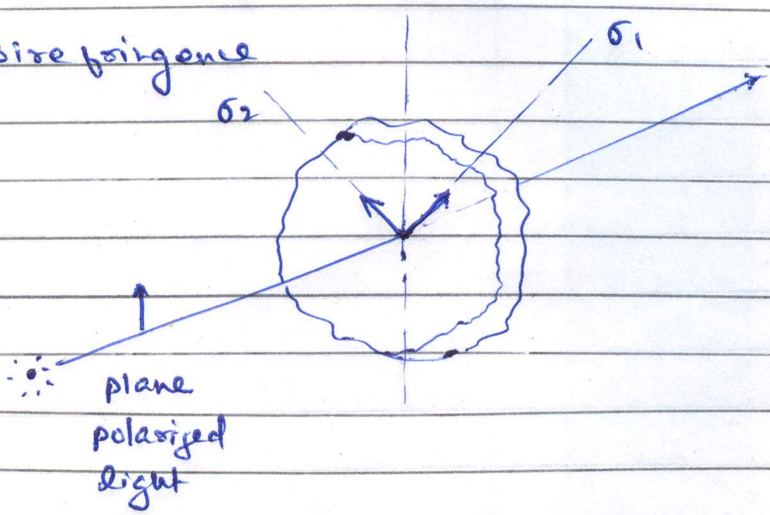
Photo-elasticity

$$\text{Index of refraction} = \frac{\text{speed of light in vacuum}}{\text{speed of light in material}}$$

Many transparent non-crystalline material that are optically isotropic when free of stress becomes optically anisotropic (temporary doubly refractive) when they are subjected to stresses. This birefringent (changing refractive index upon loading) material characteristic is used in the method of photo-elasticity.

Birefringence



When a polarized light passes through a stressed material, the light wave gets resolved into two mutually perpendicular components in the direction of material principal stresses.

The two wavefronts travel at different velocities, subjecting the stressed material having two different indices of refraction (n_1 and n_2). Maxwell noted that the changes in refraction indices are linearly proportional to the loads. For 2-D case the stress-optics (Brewster's) Law

$$n_1 - n_0 = c_1\sigma_1 + c_2\sigma_2 \quad (\ldots + c_2\sigma_3 \text{ in 3D})$$

$$n_2 - n_0 = c_1\sigma_2 + c_2\sigma_1 \quad (\ldots + c_2\sigma_3 \text{ in 3D})$$

σ_1, σ_2 : principal stresses

n_1, n_2 : refraction indices in σ_1 and σ_2 direction

n_0 : refraction index of unstressed material

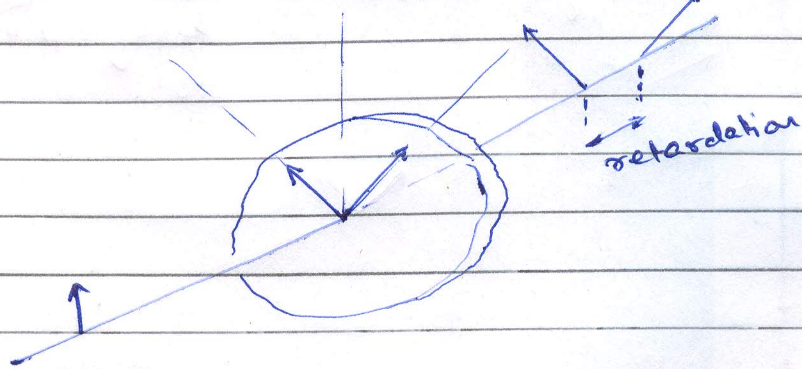
c_1, c_2 : stress-optics constants

Eliminating n_o to get

$$n_2 - n_1 = (C_2 - C_1)(\sigma_1 - \sigma_2) = C(\sigma_1 - \sigma_2)$$

where $C = C_1 - C_2$ is relative stress-optic coefficient, first expressed in terms of brewsters (1 brewster $= 10^{-12}\ m^2/N$)

The two resolved light components travel through the material at different velocities, therefore, they emerge from the plate at different times. In other words one component retards in time compared to the other.



The relative linear phase shift also known as retardation can be computed as

$$\delta = h(n_2 - n_1)$$

where h is material thickness.

or relative angular phase shift

$$\Delta = \frac{2\pi}{\lambda}\delta = \frac{2\pi h}{\lambda}(n_2 - n_1)$$

or $$\sigma_1 - \sigma_2 = \frac{\delta}{hC} = \frac{\lambda}{2\pi h C}\Delta$$

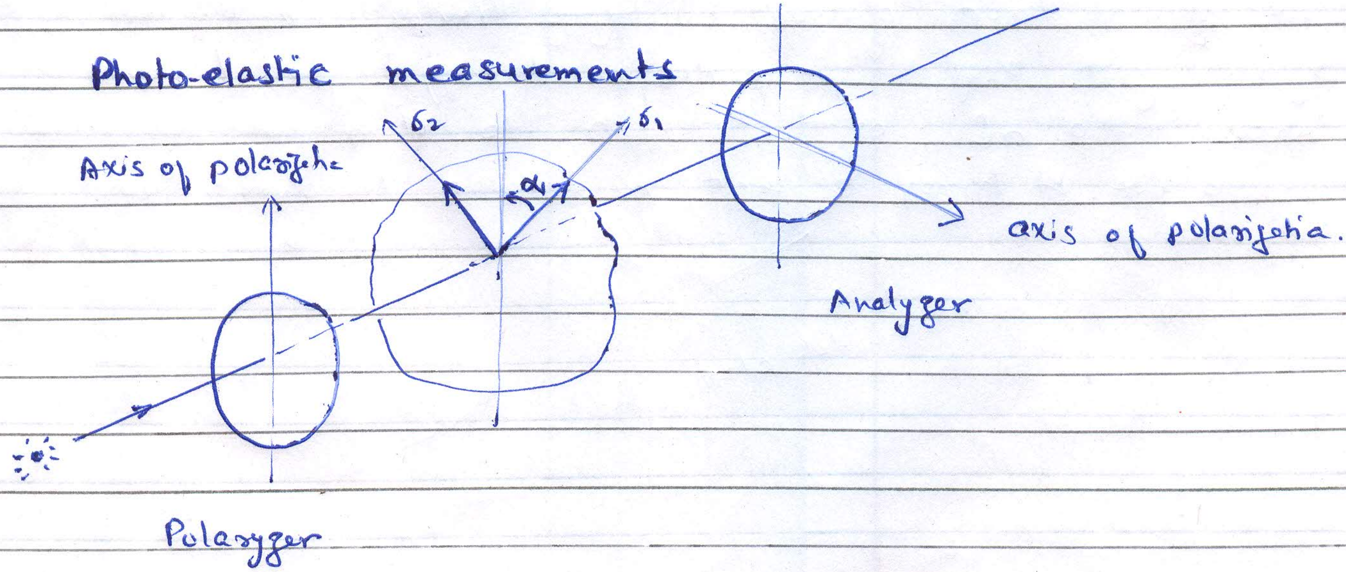
Replacing $\frac{\Delta}{2\pi} = N$ ... fringe order

and $\frac{\lambda}{C} = f_\sigma$ .... fringe constant

to get

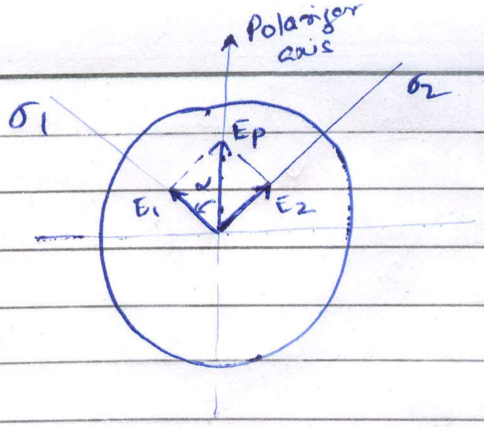
$$\sigma_1 - \sigma_2 = \frac{N f_\sigma}{h}$$

Photo-elastic measurements



Consider a case when plane stressed model is inserted into the field of plane polariscope. Let the principal stress direction (of σ_1) makes an ~~go~~ angle α from the polarization axis.

Plane polarizer resolves an incident light wave into components which vibrate parallel and perpendicular to the axis of the polarizer. The parallel component is transmitted and the component perpendicular to the polarizer axis is absorbed. Initial phase of the wave is not important here, therefore, the plane polarized light beam emerging from the polarizer can be represented as $E_p = K \cos \omega t$

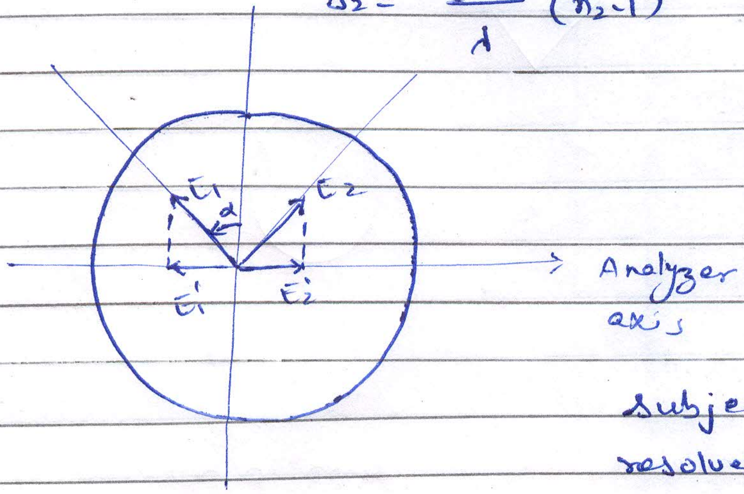


The light wave emerging from the material (stressed) resolved into two components E_1 and E_2.

$$E_1 = K \cos\alpha \cos(\omega t - \Delta_1)$$
$$E_2 = K \sin\alpha \cos(\omega t - \Delta_2)$$

where Δ_1 and Δ_2 are developed phase shifts.

$$\Delta_1 = \frac{2\pi h}{\lambda}(n_1 - 1)$$
$$\Delta_2 = \frac{2\pi h}{\lambda}(n_2 - 1)$$



After leaving the material the light waves E_1 and E_2 propagate without further change and enter the analyzer, subjecting E_1 and E_2 to get resolved further into vertical and horizontal components. Vertical components get absorbed and only the horizonal components in the direction of analyzer axis passes through. Therefore, the emerging light from the analyzer

$$E = E_2' - E_1' = E_2 \cos\alpha - E_1 \sin\alpha$$

or $$E = K \sin 2\alpha \sin \frac{\Delta_2 - \Delta_1}{2} \sin\left(\omega t - \frac{\Delta_2 + \Delta_1}{2}\right)$$

Therefore, light intensity emerging from the analyzer

$$I = \left(K \sin 2\alpha \sin \frac{\Delta_2 - \Delta_1}{2}\right)^2 = K \sin^2 2\alpha \sin^2 \frac{\Delta}{2}$$

$$K = k^2, \quad \Delta = \Delta_1 - \Delta_2$$

or $\Delta = \frac{2\pi h}{\lambda}(n_2 - n_1) = \frac{2\pi h c}{\lambda}(\sigma_1 - \sigma_2)$

Clearly $I = 0$ if $\sin^2 2\alpha = 0$ (function of principal stress direction) or when $\sin^2 \Delta/2 = 0$ (related to principal stress difference).

Effect of principal stress direction

when $2\alpha = n\pi$ ... $n = 0, 1, 2 \ldots$ $I = 0$, ie. light intensity vanishes when one of the principal stress directions coincides with the polarizer axis. When the entire stress field is observed in the polariscope, a fringe pattern is observed. The friges are loci of points where the principal stress directions (σ_1 or σ_2) coincide with the axis of polarizer. The fringe pattern produced by $\sin^2 2\alpha$ term is called isoclinic fringes.

Effect of Principal stress difference.

when $\frac{\Delta}{2} = n\pi$ ... $n = 0, 1, 2 \ldots$ $I = 0$, ie. light intensity vanishes when the principal stress-difference is zero or sufficient to produce an integral number of wavelengths of retardation ($n = 1, 2, 3 \ldots$). The fringes produced by $\sin^2 \frac{\Delta}{2}$ term is called isochromatic fringe pattern.

Consider $\Delta = \frac{2\pi h c}{\lambda}(\sigma_1 - \sigma_2)$

Light intensity extinguishes when $\frac{hc}{\lambda}(\sigma_1 - \sigma_2) = n$

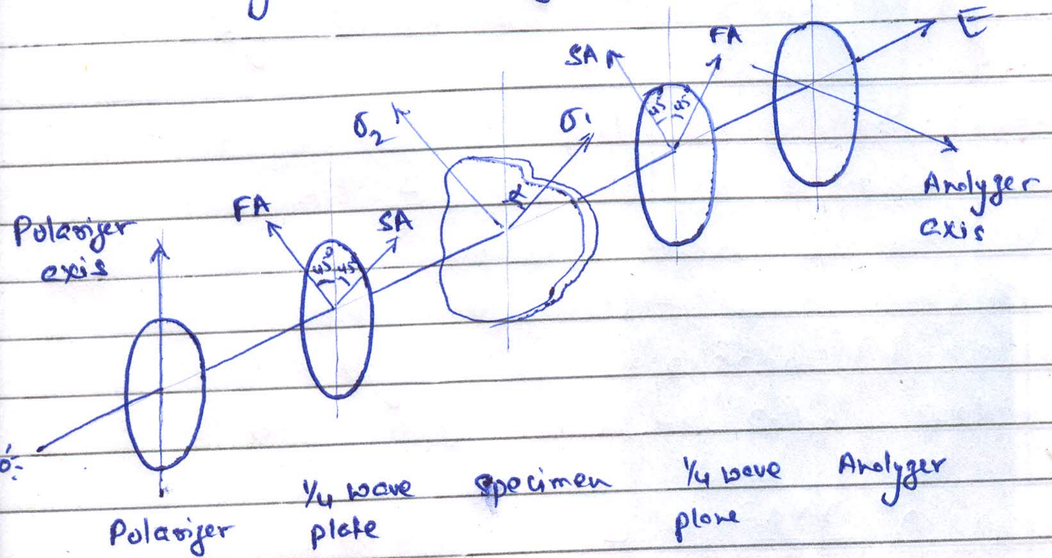
... $n = 1, 2, 3 \ldots$

or $n = 0$

$\sigma_1 = \sigma_2$

When a model is viewed in monochromatic light the fringe pattern appears as a series of dark bands. However with a white light the isochromatic fringe pattern is a series of colored bands. A black fringe appears only when $\sigma_1 - \sigma_2 = 0$. With non-zero value of $\sigma_1 - \sigma_2$, only one wave length at a time can get extinct, resulting in a complementary color to that wave length instead of dark band. e.g. when $\sigma_1 - \sigma_2$ produces the extinction of green, the complementary color red appears in the isochromatic fringe pattern. At higher levels of principal stress difference, several wave lengths can be extinguished simultaneously, therefore the fringes become pale and very difficult to analyze.

By using circular polariscope isoclinic fringes can be removed leaving behind only isochromatic fringes.



FA : fast axis
SA : slow axis
$v_{FA} > v_{SA}$
v : velocity of light

$$E = k \sin \frac{\Delta}{2} \sin\left(\omega t + 2\alpha - \frac{\Delta}{2}\right)$$

Intensity of light $I = K \sin^2 \frac{\Delta}{2}$ ... $K = k^2$

Calibration Method

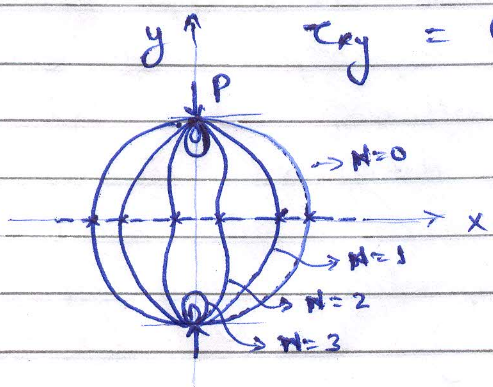
To determine the accurate stress distribution Calibration experiment is performed to get the fringe constant f_σ.

In a circular disk subjected to diametral compression the stress distribution along horizontal axis $(y=0)$ is given by

$$\sigma_{xx} = \sigma_1 = \frac{2P}{\pi h D}\left(\frac{D^2-4x^2}{D^2+4x^2}\right)^2$$

$$\sigma_{yy} = \sigma_2 = -\frac{2P}{\pi h D}\left(\frac{4D^4}{(D^2+4x^2)^2}-1\right)$$

$$\tau_{xy} = 0$$



$$\sigma_1-\sigma_2 = \frac{8P}{\pi h D}\left[\frac{D^4-4D^2x^2}{(D^2+4x^2)^2}\right] = \frac{N f_\sigma}{h}$$

$$\text{or } f_\sigma = \frac{8P}{\pi D N}\left[\frac{D^4-4D^2x^2}{(D^2+4x^2)^2}\right]$$

Calculate f_σ for variant N (fringe order) at a given load P and average out the value.

or at $x=0$, $y=0$ (center point)

$$f_\sigma = \frac{8}{\pi D}\left(\frac{P}{N}\right)$$

Fringe order at the center of the disk can be plotted with P to get f_σ.