Thrust, T

Equivalent velocity, u_{eq}

Specific impulse, I_{sp}

Mission

1) Velocity increment, Δu

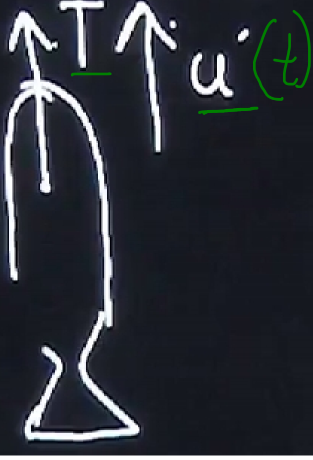
increase in the velocity provided by a rocket engine

2) Range, h

Distance achieved by burning a rocket

Velocity increment, $\boxed{\Delta u}$.

No gravity
No aerodynamic force } Interplanetary space mission



u – instantaneous velocity of the rocket

$$T = M\frac{du}{dt}$$

Newton's law

$M(t)$ Instanteous mass of the rocket

$$T = \dot{m}\, u_{eq} = M \frac{du}{dt}$$

$$-\frac{dM}{dt} = \dot{m}$$

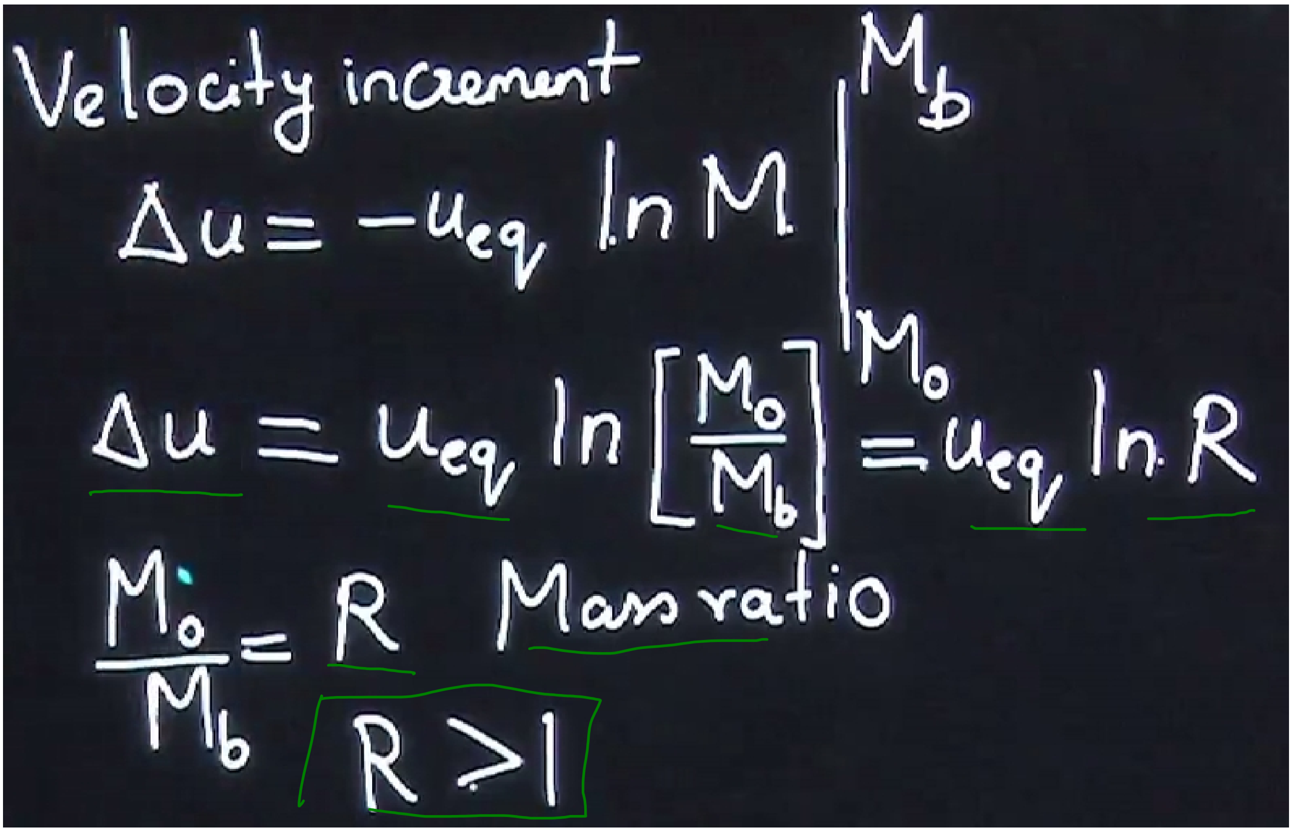
$$\boxed{-\frac{dM}{\cancel{dt}}\, u_{eq} = M \frac{du}{\cancel{dt}}}$$

$$\int_{t=0}^{t=t_b} du = -\int_{t=0,\ M=M_0}^{t=t_b,\ M=M_b} u_{eq} \frac{dM}{M}$$

M_0 – Initial mass of rocket

M_b – Burnout mass, $M_b < M_0$

t_b – Burntime of rocket


Velocity increment
$\Delta u = -u_{eq} \ln M \Big|_{M_0}^{M_b}$
$\Delta u = u_{eq} \ln \left[\frac{M_0}{M_b} \right] = u_{eq} \ln R$
$\frac{M_0}{M_b} = R$ Mass ratio
$\boxed{R > 1}$

$$\boxed{\Delta u = g_e I_{sp} \ln R}$$

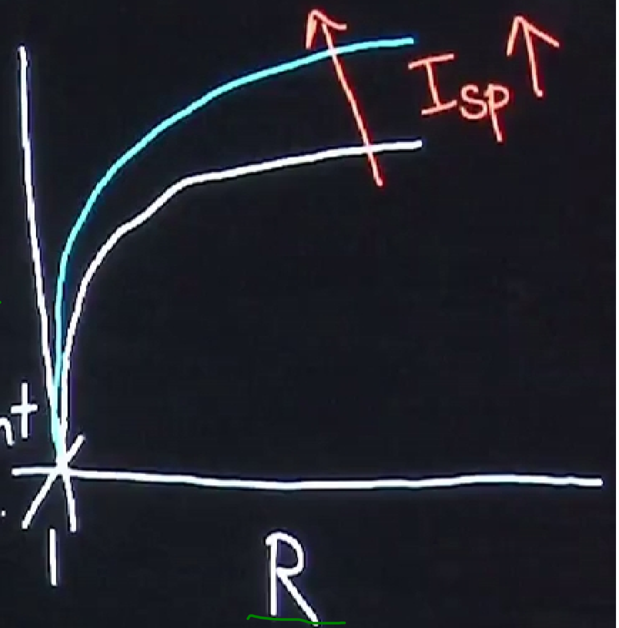
$\Delta u \propto I_{sp}$

$\propto \ln R$

$R \uparrow = \dfrac{M_0}{M_b}$

$M_0 \uparrow$ – More propellant

$M_b \downarrow$ – Efficient structure



$M_b = 0.1 M_0 \Rightarrow R = 10$

$I_{sp} = 300$ s

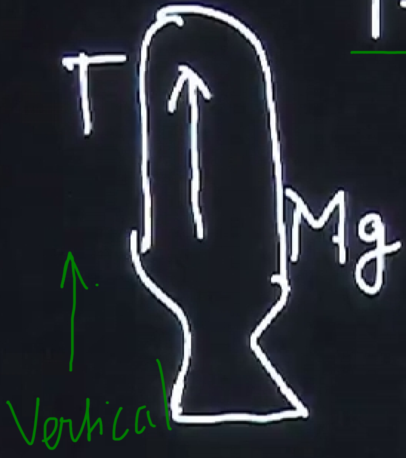
$$\Delta u = g_e I_{sp} \ln R = \underbrace{9.81}_{10} \times 300 \times \underbrace{\ln 10}_{\sim 2.3}$$

$$\Delta u \sim 6.9 \times 10^3 \text{ m/s} = \boxed{6.9 \text{ km/s}}$$

LEO orbit, $h \sim 200$ KM, $\Delta u \sim 7$ km/s

1) Include gravity

$$T - Mg = M\frac{du}{dt}$$



$g(h)$ gravity — constant

$-g_e$ (at sealevel)

$$\dot{m}u_{eq} - Mg = M\frac{du}{dt}$$

$$-\frac{dM}{dt}u_{eq} - Mg = M\frac{du}{dt}$$

$$\Delta u = u_{eq}\ln R - \underbrace{g_e t_b}_{\text{gravity}}$$ ← lower limit of t_b

$\Delta u \propto I_{sp}$

$\propto \ln R$

$\uparrow \Delta u \rightarrow t_b \downarrow \rightarrow \dot{m} \uparrow \rightarrow \frac{du}{dt} \uparrow$

stress on the structure

$\Delta u \to$ with/without gravity

h

Height reached by the rocket at the end of burnout, $h_{b_{t_b}}$ h_{max}

$$u = \frac{dh}{dt}, \quad h_b = \int_{h=0}^{h_b} dh = \int_{t=0}^{t_b} u(t)\, dt \quad \text{— (1)}$$

$$\boxed{u(t) = u_{eq} \ln\left[\frac{M}{M_0}\right] - g_e t} \quad \text{— (2)}$$

$$M(t) = M_0 - \frac{t}{t_b}(M_0 - M_b) \quad \text{— (3)}$$

$\dot{m} \to$ constant

mass

$$M(t) = M_0\left[1 - \frac{t}{t_b}\left(1 - \frac{1}{R}\right)\right] \quad (3)$$

$$h_b = -\int_{t=0}^{t=t_b} u_{eq} \ln\left[1 - \frac{t}{t_b}\left(1 - \frac{1}{R}\right)\right] dt - \int_{t=0}^{t_b} g_e\, t\, dt$$

$$\boxed{h_b = -u_{eq} t_b \frac{\ln R}{R-1} + u_{eq} t_b - \tfrac{1}{2} g t_b^2}$$

$$u_b = \Delta u = u_{eq} \ln R$$

h_{max} is the max height reached by the rocket

At $h = h_b$

$$\underbrace{M_b g_e h_b}_{PE} + \tfrac{1}{2} \underbrace{M_b u_b^2}_{KE} = \underbrace{M_b g_e h_{max}}_{PE}$$

$$h_{max} = h_b + \tfrac{1}{2} \frac{u_b^2}{g_e}$$

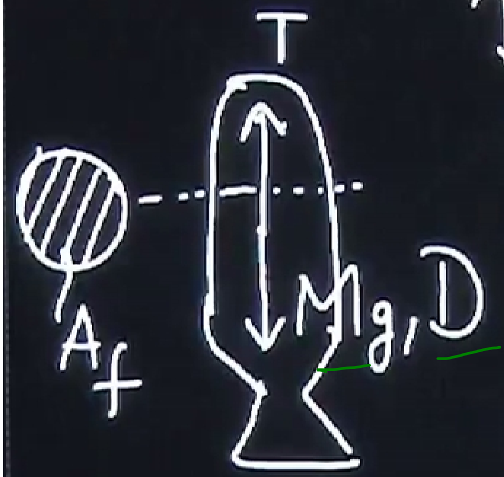
$$h_{max} = \frac{u_{eq}^2 (\ln R)^2}{2 g_e} - u_{eq} t_b \left(\frac{R \ln R}{R-1} - 1 \right)$$

$h_{max} \uparrow$, $u_{eq} \uparrow$, $R \uparrow$, $t_b \downarrow$

Include aerodynamic forces

Numerically

D - Drag force



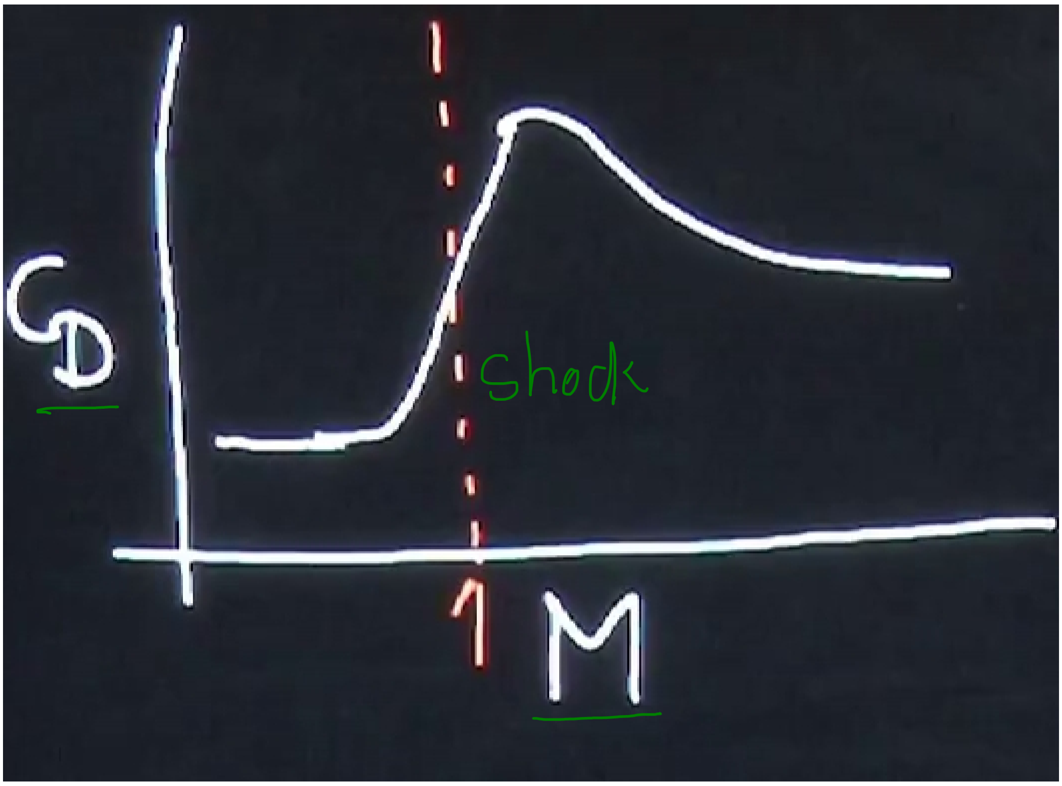
$$T - Mg - D(u) = M \frac{du}{dt}$$

$$D = C_D \tfrac{1}{2} \rho u^2(t) A_f$$

C_D - Drag coefficient

ρ - Density $(h \to t)$

A_f - Cross section area



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