

# Fork and D/y/d Connected Transformer Supplied 12-Pulse Uncontrolled Converter fed Retrofit Vector Controlled Induction Motor Drive: A Comparative Study

Aekamdeep Kaur  
Department of Electrical Engineering,  
Giani Zail Singh Campus College of  
Engineering and Technology MRSPTU  
Bathinda, India  
aekamdeep897@gmail.com

Yugal Gupta  
Department of Electrical Engineering,  
Indian Institute of Technology Kanpur  
Kanpur, India  
yugalgupta96@gmail.com

Sumit Ghatak Choudhuri, SMIEEE  
Department of Electrical Engineering,  
Indian Institute of Technology Roorkee  
Roorkee, India  
sgceefee@iitr.ac.in

**Abstract**— This paper presents a comparative investigation when a 12-pulse, uncontrolled converter fed retrofit Vector Controlled Induction Motor Drive (VCIMD) is supplied from D/y/d and Fork transformers, respectively. The study proves that Fork configuration results into reduction of total transformer magnetic rating when compared to D/y/d configuration, at almost same power quality performance level, observed on Full Load (FL) and Light Load (LL), respectively. VCIMD performance is compared under starting and load perturbation modes suitable for various industrial applications. The investigation is realised in MATLAB environment using Simulink and Sim Power System (SPS) toolboxes in Discrete Time Frame (DTF).

**Keywords**— D/y/d, Fork, Retrofit, Vector Controlled Induction Motor Drive (VCIMD).

## I. INTRODUCTION

Three- phase, squirrel-cage induction motors have been widely used for fixed-speed applications due to their simplicity, compactness, ruggedness, reliability, high efficiency, low cost and lesser maintenance. Inherent advantages of adjustable frequency operation can be realised using a suitable control. VCIMD used for variable speed applications show dynamics similar to that of separately excited dc motor. The technique uses two decoupled control signals, one responsible for control of excitation and other for control of torque [1-6].

A block schematic of VCIMD is shown in Fig.1. The system consists of a three-phase, Voltage Source Inverter (VSI), a three-phase, squirrel cage induction motor and necessary control blocks. Speed error is processed to generate reference torque command ( $T^*$ ). A suitable value of excitation reference ( $i_{nr}^*$ ) is generated from shaft speed. The VSI is operated in current controlled mode.

Non-Linear loads such as Power Electronic Converters, draw non-sinusoidal currents from supply and affect other users [7-12]. Techniques based on Pulse multiplication, realised through suitable transformer connection, are discussed in [13-16] for power quality improvement of retrofit loads.

This paper targets towards a comparative investigation between Fork and D/y/d transformer connections and also proves that for the same VCIMD, fork results into lower

rating magnetics than that of D/y/d with similar performance level.

Sections II and III explain design of D/y/d and Fork transformers, respectively for retrofit VCIMD. Drive dynamics and supply end power quality details are presented in Section-IV. The detailed mathematical formulation for calculating magnetic ratings of Inter-Phase Reactor (IPR) and power transformer is also presented in this section. The result prove that for the same drive ratings, the total magnetic rating of fork configuration is lesser as compared to D/y/d.

## II. DELTA/STAR/DELTA CONNECTED TRANSFORMER SUPPLYING 12-PULSE, AC-DC, UNCONTROLLED CONVERTER FED RETROFIT VCIMD

### A. Rating Computation for D/y/d Transformer:

Fig.2 shows the block schematic of D/y/d connected transformer, supplying 12-pulse, uncontrolled converter fed VCIMD. As the transformer windings are isolated, inter-converter currents are not possible. IPR connected between dc positive terminals of the converters neutralise instantaneous voltage difference between the two converters at the output port.

For a six-pulse, uncontrolled converter, the DC-Link output voltage,  $V_d$ , expressed as a function of transformer secondary line voltage,  $V_L$  is,

$$V_d = \frac{3\sqrt{2}}{\pi} V_L \quad (1)$$

For design simplicity, it is assumed that the load current drawn from DC link ( $i_d$ ) is constant in nature ( $I_d$ ) due to highly inductive nature of the load. Fig.3 shows respective current waveforms at secondary and primary side of the transformer and the resultant line current drawn from utility. Currents  $i_1, i_2, i_3$  and  $i_1', i_2', i_3'$  are respective input currents to the 12-pulse, uncontrolled converter. Phase currents of the secondary delta winding,  $i_I, i_{II}$  and  $i_{III}$  can be obtained as,

$$i_I = \frac{i_1 - i_2}{3}, i_{II} = \frac{i_2 - i_3}{3} \text{ and } i_{III} = \frac{i_3 - i_1}{3} \quad (2)$$







zero while  $i_2'$  and  $i_3'$  are negative and positive, respectively,  
 $v_{p0} = v_{c'b'}$  and  $v_{q0} = v_{c'b'}$  (16)

Therefore,  $v_{pq}$  can be mathematically expressed as,

$$v_{pq} = 0.732V_L \sin(\omega t + 165^\circ) \quad (17)$$

Similarly, the same computation can be extended to other intervals, respectively which is tabulated in Table I.

TABLE I

VOLTAGE ACROSS INTER-PHASE REACTOR AT VARIOUS INTERVALS

Sub-interval	$v_{pq}$	$\omega t$	$v_{pq}$
0°–30°	$0.732V_L \sin(\omega t + 165^\circ)$	0°	$0.189V_L$
		30°	$-0.189V_L$
30°–60°	$0.732V_L \sin(\omega t - 45^\circ)$	30°	$-0.189V_L$
		60°	$0.189V_L$
60°–90°	$0.732V_L \cos(\omega t + 15^\circ)$	60°	$0.189V_L$
		90°	$-0.189V_L$
90°–120°	$0.732V_L \sin(-\omega t - 75^\circ)$	90°	$-0.189V_L$
		120°	$0.189V_L$
120°–150°	$0.732V_L - \cos(\omega t + 135^\circ)$	120°	$0.189V_L$
		150°	$-0.189V_L$
150°–180°	$0.732V_L \sin(-\omega t - 15^\circ)$	150°	$-0.189V_L$
		180°	$0.189V_L$

Resultant  $v_{pq}$  waveform is shown by Curve-I in Fig.4

Area, under Curve-I for the first quarter cycle is,

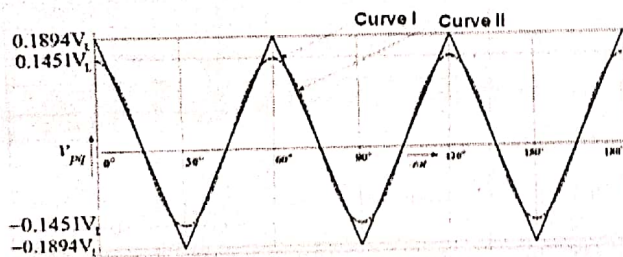


Fig.4. Voltage across Inter-Phase Reactor for D/y/d transformer topology: (A) Actual Voltage  $v_{pq}$  shown by Curve-I, (B) Approximated Sinusoidal voltage  $v_{pq}$  shown by Curve-II

$$\int_0^{15^\circ} 0.732V_L \sin(\omega t + 165^\circ) d\omega t = 0.018V_d \quad (18)$$

As power transformers are normally rated for sinusoidal voltages and currents, area under Curve-I, is therefore equated with area under an approximated sinusoidal waveform shown by Curve-II, overlapped in Fig.4. Area under Curve-II, having RMS, 'V' in the first quarter cycle is,

$$\int_0^{15^\circ} \sqrt{2}V \cos(6\omega t) d\omega t = 0.2357V \quad (19)$$

where, ' $\omega$ ' refers to the frequency of AC mains in rad/sec. Equating (18) with (19) yields,  $V = 0.076V_d$  (20)

Peak of the approximated Curve-II, computes to  $0.1451V_L$ , marked in Fig.4. Rating of IPR can be computed as,

$$VA_{IPR} = \left[ \frac{V \frac{I_d}{2} + V \frac{I_d}{2}}{2} \right] = 0.076V_d \cdot \frac{I_d}{2} = 0.038P_d \quad (21)$$

Hence, the combined total rating of D/y/d and IPR amounts to  $1.0671P_d$ , obtained by adding (8) and (21).

### III. FORK CONNECTED AUTO-TRANSFORMER SUPPLYING 12-PULSE, UNCONTROLLED CONVERTER FED RETROFIT VCIMD

#### A. Rating Computation for Fork Auto-transformer:

Fig.5 shows the block schematic for Fork connected auto-transformer supplying 12-Pulse, uncontrolled converter fed Retrofit VCIMD.  $K_1$ ,  $K_2$  and  $K_3$  refer to the respective tapings. As secondary windings are not isolated, there may be possibility of inter-converter current paths. To prevent such a situation, IPRs are connected in between respective positive terminals and in between respective negative terminals of the converters. Winding connections are realised such that both secondary windings getting connected to converters I and II have respective phase voltages  $15^\circ$  lagging and leading w.r.t. the main supply. AC supply voltages to transformer primary can be expressed as,

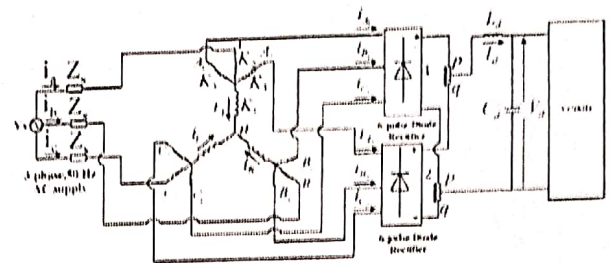


Fig.5. Block Schematic of Fork connected Auto-Transformer supplying 12-Pulse Uncontrolled Converter fed Retrofit VCIMD

$$v_A = \sqrt{2}V_s \sin(\omega t) \quad (22)$$

$$v_B = \sqrt{2}V_s \sin(\omega t - 120^\circ) \quad (23)$$

$$v_C = \sqrt{2}V_s \sin(\omega t + 120^\circ) \quad (24)$$

where, ' $V_s$ ' refers to rms AC supply phase voltage. Fig.6 relates  $K_1$ ,  $K_2$  and  $K_3$  as,

$$V_A = K_1V_A + K_2V_A \text{ and } V_{A_1} = K_1V_A - K_3V_B \quad (25)$$

Solving,  $K_1 = 0.1865$ ,  $K_2 = 0.1835$  and  $K_3 = 0.2988$

DC-link voltage,  $V_d$  can be expressed using (1) as,

$$V_d = \frac{(3\sqrt{6})V_s}{\pi} \quad (26)$$

Therefore,

$$V_{zlg} = K_3V_s = 0.1277V_d \quad (27)$$

$$V_{long} = K_1V_s = 0.3491V_d \quad (28)$$

$$V_{short} = K_2V_s = 0.0784V_d \quad (29)$$

Fig.7 shows respective winding currents of Fork connection.

Balancing mmf for phase A gives,

$$K_2(i_{A_1} + i_{A_2} + i_A) + K_1i_A + K_3i_{C_1} + K_3i_{B_1} = 0 \quad (30)$$

$$i_A = -\frac{K_2(i_{A_1} + i_{A_2}) + K_3(i_{C_1} + i_{B_1})}{(K_1 + K_2)} \quad (31)$$

Similarly,  $i_B$  and  $i_C$  can be computed by mmf balancing as,

$$i_B = -\frac{K_2(i_{B_1} + i_{B_2}) + K_3(i_{A_1} + i_{C_1})}{(K_1 + K_2)} \quad (32)$$



$$i_c = -\frac{K_2(i_{c1} + i_{c2}) + K_1(i_{n1} + i_{n2})}{(K_1 + K_2)} \quad (33)$$

From Fig. 7, rms values of currents in zig, long and short windings respectively may be computed as,

$$I_{zig} = (i_{A1})_{rms} = 0.4082I_d \quad (34)$$

$$I_{long} = (i_A)_{rms} = 0.0609I_d \quad (35)$$

$$I_{short} = (i_{A1} + i_{A2} + i_A)_{rms} = 0.7836I_d \quad (36)$$

$$\text{Using (27) and (34), } VA_{zig} = V_{zig} \times I_{zig} = 0.0521P_d \quad (37)$$

Similarly,  $VA_{long} = 0.02126P_d$  and  $VA_{short} = 0.0618P_d$

From above the rating of Fork auto-transformer,  $VA_{FORK}$  is computed as,

$$= \frac{6 \times VA_{zig} + 3 \times VA_{long} + 3 \times VA_{short}}{2} = 0.2809P_d \quad (38)$$

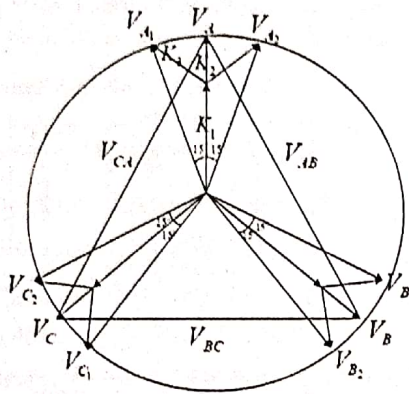


Fig. 6. Block Schematic of Fork connected Auto-Transformer supplying 12-Pulse Uncontrolled Converter fed Retrofit VCIMD

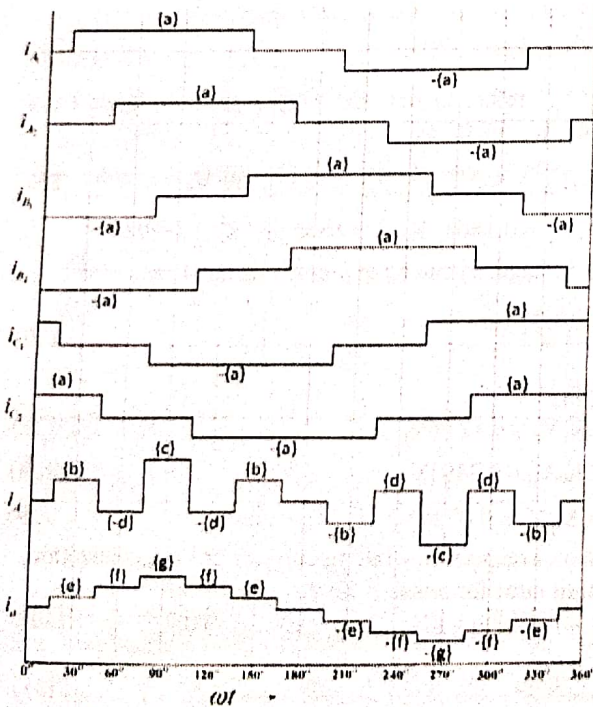


Fig. 7 Current Waveforms for Fork Auto-Transformer Supplying 12-pulse, Uncontrolled Converter Fed Constant Current Load  $\{a\}=0.5I_d$   
 $\{b\}=0.0576I_d, \{c\}=0.1153I_d, \{d\}=0.0341I_d, \{e\}=0.576I_d, \{f\}=0.9659I_d, \{g\}=1.1153I_d$

### B. Design for Inter-Phase Reactor

Fig. 8 shows connections of IPR between positive terminals of the two converters. Diodes  $D_5$  and  $D_7$  conduct during  $0^\circ - 15^\circ$ . Therefore,



Fig. 8. IPR Connected Between Positive Terminals of Converters

$$v_{pn} = v_{c1}, v_{qn} = v_{c2} \quad (39)$$

$$v_{c1} = V_s \angle 135^\circ, v_{c2} = V_s \angle 105^\circ \quad (40)$$

Resultant voltage across the IPR ( $v_{pq}$ ) can be computed as,

$$v_{pq} = 0.7319V_s \sin(\omega t - 150^\circ) \quad (41)$$

Similarly, the same computation can be extended to other intervals respectively. Table II tabulates the resultant data.

TABLE II

VOLTAGE ACROSS INTER-PHASE REACTOR AT VARIOUS INTERVALS

Interval	$v_{pq}$	$\omega t$	$v_{pq}$
$0^\circ - 15^\circ$	$0.7318V_s \sin(\omega t - 150^\circ)$	$0^\circ$	$-0.3659V_s$
		$15^\circ$	$-0.5174V_s$
$15^\circ - 45^\circ$	$1.999V_s \sin(\omega t - 30^\circ)$	$15^\circ$	$-0.5173V_s$
		$45^\circ$	$-0.5173V_s$
$45^\circ - 75^\circ$	$0.73199V_s \sin(\omega t + 90^\circ)$	$45^\circ$	$0.5173V_s$
		$75^\circ$	$0.1894V_s$
$75^\circ - 105^\circ$	$0.73199V_s \sin(\omega t + 90^\circ)$	$75^\circ$	$0.1894V_s$
		$105^\circ$	$-0.1894V_s$
$105^\circ - 135^\circ$	$0.73199V_s \sin(\omega t + 90^\circ)$	$105^\circ$	$-0.1894V_s$
		$135^\circ$	$-0.5175V_s$
$135^\circ - 165^\circ$	$1.999V_s \sin(\omega t - 150^\circ)$	$135^\circ$	$-0.5173V_s$
		$165^\circ$	$-0.5173V_s$
$165^\circ - 195^\circ$	$0.73199V_s \sin(\omega t - 30^\circ)$	$165^\circ$	$-0.5173V_s$
		$195^\circ$	$0.1894V_s$
$195^\circ - 225^\circ$	$0.73199V_s \sin(\omega t - 30^\circ)$	$195^\circ$	$0.1894V_s$
		$225^\circ$	$-0.1894V_s$
$225^\circ - 255^\circ$	$0.73199V_s \sin(\omega t - 30^\circ)$	$225^\circ$	$-0.1894V_s$
		$255^\circ$	$-0.5175V_s$
$255^\circ - 285^\circ$	$1.999V_s \sin(\omega t + 90^\circ)$	$255^\circ$	$-0.5175V_s$
		$285^\circ$	$0.5175V_s$
$285^\circ - 315^\circ$	$0.73199V_s \sin(\omega t - 150^\circ)$	$285^\circ$	$0.5175V_s$
		$315^\circ$	$0.1894V_s$
$315^\circ - 345^\circ$	$0.73199V_s \sin(\omega t - 150^\circ)$	$315^\circ$	$0.1894V_s$
		$345^\circ$	$-0.1894V_s$
$345^\circ - 360^\circ$	$0.73199V_s \sin(\omega t - 150^\circ)$	$345^\circ$	$-0.1894V_s$
		$360^\circ$	$-0.3659V_s$

Resultant  $v_{pq}$  therefore, is obtained as shown in Fig.9

Area under Curve-I for the positive half cycle is computed as,

$$= \int_{30^\circ}^{90^\circ} v_{pq} d\omega t = 0.12077V_d \quad (42)$$

As transformers are normally rated for sinusoidal voltages and currents, area under curve-I,  $v_{pq}$  is compared to that of Curve-II with rms 'V'. Area under Curve-II in the positive half cycle is computed as,



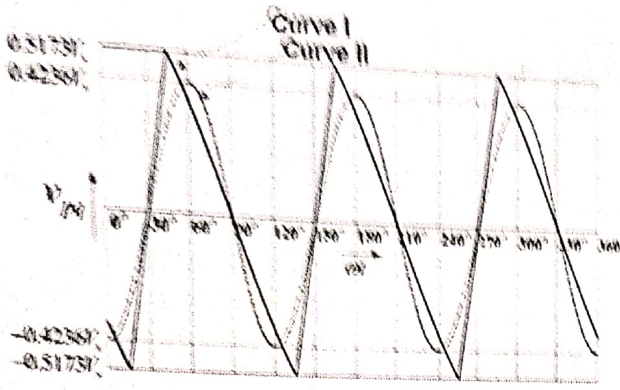


Fig.9. Voltage across IPR for Fork transformer supplying 12-Pulse Uncontrolled Converter fed Non-Linear Load  
Curve I: Actual Voltage, Curve-II: Approximated Sinusoidal Voltage

$$= \int_{-\pi/2}^{\pi/2} \sqrt{2}V \sin(3(\omega t - 30^\circ)) d\omega t = 0.942V \quad (43)$$

Equating equations (42) and (43),

$$V = 0.128V_d \quad (44)$$

Peak of approximated sinusoid signal computes to 0.4236V, as per marked in Fig.9. Rating of IPR is,

$$VA_{IPR} = \left[ \frac{V \frac{I_d}{2} + V \frac{I_d}{2}}{2} \right] = 0.128V_d \frac{I_d}{2} = 0.06405P_d \quad (45)$$

$$\text{Gross IPR rating amounts to, } VA_{IPR(TWO)} = 0.1281P_d \quad (46)$$

Henceforth, the combined rating of Fork connected auto-transformer and two IPRs amount to  $0.4089 P_d$ , as obtained by adding (38) and (46).

#### IV. RESULTS AND DISCUSSION

To improve the quality of current drawn from AC mains, a comparative investigation has been carried out between D/y/d and Fork topologies with main objective being to investigate which topology gives rise to lower magnetic rating. Respective topologies have been designed and simulated to manifest their power quality performance for retrofit VCIMD.

Simulation is carried out in MATLAB environment. Fig.11 shows the dynamic response for D/y/d connected, 12-Pulse, Uncontrolled Converter fed retrofit VCIMD for starting and load perturbation modes respectively. The plot consists of reference and actual speed, reference motor currents, actual motor currents, motor developed torque and load torque, utility supply, supply line current and DC-link voltage respectively. Fig.13 portrays two cycles of supply current on FL along with spectrum showing 6.12% THD. The comparison between various power quality parameters on FL and LL is tabulated in Table III.

Fork connected auto-transformer fed VCIMD is simulated in MATLAB environment. Fig.12 shows the dynamic response of the retrofit drive for starting and load perturbation modes respectively. The plot consists of reference and actual speeds, reference motor currents, actual motor currents, motor

developed torque and load torque, utility supply, supply line current and DC-link voltage respectively. Fig.14 shows two cycles of supply current waveform with necessary harmonic

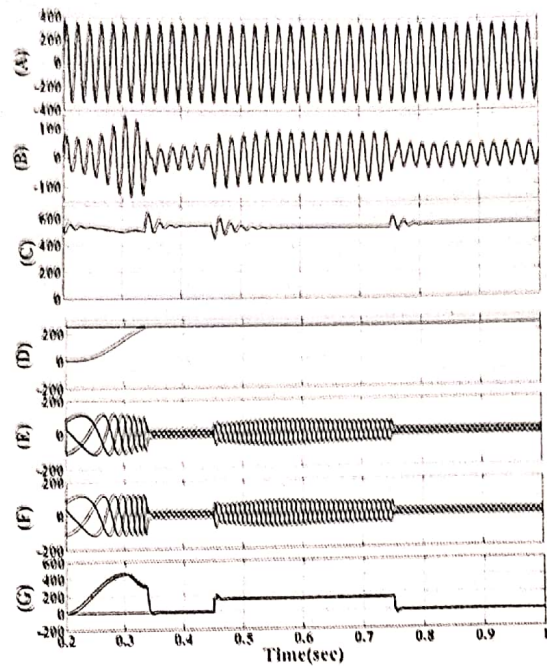


Fig.11. Dynamic Response of D/y/d, 12-pulse, Uncontrolled Converter fed retrofit VCIMD for (A) Source Voltage, (B) Source Current, (C) DC-Link Voltage, (D) Rotor Speed, (E) Reference Currents, (F) Actual Currents and (G) Developed Torque and Load Torque

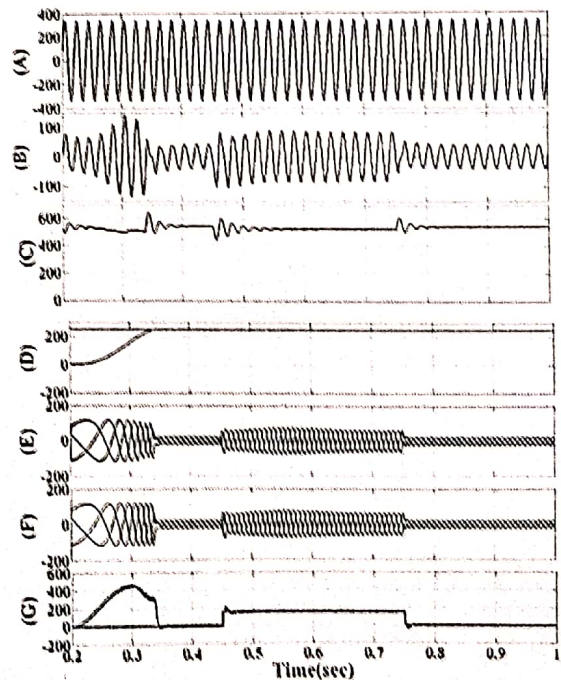


Fig.12 Dynamic Response of Fork, 12-pulse, Uncontrolled Converter fed retrofit VCIMD for (A) Source Voltage, (B) Source Current, (C) DC-Link Voltage, (D) Rotor Speed, (E) Reference Currents, (F) Actual Currents and (G) Developed Torque and Load Torque

spectrum when drive is on FL. In this case, the THD in the line current drawn from the supply mains is observed 5.84%



Necessary parameters relating to supply current power quality have been tabulated in Table III. Table IV provides the comparison between magnetic ratings.

TABLE III

COMPARISON OF POWER QUALITY INDICES OF A RETROFIT DRIVE VCIMD FED FROM 12-PULSE UNCONTROLLED CONVERTER

Topology	THD(%)		Distortion Factor		Displacement factor		Power Factor	
	LL	FL	LL	FL	LL	FL	LL	FL
D/y/d	8.30	6.12	0.99	0.99	0.98	0.97	0.97	0.97
Fork	8.14	5.84	0.99	0.99	0.98	0.97	0.97	0.96

TABLE IV

RATING OF MAGNETICS IN DIFFERENT CONVERTER TOPOLOGIES

Rating expressed as Percentage Total DC-Link Load ( $P_d$ )	D/y/d	Fork
Power Transformer	102.91%	28.09%
IPR	3.80%	12.81%
Total Magnetic Rating	106.71%	40.9%

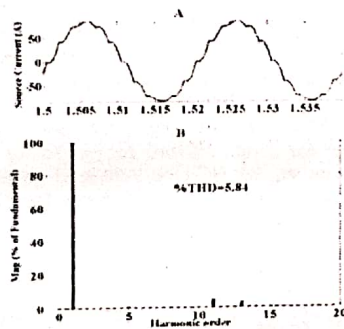


Fig. 13. (A) Supply current (B) THD analysis of supply current for D/y/d connected transformer, supplying 12-pulse uncontrolled converter fed retrofit drive VCIMD

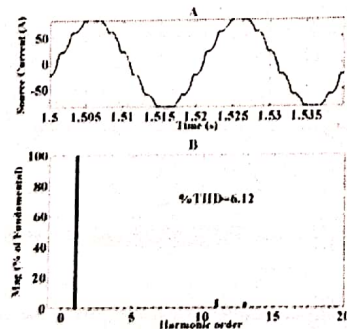


Fig. 14. (A) Supply current (B) THD analysis of supply current for Fork connected transformer, supplying 12-pulse uncontrolled converter fed retrofit drive VCIMD

## V. CONCLUSION

A comparative investigation between D/y/d and Fork connected transformer topologies supplying 12-Pulse, Uncontrolled Converter fed retrofit VCIMD has been carried out with an objective to prove that Fork connection amounts to lesser rating and saving in magnetics.

Based on design computations and MATLAB simulated results, it has been observed that the total rating for transformer and IPR amounts to 106.71% of the total DC-link load in D/y/d topology. On the contrary, Fork topology requires total 40.89% (auto-transformer and IPR) of the total DC-link load for same rating retrofit VCIMD. The comparison shows similar performance level both from drive dynamics and supply power quality aspects respectively. Therefore, use of Fork topology saves copper and leads to reduction in cost and improvement in power quality.

## VII. APPENDIX

Induction motor specifications: -

30 HP, 3-phase, 4 pole, Y-Connected, 415V, 45A, 50Hz,

$R_s = 0.251 \Omega$ ,

$R_r = 0.249 \Omega$ ,  $X_{ls} = X_{lr} = 0.439 \Omega$ ,  $X_m = 13.085 \Omega$ ,

$J = 0.305 \text{ Kg m}^2$ .

PI Controller parameters: -

$K_p = 350$ ,  $K_i = 50$ .

DC link filter parameters: -

$L_d = 2 \text{ mH}$ ,  $C_d = 1500 \mu\text{F}$

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