Indian Institute of Technology Kanpur

AE451A

Experiments in Aerospace Engineering - III

Experiment No. 4

Vibration characteristics of a slender beam

Submitted By:

Ankit Lakhiwal (180102)

ankitl@iitk.ac.in

Aerospace Engineering Department, IIT Kanpur

August 23, 2021

1. Objective

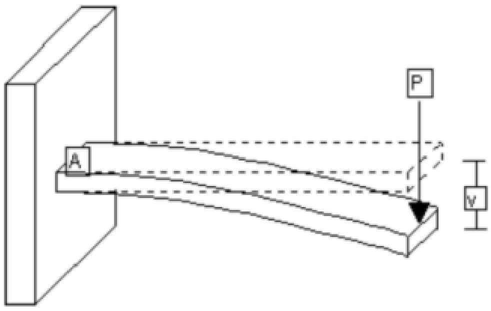
- Observe a beam vibrating under resonance, and retrieve its fundamental frequencies.
- Record acceleration and strain histories in a vibrating cantilever beam and evaluate its vibration characteristics by employing Fast Fourier Transform.

2. Introduction And Theory

Free vibration occurs when a mechanical system is set in motion with an initial input and allowed to vibrate freely. The mechanical system vibrates with a combination of frequencies, out of which one or more may be dominated. If left unperturbed, the vibration damps down to motionlessness. Some of the examples of free vibration are, pulling a child back on a swing and letting it go, or hitting a tuning fork and letting it ring.

Forced vibration is observed when a time-varying disturbance (load, displacement or velocity) is applied to a mechanical system. The disturbance can be periodic with steady-state, transient or random input. The periodic input can further be divided into harmonic and non-harmonic disturbance. The importance of this mode rises in the engineering field. Machines, motors and other industrial applications, exhibits this mode of vibrations, which may cause a serious damage of the equipment. Washing machine shaking due to an imbalance, transportation vibration caused by an engine or uneven road, or the vibration of a building during an earthquake are some of the examples of forced vibration.

A straight, horizontal cantilever beam under a vertical load will deform into a curve. When this force is removed, the beam will return to its original shape, however it's inertia will keep the beam in motion.



If there are no external driving forces and the beam is vibrating under the influence of gravitational forces, we get:

$$EI\frac{\partial^4 w}{\partial x^4} \quad = \quad -\mu\frac{\partial^2 w}{\partial t^2}$$

The above equation can be solved by variable separation to obtain:

$$y(x) = (c_1 cos\beta x + c_2 sin\beta x + c_3 cosh\beta x + c_4 sinh\beta x)(Asin\omega t + Bcos\omega t)\ \ldots\ldots\ (1)$$

$$where \quad \omega \ = \beta^2\sqrt{\frac{EI}{\rho A}}\ \ldots\ldots\ldots\ (2)$$

Where, μ is mass per unit length

E is the modulus of rigidity

I is moment of inertia

w is the displacement

x is the distance from fixed end

C1,C2,C3,C4 are constants of integration (to be determined by boundary conditions)

For a cantilever beam, the boundary conditions are following :

$$\begin{aligned} BC\,1 &\quad : \quad y(x) = 0,\ at\ x = 0\,; \\ BC\,2 &\quad : \quad \frac{dy}{dx} = 0,\ at\ x = 0\,; \\ BC\,3 &\quad : \quad \frac{d^2y}{dx^2} = 0,\ at\ x = l\,; \\ BC\,4 &\quad : \quad \frac{d^3y}{dx^3} = 0,\ at\ x = l\,; \end{aligned}$$

solving for boundary condition, we get

$$\beta \mathbf{l}\ =\ \mathbf{1.8751,\ 4.6941,\ 7.8548,\ 10.995} \ldots .(3)$$

3. Equipments'

1.Electrodynamic shaker

Image 3.(a)

2.Function generator

Image 3.(b)

3.Power amplifier

Image 3.(c)

4.Stroboscope

Image 3.(d)

4. Measurements

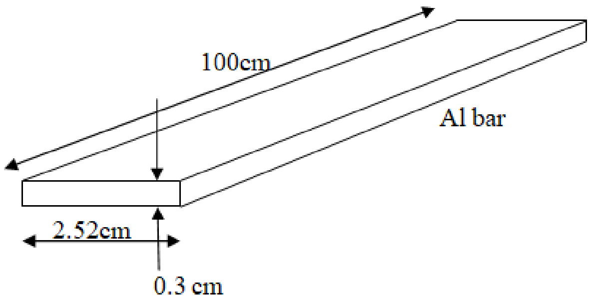


Image 4(a). Aluminium specimen

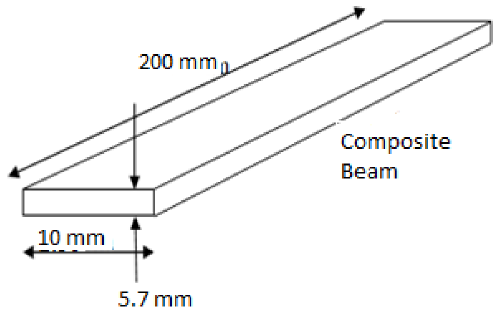


Image 4(b). Composite Specimen

1. **Forced vibration Aluminium specimen data**

S.N.	Specimen Specification	Dimension
1.	Length of the beam (L)	1m
2.	Width of the beam (b)	2.5 cm
3.	Height of the beam(h)	0.35 cm
4.	Modulus of rigidity(E)	69 GPa
5.	Density	2700 kg/m3

Table 4(a). Forced vibration Aluminium specimen table

2. **Free vibration Aluminium specimen data**

S.N.	Specimen Specification	Dimension
1.	Length of the beam (L)	200 mm
2.	Width of the beam (b)	10 mm
3.	Height of the beam(h)	2 mm
4.	Modulus of rigidity(E)	69 GPa
5.	Mass(M)	11.27 gms

Table 4(b). Free vibration Aluminium specimen data

3. **Free vibration Composite specimen data**

S.N.	Specimen Specification	Dimension
1.	Length of the beam (L)	200 mm
2.	Width of the beam (b)	10 mm
3.	Height of the beam(h)	5.7 mm
4.	Modulus of rigidity(E)	47.4 GPa
5.	Mass(M)	27.75 gms

Table 4(c). Free vibration Composite specimen data

5. Exeperimental Setup

The schematics of the experimental setup used for the demonstration of forced vibrations and natural vibrations are shown in Image 5(a), Image 5(b) respectively.

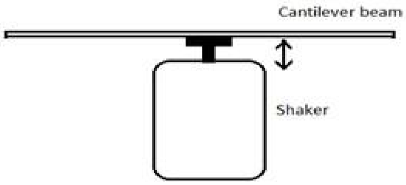


Image 5(a). Forced vibration

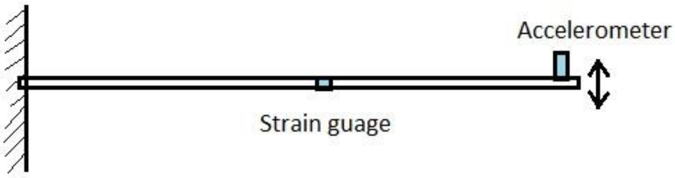


Image 5(b). Natural vibration

6. Procedure

1. **Part (A): Forced vibrations in the free-free beam – Demonstration experiment**
    - Mount the beam on electrodynamic shaker.
    - Define force amplitude by turning the knob. Make sure not to give very large amplitude which would overheat the device.
    - Gradually increase forcing frequency and observe the change in the amplitude of vibrations.
    - When the forcing frequency matches with one of the natural frequencies, resonance will be visually apparent (as shown in Figure 6(a)).

Figure 6(a) : High amplitude of vibrations at fundamental frequency

    - Switch off the lights and start stroboscope. Adjust the flickering light frequency such that the beam appears stationary.
    - Perform minor adjustments on the Stroboscope and forcing frequencies to attain maximum amplitude. Note down this fundamental frequency.
    - Repeat the process for the first three fundamental frequencies and record the mode shapes for characterization. Representative second mode shape is shown in Figure 6(b).

Figure 6(b): Second mode shape

2. **Part (B): Analyze natural vibrations in a cantilever beam by using accelerometer data**

- Mount accelerometer at the end of the cantilever beam and connect it to data acquisition system.
- Provide initial end deflection to the beam and release the beam
- Once the vibration stabilizes (after a short initial interval) then begin recording the accelerometer and strain gauge data.
- Stop recording the data before the amplitudes diminish.
- Perform FFT on the recorded data and get the free vibration characteristics.

7. Calculation

Equation for fundamental natural first, second and third frequency is,(using equation 3)

$$w_1 = (\frac{1.875}{l})^2\sqrt{\frac{EI}{\rho A}}$$

$$w_2 = (\frac{4.694}{l})^2\sqrt{\frac{EI}{\rho A}}$$

$$w_3 = (\frac{7.85}{l})^2\sqrt{\frac{EI}{\rho A}}$$

1. **For forced vibration**

$$I = \frac{bh^3}{12} = \frac{0.025 * 0.0035^3}{12} = 8.9323 * 10^{-11}\ m^4$$

$$A = \ b * h = 0.025 * 0.0035 = 8.75 * 10^{-5}\ m^2$$

$$E = \ 69 * 10^9\ Pa$$

$$\rho = \ 2700\ \frac{Kg}{m^3}$$

$$l = 1m$$

$$from\ table\ 4(a)$$

First Fundamental frequency

$$w_1 = (\frac{1.875}{1})^2\sqrt{\frac{(8.9323 * 10^{-11})(69 * 10^9)}{(2700)(8.75 * 10^{-5})}}\ s^{-1} = 17.9565\ Hz$$

Second Fundamental frequency

$$w_2 = (\frac{4.694}{1})^2\sqrt{\frac{(8.9323 * 10^{-11})(69 * 10^9)}{(2700)(8.75 * 10^{-5})}}\ s^{-1} = 112.5398\ Hz$$

similiarly we can solve for rest of the frequencies

Theoritical Frequency	Value (in Hz)
First Fundamental frequency	17.9565
Second Fundamental frequency	112.5398

2. **Free vibration**

   1. **For aluminium specimen**

$$I = \frac{bh^3}{12} = \frac{0.01 * 0.002^3}{12} = \ 6.667 * 10^{-12} \ m^4$$
$$A = \ b * h = 0.01 * 0.002 = 2 * 10^{-5} \ m^2$$
$$E = \ 69 * 10^9 \ Pa$$
$$l = 0.2m$$
$$\rho = \ \frac{M}{V} = \frac{M}{A * l} = \frac{11.27 * 10^{-3}}{(2 * 10^{-5}) * 0.2} = 2817.5 \ \frac{Kg}{m^3}$$
$$from \ table \ 4(b)$$

First Fundamental frequency

$$w_1 = (\frac{1.875}{1})^2 \sqrt{\frac{(6.667 * 10^{-12})(69 * 10^9)}{(2817.5)(8.75 * 10^{-5})}} \ s^{-1} = 251.1160 \ Hz$$

similiarly we can solve for rest of the frequencies

Theoritical Frequency	Value (in Hz)
First Fundamental frequency	251.1160

2. **For composite specimen**

$$I = \frac{bh^3}{12} = \frac{0.01 * 0.0057^3}{12} = \ 1.5432 * 10^{-10} \ m^4$$
$$A = \ b * h = 0.01 * 0.0057 = 5.7 * 10^{-5} \ m^2$$
$$E = \ 47.4 * 10^9 \ Pa$$
$$l = 0.2m$$
$$\rho = \ \frac{M}{V} = \frac{M}{A * l} = \frac{27.75 * 10^{-3}}{(5.7 * 10^{-5}) * 0.2} = 2434.21 \ \frac{Kg}{m^3}$$
$$from \ table \ 4(c)$$

First Fundamental frequency

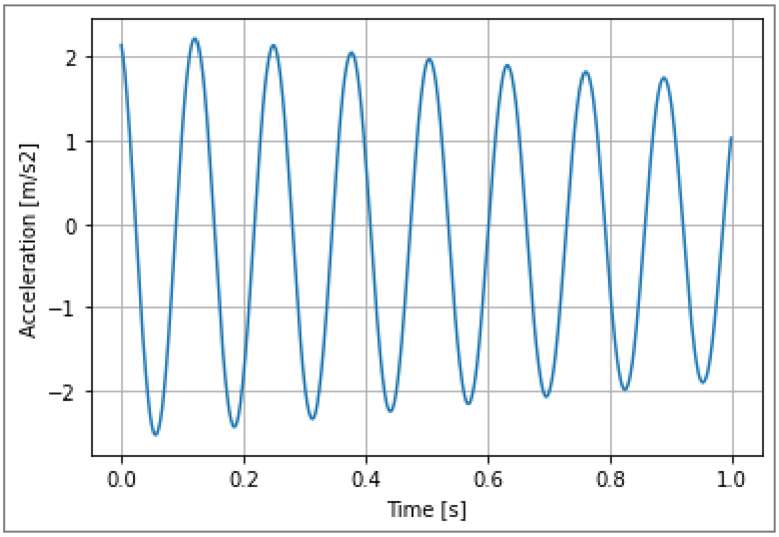
$$w_1 = (\frac{1.875}{1})^2 \sqrt{\frac{(1.5432 * 10^{-10})(47.4 * 10^9)}{(2434.21)(5.7 * 10^{-5})}} \ s^{-1} = 638.1707 \ Hz$$

similiarly we can solve for rest of the frequencies

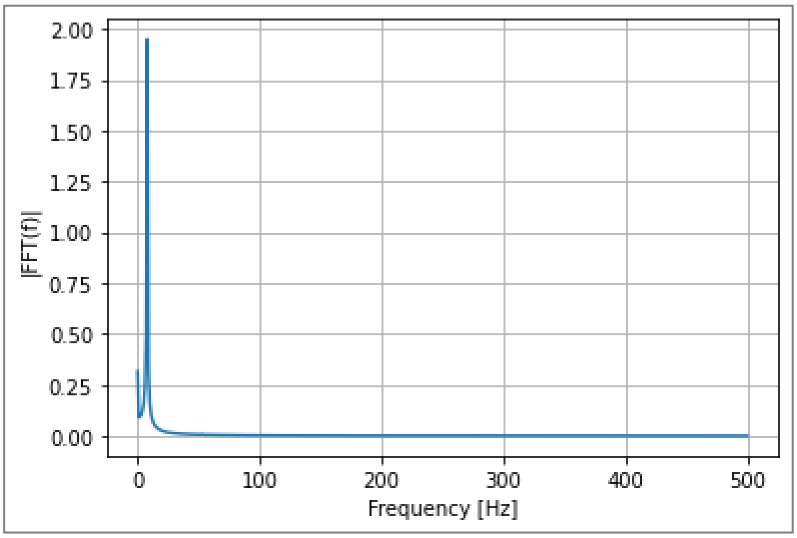
Theoritical Frequency	Value (in Hz)
First Fundamental frequency	638.17074

8. Data Analysis

1. **Forced Vibration Data Analysis**
    - Acceleration vs Time graph



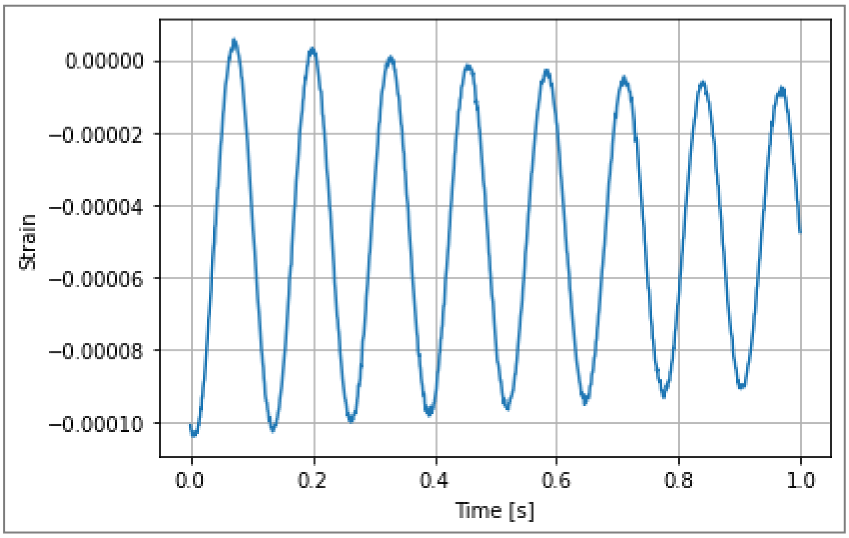
- FFT of accelaration vs Frequency Graph



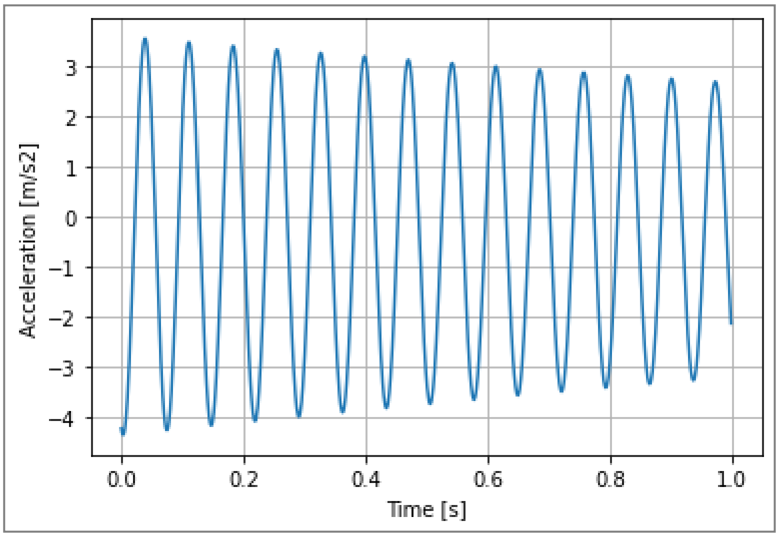
First Fundamental frequency = 8.016 Hz

Second Fundamental frequency= 53.10 Hz

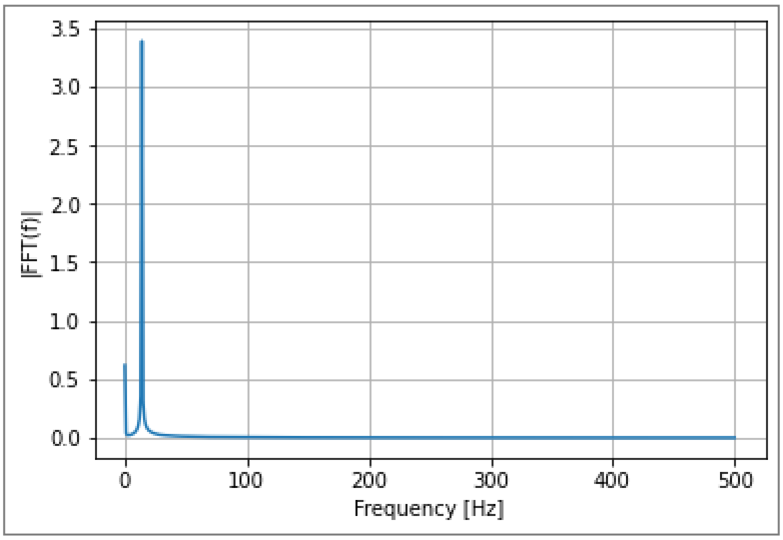
- Strain vs Time Graph



2. **Free Vibration Data Analysis**
    1. Aluminium Beam
        - Acceleration vs Time graph

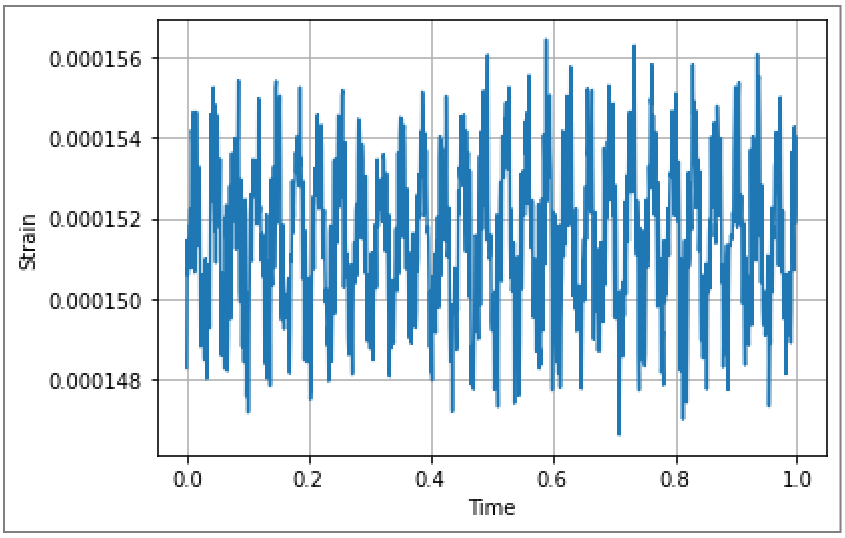


- FFT of accelaration vs Frequency Graph



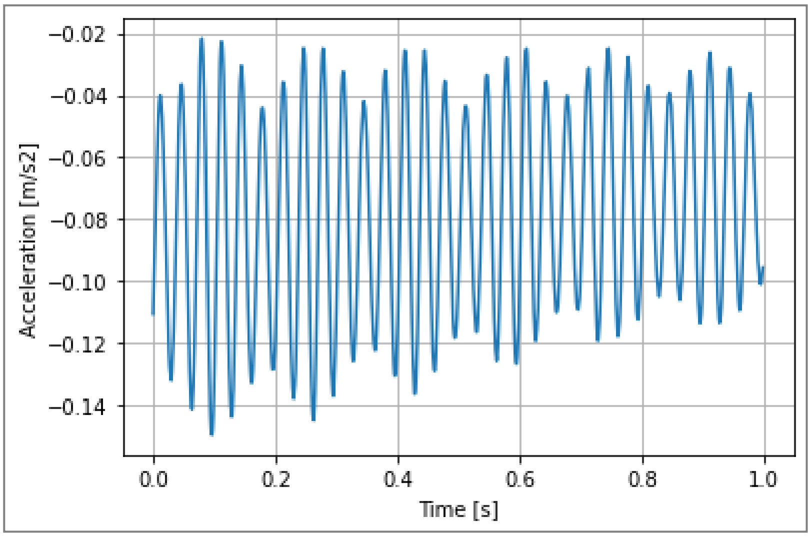
First Fundamental frequency = 14.02 Hz

- Strain vs Time Graph

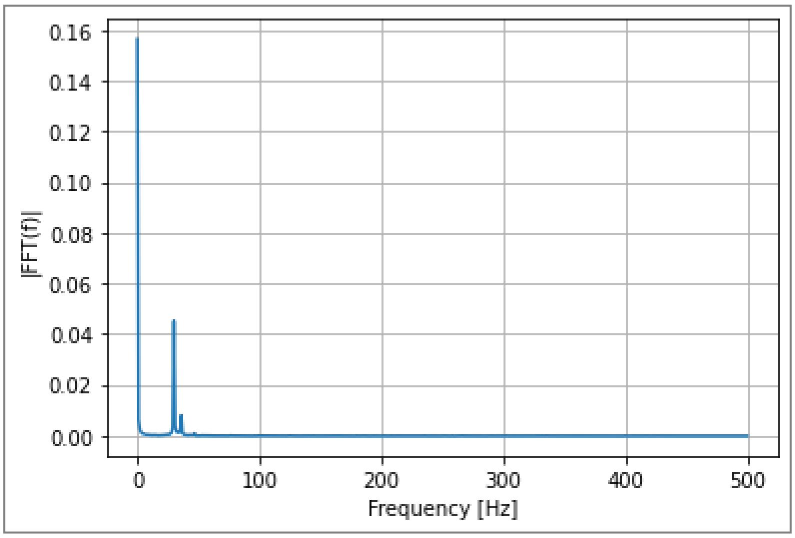


2. Composite Beam

- Acceleration vs Time graph

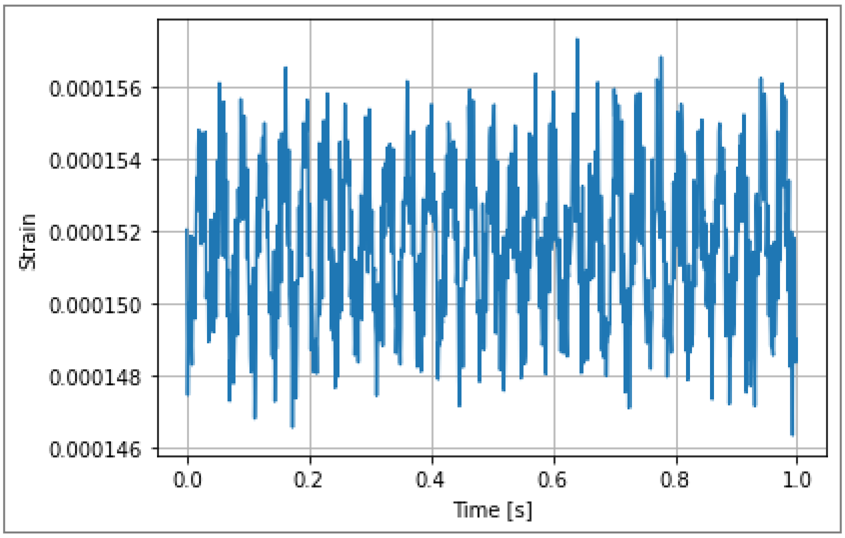


- FFT of accelaration vs Frequency Graph



First Fundamental frequency = 30.06 Hz

- Strain vs Time Graph



9. Error Analysis

$$\%\ Error\ =\ \frac{|Fth - Fexp|}{Fth} * 100\%$$

1. **Forced Vibration**

Mode	Theoritical Frequency	Experimental Frequency	% Error
First Fundamental frequency	17.9565	8.016	55.34
Second Fundamental frequency	112.5398	53.10	52.81

2. **Free Vibration**
    - **Aluminium Beam**

Mode	Theoritical Frequency	Experimental Frequency	% Error
First Fundamental frequency	251.1160	14.02	94.41

- **Composite Beam**

Mode	Theoritical Frequency	Experimental Frequency	% Error
First Fundamental frequency	638.17074	30.06	95.29

10. Results and Discussion

- Beam deflection increase at its natural frequency
- First peak in FFT graph shows specimen first overtone frequency
- We are getting error in theoritical and experimental data

11. Source of Error and Precautions

1. **Source of Error**
    - Calibration error
    - Specimen dimension measuring error
    - Accelerometer or Strain gauge sampling efficiency
2. **Precautions**
    - Check equipments and calibrate properly
    - Don't touch beam during oscillation
    - Don't increase frequency rapidly in function generator, it can overheat shaker.