Indian Institute of Technology Kanpur

AE451A

Experiments in Aerospace Engineering - III

Experiment No. 3

Full-field stress analysis using photo-elasticity method

Submitted By:

Ankit Lakhiwal (180102)

ankitl@iitk.ac.in

Aerospace Engineering Department, IIT Kanpur

August 16, 2021

1. Objective

- To perform experimental stress measurements on an epoxy beam subjected to four-point bending using photoelasticity.
- To measure the normal stress variation across the cross-sectional area of a beam, and compare the obtained data to the results predicted by beam theory.

2. Introduction and Theory

PhotoElasticity

$$Index\ of\ refraction\ =\frac{Speed\ of\ light\ in\ vaccum}{Speed\ of\ light\ in\ material}$$

Many transparent non-crystallize material that are optically isotropic when free of stress becomes optically an isotropic (temporary double refrective) when they are subjected to stress. The birefringent (changing refractive index upon loading) material characterstic is used in the method of photoelasticty.

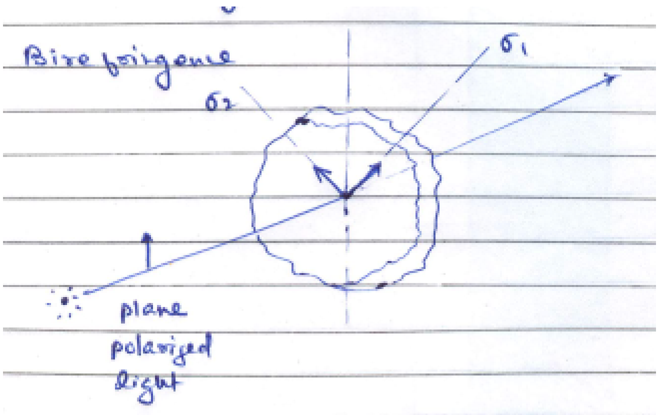


Image 1.

When a polarized light passes through a stressed material the light wave gets resolved into two mutually perpendicular components in the direction of the material principle stress.

The two wavelength travel at different velocities subjectively the stressed material having two different indices of refraction (n1 and n2). maxwell noted that the charges in the refrective indices are linearly proportional to the loads.For 2-D case the stress-optics (Brewster's) law

$$\begin{aligned} n_1 - n_0 &= C_1\sigma_1 + C_2\sigma_2(\ldots + C_2\sigma_3\ in\ 3D) \\ n_2 - n_0 &= C_1\sigma_2 + C_2\sigma_1(\ldots + C_2\sigma_3\ in\ 3D) \end{aligned}$$

$$\begin{array}{ccc} Where: & & \\ \sigma_1, \sigma_2 & = & Principal\ stress \\ n_1, n_2 & = & refrective\ indices\ in\ \sigma_1\ and\ \sigma_2\ direction \\ n_0 & = & refrective\ index\ of\ unstressed\ material \\ C_1, C_2 & = & Stress - optics\ constants \end{array}$$

eliminating n0 to get

$$n_2 - n_1 = (C_2 - C_1)(\sigma_1 - \sigma_2) = C(\sigma_1 - \sigma_2)$$

where C = C1 - C2 is relative stress-optics coeeficient first expressed in terms of brewsters

$$1\ brewster = 10^{-12}\frac{m^2}{N}$$

The two resolved light components travel through the medend at different velocities, therefore, they emerge from the plate at different times. In other words one components retards in time compared to the other.

The relative linear phase shift also known as retardation can be computed as

$$\delta = h(n_2 - n_1)$$

$$\Delta = \frac{2\pi}{\lambda}\delta = \frac{2\pi h}{\lambda}(n_2 - n_1)$$

$$or\ \sigma_1 - \sigma_2 = \frac{\delta}{hc} = \frac{\lambda}{2\pi hc}\Delta$$

$$where\ h\ is\ material\ thickness\ or\ relative\ angular\ phase\ shift$$

$$Replacing\ \frac{\Delta}{2\pi} = N \ldots\ fringe\ order$$

$$and\ \frac{\lambda}{c} = f_\sigma \ldots\ fringe\ constant$$

$$to\ get\ \sigma_1 - \sigma_2 = \frac{nf_\sigma}{h}$$

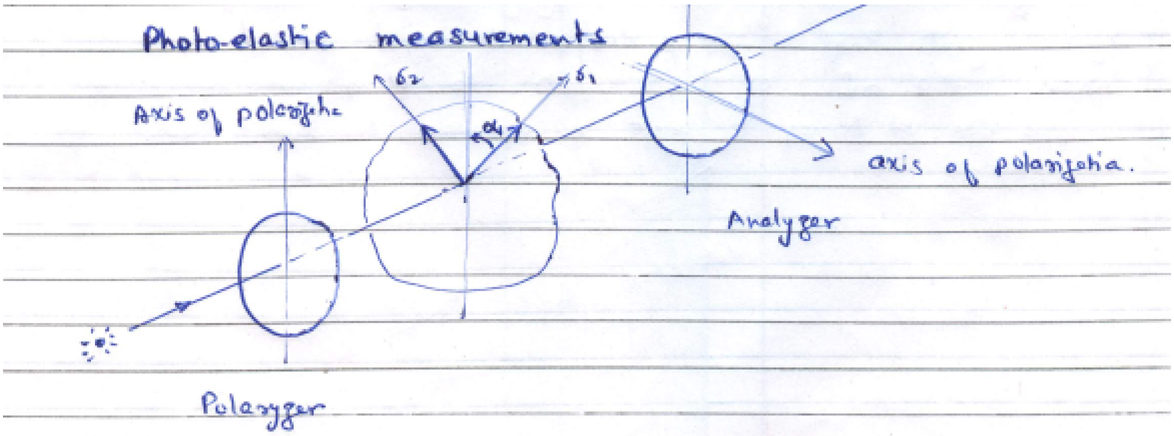


Image 2.

Consider a core when plane stresed model is inserted into the field of plane polariscope. Let the principal stress direction makes an angle polarization axis.
Plane polarizer resolves an incident light wave into Components which vibrate parallel and perpendicular to the axis of the polerizer. The parallel component is transmitted and the component perpendiculor to the polarizer axis is absorbed Initial phase of the wave is not important here, therefore the plane polasized light beem emerging from the polarizer can be represented as Ep = k cos wt

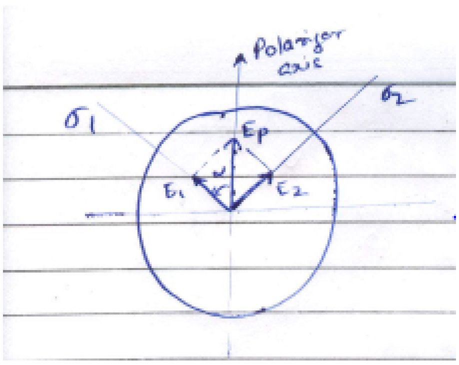


Image 3.

The light wave emerging from the material(stressed) resolved into the components E1 and E2

$$E_1 = Kcos\alpha \, cos(\omega t - \Delta_1)$$
$$E_2 = Ksin\alpha \, cos(\omega t - \Delta_2)$$
$$where \; \Delta_1 \; and \; \Delta_2 are \; developed \; phase \; shifts$$
$$\Delta_1 = \frac{2\pi h}{\lambda}(n_1 - 1)$$
$$\Delta_2 = \frac{2\pi h}{\lambda}(n_2 - 1)$$

After leaving the material the light waves E1 and E2 propagate without further change and the analyzer, subjecting E1 and E2 to get resolved further into vertical and horizontal components. Vertical components absorbed and only horizontal components in the direction of analyzer axis passes through, Therefore, the emerging light from the analyzer

$$E = E_2^{'} - E_1^{'} = E_2 cos\alpha - E_1 sin\alpha$$
$$or \; E = Ksin2\alpha \; sin(\frac{\Delta_2 - \Delta_1}{2}) \; sin(\omega t - \frac{\Delta_2 + \Delta_1}{2})$$

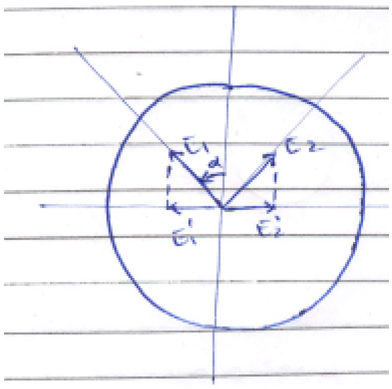


Image 4.

Therefore, light intensity emerging from the analyzer

$$I = [Ksin2\alpha \; sin(\frac{\Delta_2 - \Delta_1}{2})]^2 = Ksin^2 2\alpha \; sin^2 \frac{\Delta}{2}$$
$$\Delta = \Delta_1 - \Delta_2$$
$$or \; \Delta = \frac{2\pi h}{\lambda}(n_2 - n_1) = \frac{2\pi h}{\lambda}(\sigma_1 - \sigma_2)$$
$$clearly \; I = 0 \; if \; sin^2 2\alpha = 0 \; (function \; of \; principal \; stress \; direction)$$
$$or \; when \; sin^2 \frac{\Delta}{2} = 0 \; (related \; to \; principal \; stress \; difference)$$

Effect of principal stress direction

$$when \; 2\alpha = n\pi = 0$$
$$n = 0, 1, 2, \ldots$$

i.e. light intensity vanishes when one of the principal sterss direction coincides with the polarizer axis. When the entire stress field is observed in the poleriscope, a fringe pattern is observed. The fringes are loci of points where th e principal stress direction coincide with the axis of polarizer. The fringe pattern produced by

$$sin^2 2\alpha$$

term is called **isoclinic fringes.**
Effect of Principal stress difference.

When

$$\frac{\Delta}{2} = n\pi = 0$$
$$n = 0, 1, 2, \ldots$$

light intensity vanishes when the principal stress difference is zero or sufficient to produce an integrel number of wave lengths of retardation (n=1,2,3...). The fringes produced by

$$sin^2 \frac{\Delta}{2}$$

term is called **isochoretic fringe** pattern.

Consider

$$\Delta = \frac{2\pi hc}{\lambda}(\sigma_1 - \sigma_2)$$

light intensity extinguishes when

$$\frac{hc}{\lambda}(\sigma_1 - \sigma_2) = n$$
$$n = 1, 2, 3, \ldots.$$
$$or\ n = 0 => \ \sigma_1 = \sigma_2$$

When a model is viewed in monochromatic light the fringe pattern appears as a series of dark bands. However with á white light the isochromatic fringe pattern is a series of Colored bands. A black fringe appeors only when

$$\sigma_1 - \sigma_2 = 0$$

with non-zero value of

$$\sigma_1 - \sigma_2$$

only one wave length at c - time can get extinct, resulting in a complementary color to that have longth instead of dark band eg. when

$$\sigma_1 - \sigma_2$$

produces the extinction of green the complementary color red appears in the isochromatic fringe pattern. At higher levels of principal stress difference several wave length can be extinguished Simultaneously, therefore the fringes become pole and very difficult to analyze.
By using circulor poleriscope isoclinic fringes con be removed leaving behind only isochromatic fringes.

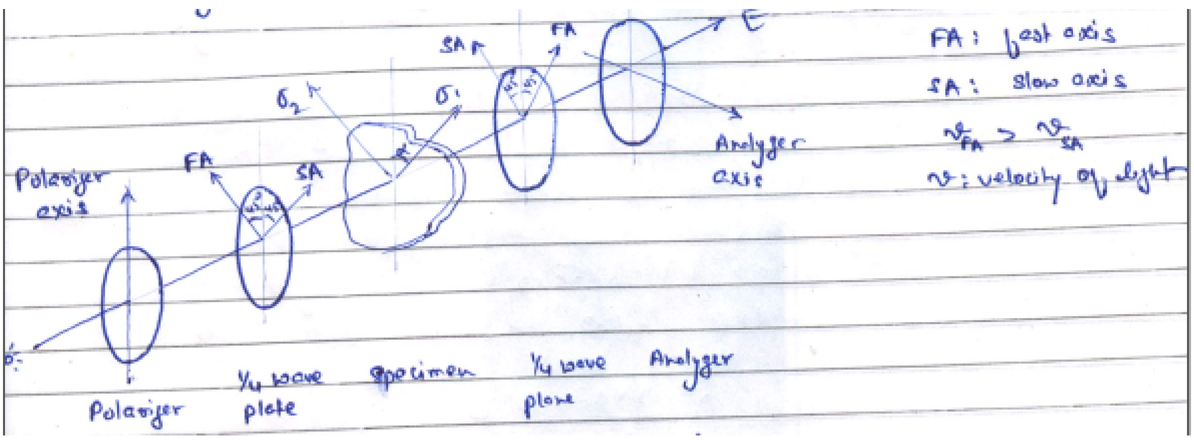


Image 5.

$$E = K sin\frac{\Delta}{2}\ sin(\omega t + 2\alpha - \frac{\Delta}{2})$$

$$intensity\, of\ light\ I = K^2 sin^2\frac{\Delta}{2}$$

Calibration Method

To determine the accurate stress distribution calibraic experiment is performed to get the fringe constant.

In a circular disk subjected to diameter compresion the stress distribution along horizontal axis (y=0) is given by

$$\sigma_{xx} = \sigma_1 = \frac{2P}{\pi hD}(\frac{D^2 - 4X^2}{D^2 + 4X^2})^2$$

$$\sigma_{yy} = \sigma_2 = -\frac{2P}{\pi hD}(\frac{4D^4}{D^2 + 4X^2} - 1)$$

$$\tau_{xy} = 0$$

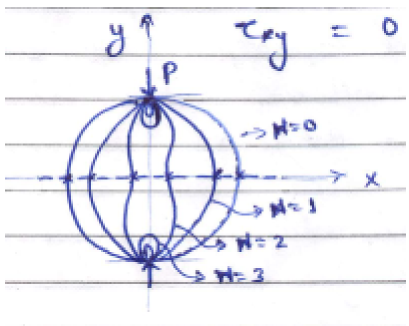


Image 6.

$$\sigma_1 - \sigma_2 = \frac{8P}{\pi hD}[\frac{D^4 - 4D^2X^2}{(D^2 + 4X^2)^2}] = \frac{Nf_\sigma}{h}$$

$$f_\sigma = \frac{8P}{\pi ND}[\frac{D^4 - 4D^2X^2}{(D^2 + 4X^2)^2}]$$

Calculate fsigma for various N (fringe order) at a given load P and average out the value

or at x=0, y=0 (center point)

$$f_\sigma = \frac{8P}{\pi ND}$$

fringe order at the center of the disk can be plotted with p to get fsigma.

3. Equipments'

1. Monochromatic light

Image 7. monochromatic light source

2. Beam Expander/Collimator and lenses

Image 8. Beam Expander *Image 9. collimator*

3. Polarizers

A polarizer is an element that converts randomly polarized light into plane-polarized light.

image 10. Polarizers

4. Quarter Wave Plates

A quarter-wave plate (Fig. 12) is a permanent wave plate that induces a phase shift δ equal to λ / 4, where λ is the wavelength of the light being used.

image 11. quarter wave plates

5. Loading fixtures

6. DSLR Camera/White Sheets to view the fringes

image 12. DSLR camera

image 13. white sheets

7. Specimen made of Epoxy (see image 14.)

4. Measurement

- **Epoxy Beam**

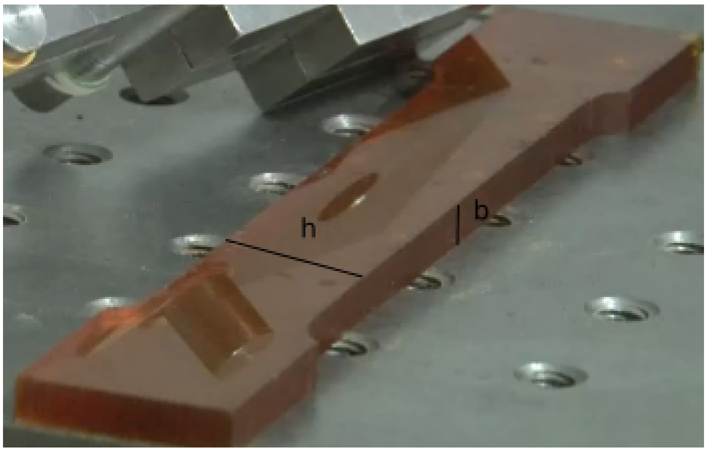
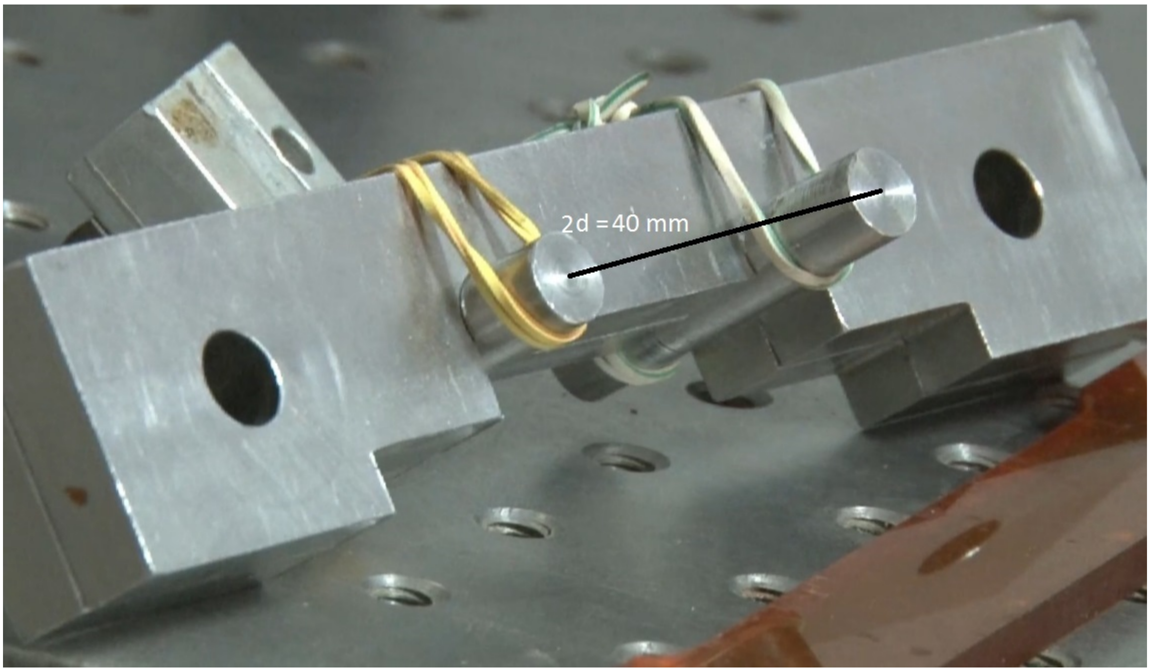


Image 14. Dog-Bone shape Epoxy Specimen

h = 19 mm

b = 5 mm

- **4 Point Beam Bending Setup**



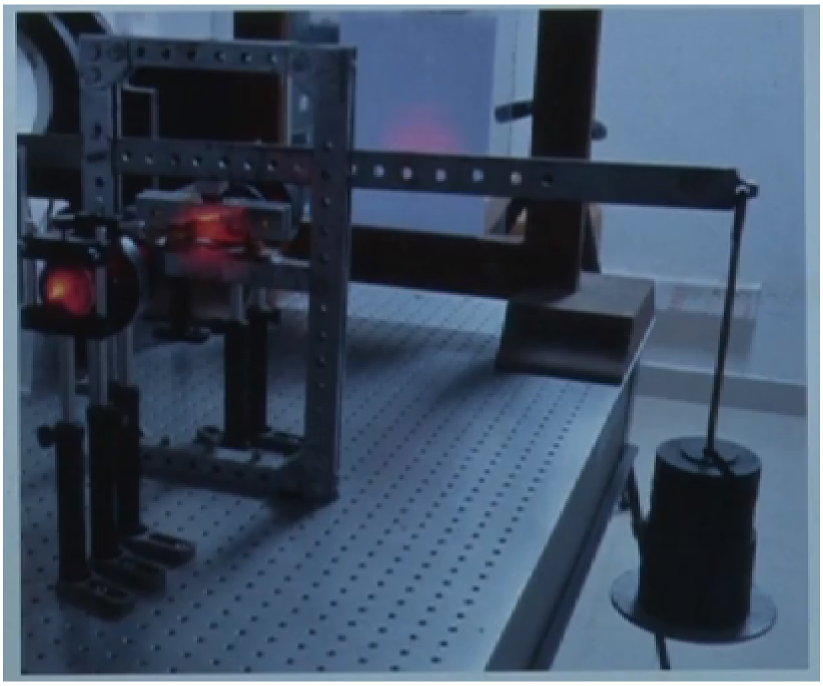


Image 15 and 16. 4 Point Bending Setup

S.N	Setup Parts Name	Mass and Dimension
1.	**Weight distribution of the rod**	1.150 Kg
2.	**Mass of the pan (Po)**	0.700 Kg
3.	**Load applied to the pan (P)**	2 Kg
4.	**P point load**	0.043 Kg
5.	**P 4pb**	0.523 Kg
6.	**Total length of the rod (L)**	745 mm
7.	**length before the simple support point (L1)**	175 mm
8.	**length from simple support to pin point (L2)**	120 mm
9.	**length from pin point to force applied on pan (L3)**	450 mm
10.	**distance from the center of symmetry (d)**	20 mm

Table 1. 4 Point Bending Setup Specification

5. Procedure

- The dimensions of the specimen needed for beam stress calculations (e.g. length of beam, cross-sectional dimensions, precise locations of supports, precise locations of applied concentrated loads, etc.) is to be measured.
- The specimen is to be placed and loaded in symmetric four point bend fixture.
- The source of monochromatic light is to be switched on and aligned in such a way that it falls on the mid-section of the specimen.
- The specimen is to be tested by applying selected loads, and photo elastic fringe patterns are captured by means of DSLR camera (or by drawing). At each fringe location, the **experimental normal stress** value is calculated by using the formula

$$\sigma_x = \frac{N f_\sigma}{b}$$

- The formula

$$\sigma_x(x,y) = \frac{M(x)y}{I}$$

  is to be used to calculate the **theoretical stress** distribution over the cross-sectional area of the
  beam , and compare the results to the photoelastic experimental measurements.

6. Experimental Setup

The figure shown below is a schematic representation of the Photo Elasticity Experiment.

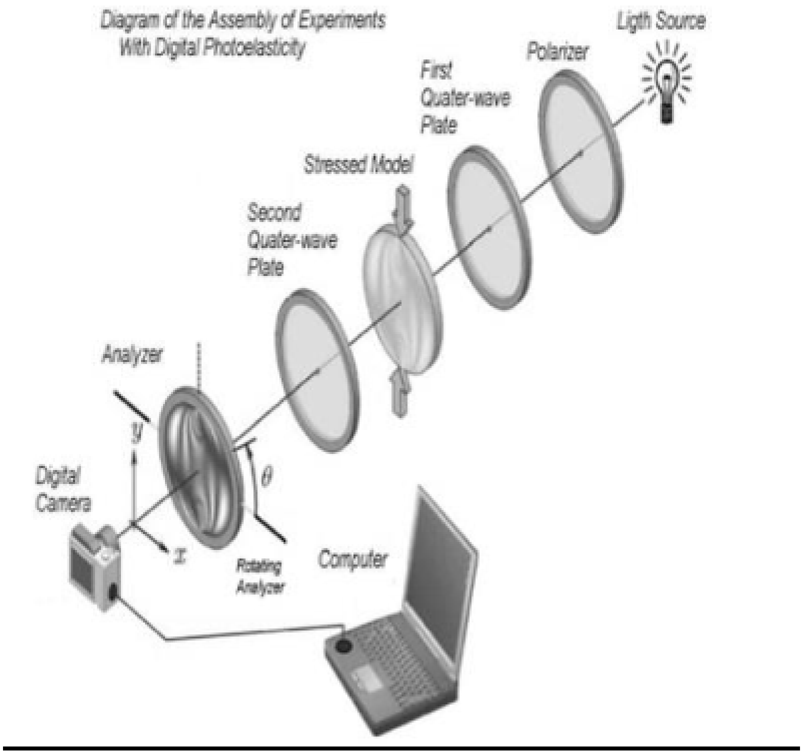


Image 17. Experimental Setup

As shown in the figure below, each two-force member transfers a force of F/2 to the epoxy beam. This results in a constant bending moment in the center region of the beam (between the inner two forces) with a value of

$$M = \frac{Fd}{2}$$

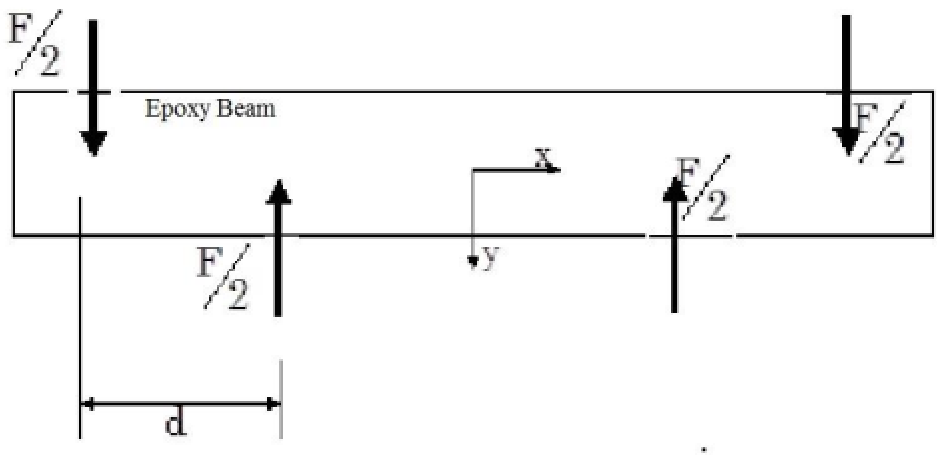


Image 18. Force distribution on the Beam

The theoretical stress distribution on any cross-sectional area in the center region of the beam is given by:

$$\sigma_x(y) = \frac{My}{I} = [\frac{Fd}{2}]y = Cy$$

where C is a constant that can be calculated. Using the recorded photo elastic fringe pattern, experimental values of stress can be calculated at various vertical positions (y). As part of this experiment, a table is to be prepared such as shown below, which lists the measured and predicted stresses at the various photo elastic fringe locations.

7. Calculation

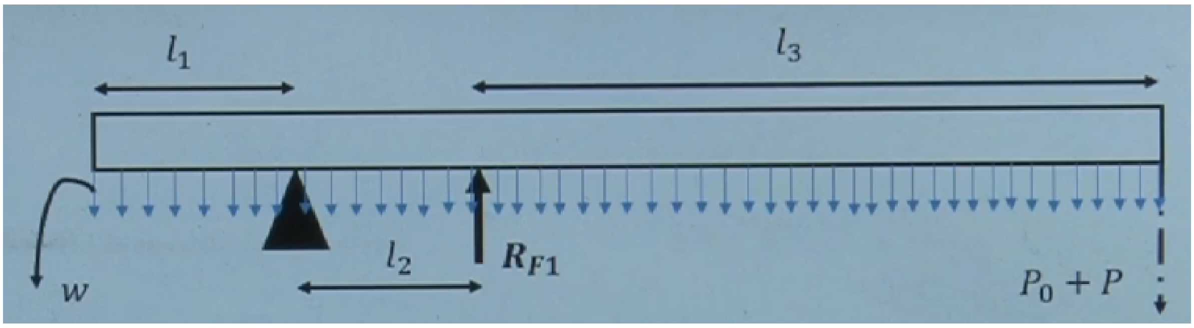


Image 12. Free body Diagram of Beam

beam mass distribution throuhout the beamlength,

$$\begin{aligned} w &= \frac{Mass\ of\ the\ beam}{L} \\ &= \frac{1.150}{745}\frac{Kg}{mm} \\ &= 0.0015\ \frac{Kg}{mm} \end{aligned}$$

Conserving the moment about the simple support,

$$\frac{wL_1^2}{2} - \frac{w(L_2+L_3)^2}{2} + (R_{F1}L_2) - (P_0+P)(L_2+L_3) = 0$$

Solving for the reaction force,

$$R_{F1} = \frac{1}{L_2}[(P_0+P)(L_2+L_3) - \frac{wL_1^2}{2} + \frac{w(L_2+L_3)^2}{2}]$$

$$= \frac{1}{120}[(0.700+2)(120+450) - \frac{0.0015*(175)^2}{2} + \frac{0.0015*(120+450)^2}{2}]\ Kg$$

$$= 14.6624\ Kg$$

Therefore, the resultant load acting on the sample,

$$F = R_{F1} + P_{point\ load} + P_{4PB}$$

$$\begin{aligned} F &= (14.6624 + 0.043 + 0.523)\ Kg \\ &= 15.2303\ Kg \end{aligned}$$

<u>Theoritical stress value</u>

$$\begin{aligned} \sigma_x(x,y) &= \frac{M(x)y}{I} \\ &= \frac{Fd}{2I_{zz}}y \end{aligned}$$

$$Where:\ I_{zz} = \frac{bh^3}{12}$$

$$= \frac{5*19^2}{12}\ mm^4$$

$$= 150.4166\ mm^4$$

From px to mm change

$$Neutral\ axis\ position = 1653\ px$$

$$y:\ i_{th}\ fringe\ position\ in\ mm$$

$$h = 19\ mm:\ specimen\ height\ in\ mm$$

$$p_{top}, p_{bottom}:\ top\ and\ bottom\ fringe\ position\ in\ px$$

$$p_i:\ position\ of\ i_{th}\ fringe$$

$$y = \frac{p_i * h}{|p_{top} - p_{bottom}|}$$

at p_i = 763 px

$$y = \frac{763 * 19}{|593 - 2439|} = 9.1504\ mm$$

$$\sigma_{th} = \frac{15.2303 * 20}{2 * 150.4166} * 9.1504\ \frac{N}{mm^2} = 9.265157\ MPa$$

Experimental stress value

$$\sigma_x = \frac{N f_\sigma}{b}$$

$$Where:\quad N = \ Order\ of\ Fringes$$

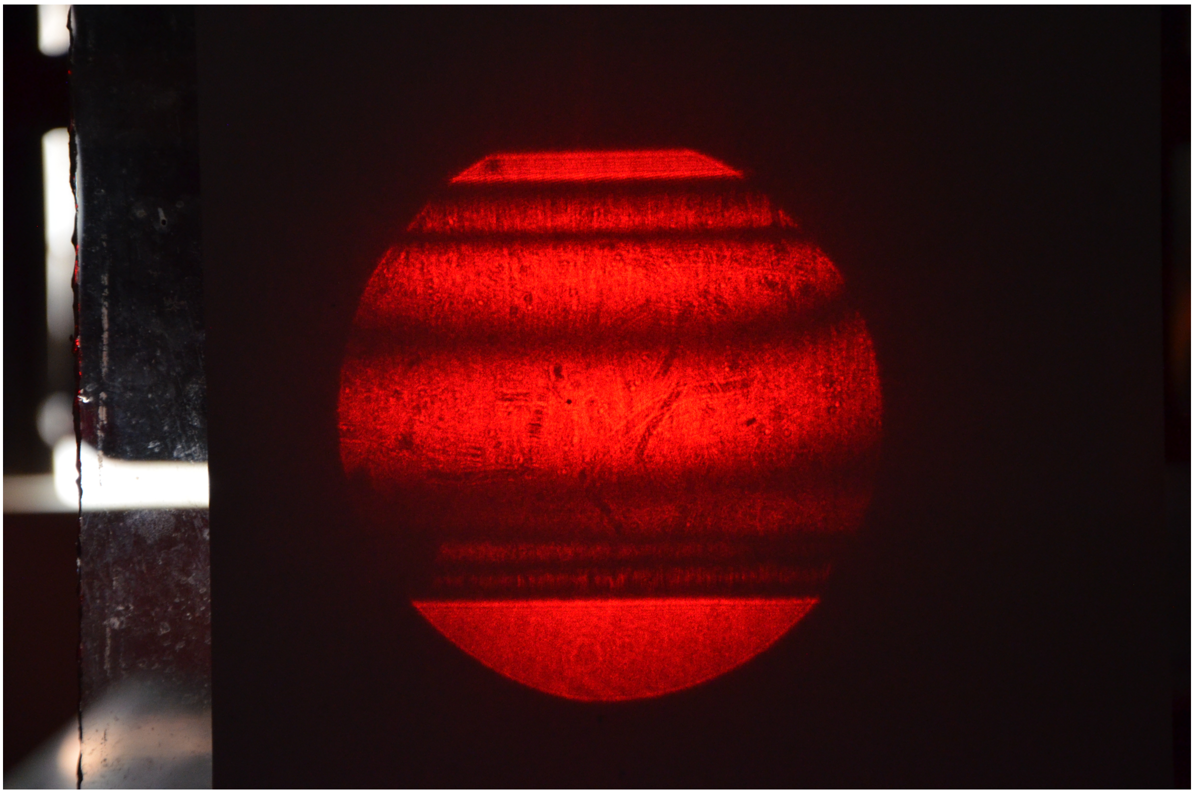
$$f_\sigma = \ Fringe\ Coefficient$$

$$f_\sigma = 12\ \frac{N}{mm}$$

For N = 2

$$\sigma_x = \frac{2 * 12}{5}\ \frac{N}{mm^2} = 4.8\ MPa$$

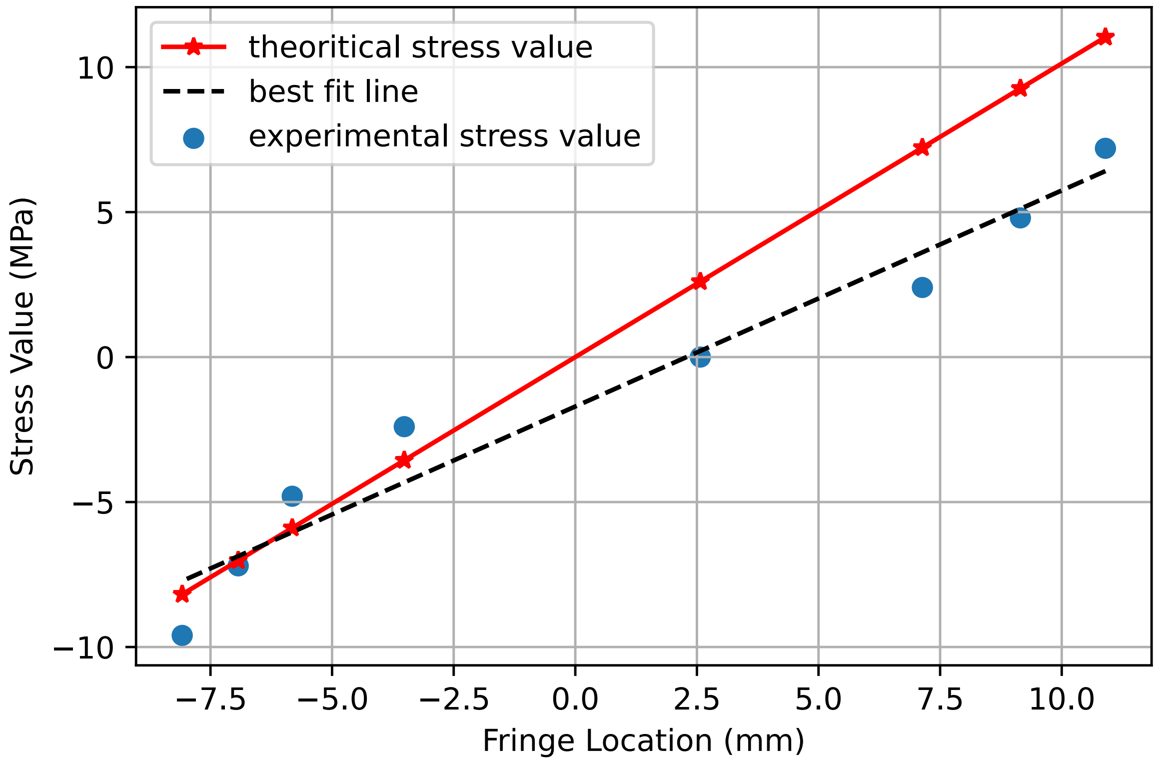
Fringe pattern



Fringe Order	Fringe Location (in px)	Fringe Location (in mm)	Theoritical stress value (MPa)	Experimental stress value (MPa)
3	593	10.8983	11.034977	7.2
2	763	9.1504	9.265157	4.8
1	959	7.1353	7.224785	2.4
0	1403	2.5703	2.602535	0
-1	1995	-3.5162	-3.560297	-2.4
-2	2219	-5.8192	-5.892180	-4.8
-3	2327	-6.9296	-7.016505	-7.2
-4	2439	-8.0811	-8.182446	-9.6

Table 2. Experimental and Theoritical stress values at different fringe location

> **Graph between Stress and Locationof fringes**



Graph 1. stress values at different fringe locations

$$\text{slope} = \frac{\mathbf{M}}{\mathbf{I_{zz}}} = \frac{\mathbf{Fd}}{\mathbf{2I_{zz}}} \frac{N}{mm^3}$$

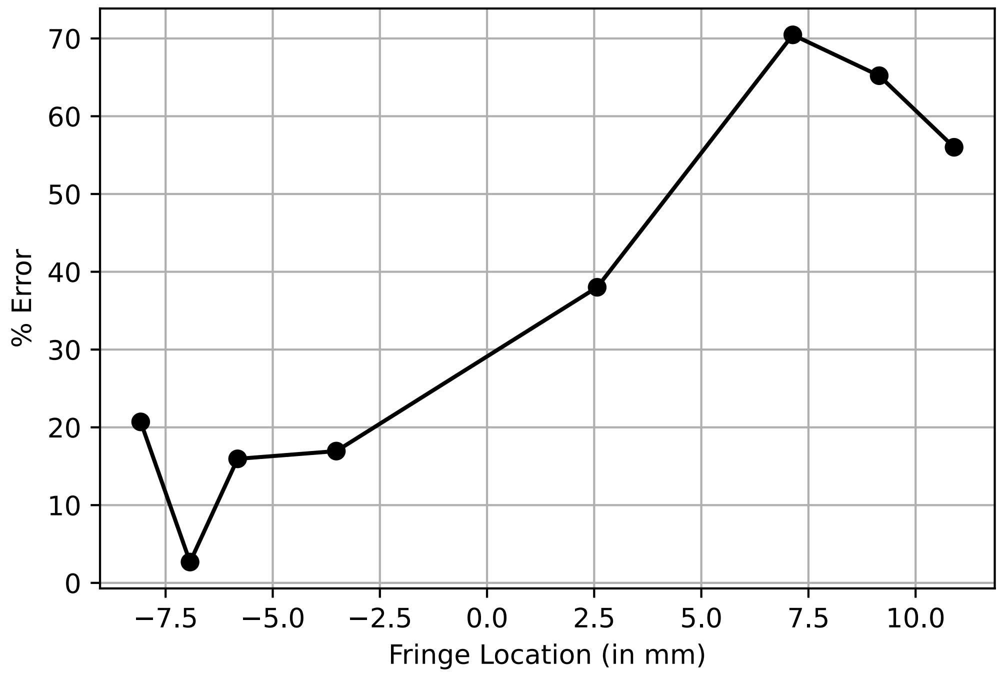
- Slope of *Theoritical stress value* and Location of fringes graph = **1.0125 N/mm3**
- Slope of *Experimental stress value* and Location of fringes graph = **0.745 N/mm3**

8. Error Analysis

	Fringe Order	% error
0	3	56.006655
1	2	65.209899
2	1	70.461971
3	0	38.007852
4	-1	16.945176
5	-2	15.950374
6	-3	2.679788
7	-4	20.702190

Table 3. Error in stress values

> **Error in stress value at different fringe location**



Graph 2. Error in stress values at differnet fringe location

9. Results

> For 2Kg mass :

- Slope of *Theoritical stress value* and Location of fringes graph = **1.0125 N/mm3**
- Slope of *Experimental stress value* and Location of fringes graph = **0.745 N/mm3**
- % mean error = 35.745 %

10. Precautions and Source of Error

> **Precautions**

- Take picture only when load pan come to equllibrium after applied load.
- Be precise during measuring location from neutral axis of a fringe.
- Correct the orientation of quarter wave plates.
- Check the equipments before experiment.

Source of Error

- Measuring fringe distance manually can produce error.
- Error in collimation of beam.
- Load may not vary linearly across the beam.