AE441A : Rocket Propulsion

DEPARTMENT OF AEROSPACE ENGINEERING
Indian Insitute of Technology Kanpur

Assignment 1

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Q. Plot the rocket trajectory (horizontal (*x*) vs. vertical (*h*) distance), rocket speed (*u* vs. *t*), rocket angle (*θ* vs. *t*) and rocket height (*h* vs. *t*) until the burn out time (*t* = *tb*). Also tabulate the burnout height (*hb*), burnout speed (*ub*), and angle of rocket at burnout (*θb*).

The rocket is fired from the ground (at t = 0 : x, h = 0) at an angle of 1 degree from the vertical (θ = 1 degree) with a non-zero initial vertical velocity 30 m/s. Given: constant equivalent exhaust velocity ueq = 3048 m/s, initial rocket mass (M0) = 15000 kg, propellant mass (Mp) = 12000 kg, burnout time (tb) = 100 s, assume constant mass burning rate (`m).

In [1]:

```
import pandas as pd
import numpy as np
import matplotlib.pyplot as plt
```

In [2]:

```
def trajectory(isDragVary,isGravityVary,isquestionE,dt):
    # initial rocket mass(kg)
    Mi = 15000
    # propellant mass(Kg)
    Mp = 12000
    # burnout mass(Kg)
    Mb = Mi - Mp
    # initial height(m)
    hi = 0
    # initial horizontal velocity(m/s)
    uxi = 0
    # initial vertical velocity(m/s)
    uyi = 30
    # initial velocity(m/s)
    ui = np.sqrt(pow(uxi,2) + pow(uyi,2))
    # equivalent velocity(m/s)
    ueq = 3048
    # initial time(sec)
    ti = 0
    # burnout time(sec)
    tb = 100
    # exhaust mass flow(Kg/s)
    mdot = -((Mb-Mi)/tb)
    # acceleration due to gravity at groung level (m/s^2)
    g0 = 9.81
    # theta from vertical
    thetai = 1*(np.pi/180)
    # earth's radius (m)
    Re = 6400000
    # # frontal cross sectional area (m2)
    Af = 1
    # gas constant (J/Kg?K)
    R = 287
    # specific heat constant
    gama = 1.4

    M_old = Mi
    h_old = hi
    u_old = ui
```

```
    theta_old = thetai
    t_old = ti
    ux_old = uxi
    uy_old = uyi
    x_old = 0

    M = []
    h = []
    u = []
    theta = []
    t = []
    ux = []
    uy = []
    x = []
    Drag = []
    G = []

    M.append(M_old)
    h.append(h_old)
    u.append(u_old)
    theta.append(theta_old)
    t.append(t_old)
    ux.append(ux_old)
    uy.append(uy_old)
    x.append(x_old)
    Drag.append(0)
    G.append(g0)

    # temprature variation with height
    def atmosTemp(h):
        if h <= 11000:
            T = 273 + (15.04 - 0.00649*h)
        elif h > 11000 and h <= 25000:
            T = 273 + (-56.46)
        elif h > 25000:
            T = 273 + (-131.21 + 0.00299*h)
        return T

    # gravity variation with height
    def g(g0,Re,h):
        return pow((Re/(Re+h)),2)*g0

    # #  dendity variation with height (kg/m3)
    def rho(h):
        return 1.2*np.exp(-2.9*pow(10,-5)*pow(h,1.15))
```

```
# Cd variation with Mach Number
def CdVariation(Mn):
    Cd = 0
    if Mn <= 0.6:
        Cd = 0.208333*pow(Mn,2) - 0.25*Mn + 0.46
    elif Mn > 0.6 and Mn <= 0.8:
        Cd = 1.25*pow(Mn,3) - 2.125*pow(Mn,2) + 1.2*Mn + 0.16
    elif Mn > 0.8 and Mn <= 0.95:
        Cd = 10.37037*pow(Mn,3) - 22.88889*pow(Mn,2) + 16.9111*Mn - 3.78963
    elif Mn > 0.95 and Mn <= 1.05:
        Cd = -30*pow(Mn,3) + 88.5*pow(Mn,2) - 85.425*Mn + 27.51375
    elif Mn > 1.05 and Mn <= 1.15:
        Cd = -20*pow(Mn,3) + 60*pow(Mn,2) - 58.065*Mn + 19.245
    elif Mn > 1.15 and Mn <= 1.3:
        Cd = 11.85185*pow(Mn,3) - 44.88889*pow(Mn,2) + 56.22222*Mn - 22.58519
    elif Mn > 1.3 and Mn <= 2:
        Cd = -0.04373178*pow(Mn,3) + 0.3236152*pow(Mn,2) - 1.019679*Mn + 1.544752
    elif Mn > 2 and Mn <= 3.25:
        Cd = 0.01024*pow(Mn,3) - 0.00864*pow(Mn,2) - 0.33832*Mn + 1.08928
    elif Mn > 3.25 and Mn <= 4.5:
        Cd = -0.01408*pow(Mn,3) + 0.191688*pow(Mn,2) - 0.86976*Mn + 1.53544
    elif Mn > 4.5:
        Cd = 0.22
    return Cd

# drag
def D(Cd,Af,rho,u):
    return pow(u,2)*0.5*Cd*Af*rho(h_old)


while t_old < tb:
    if isDragVary == 0:
        drag = 0
    else:
        if isquestionE :
            a = np.sqrt(gama*R*atmosTemp(h_old))
            Mach = u_old/a
            cd = CdVariation(Mn=Mach)
            drag = D(cd,Af,rho,u_old)
        else:
            cd = 0.1
            drag = D(cd,Af,rho,u_old)

    if isGravityVary == 0:
        gravity = g0
    else:
```

```
        gravity = g(g0,Re,h_old)

    du = ((mdot*ueq/M_old)- (drag/M_old) - gravity*np.cos(theta_old))*dt
    u_new = u_old + du

    dun = gravity*np.sin(theta_old)*dt

    dur = np.sqrt(pow(du,2)+pow(dun,2))

    dtheta = np.arctan(dun/(u_new))
    theta_new = theta_old + dtheta

    dux = dur*np.sin(theta_new)
    ux_new = ux_old + dux
    duy = dur*np.cos(theta_new)
    uy_new = uy_old + duy

    dx = ux_new*dt
    x_new = x_old + dx
    dy = uy_new*dt
    h_new = h_old + dy

    dm = -mdot*dt
    M_new = M_old + dm

    t_new = t_old + dt

    t.append(t_new)
    M.append(M_new)
    u.append(u_new)
    theta.append(theta_new)
    ux.append(ux_new)
    uy.append(uy_new)
    x.append(x_new)
    h.append(h_new)
    Drag.append(drag)
    G.append(gravity)

    theta_old = theta_new
    M_old = M_new
    ux_old = ux_new
    uy_old = uy_new
    u_old = u_new
    x_old = x_new
    h_old = h_new
    t_old = t_new
```

```
    return [x,h,u,t,theta,Drag,G,]
```

Q1. (a)

- constant acceleration due to gravity (g0) = 9.81 m/s2

- neglect drag (D = 0)

In [3]:

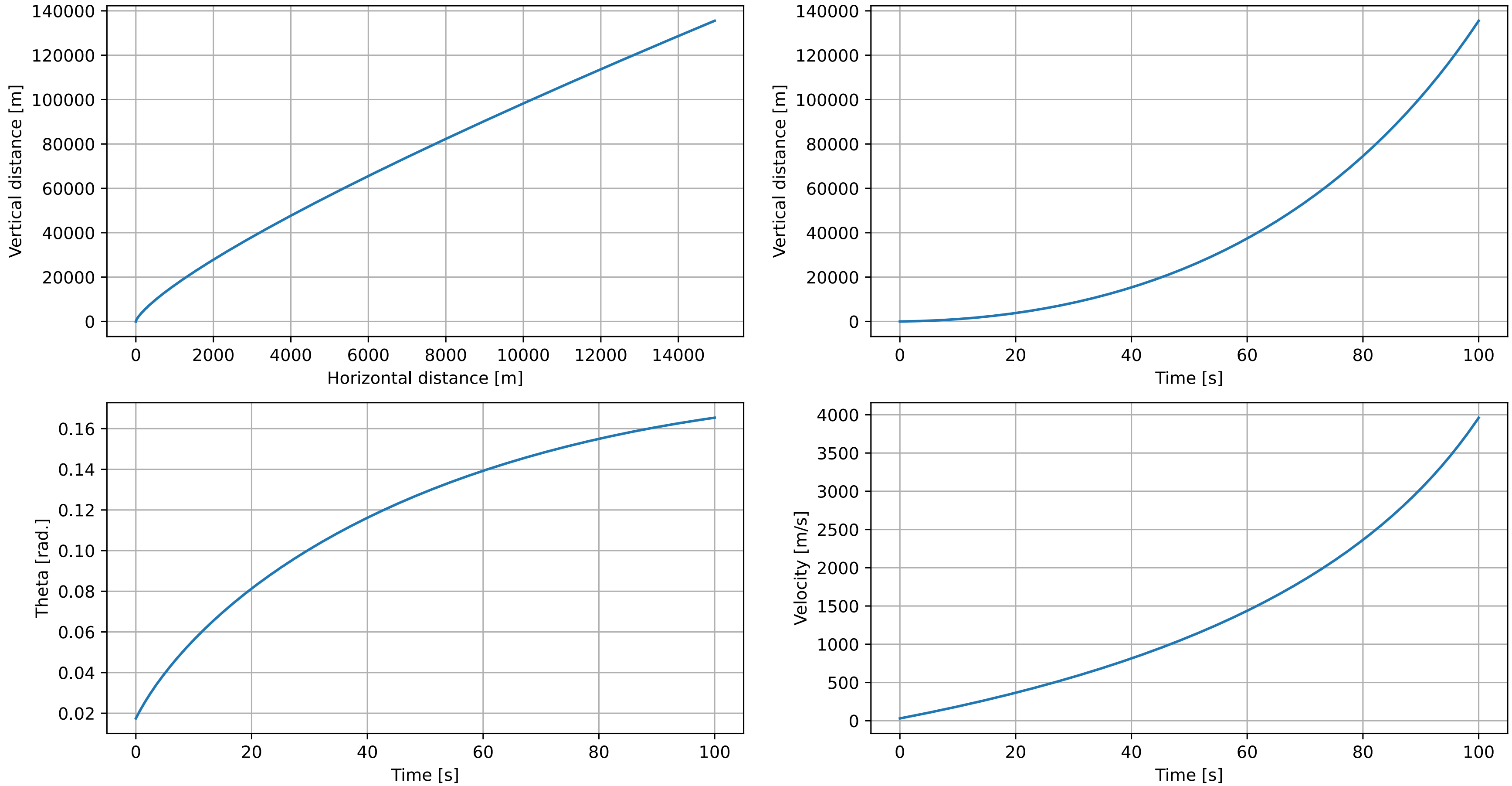
```
[xa,ha,ua,ta,thetaa,Draga,Ga] = trajectory(isDragVary=0,isGravityVary=0,isquestionE=0,dt=0.01)
```

In [4]:

```
plt.figure(figsize=(15,8));

plt.subplot(2,2,1);plt.plot(xa,ha);plt.grid();plt.xlabel("Horizontal distance [m]");plt.ylabel("Vertical distance [m]");
plt.subplot(2,2,2);plt.plot(ta,ha);plt.grid();plt.xlabel("Time [s]");plt.ylabel("Vertical distance [m]");
plt.subplot(2,2,3);plt.plot(ta,thetaa);plt.grid();plt.xlabel("Time [s]");plt.ylabel("Theta [rad.]");
plt.subplot(2,2,4);plt.plot(ta,ua);plt.grid();plt.xlabel("Time [s]");plt.ylabel("Velocity [m/s]");
```

Out[4]:

```
Text(0, 0.5, 'Velocity [m/s]')
```



Q1. (b)

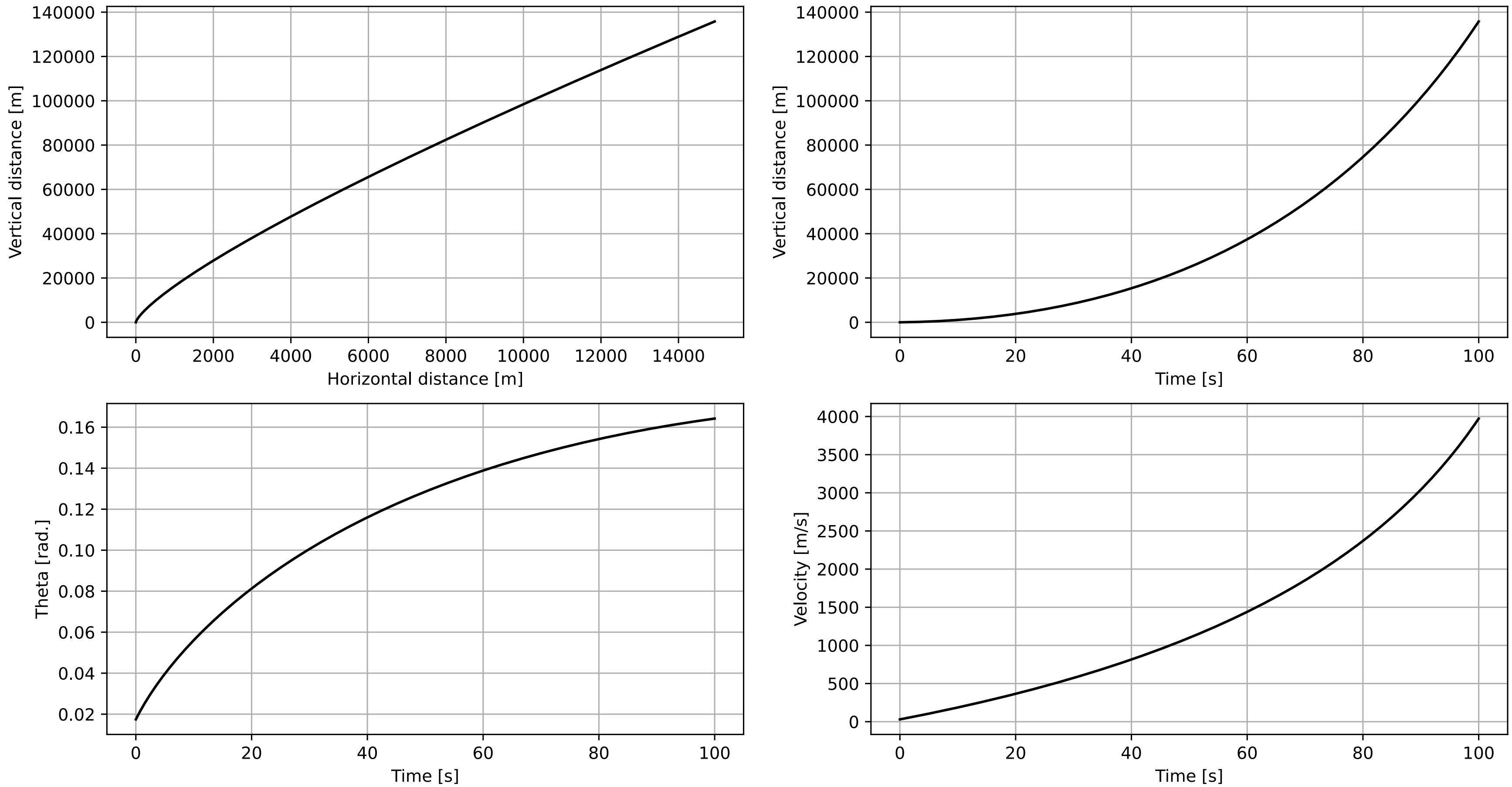
- Only acceleration due to gravity (g) varies (and D = 0):

- with height (h): g = g0 [Re/(Re + h)]2, where, Re is the earth’s radius = 6,400,00 m.

In [5]:
```
[xb,hb,ub,tb,thetab,Dragb,Gb] = trajectory(isDragVary=0,isGravityVary=1,isquestionE=0,dt=0.01)
```

In [6]:
```
```

```
plt.figure(figsize=(15,8));

plt.subplot(2,2,1);plt.plot(xb,hb,"-k");plt.grid();plt.xlabel("Horizontal distance [m]");plt.ylabel("Vertical distance [m
plt.subplot(2,2,2);plt.plot(tb,hb,"-k");plt.grid();plt.xlabel("Time [s]");plt.ylabel("Vertical distance [m]");
plt.subplot(2,2,3);plt.plot(tb,thetab,"-k");plt.grid();plt.xlabel("Time [s]");plt.ylabel("Theta [rad.]");
plt.subplot(2,2,4);plt.plot(tb,ub,"-k");plt.grid();plt.xlabel("Time [s]");plt.ylabel("Velocity [m/s]");
```



Q1. (c)

Only drag (D) varies (and g = g0): with ambient gas density (ρ) and rocket velocity (u):

$$D = \frac{1}{2} * \rho * u^2 * A_f * Cd$$

, where, CD is the coefficient of drag = 0.1 (assumed constant), Af is the frontal cross-sectional area of the rocket = 1 m2.

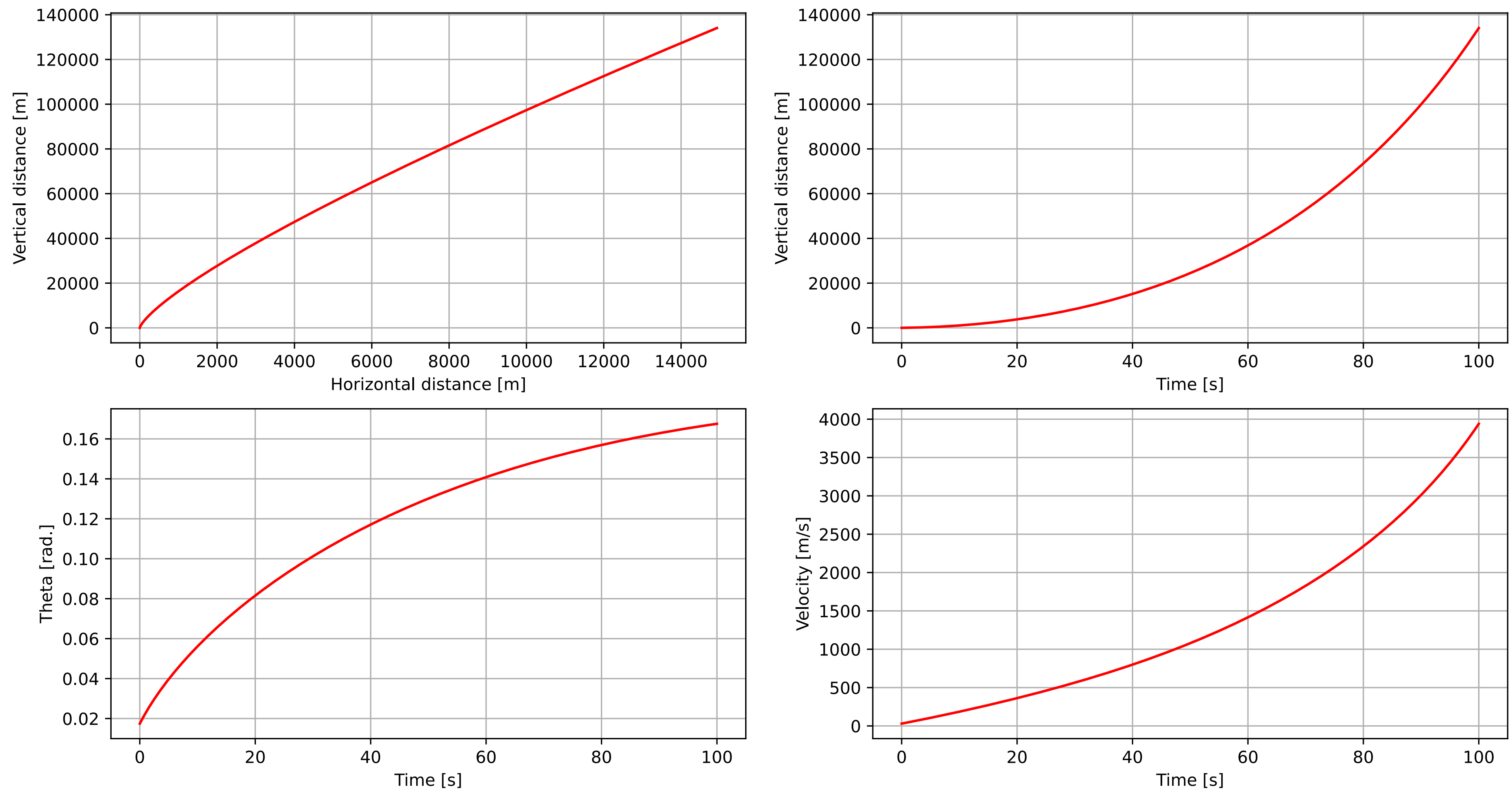
$$\rho(h) = 1.2 * e^{-2.9*10^{-5}*h^{1.15}}$$

In [7]:

```
[xc,hc,uc,tc,thetac,Dragc,Gc] = trajectory(isDragVary=1,isGravityVary=0,isquestionE=0,dt=0.01)
```

In [8]:

```
plt.figure(figsize=(15,8));

plt.subplot(2,2,1);plt.plot(xc,hc,"-r");plt.grid();plt.xlabel("Horizontal distance [m]");plt.ylabel("Vertical distance [m
plt.subplot(2,2,2);plt.plot(tc,hc,"-r");plt.grid();plt.xlabel("Time [s]");plt.ylabel("Vertical distance [m]");
plt.subplot(2,2,3);plt.plot(tc,thetac,"-r");plt.grid();plt.xlabel("Time [s]");plt.ylabel("Theta [rad.]");
plt.subplot(2,2,4);plt.plot(tc,uc,"-r");plt.grid();plt.xlabel("Time [s]");plt.ylabel("Velocity [m/s]");
```



Q1. (d)

Both g and D varies: as given in (b) and (c), respectively.

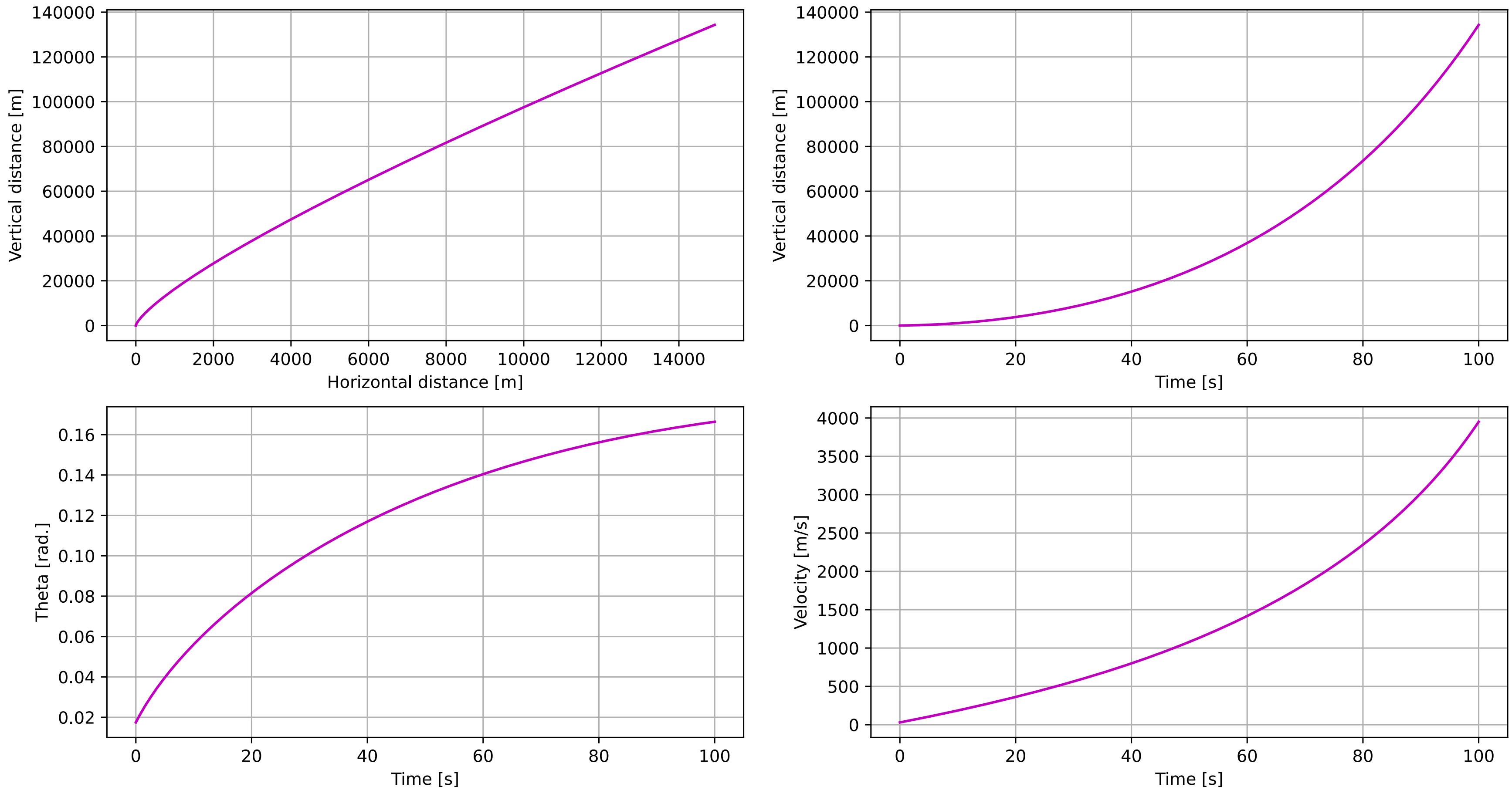
In [9]:

```
[xd,hd,ud,td,thetad,Dragd,Gd] = trajectory(isDragVary=1,isGravityVary=1,isquestionE=0,dt=0.01)
```

In [10]:

```
plt.figure(figsize=(15,8));

plt.subplot(2,2,1);plt.plot(xd,hd,"-m");plt.grid();plt.xlabel("Horizontal distance [m]");plt.ylabel("Vertical distance [m
plt.subplot(2,2,2);plt.plot(td,hd,"-m");plt.grid();plt.xlabel("Time [s]");plt.ylabel("Vertical distance [m]");
```

```
plt.subplot(2,2,3);plt.plot(td,thetad,"-m");plt.grid();plt.xlabel("Time [s]");plt.ylabel("Theta [rad.]");
plt.subplot(2,2,4);plt.plot(td,ud,"-m");plt.grid();plt.xlabel("Time [s]");plt.ylabel("Velocity [m/s]");
```



Q1. (e)

Realistic condition: CD varies with Mach number

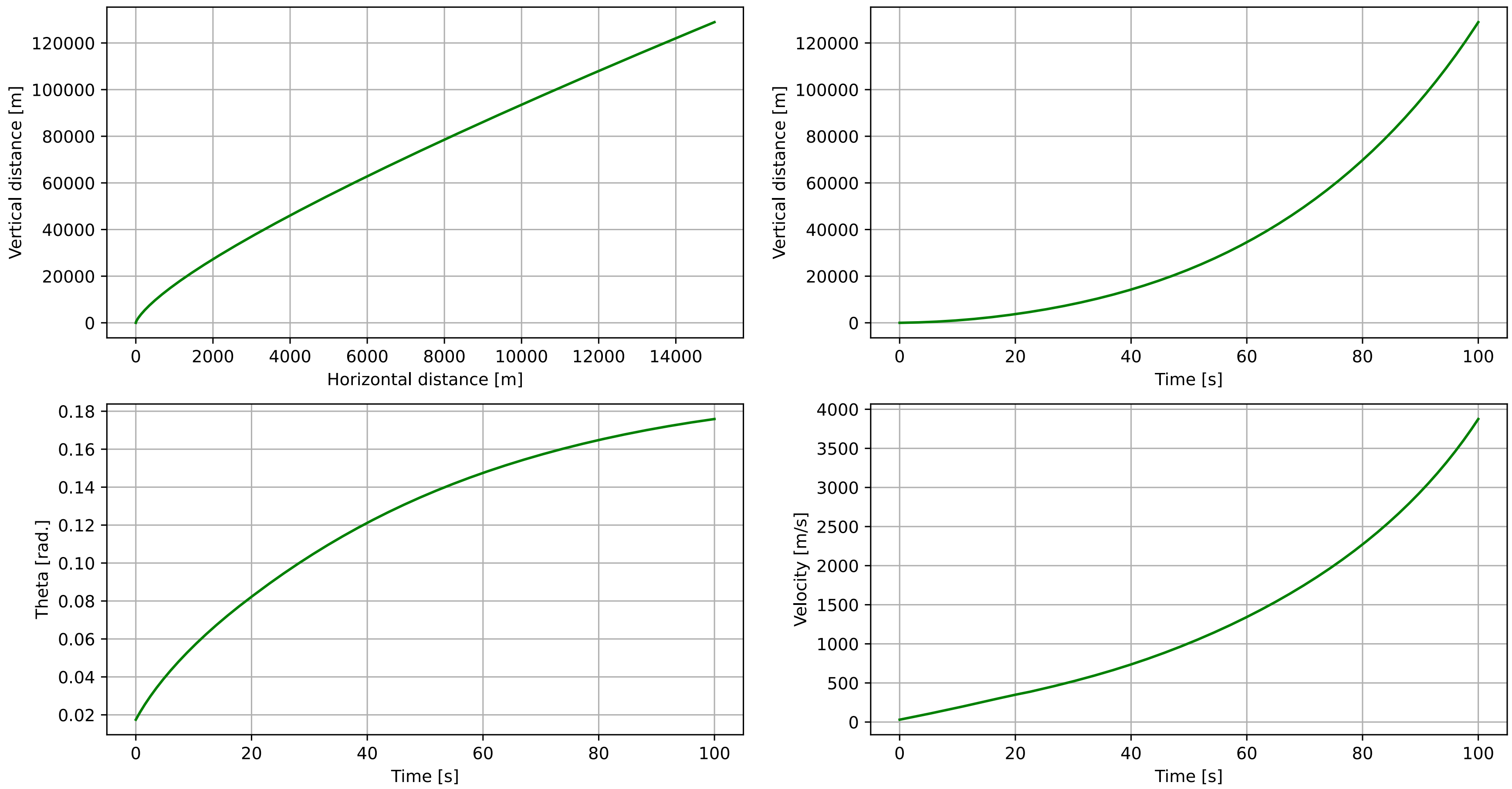
In [11]:

```
[xe,he,ue,te,thetae,Drage,Ge] = trajectory(isDragVary=1,isGravityVary=1,isquestionE=1,dt= 0.01)
```

In [12]:

```
plt.figure(figsize=(15,8));
```

```
plt.subplot(2,2,1);plt.plot(xe,he,"-g");plt.grid();plt.xlabel("Horizontal distance [m]");plt.ylabel("Vertical distance [m
plt.subplot(2,2,2);plt.plot(te,he,"-g");plt.grid();plt.xlabel("Time [s]");plt.ylabel("Vertical distance [m]");
plt.subplot(2,2,3);plt.plot(te,thetae,"-g");plt.grid();plt.xlabel("Time [s]");plt.ylabel("Theta [rad.]");
plt.subplot(2,2,4);plt.plot(te,ue,"-g");plt.grid();plt.xlabel("Time [s]");plt.ylabel("Velocity [m/s]");
```

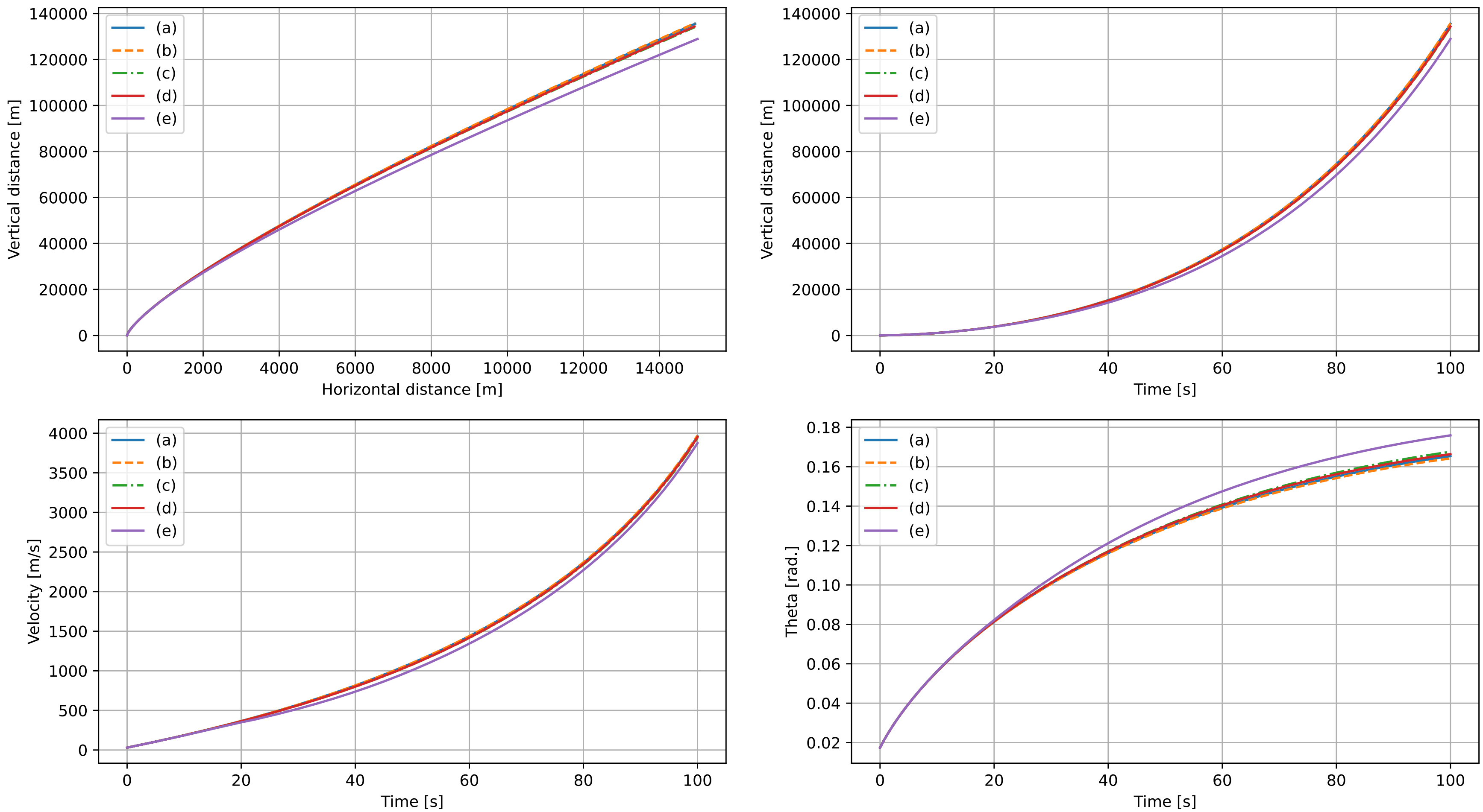


Data Comparison

In [13]:

```
plt.figure(figsize=(16,9));
plt.subplot(2,2,1);
plt.grid();
plt.xlabel("Horizontal distance [m]");plt.ylabel("Vertical distance [m]");
plt.plot(xa,ha,"-")
plt.plot(xb,hb,"--")
```

```
plt.plot(xc,hc,"-.")
plt.plot(xd,hd,"-")
plt.plot(xe,he);
plt.legend(["(a)","(b)","(c)","(d)","(e)"]);

plt.subplot(2,2,2);
plt.grid();
plt.xlabel("Time [s]");plt.ylabel("Vertical distance [m]");
plt.plot(ta,ha,"-")
plt.plot(tb,hb,"--")
plt.plot(tc,hc,"-.")
plt.plot(td,hd,"-")
plt.plot(te,he);
plt.legend(["(a)","(b)","(c)","(d)","(e)"]);

plt.subplot(2,2,3);
plt.grid();
plt.xlabel("Time [s]");plt.ylabel("Velocity [m/s]");
plt.plot(ta,ua,"-")
plt.plot(tb,ub,"--")
plt.plot(tc,uc,"-.")
plt.plot(td,ud,"-")
plt.plot(te,ue);
plt.legend(["(a)","(b)","(c)","(d)","(e)"]);

plt.subplot(2,2,4);
plt.grid();
plt.xlabel("Time [s]");plt.ylabel("Theta [rad.]");
plt.plot(ta,thetaa,"-")
plt.plot(tb,thetab,"--")
plt.plot(tc,thetac,"-.")
plt.plot(td,thetad,"-")
plt.plot(te,thetae);
plt.legend(["(a)","(b)","(c)","(d)","(e)"]);
```



Drag and Gravity Graph

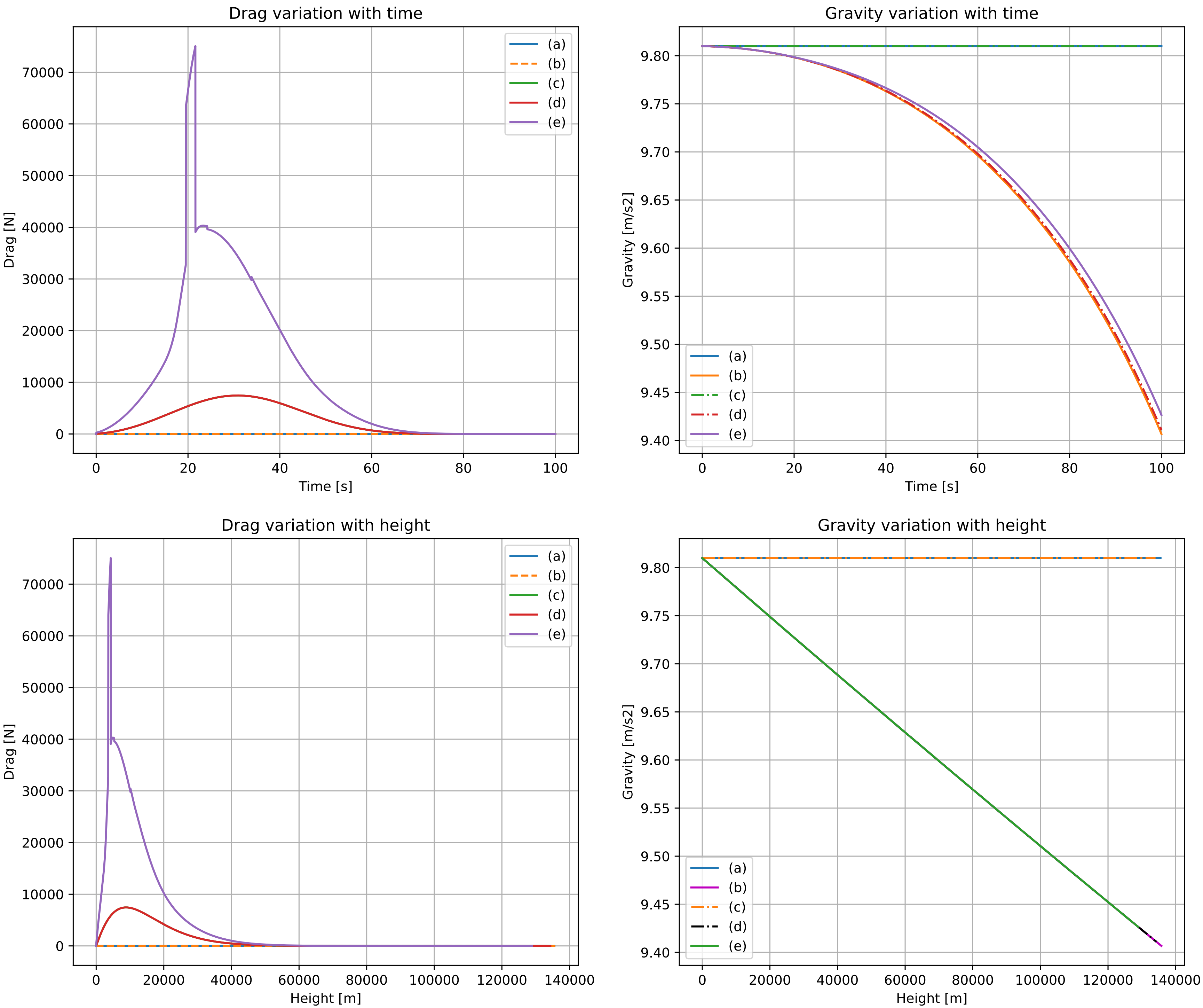
In [14]:

```
plt.figure(figsize=(15,13));

plt.subplot(2,2,1);
plt.title("Drag variation with time");
plt.grid();
plt.xlabel("Time [s]");plt.ylabel("Drag [N]");
plt.plot(ta,Draga)
plt.plot(tb,Dragb,"--")
plt.plot(tc,Dragc)
plt.plot(td,Dragd)
```

```
plt.plot(te,Drage);
plt.legend(["(a)","(b)","(c)","(d)","(e)"]);

plt.subplot(2,2,2);
plt.title("Gravity variation with time");
plt.grid();
plt.xlabel("Time [s]");plt.ylabel("Gravity [m/s2]");
plt.plot(ta,Ga,"-")
plt.plot(tb,Gb,"-")
plt.plot(tc,Gc,"-.")
plt.plot(td,Gd,"-.")
plt.plot(te,Ge);
plt.legend(["(a)","(b)","(c)","(d)","(e)"]);

plt.subplot(2,2,3);
plt.title("Drag variation with height");
plt.grid();
plt.xlabel("Height [m]");plt.ylabel("Drag [N]");
plt.plot(ha,Draga)
plt.plot(hb,Dragb,"--")
plt.plot(hc,Dragc)
plt.plot(hd,Dragd)
plt.plot(he,Drage);
plt.legend(["(a)","(b)","(c)","(d)","(e)"]);

plt.subplot(2,2,4);
plt.title("Gravity variation with height");
plt.grid();
plt.xlabel("Height [m]");plt.ylabel("Gravity [m/s2]");
plt.plot(ha,Ga,"-")
plt.plot(hb,Gb,"-m")
plt.plot(hc,Gc,"-.")
plt.plot(hd,Gd,"-.k")
plt.plot(he,Ge);
plt.legend(["(a)","(b)","(c)","(d)","(e)"]);
```

Out[14]:

```
<matplotlib.legend.Legend at 0x10a7cca0>
```



Drag variation with time
Drag [N]
Time [s]
(a)
(b)
(c)
(d)
(e)
Gravity variation with time
Gravity [m/s2]
Time [s]
Drag variation with height
Drag [N]
Height [m]
Gravity variation with height
Gravity [m/s2]
Height [m]

Burnout Data

In [15]:
```
BurnOut_data = pd.DataFrame({
    "Q1" : ("(a)","(b)","(c)","(d)","(e)"),
    "Burnout Height [m]" : (ha[-1],hb[-1],hc[-1],hd[-1],he[-1]),
    "Burnout Velocity [m/s]" : (ua[-1],ub[-1],uc[-1],ud[-1],ue[-1]),
    "Burnout Theta [rad]" : (thetaa[-1],thetab[-1],thetac[-1],thetad[-1],thetae[-1])
})
```

In [16]:
```
BurnOut_data.set_index("Q1",inplace=True)
BurnOut_data
```

Out[16]:

Q1	Burnout Height [m]	Burnout Velocity [m/s]	Burnout Theta [rad]
(a)	135534.160597	3961.668460	0.165374
(b)	135801.485837	3973.085036	0.164204
(c)	134060.341269	3939.701215	0.167559
(d)	134323.542576	3950.957755	0.166374
(e)	128937.098816	3875.243686	0.175895