Calculus Cheat Sheet for Computer Science

1 Limits

Definition:
A limit describes the value a function approaches as the input approaches a certain point.

$$\lim_{x \to c} f(x) = L$$

means that as x approaches c, $f(x)$ approaches L.

1.1 Key Properties

1. $\lim_{x \to c}[f(x) + g(x)] = \lim_{x \to c} f(x) + \lim_{x \to c} g(x)$
2. $\lim_{x \to c}[f(x)g(x)] = \lim_{x \to c} f(x) \cdot \lim_{x \to c} g(x)$
3. $\lim_{x \to c} \frac{f(x)}{g(x)} = \frac{\lim_{x \to c} f(x)}{\lim_{x \to c} g(x)} \quad (\text{if } g(x) \neq 0)$

1.2 Example

$$\lim_{x \to 2}(3x + 5) = 3(2) + 5 = 11$$

1.3 Applications

- **Algorithm Analysis:** Determine the asymptotic behavior as input size grows large

2 Derivatives

Definition:
The derivative of $f(x)$ measures the rate at which $f(x)$ changes with respect to x:

$$f'(x) = \lim_{h \to 0} \frac{f(x + h) - f(x)}{h}$$

2.1 Key Rules

1. **Constant Rule:** $\frac{d}{dx}[c] = 0$
2. **Power Rule:** $\frac{d}{dx}[x^n] = nx^{n-1}$
3. **Exponential Rule:** $\frac{d}{dx}[e^x] = e^x$
4. **Logarithmic Rule:** $\frac{d}{dx}[\ln(x)] = \frac{1}{x}$
5. **Product Rule:** $\frac{d}{dx}[uv] = u'v + uv'$
6. **Chain Rule:** $\frac{d}{dx}[f(g(x))] = f'(g(x)) \cdot g'(x)$

2.2 Examples

1. $\frac{d}{dx}[x^3] = 3x^2$
2. $\frac{d}{dx}[\sin(x)] = \cos(x)$
3. Let $f(x) = (3x + 2)^2$:
   Using the Chain Rule:
   $f'(x) = 2(3x + 2)(3) = 6(3x + 2)$

2.3 Applications

- **Optimization:** Minimize/maximize costs or performance metrics
- **Gradient Descent:** Minimize loss functions in machine learning

3 Integrals

Definition:
The integral of a function calculates the area under its curve:

$$\int f(x)\,dx$$

3.1 Common Formulas

1. $\int x^n\,dx = \frac{x^{n+1}}{n+1} + C \quad (\text{for } n \neq -1)$
2. $\int e^x\,dx = e^x + C$
3. $\int \frac{1}{x}\,dx = \ln|x| + C$

3.2 Examples

1. $\int 3x^2\,dx = x^3 + C$
2. $\int e^x\,dx = e^x + C$
3. $\int (x^2 + 3x)\,dx = \frac{x^3}{3} + \frac{3x^2}{2} + C$

3.3 Applications

- **Data Analysis:** Analyze continuous data distributions
- **Simulations:** Compute areas and physical quantities in graphics engines

4 Partial Derivatives

Definition:
Partial derivatives calculate the rate of change of a multivariable function with respect to one variable while keeping others constant:

$$\frac{\partial}{\partial x} f(x, y)$$

4.1 Example

Let $f(x, y) = x^2y + 3y^2$:

1. $\frac{\partial f}{\partial x} = 2xy$
2. $\frac{\partial f}{\partial y} = x^2 + 6y$

4.2 Applications

- **Neural Networks:** Calculate gradients for backpropagation
- **Optimization:** Optimize multivariable cost functions

5 Key Formulas Recap

1. **Derivative:** $\frac{d}{dx}[x^n] = nx^{n-1}$
2. **Integral:** $\int x^n \, dx = \frac{x^{n+1}}{n+1} + C$
3. **Product Rule:** $\frac{d}{dx}[uv] = u'v + uv'$
4. **Chain Rule:** $\frac{d}{dx}[f(g(x))] = f'(g(x)) \cdot g'(x)$
5. **L'Hôpital's Rule:**
   For indeterminate forms:
   $$\lim_{x \to c} \frac{f(x)}{g(x)} = \lim_{x \to c} \frac{f'(x)}{g'(x)}$$