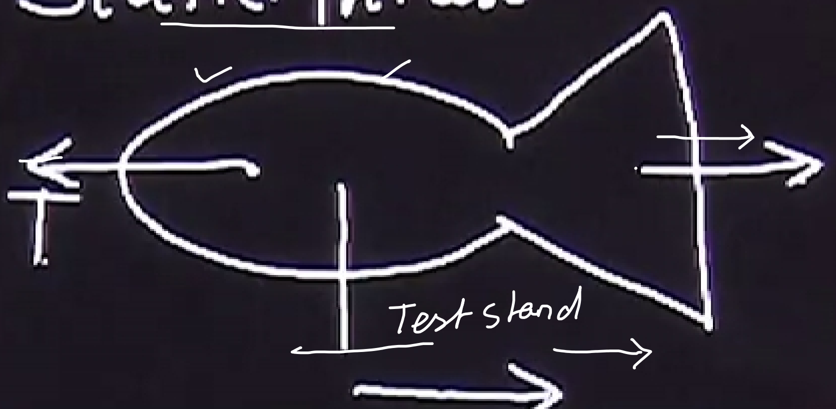
Performance parameters

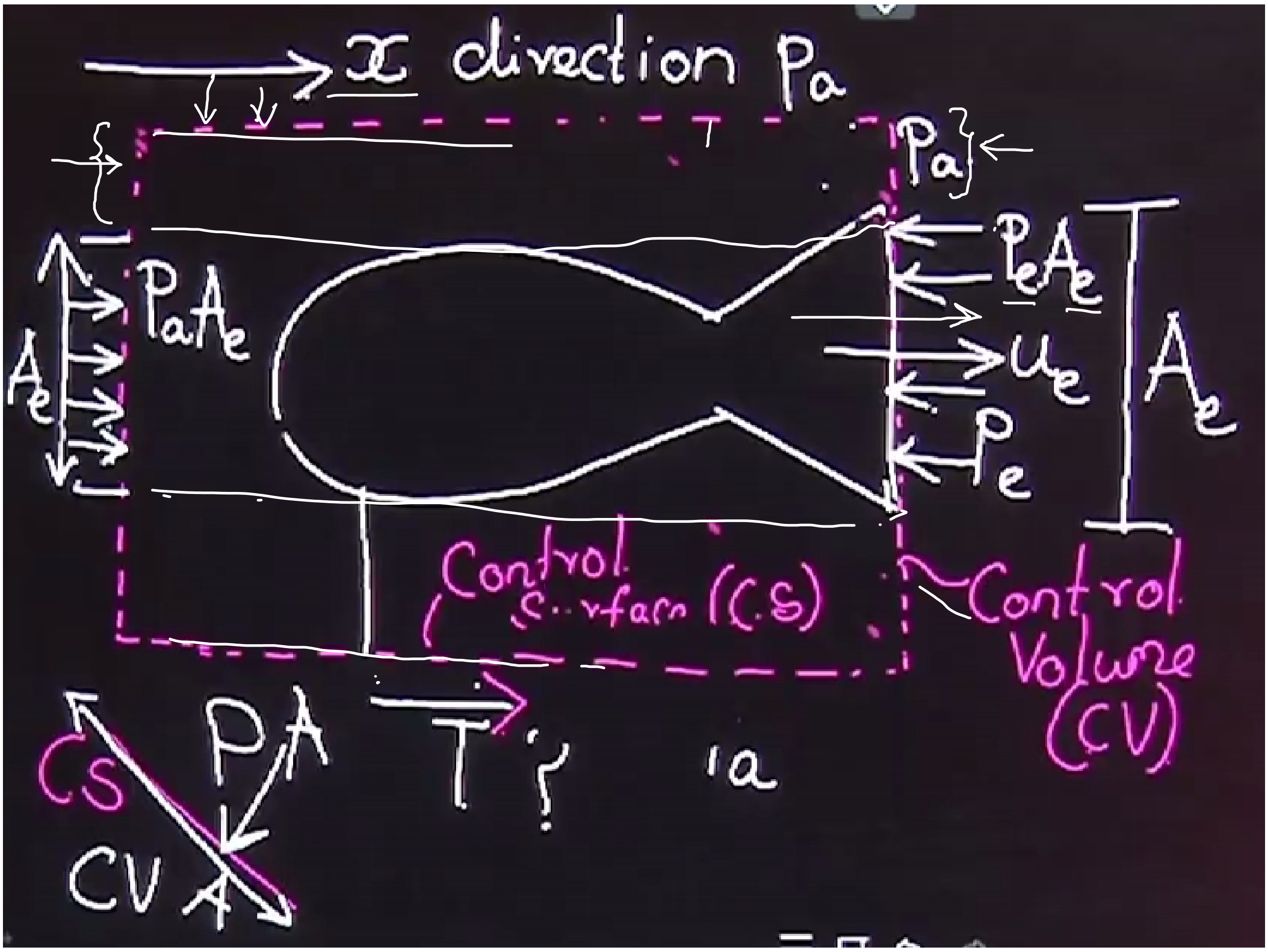
1) Thrust, T
2) Equivalent velocity, u_{eq}
3) Specific impulse, I_{sp}

1) Thrust, T.

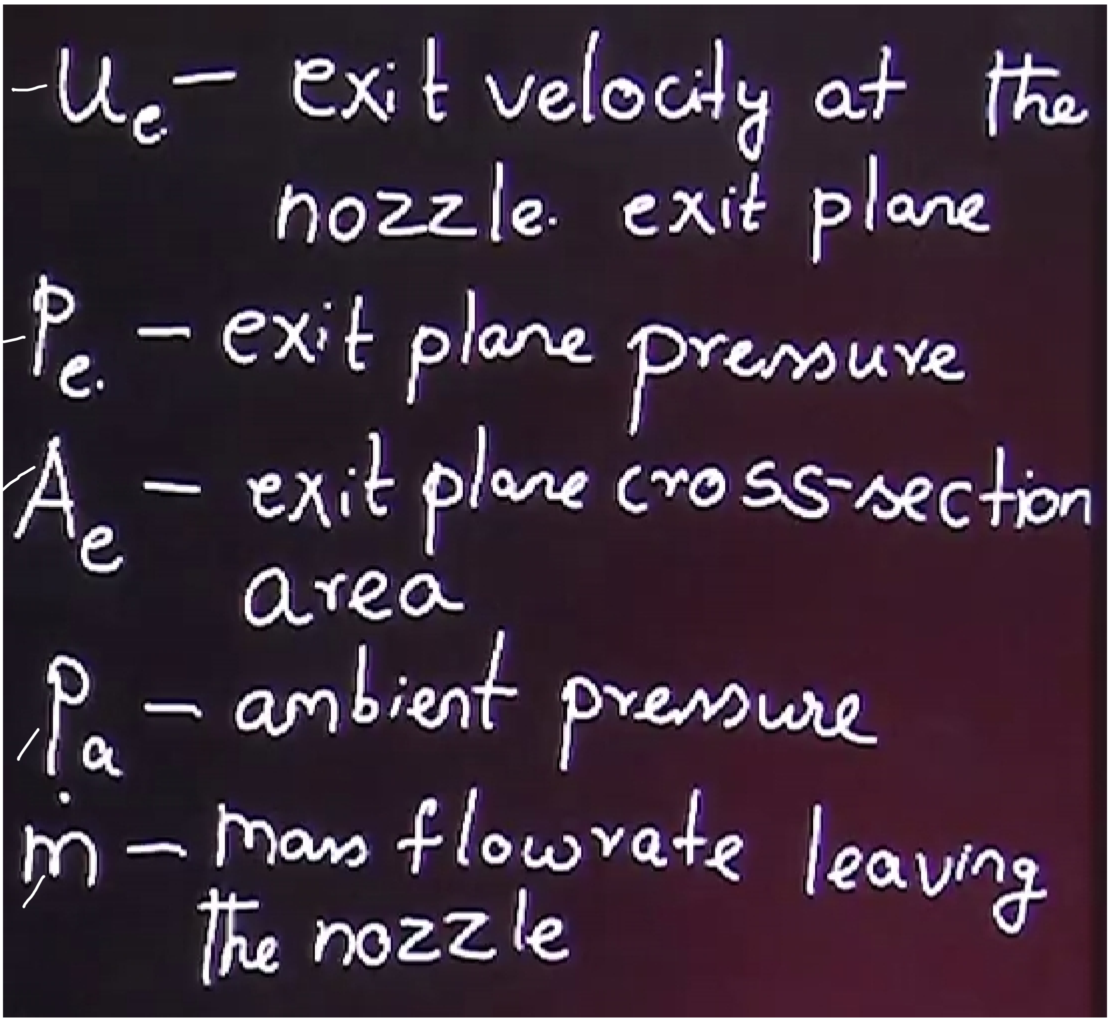
Static thrust

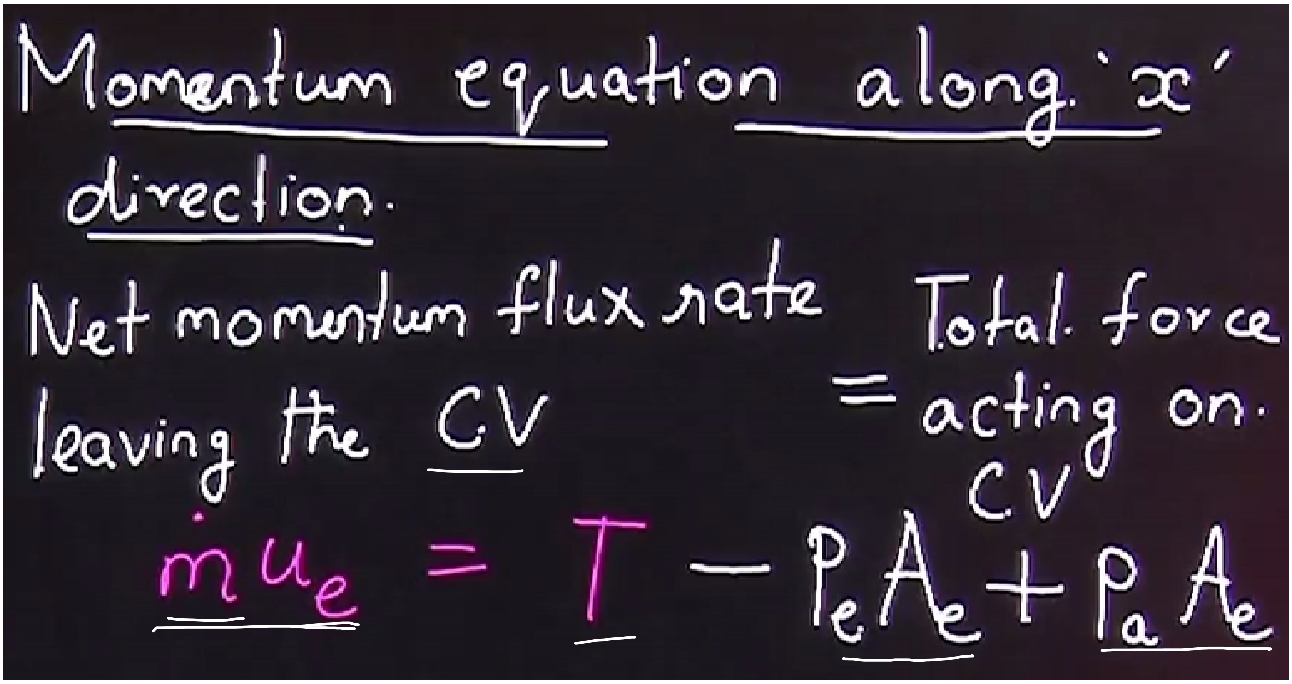


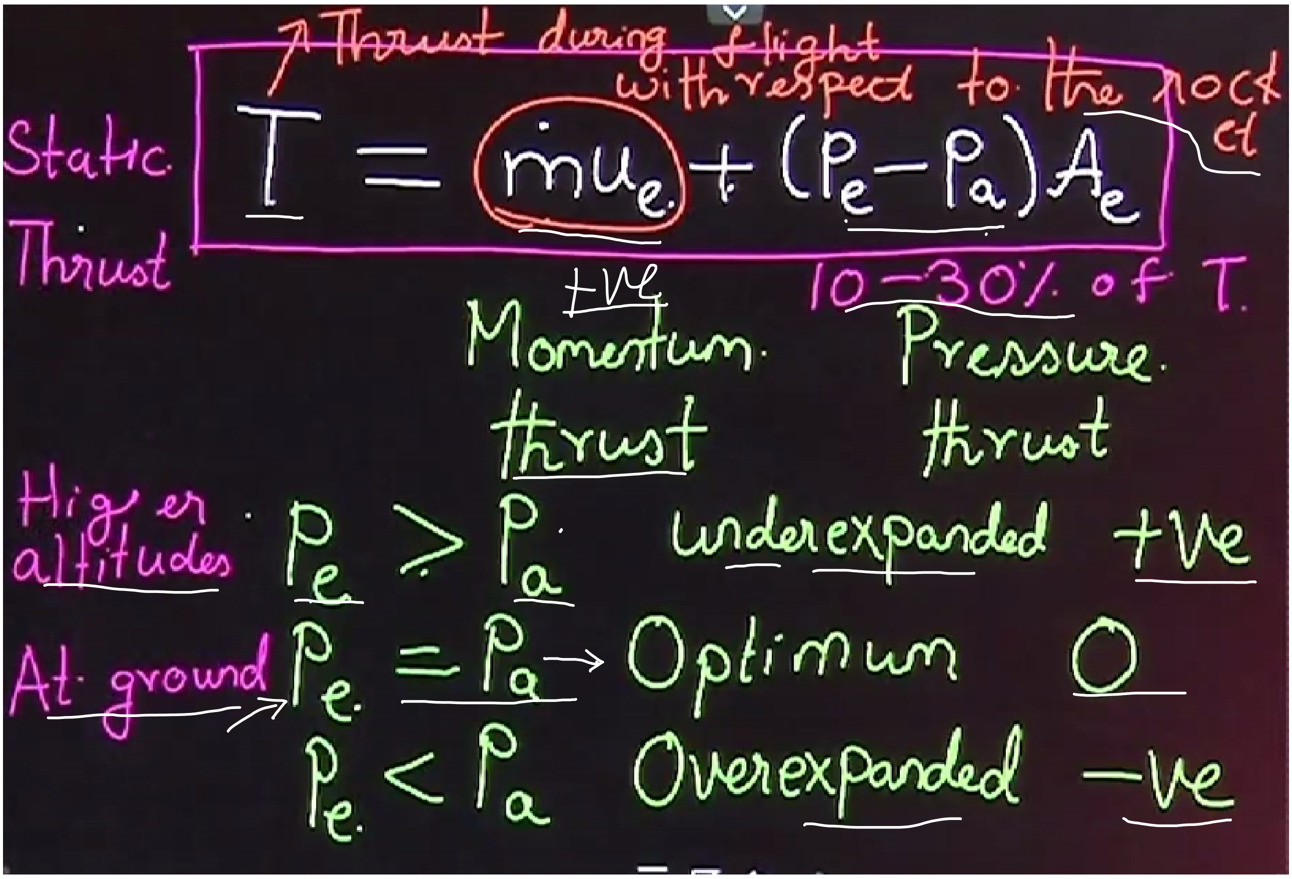
Reactive force (T)

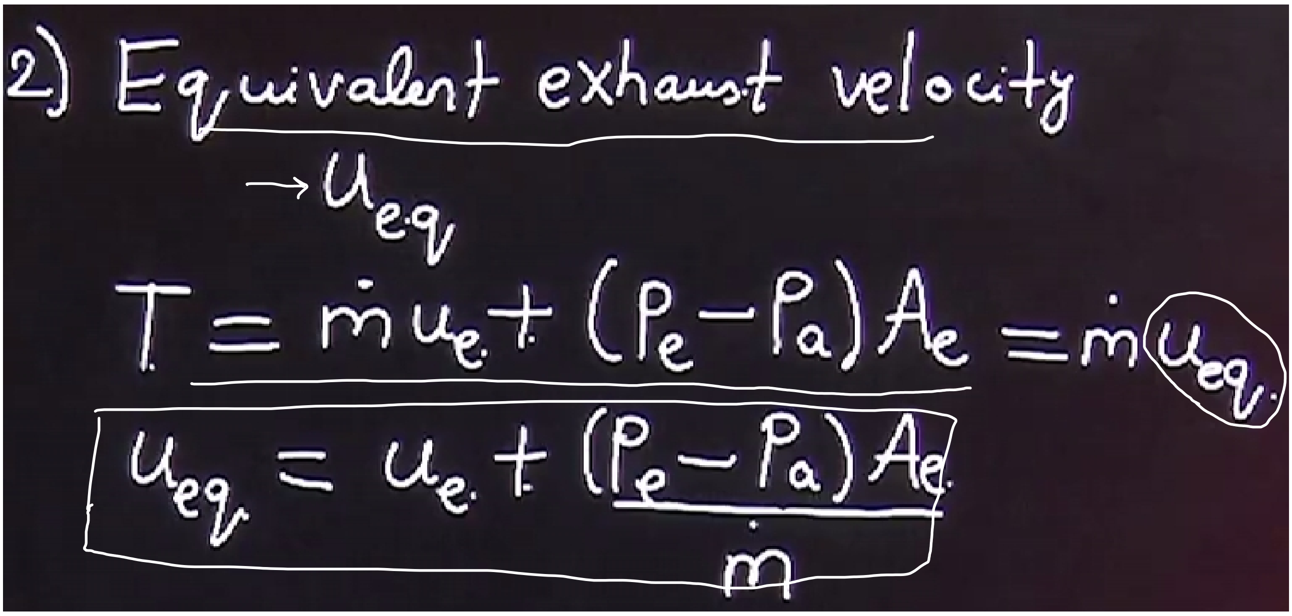


x direction P_a
P_a
$P_a A_e$
A_e
$P_e A_e$
u_e
P_e
A_e
Control Surface (C.S)
Control Volume (CV)
CS
CV
PA
A
T ?


u_e – exit velocity at the nozzle exit plane
p_e – exit plane pressure
A_e – exit plane cross-section area
p_a – ambient pressure
$\dot{m}$ – mass flowrate leaving the nozzle


Momentum equation along 'x' direction.
Net momentum flux rate leaving the CV = Total force acting on CV
$\dot{m} u_e = T - p_e A_e + p_a A_e$


Thrust during flight
with respect to the rocket
Static Thrust
$T = \dot{m}u_e + (P_e - P_a)A_e$
+ve
10–30% of T.
Momentum thrust
Pressure thrust
Higher altitudes $P_e > P_a$ underexpanded +ve
At ground $P_e = P_a$ → Optimum 0
$P_e < P_a$ Overexpanded −ve


2) Equivalent exhaust velocity
→ u_{eq}
$T = \dot{m}u_e + (P_e - P_a)A_e = \dot{m}u_{eq}$
$u_{eq} = u_e + \frac{(P_e - P_a)A_e}{\dot{m}}$

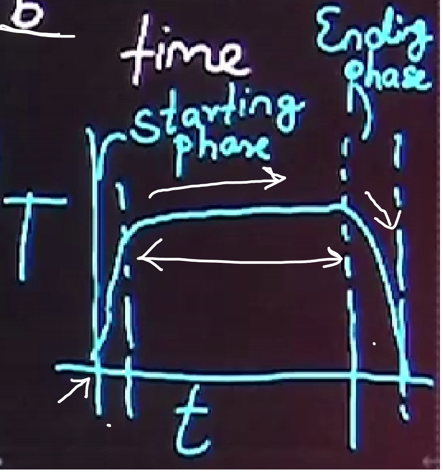
3) Specific impulse I_{sp}

Total impulse $I \equiv \int \text{Force}\, dt$

t — time

$$I = \int_{t=0}^{t_b} T\, dt \quad T \rightarrow T(t)$$

t_b – burnout time



T independent of time

$I = T\, t_b$ (related to total energy available in the propellant)

$I_{sp} \equiv \dfrac{I}{t_b M_p}$, M_p – Total mass of the propellant

$$M_p = \int_{t=0} \dot{m}\, dt$$

If $\dot{m}$ is assumed to be independent of time,

$M_p = \dot{m} t_b$,

$$\boxed{I_{sp} = \frac{T t_b}{\dot{m} t_b} = \frac{T}{\dot{m} g_e}}$$

g_e – acceleration due to gravity

g_e 9.81 m/s^2

$$\boxed{I_{sp} = \frac{T}{\dot{m} g_e}} \quad \frac{kg\, m/s^2}{kg/s \; m/s^2}$$

(s)

$$I_{sp} = \frac{T}{\dot{m}\, g_e} = \frac{\dot{m}\, u_{eq}}{\dot{m}\, g_e}$$

$$\boxed{I_{sp} = \frac{u_{eq}}{g_e}}$$

Solid rocket motor $I_{sp} \sim 250$ s

Liquid rocket engine

Kerosene + N_2O_4 $I_{sp} \sim 350$ s

$LH_2 + LO_2$ $I_{sp} \sim 450$ s