AE451 - Experiments in Aerospace Engineering-III.

Laminar and turbulent boundary layers characteristics on a flat plate

Learning objective:

To understand the features of laminar and turbulent boundary layers and characterize it based on its velocity profile, displacement thickness, momentum thickness, using a Pitot tube and a differential manometer.

Proposed Plan:

a) Take a picture of the experimental setup. Note the geometry of the flat plate leading edge, and the smoothness of the surface.

b) Take a picture of the traversing mechanism and the miniature Pitot probe.

c) Make a block diagram of the experimental setup, including the DAQ system.

d) Mount the miniature Pitot tube in the horizontal traversing mechanism. Connect the Pitot tube lead to a digital micro-manometer.

e) At one free stream wind speed at a stream-wise station, x, measure velocity profiles in the laminar and the turbulent boundary layer. The origin of x coordinate is at the flat plate leading edge, and it is increasing in the stream-wise direction. Note that you can have turbulent boundary layer at the same stream-wise station by introducing roughness or a tripping element at the leading edge of the plate.

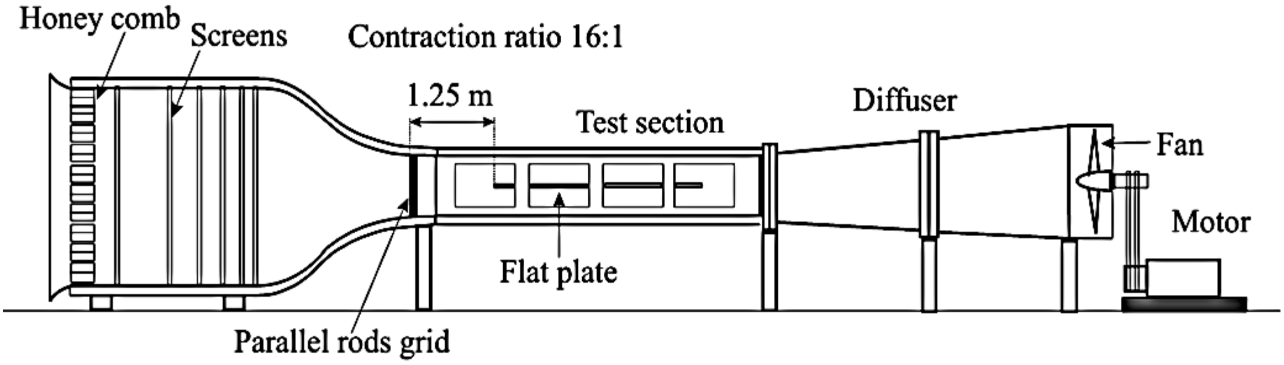
f) Plot velocity profiles and obtain wall shear stress and boundary layer thicknesses. Note the differences between laminar and turbulent boundary layer velocity profiles and understand why they are different.

Equipment's used:

- Low-speed wind tunnel
- Flat plate
- Pitot tube
- Digital differential manometer
- NI data acquisition card.
- Computer

Wind Tunnel specifications:

- Open non-return suction type
- Contraction ratio : 16:1
- Operational speed : 3 to 18 m/s.



- No of screens : 6, square mesh 1mm.
- Honeycomb : square 1ft long.
- Test section : 610 x 610mm square cross section, 10ft long.
- Motor Specs : A.C induction motor, 15hp, 50 Hz, 1445 max rpm.
- Diffuser : Square section of 10ft long.

Flat plate Specifications:

- Leading edge - Asymmetrical Modified super ellipse
- Material - Acrylic-glass
- Geometry - 2 m long, 0.61 m width, 12 mm thick
- Semi major axis, a =120mm; AR_u= 30, AR_l = 15.

Pitot tube:

- Used for measuring the total pressure of the flow.
- Static pressure is measured using a port drilled in the tunnel wall.

Digital differential manometer:

- Range – 20 mm of H_2O (= 200 Pascal which is = 5V).
- Furness FC012 digital manometer.

Data Acquisition System:

- PXI – 1073 chassis.
- NI-PXI 6251, 16 bit data acquisition card.
- BNC 2120 signal I/O board

Questions:

1) What do you understand by the probe displacement effect?
2) How can you obtain local shear stress from the velocity profile data?
3) What are displacement thickness and momentum thickness? How are they related to one another in the case of laminar and turbulent boundary layers?

4) Maximum resolution of the DAQ system?

5) Why does a laminar boundary layer become turbulent?

6) What do you understand by the transition and intermittency in a flat plate boundary layer? What is the critical Reynolds number for transition over a flat plate?

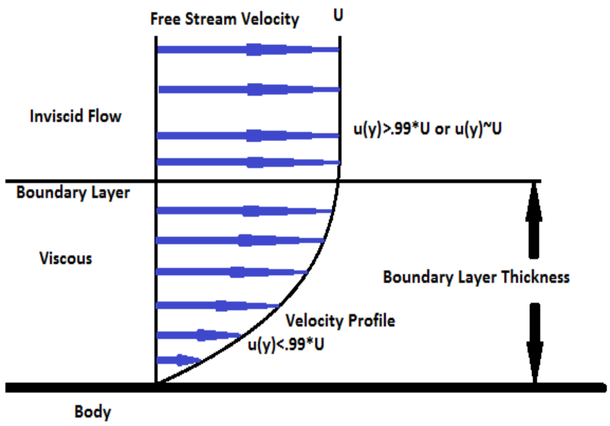
References:

1. Boundary layer theory - Schlichting
2. Turbulence - Hinze
3. A first course in turbulence - Tennekes & Lumley
4. Viscous fluid flow - F.M. White
5. Fluid mechanics - Kundu & Cohen
6. Fluid mechanics - F.M. White

Pre-requisite:

Boundary layer:

A thin layer close to the solid body where the fluid velocity increases from zero at the wall (due to no-slip condition for viscous fluids) to its maximum value at the free stream (U_∞) is called the boundary layer. The variations (i.e. velocity gradients) will be confined within this thin layer close to the wall as long as the Reynolds number is large enough or the fluid viscosity is small enough.



Features of boundary layer:

- Strong velocity gradient in the wall-normal direction exists hence shear stresses are always present though the viscosity is small, because

$$\tau = \mu\frac{\partial u}{\partial y}, \quad \text{where } \frac{\partial u}{\partial y} \text{ is the velocity gradient}$$

- $\delta \uparrow$ as - $U_\infty \downarrow$, $\nu \uparrow$, $x \uparrow$, $Re \downarrow$

  Where δ is the BL thickness, U_∞ is the free-stream velocity, ν is the kinematic viscosity, 'X' is the characteristic length and Re is the Reynolds number.

Assumptions:

- Flow is steady, incompressible and the body force act on the fluid element is neglected.
- The fluid is assumed to be a Newtonian fluid, and the value of $Re >> 1$.
- Boundary thickness is very small compared with the characteristic length of the flow $\delta << L$.

- transverse velocity**, v** << longitudinal velocity**, u.**

$$\frac{\partial \mathrm{u}}{\partial \mathrm{x}} \ll \frac{\partial \mathrm{u}}{\partial \mathrm{y}} \;,\quad \frac{\partial \mathrm{v}}{\partial \mathrm{x}} \ll \frac{\partial \mathrm{v}}{\partial \mathrm{y}}$$

Boundary layer equations:

Navier-Stokes equation completely describes the motion of a fluid element subjected to body and surface forces.

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

$$\rho \left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}\right) = X - \frac{\partial P}{\partial x} + \mu \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right)$$

$$\rho \left(\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z}\right) = Y - \frac{\partial P}{\partial y} + \mu \left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2}\right)$$

$$\rho \left(\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z}\right) = Z - \frac{\partial P}{\partial z} + \mu \left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2}\right)$$

Where, X, Y, Z are the body force acts on the fluid element in x, y, z directions, respectively. For the boundary layer flow, the NS equation is simplified to the following form by Prandtl using the above-mentioned assumptions.

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial P}{\partial x} + \gamma \frac{\partial^2 u}{\partial y^2}$$

This equation can be applied as such for a laminar flow but for a turbulent flow these instantaneous velocities have to be decomposed into mean and a fluctuating part and must be averaged over a long period of time. Doing so, we get the Reynolds Averaged Navier Stokes (RANS) equation shown below.

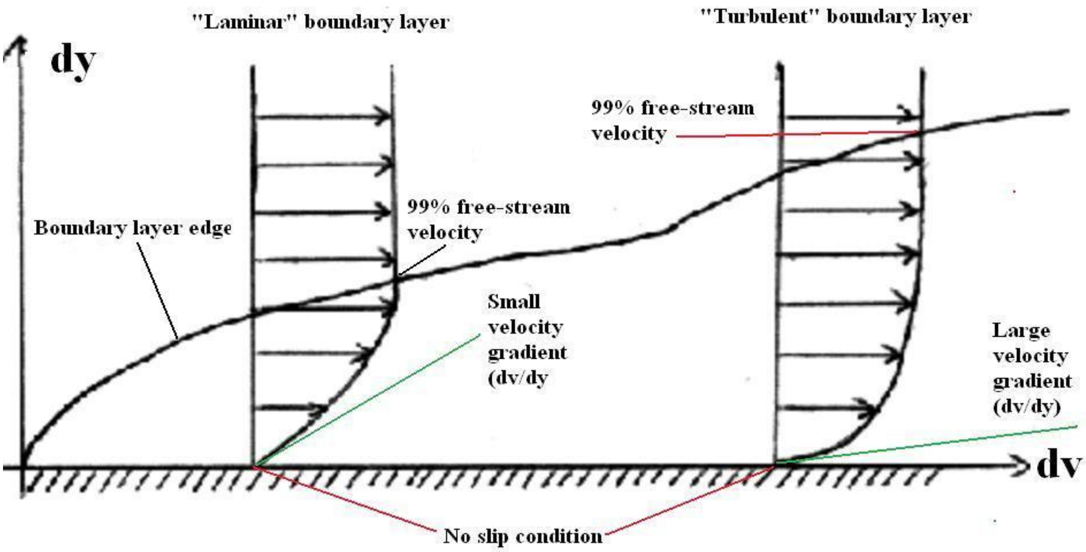
$$\frac{\partial \bar{u}}{\partial x} + \frac{\partial \bar{v}}{\partial y} = 0$$

$$\bar{u}\frac{\partial \bar{u}}{\partial x} + \bar{v}\frac{\partial \bar{u}}{\partial y} = -\frac{1}{\rho}\frac{\partial \bar{P}}{\partial x} + \frac{\partial}{\partial y}\left\{\gamma \frac{\partial \bar{u}}{\partial y} - \overline{u'v'}\right\}$$

For a proper derivation of these equations, one may refer to ‘Boundary Layer Theory’ by “Schlichting” and ‘Viscous fluid flow’ by “F. M. White”.

Laminar and turbulent boundary layer:

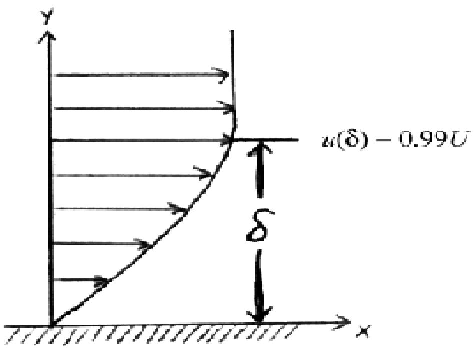
- A boundary layer can be laminar or turbulent depending on whether the flow inside the boundary layer is laminar or turbulent which in turn depends on the flow Reynolds number.
- Once the flow becomes turbulent, many changes happen in the flow. Most of the properties like skin friction, heat transfer rate gets changed which in turn affects the drag experienced by a body, the amount of heat transfer from a hot body to a cold stream, the efficiency of turbine blades, etc.
- Hence, a proper understanding of laminar and turbulent boundary layers is required, especially for aerospace engineers.



PARAMETERS:

<u>Boundary layer thickness (δ):</u>

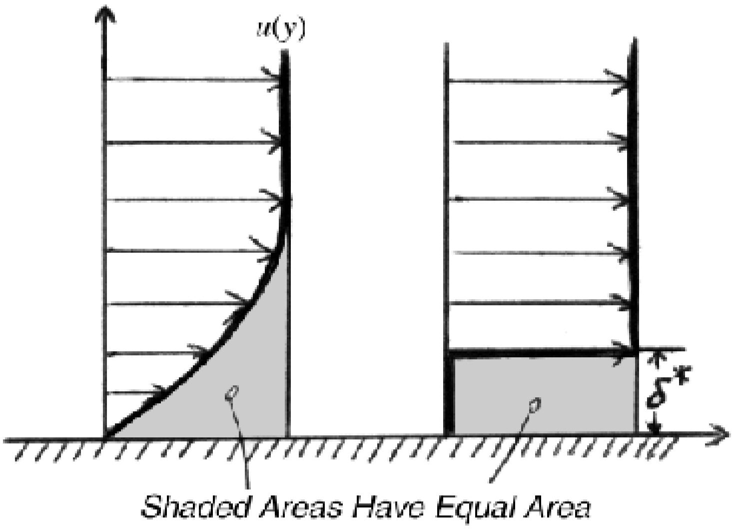
The distance from the wall where the mean velocity reaches 99% of free stream velocity.



$$\delta = y|_{@\ u=0.99*U_\infty}$$

Displacement thickness (δ*):

The displacement thickness is defined as the distance by which the wall would have to be displaced outward in a hypothetical frictionless flow so as to maintain the same mass flux as in the actual flow.



$$\boxed{\delta^* = \int_0^\infty \left(1 - \frac{u}{U_\infty}\right) dy}$$

In other words, the distance by which the streamlines of the external flow are displaced due to the presence of the body.

Momentum thickness (θ):

Distance by which the streamlines have to be shifted in a potential flow so that it produces the same momentum loss produced by the presence of a boundary layer.

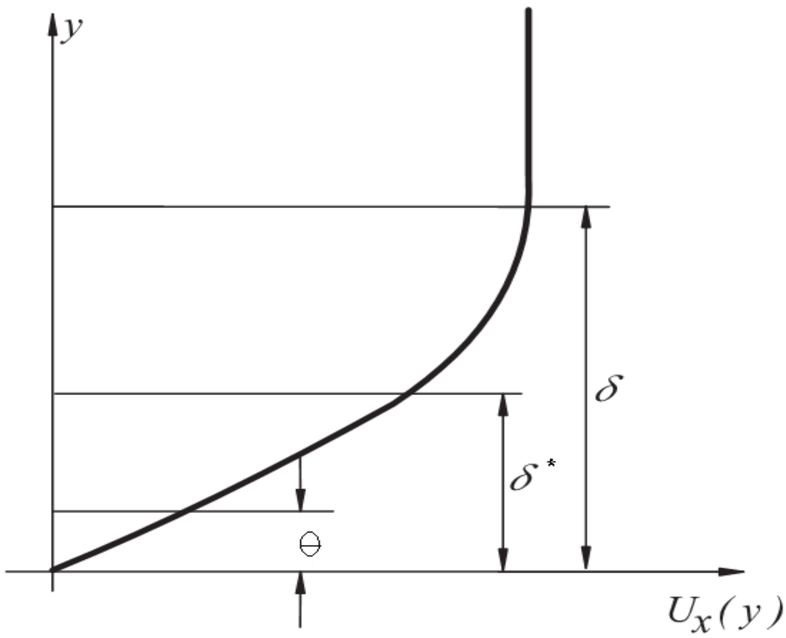
$$\boxed{\theta = \int_0^\infty \frac{u}{U_\infty}\left(1 - \frac{u}{U_\infty}\right) dy}$$

Shape factor (H):

The ratio of displacement to momentum thickness, $H = \frac{\delta^*}{\theta}$, a parameter, which quantifies the relative importance of the mass and momentum diffusion.

All these parameters are common for the both laminar and turbulent boundary layer, but their values are different depending on the wall-normal velocity profile.

Relative dimensions of these parameters:



How to calculate the boundary layer parameters experimentally?

- Obtain the velocity profile using the Pitot tube.
- Find the boundary layer thickness by using the formula, $\delta = y|_{@\ u=0.99*U_\infty}$
- Plot the profiles of $1 - \frac{u}{U_\infty}$ and $\frac{u}{U_\infty}\left\{1 - \frac{u}{U_\infty}\right\}$.
- Find the area under these two profiles. This gives displacement, δ* and momentum thickness, θ.
- These steps are common for both laminar and turbulent boundary layer.
- The calculation of skin friction coefficient (C_f) for the turbulent velocity profile differs from the laminar velocity profile. The calculation procedure for the laminar and the turbulent velocity profile is given below

LAMINAR:

- The skin friction coefficient C_f can be calculated based on the formula $C_f = \frac{\theta}{x}$.
- x – The distance between the leading edge and the measurement location.

TURBULENT:

- Fit the linear curve between $\frac{\bar{u}}{U_\infty}$ and ln($\frac{yU_\infty}{\gamma}$).

  Where, $\bar{u}$ – measured velocity at a point, U_∞ - Free-stream velocity, y- The location at which the velocity ($\bar{u}$) measured.
- The slope of the curve gives the value of C in the equation given below. From C, we can find C_f.

$$\frac{\bar{u}}{U_\infty} = C * \ln\left(\frac{yU_\infty}{\vartheta}\right) + D$$

Where, $C = \frac{1}{K}\sqrt{\frac{C_f}{2}}$ $and\ D = \sqrt{\frac{C_f}{2}}\left\{\frac{1}{K} ln\left\{\sqrt{\frac{C_f}{2}}\right\} + B\right\}$

With k=0.41 and B=5.0

THEORETICAL FORMULAS:

Parameters	Type of profile	
	Laminar (Blasius solution)	Turbulent (Prandtl approximation)
δ	$\frac{5x}{\sqrt{Re_x}}$	$\frac{0.16x}{(Re_x)^{1/7}}$
δ*	$\frac{1.72x}{\sqrt{Re_x}}$	$\frac{0.02x}{(Re_x)^{1/7}}$
θ	$\frac{0.664x}{\sqrt{Re_x}}$	$\frac{0.016x}{(Re_x)^{1/7}}$
C_f	$\frac{0.664}{\sqrt{Re_x}}$	$\frac{0.027x}{(Re_x)^{1/7}}$