Bernoulli's equation

$$\boxed{\frac{P}{\rho g} + \frac{V^2}{2g} + Z = \text{Constant}}$$

problem 1

1) water is flowing through a pipe of 5cm diameter under a pressure of $29.43\ N/cm^2$ and with mean velocity of 2 m/sec. Find the total head or total energy per unit weight of the water at a cross section, which is 5 m above the datum line.

Solution Given data

P = pressure $= 29.43\ N/cm^2$

$P = 29.43 \times 10^4\ N/m^2$

V = Velocity $= 2\ m/sec$

$Z = 5m$ (height)

Bernoulli's equation is given by

Total head = pressure head + kinetic head + potential head

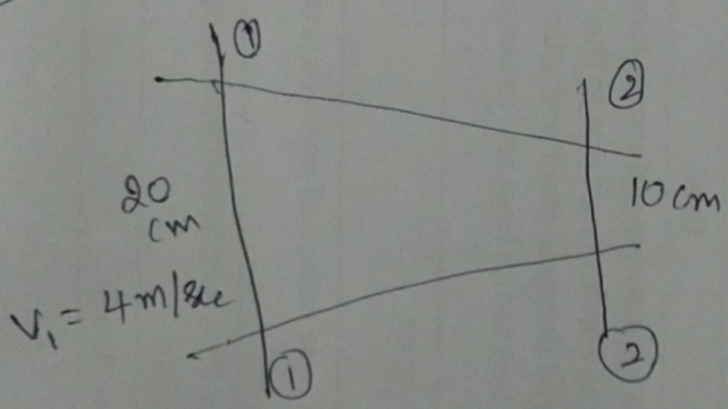
$$\frac{P}{\rho g} + \frac{V^2}{2g} + Z = \text{Constant}$$

$$\frac{29.43 \times 10^4}{1000 \times 9.81} + \frac{(2)^2}{2 \times 9.81} + 5 = 35.204\,m$$

Total head = Total energy per unit weight of the water = 35.204 m

(2) A pipe through which water is flowing, is having diameters, 20 cm and 10 cm at the cross sections 1 and 2 respectively. The velocity of water at section 1 is given 4 m/sec. Find the velocity head at sections 1 and 2 and also rate of discharge

Solution



Given data

$D_1 = 20\,cm$

$$A_1 = \frac{\pi D_1^2}{4} = \frac{\pi \times (20 \times 10^{-2})^2}{4}$$

$$\boxed{A_1 = 0.0314\ m^2}$$

$D_2 = 0.1\ m = 10\ cm$

$$A_2 = \frac{\pi D_2^2}{4} = \frac{\pi \times (0.1)^2}{4}$$

$$\boxed{A_2 = 0.00785\ m^2}$$

$V_1 = 4\ m/sec$

(i) <u>velocity head at section 1</u> $(V_1^2/2g)$

$$\frac{V_1^2}{2g} = \frac{(4)^2}{2\times 9.81} = 0.815\ m$$

ii) Velocity head at section 2 $(V_2^2/2g)$

$\frac{V_2^2}{2g}$, here V_2 is not known.

To get V_2, Continuity equation is applied

$$A_1 V_1 = A_2 V_2$$

$$(0.0314)(4) = (0.00785)\, V_2$$

$$\boxed{V_2 = 16\ m/sec}$$

$$\frac{V_2^2}{2g} = \text{Velocity head at Section 2}$$

$$\frac{(16)^2}{2 \times 9.81} = 83.047 \text{ m/sec}$$

$$\boxed{V_2 = 83.047 \text{ m/sec}}$$

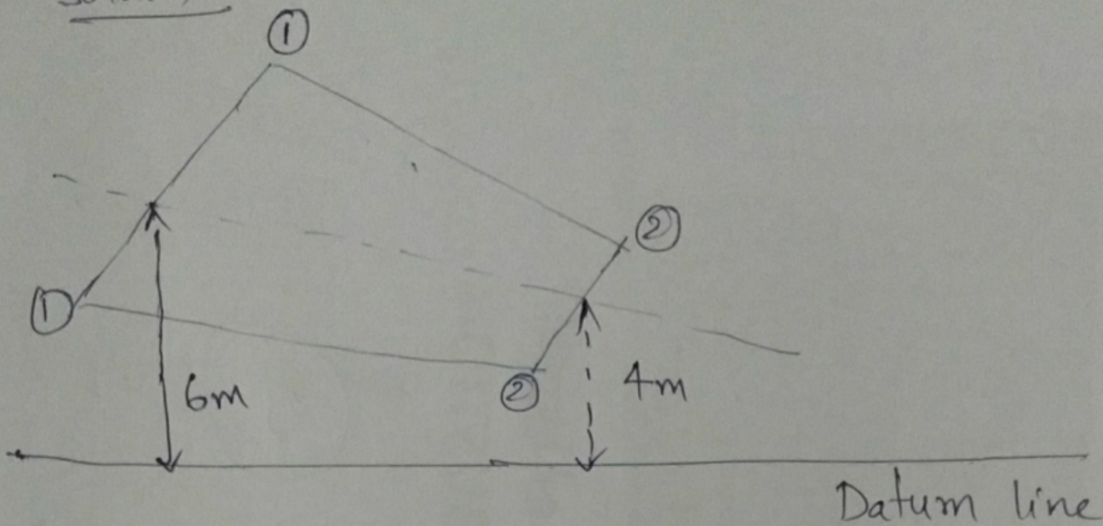
(iii) Rate of discharge

$$Q = A_1 V_1$$

$$\boxed{Q = 0.0314 \times 4 = 0.1256 \text{ m}^3/\text{sec}}$$

(3) The water is flowing through a pipe having diameters 20 cm and 10 cm at Section 1 and 2 respectively. The rate of flow through pipe is 35 litres/sec. The Section 1 is 6 m above datum and Section 2 is 4 m above datum. If the pressure at Section 1 is 39.24 N/cm^2, Find the pressure intensity at section 2

Solution



$D_1 = 20\,cm = 0.2\,m$

$A_1 = \dfrac{\pi (0.2)^2}{2} = 0.0314\ m^2$

$D_2 = 0.1\,m$

$A_2 = \dfrac{\pi (0.1)^2}{2} = 0.00785\ m^2$

$P_1 = 39.24\ N/cm^2$

$P_1 = 39.24 \times 10^4\ N/m^2$

$Z_1 = 6\,m$

$Z_2 = 4\,m$

$P_2 = ?$

$Q = 35\ lit/sec = \dfrac{35 \times 10^{-3}}{} = 0.035\ \dfrac{m^3}{sec}$

To find V_1 and V_2

$$V_1 = \frac{Q}{A_1} = \frac{0.035}{0.0314} = 1.114 \text{ m/sec}$$

$$V_2 = \frac{Q}{A_2} = \frac{0.035}{0.00785} = 4.456 \text{ m/sec}$$

Applying Bernoulli's equation at section 1 and 2, we get

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + Z_2$$

$$\left(\frac{39.24 \times 10^4}{1000 \times 9.81}\right) + \frac{(1.114)^2}{2 \times 9.81} + 6 =$$

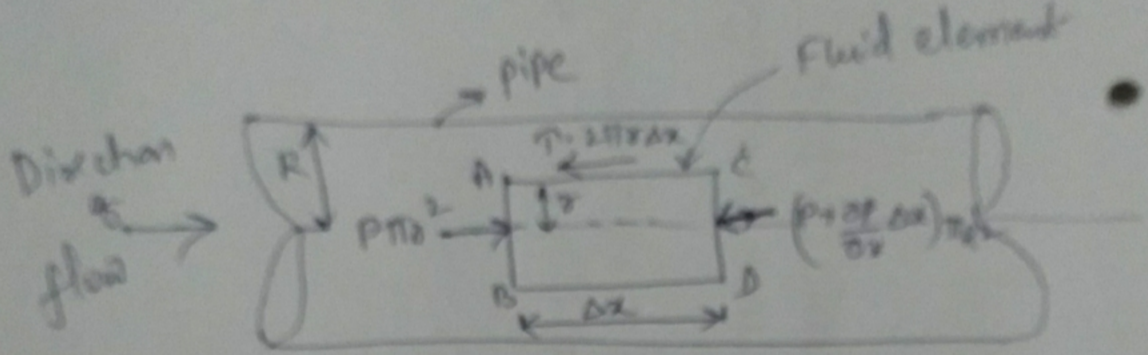
$$= \frac{P_2}{1000 \times 9.81} + \frac{(4.456)^2}{2 \times 9.81} + 4$$

$$\boxed{P_2 = 40.27 \times 10^4 \text{ N/m}^2}$$

Hagen poiseuille formula

Flow through circular pipe

Consider a horizontal pipe of radius R. The viscous fluid (laminar) is flowing from left to right in the pipe. as shown in fig



Then the forces acting on the fluid element are

(1) The pressure force on face AB $= p\pi r^2$

(2) The pressure force on face CD $= \left(p+\frac{\partial p}{\partial x}\Delta x\right)\pi r^2$

(3) The shear force on the surface of fluid element $= \tau \cdot 2\pi r \Delta x$

As there is no acceleration, hence summation of all forces in the direction of flow must be zero.

$$p\pi r^2 - \left(p + \frac{\partial p}{\partial x}\Delta x\right)\pi r^2 - \tau 2\pi r \Delta x = 0$$

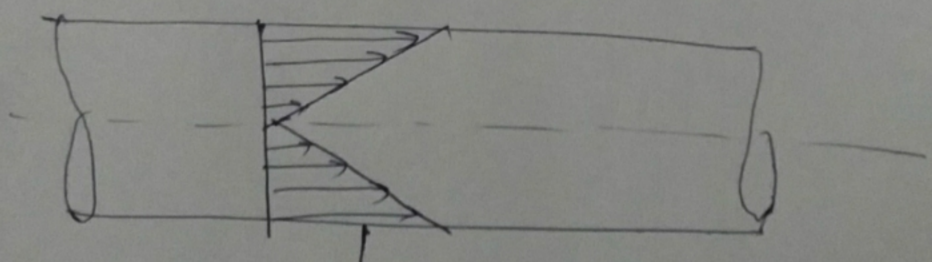
$$-\frac{\partial p}{\partial x}\Delta x \pi r^2 - \tau 2\pi r \Delta x = 0$$

$$\frac{-\partial p}{\partial x} r - 2\tau = 0$$

$$\boxed{\tau = -\frac{\partial p}{\partial x} \, r/2}$$

The shear stress τ across a section varies with r as $\frac{\partial p}{\partial x}$ across a section is constant

Hence shear stress distribution across a section is linear as shown in fig



Shear stress distribution.