GRAPHICAL SOLUTION METHOD

3.1 GRAPHICAL PROBLEM

In the previous section, we have looked at some models called linear programming models. In each case, the model had a function called an objective function, which was to be maximized or minimized while satisfying several conditions or constraints.

If there are only two variables, one can use a graphical method of solution. Let us begin with the set of constraints and consider them as a system of inequalities. The solution of this system of inequalities is a set of points, S. Each point of the set S is called a feasible solution. The objective function can be evaluated for different feasible solutions and the maximum or minimum values obtained.

Graph (Linear): A linear graph consists of a number of nodes or junction points, each joined to some or all of the others by arcs or lines.

3.2: METHODS FOR SOLVING GRAPHICAL PROBLEM

There are three methods of solving graphical problem. They are

i. General Graphical Method
ii. Corner Point Solution, and
iii. Computer Solution Method

3.2 STEPS FOR SOLVING GNERAL GRAPHICAL PROBLEM

The various steps for solving Graphical problems are as follows

- ✍ Formulate te problem with mathematical form by
    - o Specifying the decision variables
    - o Identifying the objective function
    - o Writing the constraint equations
- ✍ Plot the constraint equation on a graph
- ✍ Identify the area of feasible solution
- ✍ Locate the corner points of the feasible region
- ✍ Plot the objective function
- ✍ Choose the points where objective functions have optimal values

3.4 CORNER POINT SOLUTION

The search procedure adopted in can be simplified by taking advantage of the first characteristics of feasible solution and the objective function

The following statements are of fundamental importance in linear programming

- ✍ The solution set for a group of linear inequalities is a convex set. Therefore the area of feasible solution for a linear programming problem is a convex set

- Given a linear objective function linear programming problem , the optimal solution will always include a corner point in the area of feasible solution.

Thus the corner point method for solving linear programming problem has the following steps

- Step !: Graphically identify the area of feasible solution
- Step 2: Determine the co ordinates of each corner point on the area of feasible solution
- Step 3: Substitute the co ordinates of the corner point in the objective function to determine the corresponding value of Z
- Step 4: An optimal solution occurs in a maximization problem at the corner point yielding the highest value of Z and in a minimization problem at the corner point yielding the lowest value of Z

3.5 COMPUTER SOLUTION METHOD

In actual application, LP problems are solved by computer methods as today much efficient computer softwares are readily available. The person who knows in detail about the LP problem can use these softwares effectively. Yet, following guidelines will be helpful for the person who wants to use computer softwares

- One should fully understand the LP model
- One may be able to make assumptions
- Have necessary skill to formulate the problem
- Ability to arrange solutions using a computer
- Capable of interpreting the output from such packages

SOLVED EXAMPLES OF GRAPHICAL PROBLEM

Example 3. 1

Maximize: $Z = 4x + 5y$

Subject to:

$2x + 5y \leq 25$

$6x + 5y \leq 45$

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$x \geq 0 , y \geq 0$

SOLUTION

To solve the above linear programming model using the graphical method, we shall turn each constraints inequality to equation and set each variable equal to zero (0) to obtain two (2) coordinate points for each equation (i.e. using double intercept form). Having obtained all the coordinate points, we shall determine the range of our variables which enables us to know the appropriate scale to use for our graph. Thereafter, we shall draw the graph and join all the coordinate points with required straight line.

$2x + 5y = 25$ [Constraint 1]

When $x = 0$, $y = 5$ and when $y = 0$, $x = 12.5$.

$6x + 5y = 45$ [Constraint 2]

When $x = 0$, $y = 9$ and

when $y = 0$, $x = 7.5$.

Minimum value of x is $x = 0$.

Maximum value of x is $x = 12.5$.
Range of x is $0 \leq x \leq 12.5$.
Minimum value of y is $y = 0$.
Maximum value of y is $y = 9$.

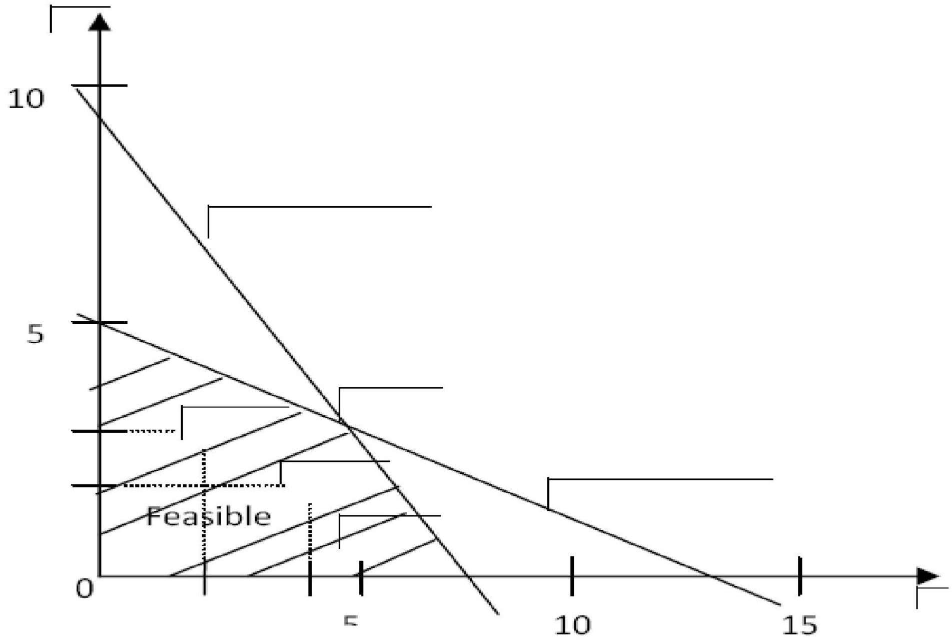

Fig.3.1

The constraints give a set of feasible solutions as graphed above. To solve the linear programming problem, we must now find the feasible solution that makes the objective function as large as possible. Some possible solutions are listed below:

Table 3.1

Feasible solutions (A point in the solution set of the system)	Objective function Z= 4x + 5y
(2,3)	4(2)+5(3) = 8 + 15 = 23
(4,2)	4(4)+5(2) = 16 + 10 = 26
(5, 1)	4(5)+5(1) = 20 + 5 = 25
(7, 0)	4(7)+5(0) = 28 + 0 = 28
(0, 5)	4(0)+5(5) = 0 + 25 = 25

In this list, the point that makes the objective function the largest is (7,0) . But, is this the largest for all feasible solutions? How about (6,1)? or (5,3)? It turns out that (5,3) provide the maximum value: 4(5) + 5(3) = 20 +15 = 35 .

Hence, maximum profit at point (5,3) and it is the objective functions which have optimal values

Example 3. 2

Find the corner points for:

$2x + 5y \leq 25$

$6x + 5y \leq 45$

$x \geq 0$, $y \geq 0$

This is the set of feasible solution for Example 6 .

SOLUTION:

The graph for Example 3.1 is repeated here and shows the corner points.

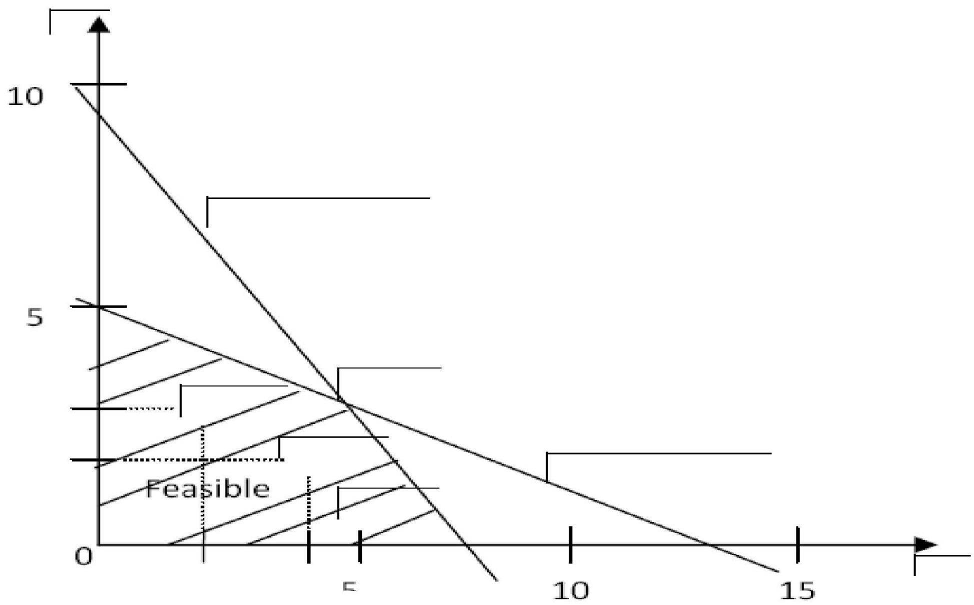

Fig.3.2

Some corner points can usually be found by inspection. In this case, we can see $A = (0,0)$ and $D = (0,5)$. Some corner points may require some work with boundary lines (uses equations of boundaries not the inequalities giving the regions).

Point C:

System: $2x + 5y = 25$ … (1)

$6x + 5y = 45$ … (2)

(1) − (2) ® $-4x = -20$

$\Rightarrow \quad x = 5$.

If $x = 5$, then from (1) or (2) :

$y = 3$.

Point B:

System: $y = 0$ … (1)

$6x + 5y = 45$ … (2)

Solve by substitution:

$\Rightarrow \quad 6x + 5(0) = 45 = 7.5$

The corner points for example 7 are: (0,0) , (0,5) , (7.5,0) and (5,3) .

Convex sets and corner points lead us to a method for solving certain linear programming problems.

Example 3.3

Solve the following linear programming problem:

Minimize: $Z = 60x + 30y$

Subject to:

$$2x + 3y \geq 120$$
$$2x + y \geq 80$$
$$x \geq 0 \, , y \geq 0 \, .$$

SOLUTION:

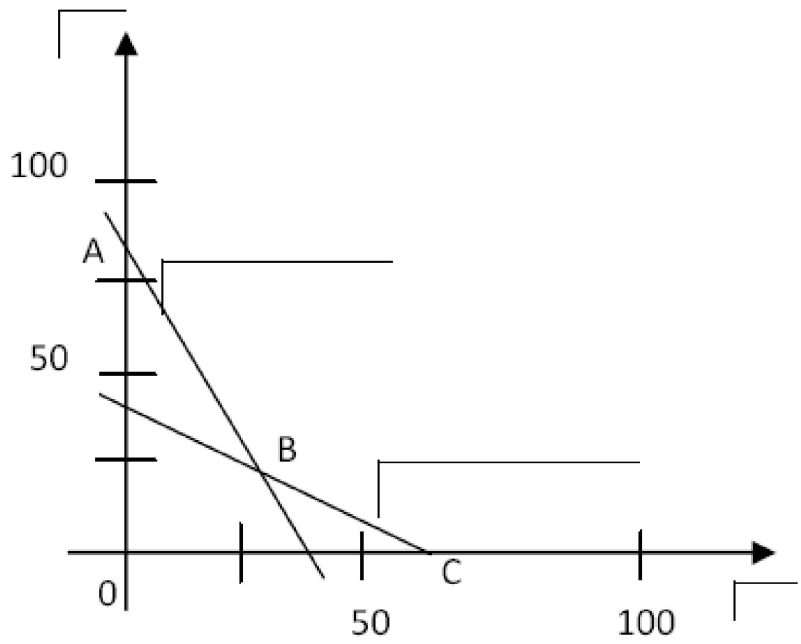

Fig.3.3

Corner points $A = (0,80)$ and $C = (60,0)$ are found by inspection.

Point B:

System: $2x + 3y = 120$ … ……………………………………..(1)

$2x + y = 80$ ………………………………………………… (2)

$$(1) - (2) = 2y = 40$$
$$y = 20 \, .$$

Substitute for $y = 20$ in (2) :

$$2x + 20 = 80 \, .$$
$$2x = 60 \, .$$

$x = 30$.

Point B: $(30,20)$.

Table 3.2

Extreme Values

Corner point	**Objective function Z = 60x + 30 y**
(0, 80)	60(0) + 30(80) = 2400
(30, 20)	60(30) + 30(20) = 2400
(60, 0)	60(60) + 30(0) = 3600

From the table above, there are two minimum values for the objective function: A = (0,80) and $B = (30,20)$. In this situation, the objective function will have the same minimum value (2,400) at all points along the boundary line segment A and B.

Example 3.4

Maximize: $z = 2x + 3y$

Subject to:

$x + 2y \le 40$

$6x + 5y \le 150$

$x \ge 0$, $y \ge 0$

SOLUTION

The feasible area is defined by the constraints as shown in the figure below in fig 3.4.

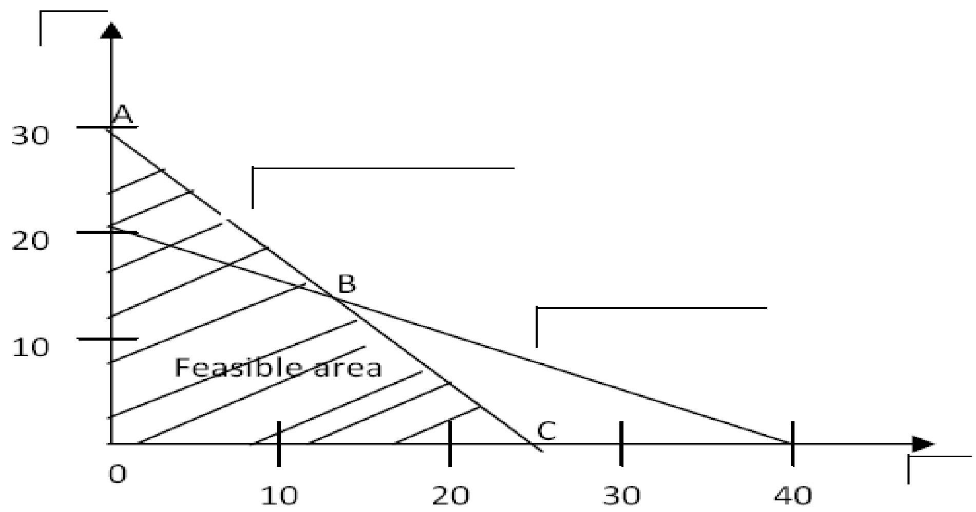

Fig.3.4

Suppose that in addition to the existing constraints, the company is contracted to produce at least 30 units each week. This additional constraint can be written as: $x + y \geq 30$. As a boundary solution, the constraint would be: $x + y = 30$, $(x = 0, y = 30)(x = 30, y = 0)$. The three structural constraints are shown in the figure below in fig 3.5.

This case presents the manager with demands which cannot simultaneously be satisfied.

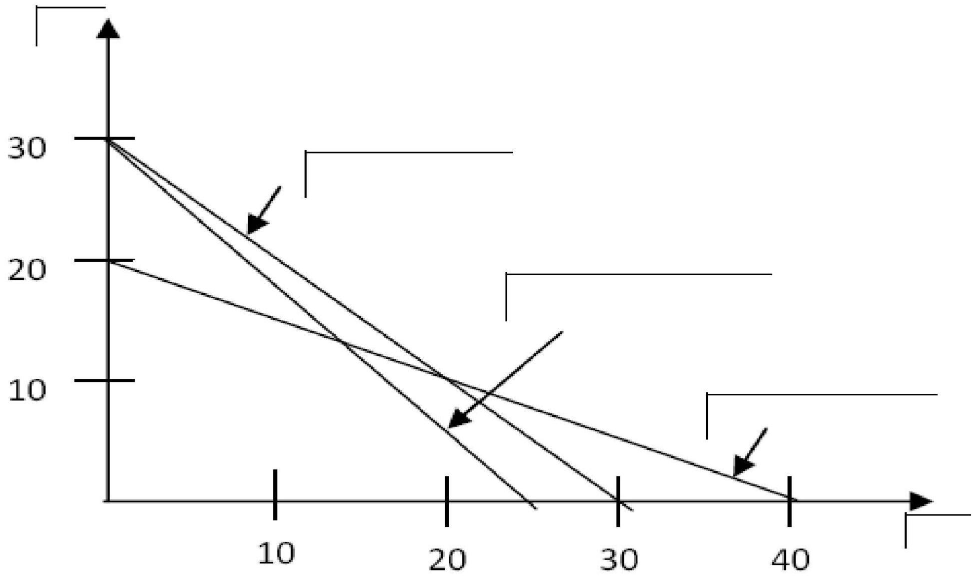

Fig.3.5

Example 3.5
Minimize: $z = 600x + 900y$
Subject to:

$$40x + 60y \geq 480$$
$$30x + 15y \geq 180$$
$$x \geq 0 , y \geq 0$$

SOLUTION
If we let $z =$ Rs 8100 , then:
$8100 = 600x + 900y$, $(x = 0, y = 9)(x = 13.5, y = 0)$.
The resultant trial cost is shown in the figure below.

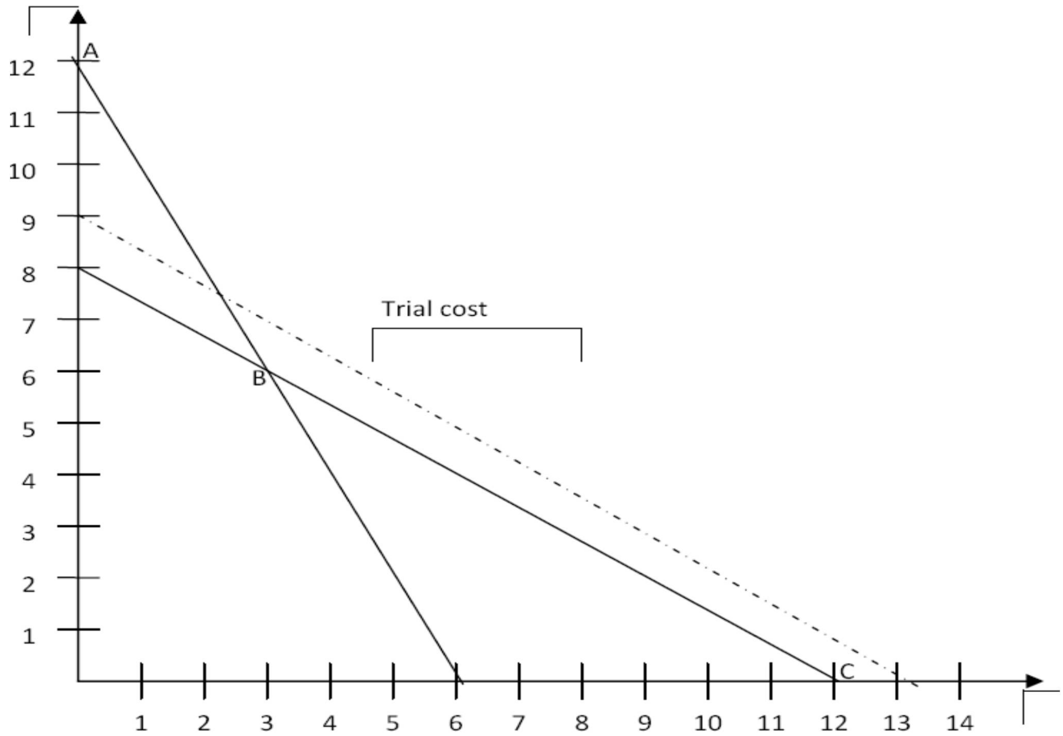

Fig.3.6

This line is parallel to the boundary line BC. The lowest acceptable cost solution will be coincidental with the line BC making point B, point C and any other points on the line BC optimal. Multiple optimum solutions present the manager with choice and hence some flexibility.

Example 3.6 (116): Solve the following problem graphically:

$$\text{Maximize } Z = -x_1+4x_2,$$

$$\text{Subject to } -3x_1+x_2 \leq 6,$$

$$x_1+2\,x_2 \leq 4\ ,$$

$$x_2 \leq -3,$$

No lower bound constraint for x_1.

Solution

: The third constraint can be re-written as $-\,x_2 \geq 3$.The solution space satisfying the constraints is shown shaded in **fig 1.1.**

FIG(1.1)

The co-ordinates of the two vertices of the region of solution are:

A(-3,-3) and B(10,-3).value of the objective function $Z = -x_1 + 4x_2$ at these vertices are Z(A) = 3-12=-9, Z(B)= -10-12=-22.
Thus the maximum value of Z occurs at A. Hence the solution to the problem is

$$x_1 = -3,\ x_2 = -3;\ Z_{max} = -9.$$

QUESTION BANK: 3.1

1. Maximize $Z = 3x1 + 4x2$ by using graphical method

   Subject to
   $6x1 + 12\,x2 \le 1200$
   $0.8x1 + 0.5\,x2 \le 60$
   $0.3x1 + 0.4\,x2 \le 50$
   $x1 \ge 0,\ x2 \ge 0$

2. Maximize $Z = 5x1 + 3x2$ by using graphical method
   Subject to
   $3x1 + 5x2 = 15$
   $5x1 + 2\,x2 = 10$
   $x1 \ge 0,\ x2 \ge 0$

3. Solve graphically the following linear programming problem
   Minimize, $Z = 2500x1 + 3500x2$
   Subject to
   $50x1 + 60x2 \ge 2500$
   $100x1 + 60\,x2 \ge 3000$
   $x1 \ge 0,\ x2 \ge 0$
4. Maximize $6A + 5B$
   Subject to
   $3A + 4B \le 120$
   $3A + 1B \le 40$
   $A + B = 30$
   $A \ge 0,\ B \ge 0$