



## STATISTICS, LAPLACE TRANSFORM AND NUMERICAL METHODS (MA231TB)

### UNIT 1: STATISTICS

#### TUTORIAL SHEETS – 1

##### Objective type Questions:

1. Formula for calculating moment about the origin is \_\_\_\_\_.
2. The relation between second order moment about mean and moments about any arbitrary point is \_\_\_\_\_.
3. For a distribution the mean is 12 and the variance is 19. Then the second moment about the mean is \_\_\_\_\_.
4. The first four moments of a distribution about the value 4 of the variables are -1.5, 17, -30 and 108. The third moment about the mean is \_\_\_\_\_.

##### Descriptive Questions:

1. From the following frequency distribution compute first four moments about the mean and also find the measures  $\beta_1$  and  $\beta_2$ .

|   |   |   |    |    |    |
|---|---|---|----|----|----|
| x | 4 | 8 | 12 | 16 | 20 |
| f | 2 | 2 | 1  | 4  | 1  |

2. The first four moments of distribution about the value 5 of the variable are 2, 20, 40 and 50. Calculate mean,  $\mu_2$ ,  $\mu_3$ ,  $\mu_4$ ,  $\beta_1$  and  $\beta_2$ .
3. From the following frequency distribution compute first four moments about the mean and also find the measures  $\beta_1$  and  $\beta_2$ .

|   |   |   |    |    |    |    |
|---|---|---|----|----|----|----|
| x | 1 | 2 | 3  | 4  | 5  | 6  |
| f | 1 | 6 | 13 | 25 | 30 | 22 |

4. The distance (in km) of 60 engineers from their residence to their place of work were found as follows:

|                  |     |      |       |       |       |       |
|------------------|-----|------|-------|-------|-------|-------|
| Distance (in km) | 0-5 | 5-10 | 10-15 | 15-20 | 20-25 | 25-30 |
| No. of Engineers | 8   | 11   | 15    | 12    | 9     | 5     |



## TUTORIAL SHEETS – 2

### Objective type Questions:

1. The normal equations to fit a straight line of the form  $P = aW + b$  is \_\_\_\_.
2. For a statistical data with  $n = 5$ ,  $\sum x = 30$ ,  $\sum y = 40$ ,  $\sum x^2 = 220$  and  $\sum xy = 266$ , the straight line of best fit by the method of least squares is \_\_\_\_.
3. In a tensile test of a metal bar, the following observations were made, where  $x$  is the load and  $y$  is the elongation and the following pairs of values of  $x$  and  $y$  are obtained: (1, 14), (2, 27) and (5, 68). Using the method of least squares, find a linear law connecting  $y = a + bx$ . The value of  $y$  when  $x = 7$  is \_\_\_\_\_.

### Descriptive Questions:

1. A simply supported beam carries a concentrated load  $P$  at its mid-point. Corresponding to various values of  $P$  the maximum deflection  $Y$  is measured and is given in the following table.

|     |      |      |      |      |      |      |
|-----|------|------|------|------|------|------|
| $P$ | 100  | 120  | 140  | 160  | 180  | 200  |
| $Y$ | 0.45 | 0.55 | 0.60 | 0.70 | 0.80 | 0.85 |

Fit a law of the form  $Y = a + bP$  and hence estimate  $Y$  when  $P$  is 150.

2. Fit a second-degree parabola  $y = ax^2 + bx + c$  in the least square sense for the following data and hence estimate  $y$  at  $x = 6$ .

|     |    |    |    |    |    |
|-----|----|----|----|----|----|
| $x$ | 1  | 2  | 3  | 4  | 5  |
| $y$ | 10 | 12 | 13 | 16 | 19 |

3. The following table gives the results of the measurements of train resistances;  $V$  is the velocity in miles per hour,  $R$  is the resistance in pounds per ton. By the method of least squares find the best values of  $a$ ,  $b$ ,  $c$  for the equation  $R = a + bV + cV^2$  which fits the data

|     |     |     |      |      |      |     |
|-----|-----|-----|------|------|------|-----|
| $V$ | 20  | 40  | 60   | 80   | 100  | 120 |
| $R$ | 5.5 | 9.1 | 14.9 | 22.8 | 33.3 | 46  |

Represent the required values in tabular form and also extrapolate the value of  $R$  when  $V = 125$ .



## TUTORIAL SHEETS – 3

### **Objective type Questions:**

1. The equations of regression lines are  $y = 0.5x + a$  and  $x = 0.4y + b$ . The correlation coefficient is \_\_\_\_\_.
2. Regression coefficient of  $y$  on  $x$  is 0.7 and that of  $x$  on  $y$  3.2. Is the correlation coefficient  $r$  consistent?
3. The following data regarding the heights ( $y$ ) and the weights ( $x$ ) of twelve college students are given  $\sigma_x = 16.8$ ,  $\sigma_y = 10.8$ ,  $\sum(x - \bar{x})(y - \bar{y}) = 2020$ . For the given data the regression coefficient of  $x$  on  $y$  is \_\_\_\_\_.
4. If  $y = x + 1$  and  $x = 3y - 7$  are the two lines of regression then  $\bar{x} =$  \_\_\_\_\_ and  $\bar{y} =$  \_\_\_\_\_ and  $r =$  \_\_\_\_\_.

### **Descriptive Questions:**

1. Obtain the lines of regression and hence find the coefficient of correlation for the data

|     |   |   |    |    |    |    |    |
|-----|---|---|----|----|----|----|----|
| $x$ | 1 | 2 | 3  | 4  | 5  | 6  | 7  |
| $y$ | 9 | 8 | 10 | 12 | 11 | 13 | 14 |

2. The following data gives the age of husband ( $x$ ) and the age of wife ( $y$ ) in years. Form the two regression lines and calculate the age of husband corresponding to 16 years age of wife

|     |    |    |    |    |    |    |    |    |    |    |
|-----|----|----|----|----|----|----|----|----|----|----|
| $x$ | 36 | 23 | 27 | 28 | 28 | 29 | 30 | 31 | 33 | 35 |
| $y$ | 29 | 18 | 20 | 22 | 27 | 21 | 29 | 27 | 29 | 28 |

3. The following results were obtained from records of correlation data of age ( $x$ ) and blood pressure ( $y$ ): variance of  $y$  is 16, equation of lines of regression of  $y$  on  $x$  is  $5y = 8x - 17$  and  $x$  on  $y$  is  $5x = 2y + 14$ . Determine
  - (a) the mean values of  $x$  and  $y$
  - (b) coefficient of correlation between  $x$  and  $y$
  - (c) standard deviation of  $x$ .
4. A computer operator, while calculating correlation coefficient between two variables  $x$  and  $y$  from 20 pairs of observations obtained the following results.
$$n = 20, \sum x = 100, \sum y = 120, \sum xy = 700, \sum x^2 = 640, \sum y^2 = 860$$



On further verification it was found that two pairs of values were entered wrongly. They were copied as (8,10) and (4,6) while the correct values were (10,8) and (14,16) respectively.

5. Find the rank correlation for the following data

$$X: 14 \ 17 \ 28 \ 17 \ 16 \ 13 \ 24 \ 25 \ 18 \ 31$$

$$Y: 0.9 \ 1.1 \ 1.6 \ 1.3 \ 1.0 \ 0.8 \ 1.5 \ 1.4 \ 1.2 \ 2.0$$

6. Find the rank correlation for the following data

**Two judges gave the following ranks to 11 girls in a beauty contest:**

| Girl    | 1 | 2 | 3 | 4 | 5 | 6  | 7  | 8  | 9  | 10 | 11 |
|---------|---|---|---|---|---|----|----|----|----|----|----|
| Judge A | 3 | 4 | 1 | 2 | 5 | 10 | 11 | 7  | 9  | 8  | 6  |
| Judge B | 2 | 4 | 3 | 1 | 7 | 9  | 6  | 11 | 10 | 5  | 8  |

7. Find the rank correlation for the following data

**Rank by Judge**

$$A: 1 \ 6 \ 5 \ 10 \ 3 \ 2 \ 4 \ 9 \ 7 \ 8$$

$$B: 3 \ 5 \ 8 \ 4 \ 7 \ 10 \ 2 \ 1 \ 6 \ 9$$

$$C: 6 \ 4 \ 9 \ 8 \ 1 \ 2 \ 3 \ 10 \ 5 \ 7$$

8. Determine the equation of the regression plane connecting  $x_1$ ,  $x_2$  and  $y$  estimate  $y$  at  $x_1 = 1.8$ ,  $x_2 = 112$

|                               |     |     |     |     |      |     |     |     |     |      |
|-------------------------------|-----|-----|-----|-----|------|-----|-----|-----|-----|------|
| Diffusion time (hrs)<br>$x_1$ | 1.5 | 2.5 | 0.5 | 1.2 | 2.6  | 0.3 | 2.4 | 2   | 0.7 | 1.6  |
| Sheet-resistance<br>$x_2$     | 66  | 87  | 69  | 141 | 93   | 105 | 111 | 78  | 66  | 123  |
| Current gain<br>$y$           | 5.3 | 7.8 | 7.4 | 9.8 | 10.8 | 9.1 | 8.1 | 7.2 | 6.5 | 12.6 |

