S_1-row or x_1-row =$(\frac{1}{2})$[1 2 4 1 0 0 0 2000]

=[1/2 1 2 1/2 0 0 0 1000]

New S_2-row = [2 1 1 0 1 0 0 3600]

-(1)[$1/2$ 1 2 $1/2$ 0 0 0 1000]

=[3/2 0 -1 -1/2 1 0 0 2600]

New S_3-row = [1 2 2 0 0 1 0 2400]

-(2)[$1/2$ 1 2 $1/2$ 0 0 0 1000]

=[0 0 -2 -1 0 1 0 400]

New S_4-row = [1 0 0 0 0 0 1 30]

-(0)[$1/2$ 1 2 $1/2$ 0 0 0 1000]

=[1 0 0 0 0 0 1 30]

New Z-row = [-16 -17 -10 0 0 0 0 0]

-(-17)[$1/2$ 1 2 $1/2$ 0 0 0 1000]

=[15/2 0 24 17/2 0 0 0 17000]

Table 2: (optimal solution):

Basic	x_1	x_2	x_3	S_1	S_2	S_3	S_4	Sol.
x_2	1/2	1	2	1/2	0	0	0	1000
S_2	3/2	0	-1	-1/2	1	0	0	2600
S_3	0	0	-2	-1	0	1	0	400
S_4	1	0	0	0	0	0	1	30
Max Z	15/2	0	24	17/2	0	0	0	17000

The optimal solution :

x_2=1000 , $S_2 = 2600$, $S_3 = 400, S_4 = 30$ x_1 , x_2 $S_1 = 0$ /

Z=17000

Example 6:

Use the simplex method to solve the (LP) model:

$$max\ Z = 3x_1 + 5x_2 + 4x_3$$

Subject to

$$2x_1 + 3x_2 \leq 8$$

$$2x_1 + 5x_2 \leq 10$$

$$3x_1 + 2x_2 + 4x_3 \leq 15$$

$$x_1, x_2,\ x_3 \geq 0$$

Solution:

$$max\ Z - 3x_1 - 5x_2 - 4x_3 = 0$$

Subject to

$$2x_1 + 3x_2 + S_1 \leq 8$$

$$2x_1 + 5x_2 + S_2 \leq 10$$

$$3x_1 + 2x_2 + 4x_3 + S_3 \leq 15$$

$$x_1, x_2,\ x_3, S_1,\ S_2, S_3 \geq 0$$

Table 1:

Basic	x_1	x_2	x_3	S_1	S_2	S_3	Sol.
S_1	2	3	0	1	0	0	8
S_2	2	5	0	0	1	0	10
S_3	3	2	4	0	0	1	15
Max Z	-3	-5	-4	0	0	0	0

$\frac{8}{3} = 2.7$

$\frac{10}{5} \boxed{= 2}$

$\frac{15}{2} = 7.5$

New S_2-row or x_2-row =$(\frac{1}{5})$[2 5 0 0 1 0 10]

=[2/5 1 0 0 1/5 0 2]

New S_1-row = [2 3 0 1 0 0 8]

-(3)[$2/5$ 1 0 0 $1/5$ 0 2]

=[4/5 0 0 1 -3/5 0 2]

New S_3-row = [3 2 4 0 0 1 15]

-(2)[$2/5$ 1 0 0 $1/5$ 0 2]

=[11/5 0 4 0 -2/5 1 11]

New Z-row = [-3 -5 -4 0 0 0 0]

-(-5)[$2/5$ 1 0 0 $1/5$ 0 2]

=[-1 0 -4 0 1 0 10]

Table 2:

Basic	x_1	x_2	x_3	S_1	S_2	S_3	Sol.
S_1	4/5	0	0	1	-3/5	0	2
x_2	2/5	1	0	0	1/5	0	2
S_3	11/5	1	4	0	-2/5	1	11
Max Z	-1	0	-4	0	1	0	10

New S_3-row or x_3-row =$(\frac{1}{4})$[11/5 0 4 0 -2/5 1 11]

=[11/20 0 1 0 -1/10 1/4 11/4]

New S_1-row = [4/5 0 0 1 -3/5 0 2]

-(0)[$11/20 \quad 0 \;\; 1 \quad 0 \quad -1/10 \quad 1/4 \quad 11/4$]

=[4/5 0 0 1 -3/5 0 2]

New x_2-row = [2/5 1 0 0 1/5 0 2]

New Z-row = [-1 0 -4 0 1 0 10]

-(-4)[$11/20 \quad 0 \;\; 1 \quad 0 \quad -1/10 \quad 1/4 \quad 11/4$]

=[6/5 0 0 0 3/5 1 21]

Table 3: (optimal solution):

Basic	x_1	x_2	x_3	S_1	S_2	S_3	Sol.
S_1	4/5	0	0	1	-3/5	0	2
x_2	2/5	1	0	0	1/5	0	2
x_3	11/20	0	1	0	-1/10	1/4	11/4
Max Z	6/5	0	0	0	3/5	1	21

The optimal solution :

x_2=2 ,

$x_3 = 11/4$,

$S_1 = 2$,

Z=21

$x_1 = 0$, $S_2 = 0$, $S_3 = 0$,