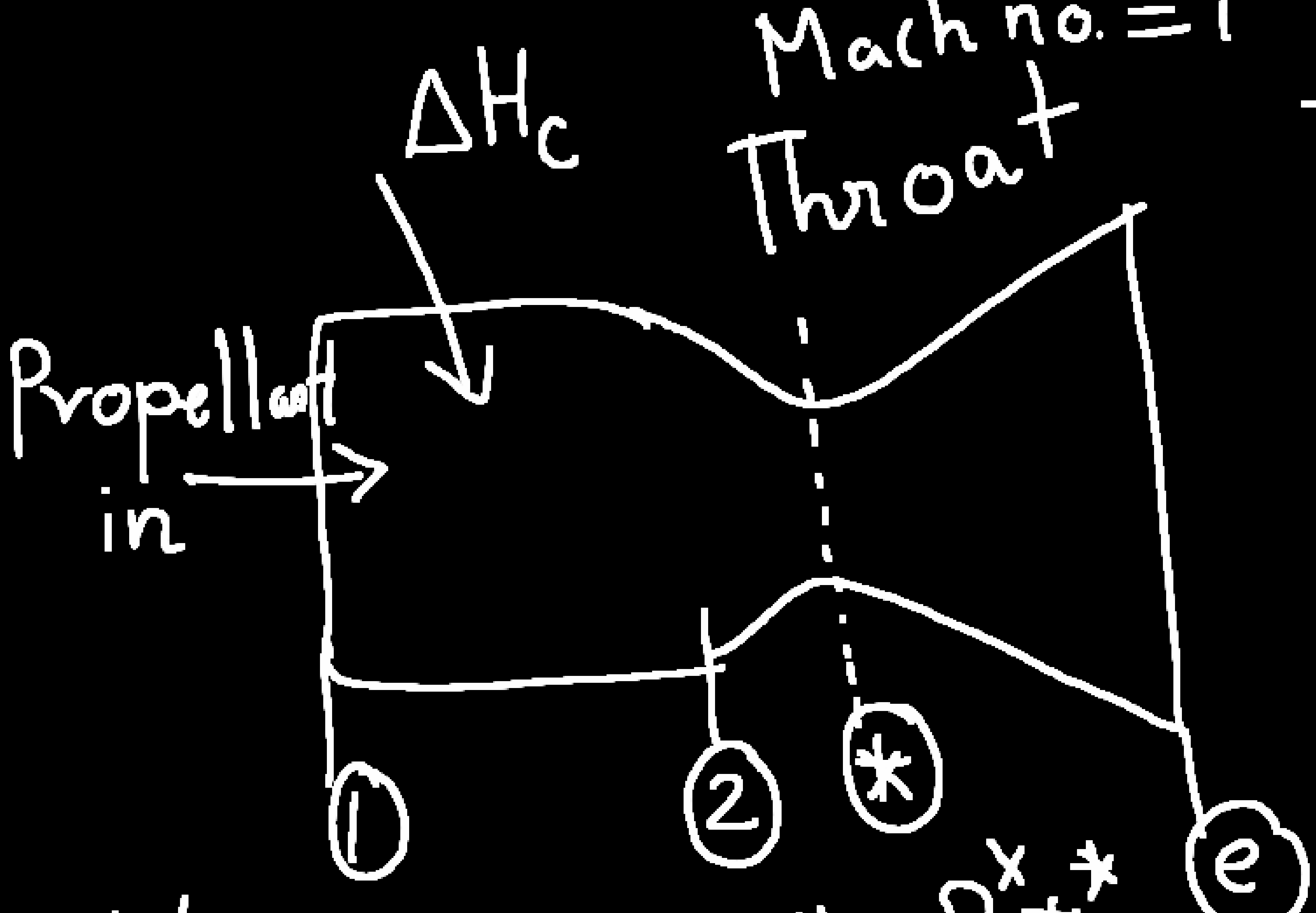
Thrust chambers

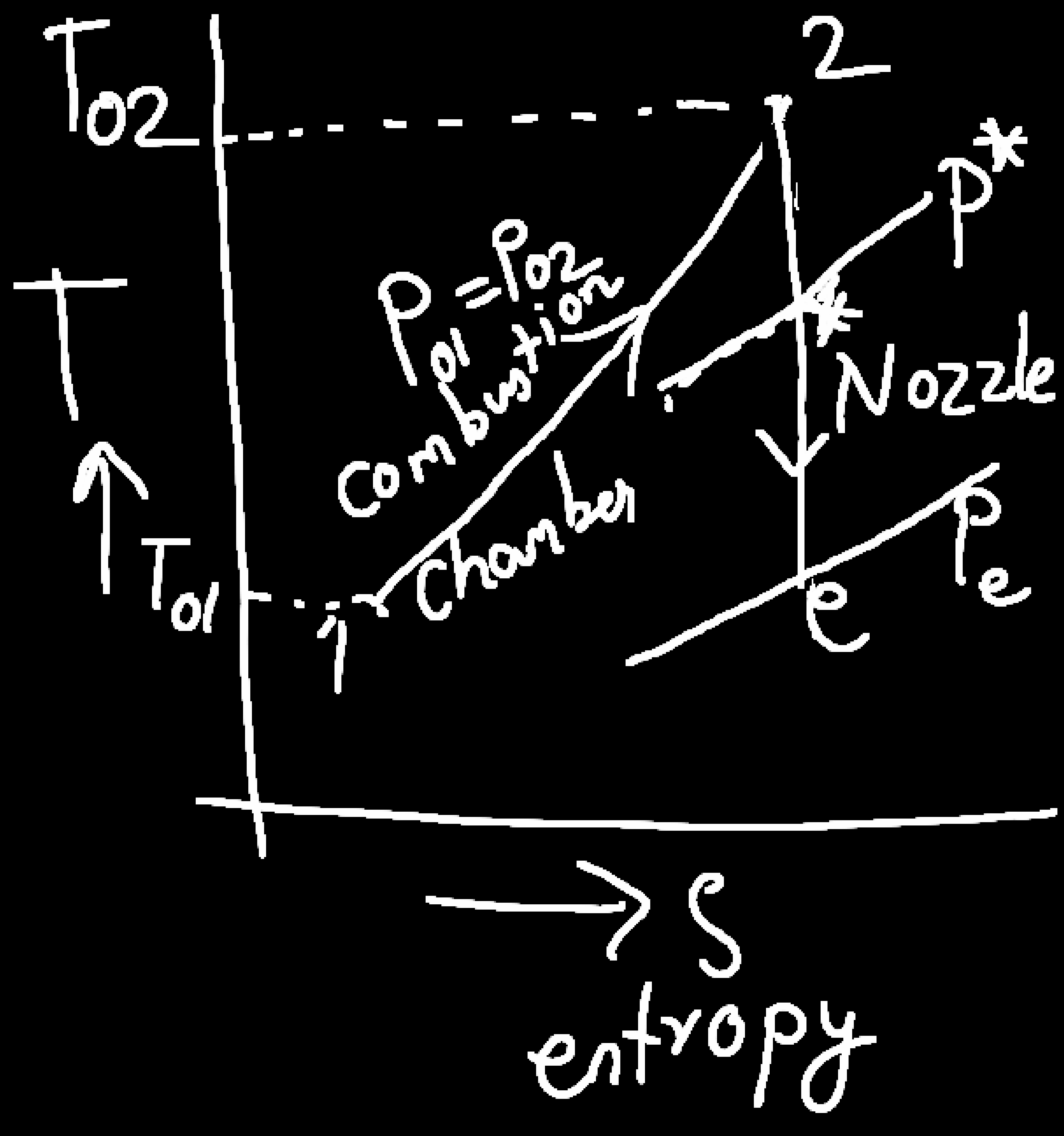
$$T = \dot{m} u_e + (P_e - P_a) A_e$$



Static → P_1 T_1 P_2 T_2 P^* T^* P_e, T_e

→ P_{01} T_{01} P_{02} T_{02} P_{0e}, T_{0e}

Stagnation quantities



T_{02} – temperature of the propellant after combustion (same T_{ad})

$$T_{02} = T_{0e} = T_e + \frac{u_e^2}{2c_p}$$

$$u_e = \sqrt{2c_p(T_{02} - T_e)} = \sqrt{2c_pT_{02}\left[1 - T_e/T_{02}\right]}$$

$$u_e = \sqrt{2c_pT_{02}\left[1 - \left(P_e/P_{02}\right)^{\frac{\nu-1}{\nu}}\right]}$$

ν – ratio of specific heat capacities

$$C_p = \frac{\gamma R}{\gamma - 1}$$, R - Characteristic gas Constant

$$= \frac{\gamma}{\gamma - 1} \cdot \frac{R_u}{M_W}$$

R_u - Universal gas constant

8314 J/Kg K

M_W - Malar mass of the gas

$$u_e = \sqrt{\frac{2\gamma R_u}{\gamma - 1} \frac{T_{02}}{M_W} \left[1 - \left(\frac{P_e}{P_{02}} \right)^{\frac{\gamma - 1}{\gamma}} \right]}$$

$T_{02}/M_W \rightarrow$ maximized for large u_e

$LH_2 - LO_2 \longrightarrow$ fuel rich mixture

in order to maximize T_{02}/M_w

$\dot{m} \rightarrow$ mass flowrate of the propellant

$$\dot{m} = \rho^* A^* u^* = \rho^* A^* \sqrt{\gamma R T^*}$$

$$\rho^*/\rho_{02} = \left(2/\gamma+1\right)^{1/\gamma-1}, \quad T^*/T_{02} = 2/\gamma+1$$

$$\dot{m} = \frac{P_{o2} A^*}{\sqrt{R T_{o2}}} \sqrt{\gamma \left(\frac{2}{\gamma + 1} \right)^{\left(\frac{\gamma+1}{\gamma-1}\right)}}$$

$$T = \dot{m} u_e + (P_e - P_a) A_e$$

$$\frac{I}{A^* P_{02}} = \sqrt{\frac{2\gamma^2}{(\gamma-1)}\left(\frac{2}{\gamma+1}\right)^{\left(\frac{\gamma+1}{\gamma-1}\right)}\left[1-\left(P_e/P_{02}\right)^{\frac{\gamma-1}{\gamma}}\right]}$$

$$+\left(P_e/P_{02} - P_a/P_{02}\right) A_e/A^*$$

1) Characteristic velocity C^*

2) thrust coefficient, C_T

Characteristic velocity

$$C^* = \frac{P_{02} A^*}{\dot{m}} \quad \left[\begin{array}{l}\text{Actual/real or} \\ \text{ideal rockets}\end{array}\right]$$

$$\left(\begin{array}{l}\text{dimensions} \\ \text{of velocity}\end{array}\right) = \sqrt{RT_{02}}\sqrt{\frac{1}{\gamma}\left(\frac{\gamma+1}{2}\right)^{\left(\frac{\gamma+1}{\gamma-1}\right)}}$$

C^* is a function $\left[\text{Ideal rocket}\right]$ of stagnation temperature T_{02} at the end of combustion.

Thrust coefficient

(Dimensionless)

$$C_T = \frac{T}{P_{02} A^*}$$

[Valid for real & ideal rockets]

$$= \sqrt{\frac{2\nu^2}{\nu-1}\left(\frac{2}{\nu+1}\right)^{\left(\frac{\nu+1}{\nu-1}\right)}\left[1-\left(P_e/P_{02}\right)^{\nu-1/\nu}\right]} + \left(P_e/P_{02} - P_a/P_{02}\right) A_e/A^*$$

represents the performance of the nozzle.

$$T = \dot{m}\, c^* C_T$$

$$I_{sp} = \frac{T}{\dot{m}\, g_e} = \frac{c^* C_T}{g_e}$$

<u>Efficiency</u>

$$\eta_{c^*} = \frac{c^*_{real}}{c^*_{ideal}}$$

Performance of Combustion chamber

$$\eta_{C_T} = \frac{C_{T\,real}}{C_{T\,ideal}}$$

Nozzle

IIT Kanpur

Example: Thrust chamber performance

Rocket propulsion by Ramamurthi. K. 2012, Third Edition, Macmillan Publishers, India

Example 3.1 from Rocket propulsion by. K Ramamurthi

(1)

Combustion chamber temperature (burnt products) $T_{02} = 2000\,K$.

Pressure $P_{02} = 15\,MPa$

$\gamma = 1.32$. Molecular mass of burnt gas $M_{Wp} = 22\,kg/kmol$

Ambient pressure $P_a = 0.1\,MPa$

Throat area $A^* = 0.1\,m^2$

Calculate i) Exit velocity u_e, ii) Chacteristic velocity c^*, iii) Ideal thrust coefficient C_T, iv). Specific impulse, v). Thrust generated. , vi). Exit Area rate A_e/A^* for correctly expanded nozzle condition.

i) $$u_e = \sqrt{2 C_p T_{02} \left[1 - \left(\frac{P_e}{P_{02}}\right)^{(\gamma-1)/\gamma}\right]}$$

IIT Kanpur

Example: Thrust chamber performance

Rocket propulsion by Ramamurthi. K. 2012, Third Edition, Macmillan Publishers, India

condition

i) $$u_e = \sqrt{2 C_p T_{02}\left[1 - \left(\frac{P_e}{P_{02}}\right)^{(\gamma-1)/\gamma}\right]}$$

$$C_p = \frac{\gamma \cdot R_u}{(\gamma-1) M_{w_p}} = \frac{1.32 \times 8.314 \times 10^3}{(1.32-1) \times 22} = 1558.875 \ J/kgK$$

$$u_e = \sqrt{2 \times 1558.875 \times 2000 \left[1 - \left(\frac{0.1}{.15}\right)^{0.32/1.32}\right]}$$

$\boxed{u_e = 2094 \ m/s}$ Exhaust velocity

ii) $$C^*_{ideal} = \sqrt{R T_{02}} \sqrt{\gamma^{-1}\left(\frac{2}{\gamma+1}\right)^{-\frac{\gamma+1}{\gamma-1}}}$$

$$R = \frac{R_u}{M_{w_p}} = 377.9 \ J/kg$$

$$= \sqrt{377.9 \times 2000} \sqrt{1.32^{-1}\left(\frac{2}{2.32}\right)^{-2.32/.32}}$$

$\boxed{C^*_{ideal} = 1296 \ m/s}$ Characteristic velocity

IIT Kanpur

Example: Thrust chamber performance

Rocket propulsion by Ramamurthi. K. 2012, Third Edition, Macmillan Publishers, India

iii) $$C_f = \sqrt{\frac{2\gamma^2}{\gamma-1}\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{\gamma-1}}\left[1-\left(\frac{P_e}{P_{02}}\right)^{\frac{(\gamma-1)}{\gamma}}\right]} + \left(\frac{P_e - P_0}{P_0}\right)\frac{A_e}{A^*}$$

We have to find A_e.

For isentropic, correctly expanded Nozzle,

$$\frac{P_{02}}{P_e} = \left(1 + \frac{\gamma-1}{2} M_e^2\right)^{\gamma/\gamma-1}$$

$$\left[\left(\frac{P_{02}}{P_e}\right)^{\frac{\gamma-1}{\gamma}} - 1\right]\frac{2}{\gamma-1} = M_e^2$$

$$M_e = \sqrt{\left(\frac{2}{\gamma-1}\right)\left(\left(\frac{P_{02}}{P_e}\right)^{\frac{\gamma-1}{\gamma}} - 1\right)}$$

$$= \sqrt{\frac{2}{0.32}\left[\left(\frac{15}{0.1}\right)^{\frac{.32}{1.32}} - 1\right]}$$

$$M_e = 3.848$$

$$\frac{A_e}{A^*} = \frac{1}{M_e^2}\left[\frac{2}{\gamma+1}\left(1 + \frac{\gamma-1}{2} M_e^2\right)\right]^{\frac{\gamma+1}{\gamma-1}}$$

[from Gas dynamics part]

$$= \frac{1}{3.848^2}\left[\frac{2}{2.32}\left(1 + \frac{0.32}{2} \times 3.848^2\right)\right]^{\frac{2.32}{0.32}}$$

IIT Kanpur

Example: Thrust chamber performance

Rocket propulsion by Ramamurthi. K. 2012, Third Edition, Macmillan Publishers, India

vi) $\boxed{\dfrac{A_e}{A^*} = 153.72}$

$$\therefore C_f = \sqrt{\frac{2\times 1.32^2}{0.32}\left(\frac{2}{2.32}\right)^{\frac{2.32}{.32}}\left[1-\left(\frac{0.1}{15}\right)^{\frac{0.32}{1.32}}\right]} + (0)\frac{A_e}{A^*}$$

$\boxed{C_f = 1.616}$ Thrust Coefficient

iv) Specific Impulse $I_{sp} = \dfrac{C^* \times C_f}{g} = \dfrac{1296\times 1.616}{9.81} = 213\ s$

v) Thrust $F = \dot{m}C^* \times C_f = C_f P_{o2} A^* = 1.616\times 15\times 10^6 \times 0.1$

$\boxed{F = 2.424\ MN}$