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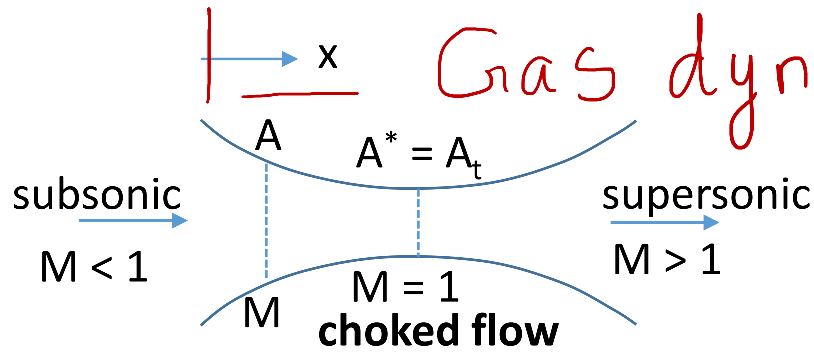
1

Gas dynamics

Nozzle



convergent divergent nozzle,
A = A(x)

- Such a flow, where area varies as A = A(x), but where it is assumed that P, ρ, T and u (flow properties) are still functions of x only, is defined as **quasi-one-dimensional flow**
- Flow through a duct with varying cross-sectional area, A = A(x)

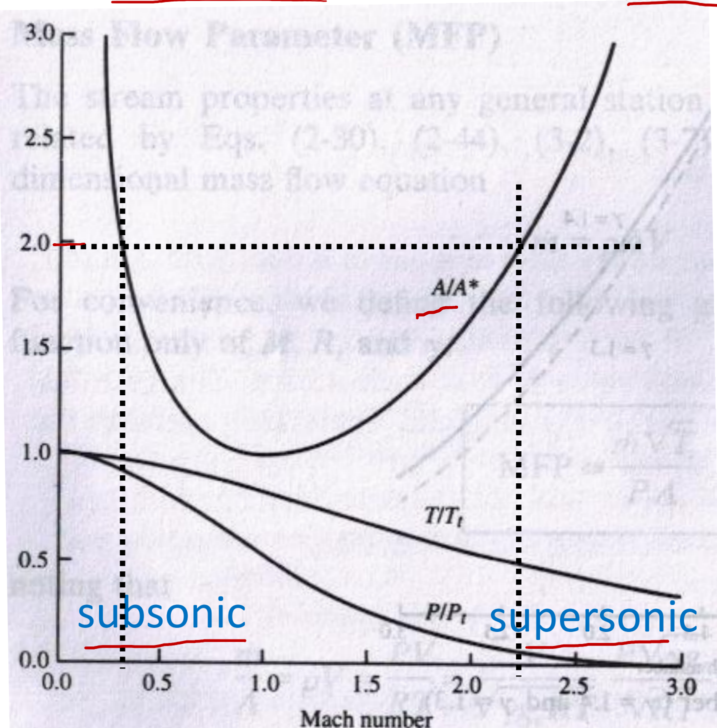


Fig. 3.10 A/A^*, T/T_t, and P/P_t, versus Mach number (γ = 1.4) (Elements of Gas Turbine Propulsion by Jack D. Mattingly, McGraw Hill India Pvt. Ltd., New Delhi, 2015)

Area-ratio

$$\frac{A}{A^*} = \frac{1}{M}\left[\frac{1+\frac{1}{2}(\gamma-1)M^2}{\frac{1}{2}(\gamma+1)}\right]^{\frac{\gamma+1}{2(\gamma-1)}}$$

Throat area

A^* = area at which flow is sonic (M = 1)

M=1

Min cross section area in the nozzle

- Isentropic, quasi-1D flow
- There will be two solutions for M for a given A/A^*, one subsonic and the other supersonic

$$\dot{m} = \frac{A^* p_{02}}{\sqrt{RT_{02}}}\sqrt{\gamma\left(\frac{2}{\gamma+1}\right)^{(\gamma+1)/(\gamma-1)}}$$

mass flow-rate

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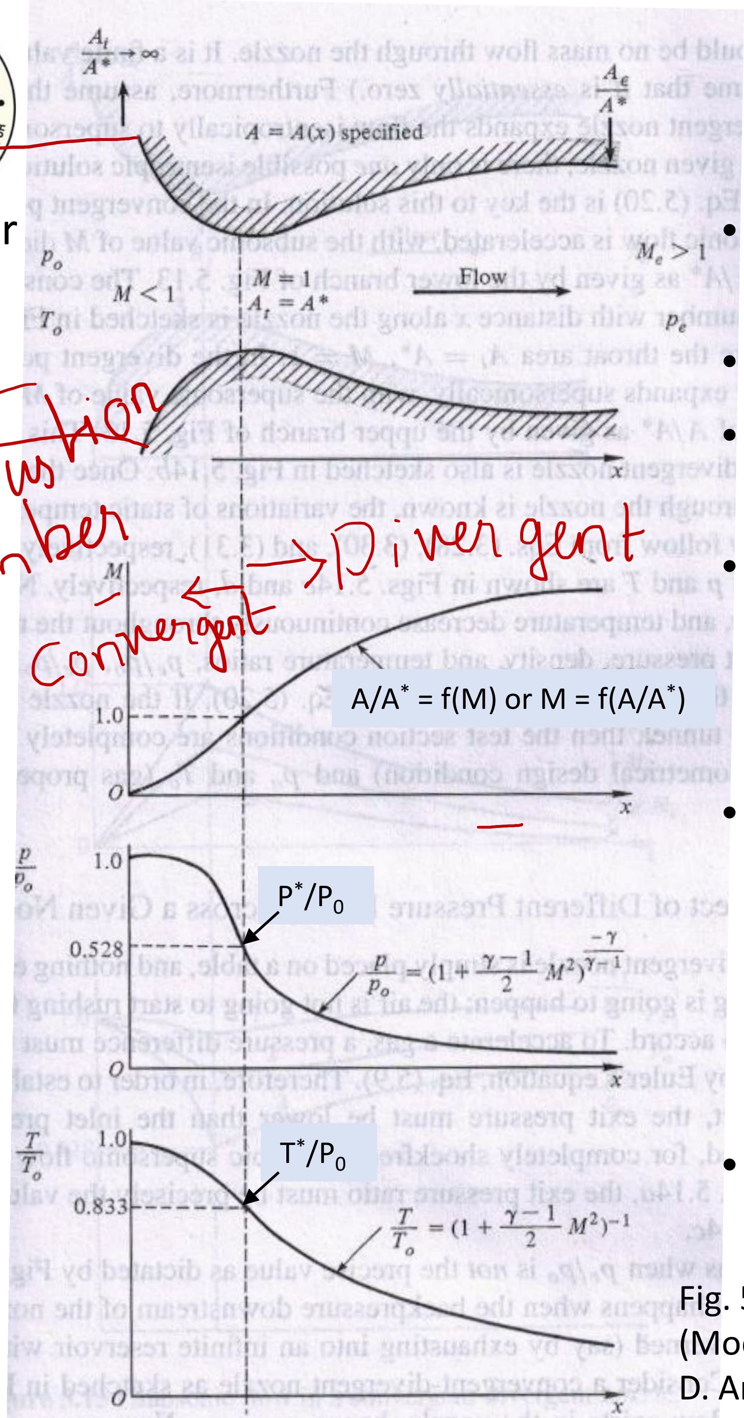
Area Variation



- Consider a convergent divergent nozzle with known variation of area: $[A/A^*](x)$ A/A^*
- Flow is isentropic, steady and quasi-1D
- Stagnation quantities are known: at combustion chamber $P \approx P_0$, $T \approx T_0$, $M_i \approx 0$ Known
- In the convergent portion, the flow is subsonic ($M < 1$), at the throat it is sonic ($M = 1$) and in the divergent portion it is supersonic ($M > 1$), $M(x)$ can be obtained from $[A/A^*](x)$
- Once variation of $M(x)$ is obtained, $[P/P_0](x)$ and $[T/T_0](x)$ can be obtained

$$\frac{T_0}{T} = 1 + \frac{\gamma - 1}{2} M^2, \quad \frac{P_0}{P} = \left(\frac{T_0}{T}\right)^{\frac{\gamma}{\gamma - 1}}$$

Isentropic relation

- **note: exit conditions, T_e/T_0, P_e/P_0, M_e, u_e will depend only on A_e/A_t**

Fig. 5.14 Isentropic supersonic nozzle flow (Modern Compressible Flow: with Historical Perspective by John D. Anderson, Third Edition, McGraw-Hill, New Delhi, 2014)

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Under, Correct and Over expansion

- An **under-expanded** (case I) nozzle has exit pressure greater than the ambient pressure ($P_e > P_a$), because the exit area is too small as required for the correct expansion +ve pr. thrust
- A **correctly-expanded** (case H) nozzle has exit pressure matching the ambient ($P_e = P_a$) No pressure thrust Max thrust
- An **over-expanded** (case G, cases between H and F) nozzle has lower exit pressure than the ambient ($P_e < P_a$) as it has exit area too large for optimum -ve
- As P_a is further increased (cases F, E and D), normal shock will sit in the divergent portion so that $P_e = P_a$
- Further increase in P_a will result in subsonic flow throughout the nozzle (cases C, B and A), for case C, the throat is sonic

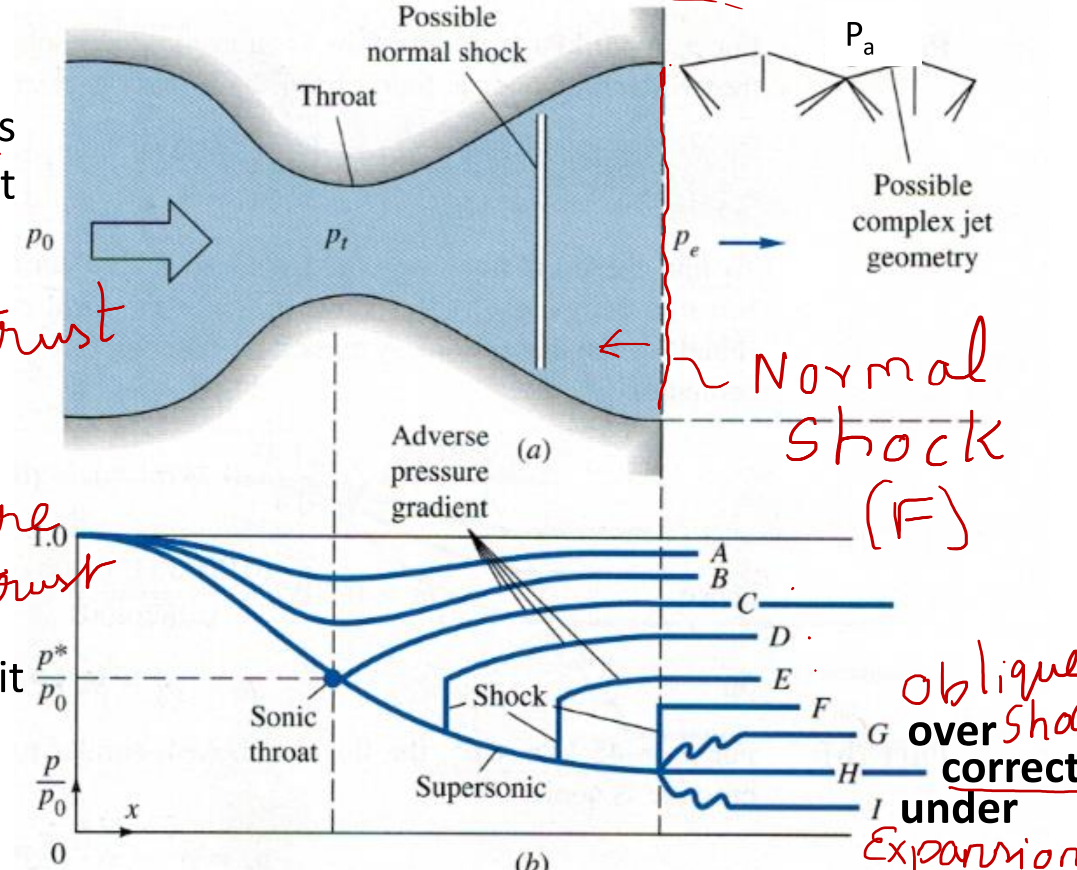

Fig. 9.12 Operation of Convergent-Divergent nozzle (Fluid Mechanics by Frank M. White, Fifth Edition, McGraw-Hill, New Delhi, 2003)

- Practical rockets operate in regimes (G – H – I)
- **Note:** normal shock patching idea is idealized: downstream of the shock, the nozzle flow has an adverse pressure gradient, which may lead to boundary layer separation

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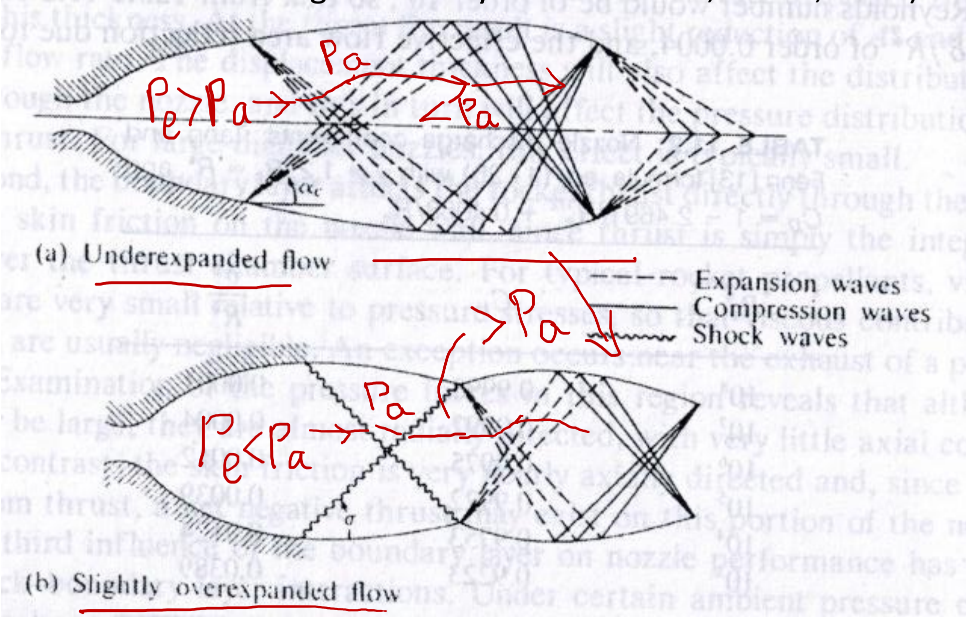
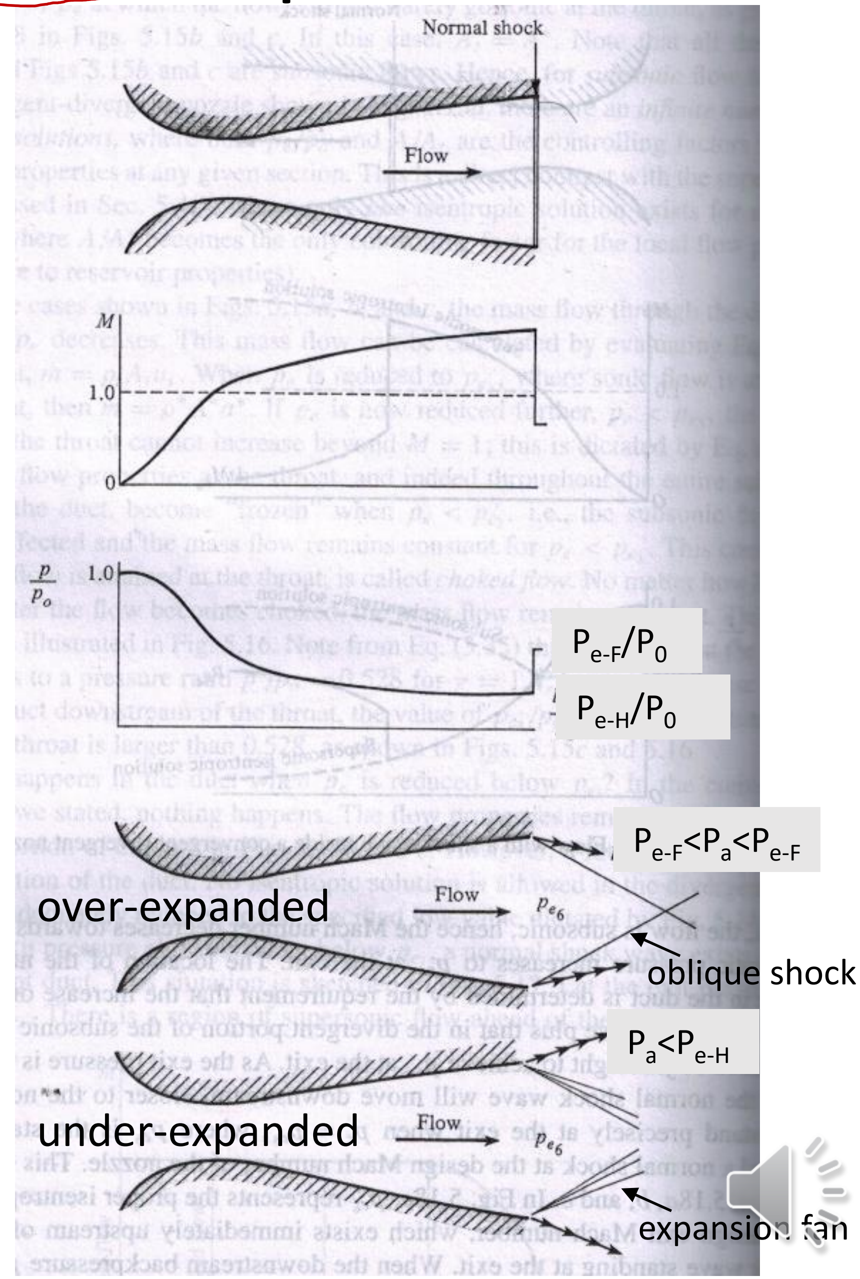
Over and under-expanded nozzle

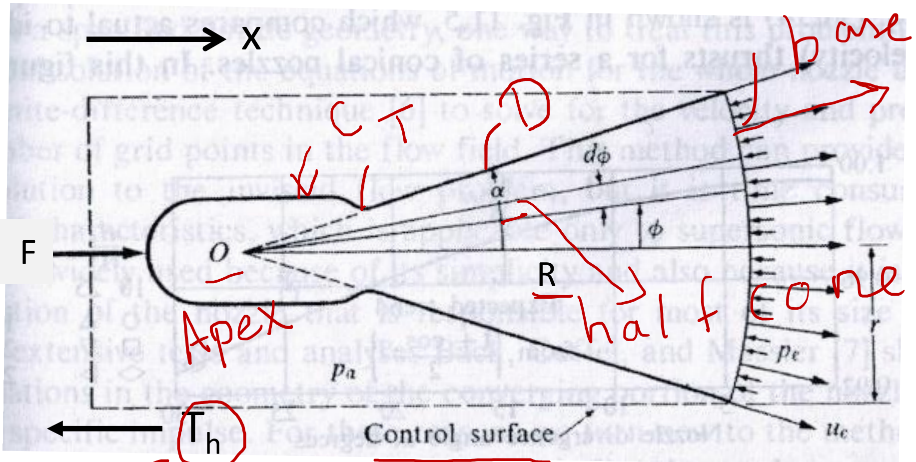
Fig. 11.17 Exhaust jet behavior.
(Mechanics and Thermodynamics of Propulsion by Philip Hill and Carl Peterson, Second Edition, Dorling Kindersley India Pvt. Ltd., Noida, 2010)





- For under-expanded case, pressure adjustment occurs beyond exit plane in the form of expansion waves (exit Mach wave angle α_e)
- For over expanded case, pressure adjustment occurs beyond exit plane in the form of oblique shock (exit shock angle σ)

Fig. 5.18 Flow with shock and expansion waves at the exit of a convergent-divergent nozzle.
(Modern Compressible Flow: with Historical Perspective by John D. Anderson, Third Edition, McGraw-Hill, New Delhi, 2014)



Conical Nozzle

$A_e = \pi r^2$

Fig. 11.4 Spherically symmetric nozzle flow. (Mechanics and Thermodynamics of Propulsion by Philip Hill and Carl Peterson, Second Edition, Dorling Kindersley India Pvt. Ltd., Noida, 2010)

T Thrust

- Converging nozzle section and throat contour is not very critical to performance, and any wall curve is acceptable, pressure gradients (favorable) are high in these two regions and flow adheres to the walls. Principal difference in different nozzle configurations is found in the diverging-supersonic section
- The conical nozzle is the oldest and perhaps the simplest configuration. It is relatively easy to fabricate and is still used today in many small nozzles
- Consider the flow out of the conical nozzle and assume that the streamlines in the expanding part of the nozzle are straight lines that all intersect at point O
- Control surface passes through the spherical segment (of radius R) over which exhaust properties are constant,
- No mass crosses the control surface except the spherical exhaust segment

$$\boxed{T = \overbrace{(P_e - P_a)A_e}^{\text{Pressure}} + \int_{CS} \underbrace{u_x \rho(\boldsymbol{V}\cdot\boldsymbol{n})dA}_{\text{Momentum}}}$$

A_e is the plane cross sectional area: $\mathbf{A_e = \pi r^2}$

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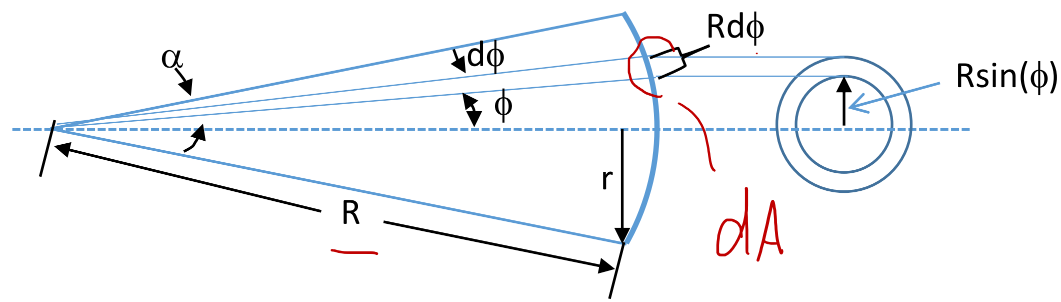
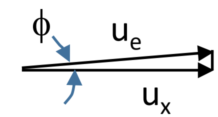
Conical Nozzle





$$dA = 2\pi R \sin(\phi)\, Rd\phi$$

$$\boldsymbol{V}\cdot\boldsymbol{n} = u_e$$

$$u_x = u_e \cos(\phi)$$

$$T = \int_{CS} u_x \rho(\boldsymbol{V}\cdot\boldsymbol{n})dA + (P_e - P_a)A_e$$

$$T = \int_0^{\alpha} u_e \cos(\phi)\, \rho_e(u_e) 2\pi R \sin(\phi)\, Rd\phi + (P_e - P_a)A_e$$

$$T = 2\pi R^2 \rho_e u_e^2 \int_0^{\alpha} \cos(\phi)\sin(\phi)\, d\phi + (P_e - P_a)A_e$$

$$T = 2\pi R^2 \rho_e u_e^2 \left[\frac{-\cos^2(\phi)}{2}\right]_0^{\alpha} + (P_e - P_a)A_e$$

$$\boxed{T = 2\pi R^2 \rho_e u_e^2 \frac{1-\cos^2(\alpha)}{2} + (P_e - P_a)A_e}$$

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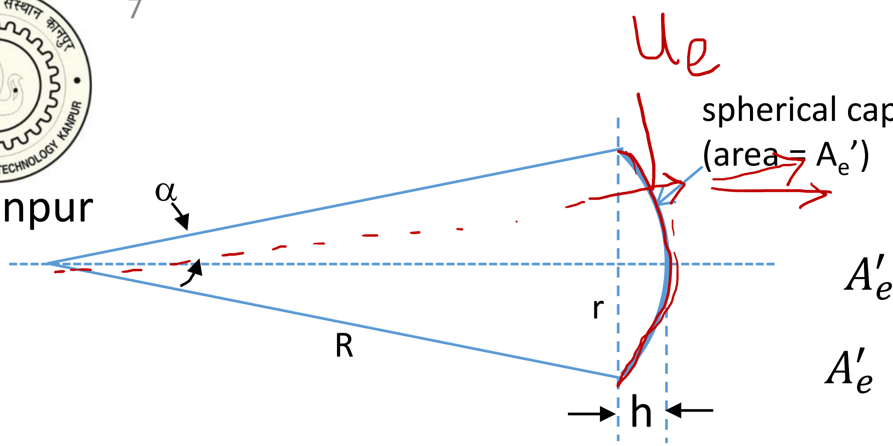
Conical Nozzle



$$A_e' = \int_0^{\alpha} dA$$

$$A_e' = \int_0^{\alpha} 2\pi R \sin(\phi)\, R d\phi = 2\pi R^2 \int_0^{\alpha} \sin(\phi)\, d\phi$$

$$A_e' = 2\pi R^2[-\cos(\phi)]_0^{\alpha} = 2\pi R^2[1-\cos(\alpha)]$$

surface area (A_e') of the spherical cap is given by: $2\pi Rh$

$h = R - R\cos(\alpha)$ $\qquad$ $A_e' = 2\pi R^2[1-\cos(\alpha)]$

propellant mass flow rate: $\dot{m} = \rho_e u_e A_e' = \rho_e u_e 2\pi R^2[1-\cos(\alpha)]$

$$\boxed{T = \dot{m} u_e \boxed{\frac{1+\cos(\alpha)}{2}} + (P_e - P_a)A_e}$$

note: the thrust contribution **from velocity term** reduces by: correction factor: $\lambda = \frac{1+\cos(\alpha)}{2}$, the pressure term is not affected

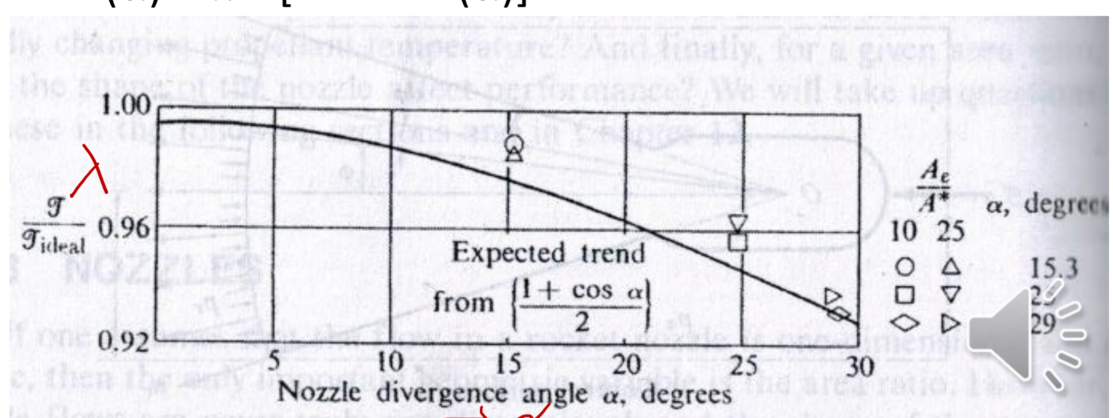
$A_e = \pi r^2$ $\qquad$ $R = R\sin(\alpha)$ $\qquad$ $A_e = \pi R^2 \sin^2(\alpha) = \pi R^2[1-\cos^2(\alpha)]$

$$A_e = \frac{1+\cos(\alpha)}{2} A_e'$$

spherical cap area

$$\boxed{T = \frac{1+\cos(\alpha)}{2}[\dot{m} u_e + (P_e - P_a)A_e']}$$

Fig. 11.5 Radial flow thrust loss of conical nozzles. (Mechanics and Thermodynamics of Propulsion by Philip Hill and Carl Peterson, Second Edition, Dorling Kindersley India Pvt. Ltd., Noida, 2010)



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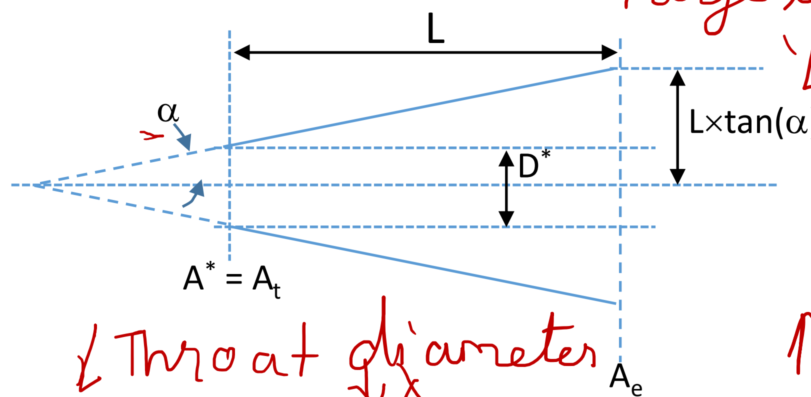
Conical Nozzle

large α → small α

→ given A_e/A^*, required large length

- The problem with large cone angles is reduced thrust
- The problem with small cone angles is excessive nozzle length.

$$\boxed{\frac{A_e}{A^*} = \left(\frac{D^* + 2L\tan(\alpha)}{D^*}\right)^2 \qquad L = D^* \frac{\sqrt{\frac{A_e}{A^*}} - 1}{2\tan(\alpha)}}$$



↓ Throat diameter ↓ α

- For an area ratio (A_e/A^*) of 100, we would have $L/D^* = 7.8$ for $\alpha = 30°$ and 16.8 for $\alpha = 15°$
- Throat diameter (D^*) will be fixed by the combustion chamber condition so that the nozzle length (and mass) is strongly dependent on α. Reducing α from 30° to 15° would more than double the mass of the divergent part of the nozzle. Optimum α is typically between 12° and 18° and it is a compromise, which, depends on the specific application and flight path
- A better way to minimize divergence loss is to use shaped nozzle, so as to provide close to uniform parallel flow at the exit. **Method of characteristics** (an analytical method) is widely used for determining the practical shapes for the nozzles.

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Shaped Nozzle

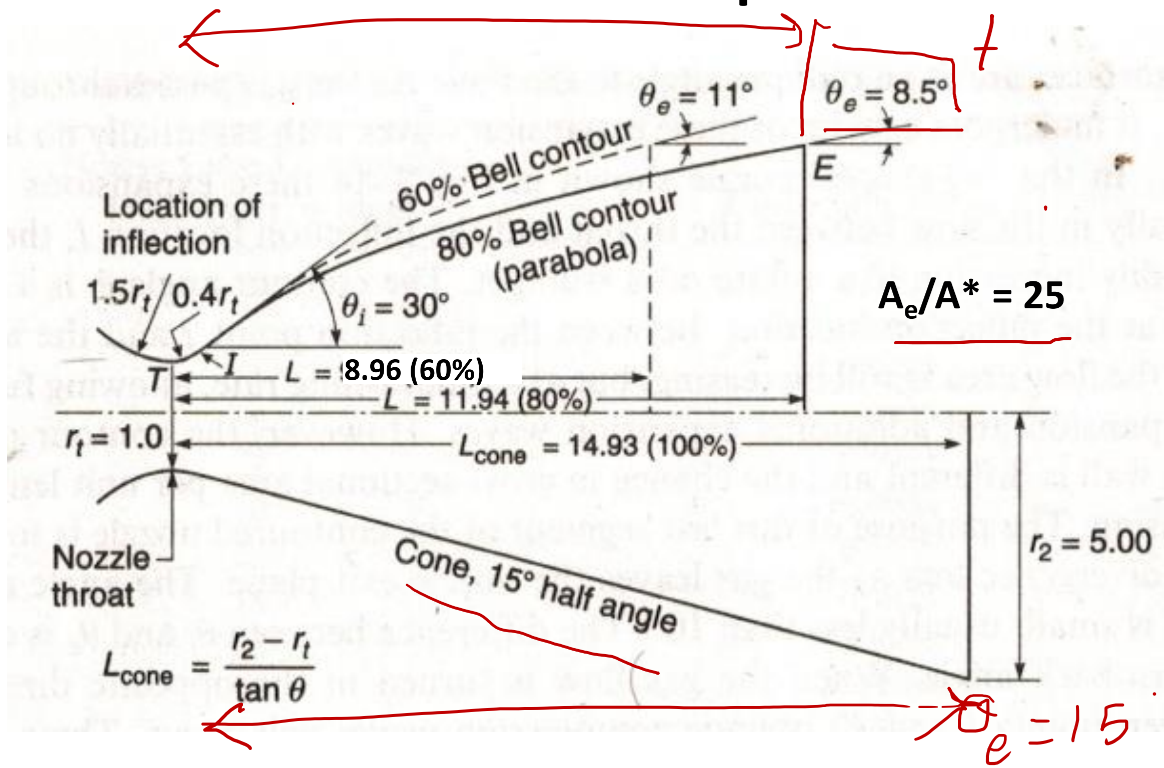

Fig. 3.14 Comparison sketches of nozzle inner wall surfaces for a 15° conical nozzle, an 80% and 60% length bell nozzles (w.r.t. conical), all at an area ratio of 25. (Rocket Propulsion Elements by George P. Sutton and Oscar Biblarz, Seventh Edition, Wiley India Pvt. Ltd., New Delhi, 2014)

note: correction factors (λ) for conical, 80% bell, 60% bell are 0.983, 0.987, 0.968, respectively

- Bell shaped is probably the most common nozzle shape. It has a high angle expansion (θ_i = 20° to 50°) right behind the nozzle throat, followed by gradual decrease of the nozzle contour slope so that at the nozzle exit the divergence angle is small (θ_e < 10°)
- Large θ_i is possible because the high relative pressure, the large pressure gradient, and the rapid expansion of the working fluid do not allow separation in this region.
- Lower θ_e (w.r.t θ_i) requires flow to turn in opposite direction, resulting in multiple oblique compression waves, causing losses. By carefully determining the wall contour (method of characteristics) this loss can be minimized

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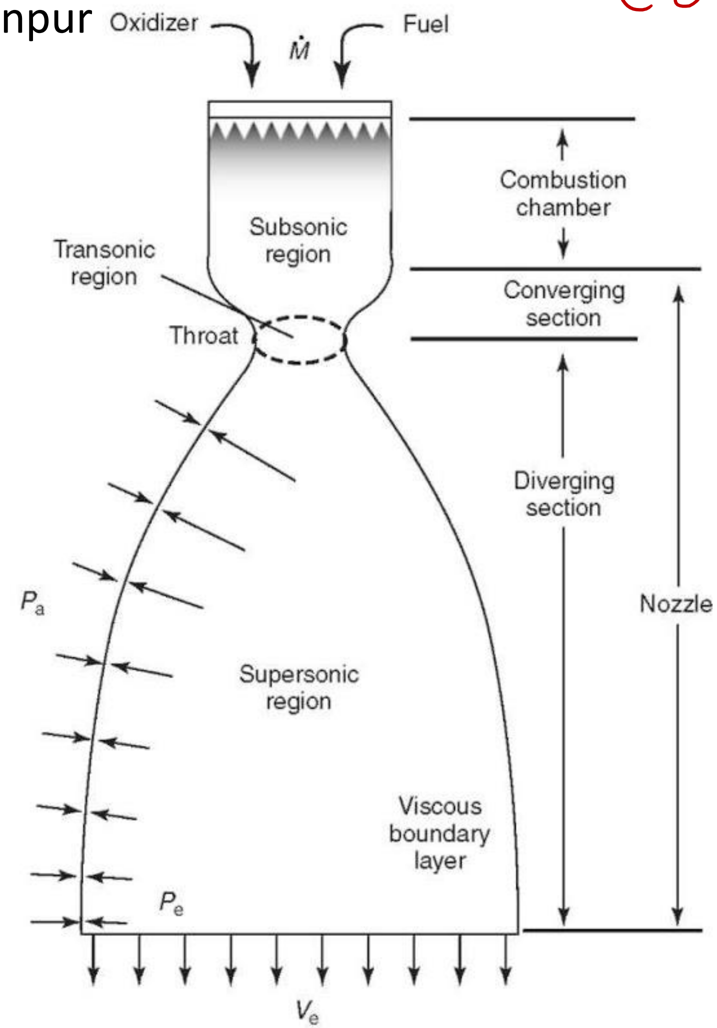
Bell Shaped Nozzle

Most Common



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http://www.k-makris.gr/RocketTechnology/Nozzle_Design/Pics/nozzle1.jpg