Kinematics of flow ~~and~~

Kinematics is defined as that branch of science which deals with motion of particles without considering the forces causing the motion.

Types of fluid flow

The fluid flow is classified as

(i) Steady and unsteady flows
(ii) Uniform and non-uniform flows
(iii) Laminar and turbulent flows
(iv) compressible and Incompressible flows
(v) Rotational and irrotational flows
vi) One, two and three - dimensional flows.

Steady and unsteady flows

Steady flow is defined as that type of flow in which the fluid characteristics like Velocity, pressure, density etc at a point donot change with time.

mathematically, we have

$$\left(\frac{\partial v}{\partial t}\right)_{x_0, y_0 z_0} = 0$$

$$\left(\frac{\partial p}{\partial t}\right)_{x_0 y_0 z_0} = 0$$

where $x_0\ y_0\ z_0$ is a fixed point in fluid field.

<u>Unsteady flow</u> :- It is that type of fl[o]w in which the velocity, pressure, density at point changes with respect to time.

Mathamatically, for unsteady flow

$$\left(\frac{\partial v}{\partial t}\right)_{x_0 y_0 z_0} \neq 0 , \quad \left(\frac{\partial p}{\partial t}\right)_{x_0 y_0 z_0} \neq 0$$

$$\left(\frac{\partial \rho}{\partial t}\right) \neq 0$$

(ii) Uniform and Non-Uniform flows

Uniform flow is defined as that type of flow in which the velocity at any given time does not change with respect to space (ie length of direction of flow).

Mathematically, for Uniform flow

$$\left(\frac{\partial V}{\partial s}\right)_{t=\text{constant}} = 0$$

where ∂V = change in velocity

∂s = length of flow in the direction S.

Non Uniform flow :- Type of flow in which velocity at any given time changes with respect to space.

Mathematically, for Non-Uniform flow

$$\left(\frac{\partial V}{\partial s}\right)_{t=\text{Constant}} \neq 0$$

iii) laminar and turbulent flow

laminar flow is defined as that type of

flow in which the fluid particles move along well defined path or stream lines and all the stream lines are stright and parallel. Thus the particles move in layer gliding smoothly over the adjacent layer. This type of flow is also called stream line flow or viscous flow.

Turbulent flow :- Turbulent flow is that type of flow in which the fluid particles move in a zig zag way. Due to the movement of fluid particles in a zig zag way, the eddies formation take place which are responsible for high energy loss.

For a pipe flow, the type of flow is determined by a non dimensional number called the Raynold number

$$N_{Re} = \frac{DV\rho}{\mu}$$

where D = Diameter of pipe
V = Velocity of fluid
ρ = Density of fluid
μ = Viscosity of fluid.

If the Raynold number is less than 2000 the flow is called laminar.

If the Raynold number is more than 4000, it is called turbulent flow.

If the Raynold number is lies between 2000 and 4000, the flow may be laminar or turbulent

iv) <u>Compressible and Incompressible flows</u>

<u>Compressible flow</u> :- Compressible flow is that type of flow in which the density of fluid changes from point to point.

ie Density is not constant for the fluid.

Mathematically, for compressible flow

$$\rho \neq \text{constant}$$

Ex :- gases are compressible.

Incompressible flow :- Incompressible flow is that type of flow in which the density is constant for the fluid flow.

Mathematically, for incompressible flow,

$$\rho = \text{Constant}$$

Ex:- liquids are Incompressible

(v) Rotational and Irrotational flow

Rotional flow is that flow in which the fluid particles while flowing along streamlines, also rotate about their own axis.

Irrotation flow :- If the fluid particles while flowing along streamlines, do not rotate about their own axis.

vi) one, two and Three dimensional flows

one dimensional flow :- It is the type of flow in which the flow parameter such as Velocity is a function time and one space coordinate only

For steady one dimensional flow, the velocity is a function of one space coordinate only.

$$V = f(x)$$

Two dimensional flow :- For steady, two dimenal flow, $\boxed{V = f(x, y)}$

Velocity is function of two space coordinate.

Three dimensional flow

For steady three dimension flow, the velocity is function of three mutual perpendicular directions $V = f(x, y, z)$.

Rate of flow or Discharge (Q)

$$Q = \frac{\text{quantity of fluid}}{\text{Second.}}, \quad m^3/sec$$

Consider a liquid flowing through a pipe

$$Q = A \times V$$

where Q = Discharge of fluid

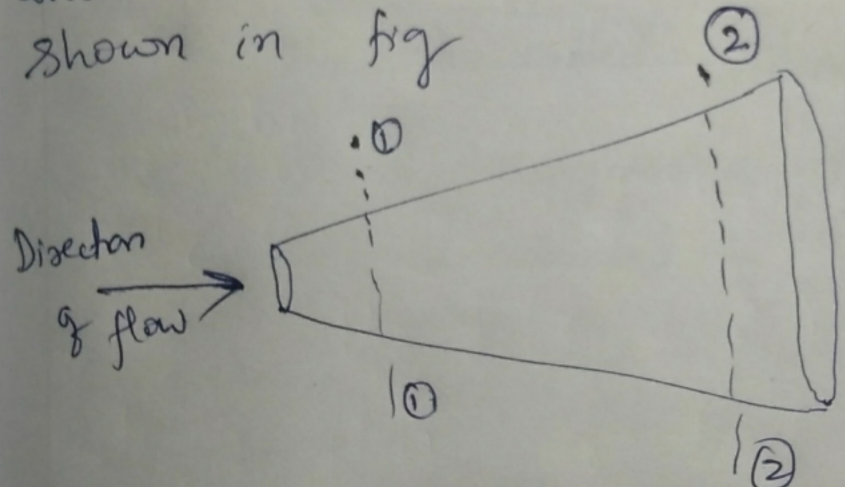
A = Cross-sectional area of pipe

V = Average Velocity of fluid across the Section

Continuity equation

The Continuity equation is based on the principle of Conservation of mass.

Consider two Cross sections of a pipe as shown in fig



let V_1 = Average Velocity at Cross-section 1-1

ρ_1 = Density at section 1-1

A_1 = Area of pipe at section 1-1

and V_2, ρ_2 A_2 are Corresponding Values at section 2-2.

Then rate of flow at section 1-1 $= \rho_1 A_1 V_1$

Rate of flow at Section 2-2 $= \rho_2 A_2 V_2$

According to law of conservation of mass

Rate of flow at section 1-1 = Rate of flow at section 2-2

$$\boxed{\rho_1 A_1 V_1 = \rho_2 A_2 V_2} \longrightarrow (1)$$

This is called Continuity equation

If the fluid is incompressible, then $\rho_1 = \rho_2$ then equation is reduced to

$$\boxed{A_1 V_1 = A_2 V_2}$$

Continuity equation for Incompressible fluid problem 1

The diameters of a pipe at the section 1 and 2 are 10 cm and 15 cm respectively. Find the discharge through the pipe if the velocity of water flowing through the pipe at section 1 is 5 m/sec. Determine also the velocity at Section 2.

Solution :-

$$D_1 = 10 \text{ cm} = 0.1 \text{ m}$$

$$D_2 = 15 \text{ cm} = 0.15 \text{ m}$$

$$A_1 = \frac{\pi D_1^2}{4}$$

$$A_1 = \frac{\pi \times (0.1)^2}{4} = 0.007854 \text{ m}^2$$

$$A_2 = \frac{\pi D_2^2}{4} = \frac{\pi \times (0.15)^2}{4}$$

$$\boxed{A_2 = 0.01767 \text{ m}^2}$$

(i) Discharge through pipe is given by

$$Q = A_1 V_1$$

$$Q = 0.007854 \times 5$$

$$\boxed{Q = 0.03927 \ m^3/sec}$$

(ii) Velocity at Section 2

$$Q = A_2 V_2$$

Here $Q_1 = Q_2 = Q$

$$V_2 = \frac{Q_2}{A_2} = \frac{0.03927}{0.01767}$$

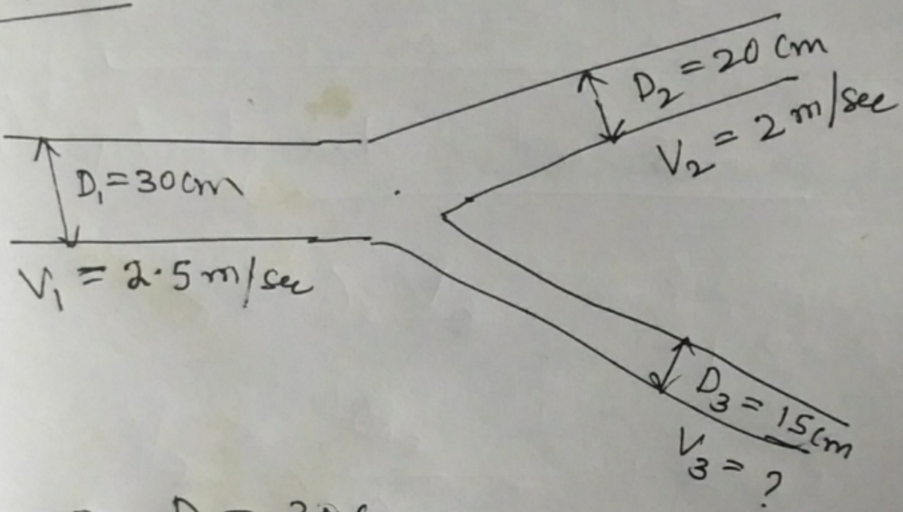
$$\boxed{V_2 = 2.22 \ m/sec}$$

2) A 30 cm diameter pipe, conveying water, branches into two pipes of diameters 20 cm and 15 cm respectively. If the average velocity in the 30 cm diameter pipe is 2.5 m/s

Find the discharge in this pipe.

(ii) Also determine the velocity in 15 cm pipe if the average velocity in 20 cm diameter pipe is 2 m/sec.

Solutions



$D_1 = 30$ cm $= 0.3$ m

$D_2 = 20$ cm $= 0.2$ m

$D_3 = 15$ cm $= 0.15$ m

$$A_1 = \frac{\pi D_1^2}{4} = \frac{\pi}{4} \times (0.3)^2 = 0.07065 \, m^2$$

$$A_2 = \frac{\pi D_2^2}{4} = \frac{\pi}{4} \times (0.2)^2 = 0.0314 \, m^2$$

$$A_3 = \frac{\pi}{4} \times 0.15 = 0.01767 \, m^2$$

(i) Dischare in pipe 1 ie Q_1

let Q_1, Q_2 and Q_3 are discharges in pipe 1, 2 and 3 respectively

Then according to Continuity equation

$$Q_1 = Q_2 + Q_3$$

(i) the discharge Q_1 is given by

$$Q_1 = A_1 V_1$$

$$Q_1 = 0.07068 \times 2.5 = 0.1767 \, m^3/sec$$

(ii) Value of V_3

$$Q_1 = Q_2 + Q_3$$

$$Q_1 = A_1 V_1 = 0.1767$$

$$Q_2 = A_2 V_2 = 0.314 \times 2 = 0.0628 \, m^3/sec$$

$Q_3 = Q_1 - Q_2$

$Q_3 = 0.1767 - 0.0628$

$$\boxed{Q_3 = 0.1139 \ m^3/sec}$$

WKT $Q_3 = A_3 V_3$

$$V_3 = \frac{Q_3}{A_3} = \frac{0.1139}{0.01767}$$

$$\boxed{V_3 = 6.44 \ m/sec}$$

↳ Required answer