

## (Permutations and Combinations)

Topics.

1. Permutations ← with out Repetitions  
with Repetitions.
2. Combinations.
3. Applications.

Note: Combinatorics is a branch of mathematics that deals with counting, arrangement and selection of objects.  $\Rightarrow$  Algorithm Analysis, Graph Theory, Cryptography, Data structure - BST, optimization, etc.

(A) (Permutations without Repetitions)

Permutations:- The arrangements of objects by taking some or all out of  $n$  different objects, are called permutations.  
(of a set)

(i) The number of all Permutations of  $n$ -different objects taking all at a time are  $n!$

(ii) The number of all Permutations of  $n$ -different objects taking some i.e. ( $1 \leq r \leq n$ ) or  $r$ -objects at a time, are  ${}^n P_r$ .

$${}^n P_r = \frac{n!}{(n-r)!}$$

Ex Consider digits 1, 2 & 3. The arrangements are

1 2 3 , 2 3 1 , 3 1 2

1 3 2 , 2 1 3 , 3 2 1

Here we see number change when order is changed. Total number of Permutations are 6. (1)

### (B) Permutations with Repetitions

• If we have  $n$ -objects (Not all different) but  $p$  objects are of 1st kind,  $q$ -objects are of 2nd type &  $r$ -objects are of 3rd kind and all remaining objects are different. then number of permutations of such  $n$ -objects taking all at a time is

$$= \frac{n!}{p! \times q! \times r!}$$

Ex How many ways are there to arrange the nine letters in word ALLAHABAD.

Sol:-  $n=9$  & 4 A's, 2 L's.

$$\begin{aligned} \text{Total number of arrangements} &= \frac{9!}{4! \times 2!} \\ &= 7,560 \text{ ways.} \end{aligned}$$

Note ①

$$\begin{aligned} nP_1 &= \frac{n!}{(n-1)!} = n \\ \downarrow \text{(increasing)} \\ nP_2 &= \frac{n!}{(n-2)!} = n \cdot (n-1) \\ nP_3 &= \frac{n!}{(n-3)!} = n \cdot (n-1) \cdot (n-2) \\ &\vdots \\ nP_n &= n! \end{aligned}$$

## Factorial function:-

Let  $n$  be a positive Integer. Then, the continued product of first  $n$  natural numbers is called factorial  $n$ . & denoted by,  $n!$  or  $\text{Ln}$ .

Therefore;  $n! = n \cdot (n-1) \cdot (n-2) \dots 3 \cdot 2 \cdot 1$

Note:- ①  $\text{Ln} = n \cdot \text{Ln}-1$

②  $0! = 1 = 1!$

Ex compute

(i)  $\frac{8!}{4! \times 3!}$

(ii)  $\frac{30!}{20!}$

(iii) LCM.  $(4!), 5!, 6$

(iv)  $\frac{1}{5!} + \frac{1}{6!} \neq \frac{1}{7!}$

Ex. convert into factorial >

(i)  $6 \cdot 7 \cdot 8 \cdot 9 = \frac{9!}{5!}$

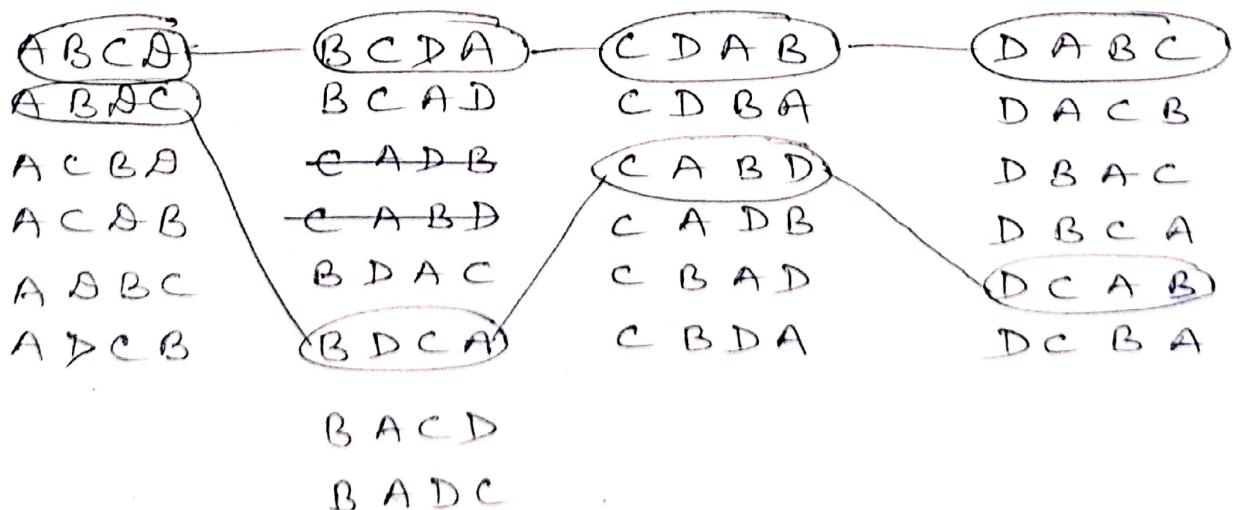
(ii)  $2 \cdot 4 \cdot 6 \cdot 8 \cdot 12 = 2^5 \cdot 5!$

(iii)  $1 \cdot 3 \cdot 5 \cdot 7 \cdot 9 \cdot 11 = \frac{11!}{2^5 \cdot 5!}$

etc.

Note:  $4P_4 = 4! = 4 \times 3 \times 2 \times 1 = 24$

ABCD



find  $x$  or  $n$ . If

$$(i) \frac{1}{4!} + \frac{1}{5!} + \frac{1}{6!} = \frac{x}{7!}$$

$$x = 259$$

$$(ii) (n+1)! = 12 \times (n-1)!$$

$$n = 3$$

$$(iii) \frac{1}{9!} + \frac{1}{10!} = \frac{n}{11!}$$

$$n = 121$$

$$(iv) (n+2)! = 1560 \cdot n$$

$$n = 30, -41.$$

### Problems based on $(nP_r)$

Ex 9 (i) Evaluate  ${}^{12}P_4$

$$11880$$

(ii)  ${}^6P_4$

$$360$$

(iii)  ${}^8P_8$

$$8! = 40,320$$

Ex 10 If  ${}^nP_4 = 20 \times {}^nP_2$  find  $n$ .

$$\frac{n!}{(n-4)!} = 20 \times \frac{n!}{(n-2)!}$$

$$n(n-1)(n-2)(n-3) = 20 \times n(n-1)$$

$$(n-2)(n-3) = 20$$

$$n^2 - 5n + 6 = 20$$

$$n^2 - 5n - 14 = 0$$

$$\Rightarrow n = 7, -2 \quad \text{ANS}$$

Ex 11 find  $r$ , if  ${}^{20}P_r = 6840$ .

$$\frac{20!}{(20-r)!} = 6840 = 20 \times 19 \times 18$$

$$\frac{20!}{(20-r)!} = \frac{20!}{17!} \Rightarrow r = 3 \quad \text{ANS}$$

Ex  $n+5 P_{n+1} = \frac{11 \cdot (n-1)}{2} \times n+3 P_n$ , find  $n$ .

$$\frac{(n+5)!}{(n+5)-(n+1))!} = \frac{11 \cdot (n-1)}{2} \times \frac{(n+3)!}{(n+3-n)!}$$

$$\frac{(n+5)(n+4) \cancel{(n+3)!}}{4!} = \frac{11 \cdot (n-1)}{2} \times \frac{\cancel{(n+3)!}}{3!}$$

$$\frac{(n+5) \cdot (n+4)}{4} = \frac{11(n-1)}{2}$$

$$n^2 + 9n + 20 = 22(n-1)$$

$$n^2 + 9n + 20 = 22n - 22$$

$$n^2 - 13n + 42 = 0 \Rightarrow n = 6, 7 \text{ ANS}$$

Ex 14 If  $9P_5 + 5 \cdot 9P_4 = 10P_n$  find  $n$ .

$$\frac{9!}{4!} + 5 \cdot \frac{9!}{5!} = \frac{10!}{(10-n)!}$$

$$2 \cdot \frac{9!}{4!} = \frac{10 \cdot 9!}{(10-n)!}$$

$$\frac{1}{4!} = \frac{5}{(10-n)!}$$

$$(10-n)! = 5 \times 4! \Rightarrow n = 5$$

Permutation

$n$  object: {Set}

All different

Not All different  
(Some Repeated identical)

Taking  $r$   
 $n$

Taking all  
 $n$

$n!$   
111111

(5)

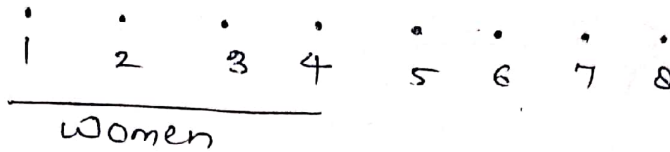




Ex 22 Each of 2 women & 3 men can occupy one chair out of 8 chairs, numbered 1 to 8.

The women can occupy any two chairs 1 to 4, then 3 men can occupy 3 chairs from rest of 6. How many ways it can be done.

Sol:-

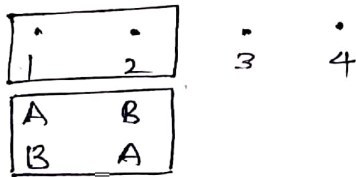


$${}^4P_2 \times {}^6P_3 = 12 \times 120 = 1440. \quad \text{ANS}$$

Ex 24 A person has four notes of rupees 1, 2, 5 & 10. denomination. The number of different sum he can form?

Ex 25 In how many ways 4 students can stand in a line such that two of them are always together.

Sol:-



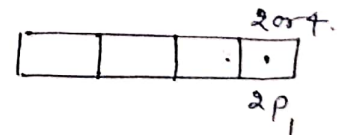
$$= {}^2P_2 \times {}^3P_2 = 2! \times 3! = 12. \quad \text{ANS}$$

Ex 31 How many four digit number can be formed by using digits 1 to 5. Also out of these how many will be even?

Sol:-  ${}^5P_4 = 120$  (All numbers).

For Even, last place will be fixed for 2 or 4.

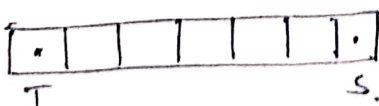
$${}^2P_1 \times {}^4P_3 = 2 \times 24 = 48.$$



Ex 29 The letters of word TUESDAY are arranged in a line that each arrangement ends with letter S. Also how many start with letter T.



$${}^6P_6 = 720.$$



$${}^5P_5 = 120 \text{ ways.} \quad \text{ANS}$$

## Combinations:-

Each of different group which can be formed by taking some or all objects (irrespective of order of arrangement) is called combinations.

The total numbers of combinations of  $n$ -distinct object by taking  $r$  at a time is denoted by  ${}^nC_r$  (where  $1 \leq r \leq n$ ).

Also, 
$${}^nC_r = \frac{n!}{r! \cdot (n-r)!}$$

Note ①  ${}^nC_0 = {}^nC_n$

②  ${}^nC_r = {}^nC_{n-r}$

Ex  ${}^nP_r = 720$  &  ${}^nC_r = 120$  find  $r$  ;  $r = 3$

Ex  ${}^{107}C_{07} = {}^{107}C_x$  find  $x$ . (since  $x = n - r$ )  $x = 20$

Ex  $x \cdot {}^{77}C_{55} = 23 \cdot {}^{78}C_{55}$  find  $x$ .

Ex 71  
253 How many ways a committee of 5 members can be selected from 6 men & 5 women, consisting 3 m & 2 ladies? ( ${}^6C_3 \times {}^5C_2 = 200$ )

Ex 72  
253 A committee of 5 is to be formed out of 6 men & 4 ladies. In how many ways it can be done. when it consist.

(1) at least 2 ladies included  $\Rightarrow$

2 ladies 3 men  $\Rightarrow {}^4C_2 \times {}^6C_3 = 120$

3 ladies 2 men  $\Rightarrow {}^4C_3 \times {}^6C_2 = 60$

4 ladies 1 man  $\Rightarrow {}^4C_4 \times {}^6C_1 = 20$

Total = 200

(2) at most 2 ladies include  $\Rightarrow$

1 lady 4 men  $\Rightarrow {}^4C_1 \times {}^6C_4 = 60$

Total = 120. ⑦



Repeatable/Non-Repeatable  $[R]^{NR}$

If there are  $n$ -different objects and we need to select  $r$ -at times. Repeation allowed, then number of ways it can be done are

$$(N)^r$$

4-digit

Ex. How many different PIN can be generated by number 0 to 9.

$$\begin{array}{cccc} 1 & 2 & 3 & 4 \\ \square & \square & \square & \square \\ 10 & 10 & 10 & 10 \end{array} = 10^4$$

Ex.23 A question paper had 10-questions. Each question has two choice true or false. How many different ways are there to solved QP.

Sol:-  $[R]^{NR} = [2]^{10} = 1024.$

Ex53 A man has 6 friends to invite. In how many ways he can send invitation to them if he has 3 servants to carry cards?

Sol:-  $3^6 = 729.$       **Ans**



## Permutations (with Repetition)

$$\frac{n!}{r! s! t!}$$

Ex 26  
232 How many ways are there to arrange nine letters of word ALLAHABAD.

Sol:-

$$\frac{9!}{4! \cdot 2!} = 7,560.$$

Ex 27  
233 How many different messages can be formed by sequences of four dashes and three dots.

$$\frac{7!}{4! \cdot 3!} = 35.$$

Ex 28  
233 Find how many ways to point 12 officers. so that 3 of them are green, 2 are pink, 2 are yellow & rest are white.

$$\frac{12!}{3! \cdot 2! \cdot 2! \cdot 5!} = 1,66,320.$$

Ex 30  
233 How many signals can be made by using 6 flags of different colours when any number of them can be hoisted at a time?

Sol:-

One-at time	${}^6P_1 = 6$
Two at time	${}^6P_2 = 30$
THREE	${}^6P_3 = 120$
4	${}^6P_4 = 360$
5	${}^6P_5 = 720$
6	${}^6P_6 = 720$
<hr/>	
Total = 1956	