UNIT-4

(Probability Theory)

* It is a branch of Mathematics that deals with study of randomness & uncertainty (in result) of an experiment. It helps us to quantify the likelihood of different outcomes..

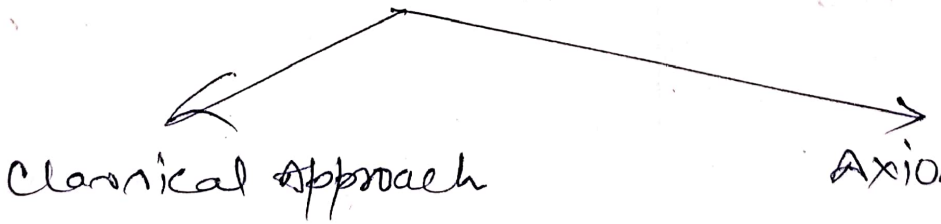
* The Probability of likelihood or chance that a particular event will occur. It quantifies uncertainty and $0 < p < 1$.

$$p = P(A) = \frac{\text{Number favourable outcomes}}{\text{Number of All possible outcomes}}$$

$$= \frac{m}{n} \quad (\text{when } n = \text{Exhaustive No. of cases} \setminus \text{\& event is equally likely / mutually Exclu})$$

Note :-

1. Above def assumes that all outcomes are equally likely.
2. $P(A) = 0 \Rightarrow$ Impossible event A.
   $P(A) = 1 \Rightarrow$ Sure / certain event A.
3. $0 < P(A) < 1$
4. Two approach



5. $P(\bar{A}) = 1 - P(A)$

   or $P(A) + P(\bar{A}) = 1$ or $p + q = 1$.

Simple

Ex What is the prob. of getting Two heads when
(1) ~~Adding~~ 2 coins tossed
(2) 3 coins tossed
(3) 4 coins tossed.

Ex What is prob. of getting sum of 8 or 9, or 1 on upper faces of two dice. when these two rolled simultaneously.

Ex What is the prob. of getting a face card when 1 card is drawn from well shuffled deck of cards.

Ex What is the prob. of getting even number in throwing a single die.

Ex BCA · what is prob. of getting a sum 17 or 18. in throwing three dice simultaneously. (4/216 Ans).

Ex BCA A ball is drawn at random from a box containing 6R, 4W, 5B. balls. Determine the prob. that drawn ball is
(1) Red (2) White (3) Black
(4) Not Red (5) Red or White*.

see ① Ques ③ use of combinations (without replacement)

Ex2 266 Two cards drawn at random from well shuffled pack of 52 cards. find prob. of getting two Aces?

$$P = \frac{4C_2}{52C_2}$$

Ex3 267 A bag contains 6W, 5B balls. find prob. that three ball drawn at random are all white.

$$\frac{6C_2}{}$$

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Ex5 / 267 Two cards are drawn from 52 cards at random. find chance that one is King and other Ace.

$$P = \frac{m}{n} = \frac{{}^4C_1 \times {}^4C_1}{{}^{52}C_2}$$

Ex6 / BCA a group containing 3 M, 2 W & 4 children. Obtain prob. that 2 children are there.

Sol:-

0 M	2 W	2 C	$= {}^3C_0 \times {}^2C_2 \times {}^4C_2$
1 M	1 W	2 C	$= {}^3C_1 \times {}^2C_1 \times {}^4C_2$
2 M	0 W	2 C	$= {}^3C_2 \times {}^2C_0 \times {}^4C_2$

$$\text{Total} = (1\times1 + 3\times2 + 3\times1)\ {}^4C_2$$
$$= 10 \times {}^4C_2$$

$$\text{Total cases} = {}^9C_4 \Rightarrow$$

$$P = \frac{m}{n} = \frac{10 \times {}^4C_2}{{}^9C_4}$$ Ans

Ex11 / BCA A Bag contain 6 R, 5 W, 4 B Balls. Two drawn at random. what is prob. that alone of them is red.

$$P = \frac{m}{n} = \frac{{}^9C_2}{{}^{15}C_2} = \frac{12}{35}.$$

Ex1 / 266 what is the chance (prob.) that a leap year selected at random will contain 53 sundays.?

Sol:- Total days = 366

divide by 7 = $\frac{366}{7}$

$= 52.\frac{2}{7}$

1. (S) M
2. M T
3. T W
4. W T
5. T F
6. F S

$P(\text{Sundy}) = \frac{2}{7}$

Classical definition of Prob.

If there are n- exhaustive, mutually exclusive and equally likely events. Out of which m are favourable of an event A. Then prob. of happening of A., denoted by P(A) & defined as.

$$P(A) = \frac{\text{favourable No. of cases of event A}}{\text{(Total or Exhaustive No. of cases of Exp.)}} = \frac{m}{n}$$

Ex 9 / 269 In a class of 13 students, 5 of them are boys & rest are girls. find prob. that two students are selected at random and both are girls.

$$P(G) = \frac{{}^{8}C_2}{{}^{13}C_2} = 0.36$$

Ex 10 / 269 A box contain 5 W, 4 B & 3 R balls. Three balls are drawn at random. What is the prob. that all are

1) White 2). Black 3) red.

4) 1 W compusory. +2 B 5) 1 B + 2 R 6) 1 W, 1 B, 1 R.

Tickets (Marked No.)

Ex 21 / 273 BCA A Bag contain tickets numbered 1 to 20. Two are drawn at random. find prob. of that both are prime numbered.

Sol:- $n = 20$

2, 3, 5, 7, 11, 13, 17, 19 (prime) = 8.

$$p = \frac{{}^{8}C_2}{{}^{20}C_2}$$

ANS.

Ex 12 / 270 BCA If lottery tickets, marked 1–30, four are drawn at random, what is the prob that 2 among them are 1 & 2.

Sol:- Since two draw are supposed to be fixed Rest are 28 tickets.

$$p = \frac{{}^{28}C_2}{{}^{30}C_4}$$

or

$$p = \frac{{}^{2}C_2 \times {}^{28}C_2}{{}^{30}C_4}$$

ANS

Multiplication theorem

Independent ↓ | Dependent.

It is applied to find joint prob. of two or more events. happening together.

$$P(A \cap B) = P(A) \cdot P(B) \quad \text{(If Independent)}$$

$$P(A \cap B) = P(A) \cdot P(B|A) \quad \text{(If dependent)}.$$

Where:-

$P(B|A)$ = conditional prob. of B given that A has already occured.

Ex 15. A class consist of 80 students, 25 Girl 10 of them rich & 20 with fair complexion. What is prob. 1. is selected & she is fair & rich.

$$\frac{20}{80} \times \frac{10}{20} \times \frac{25}{80}.$$

Ex. A problem is given to A, B, C. whose chance to solve it are $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}$ resp. what is prob that prob. will be solved.

$$P(\text{Solved}) + P(\text{Not solved}) = 1$$

$$P(\overline{\text{Solved}}) = P(\overline{A}) \cdot P(\overline{B}) \cdot P(\overline{C}).$$

Conditional probability

Conditional probability is the probability of an event A occuring given that another event B has already occured.

It is denoted and defined as

$$P(A|B) = \frac{P(A \cap B)}{P(B)} \quad ; \text{ provided } P(B) > 0.$$

where $P(A|B)$ = prob. of A given B has already occured.

$P(A \cap B)$ = prob. of A & B happening simultaneously.

Note- $P(B) \neq 0$ ie $P(B) > 0$

It is useful in decision making, Bayes theorem & machine learning & more.

Multiplication theorem It is useful to find prob. of Intersection of two events. ie both event happening ~~together~~.

$$P(A \text{ and } B) = P(A) \cdot P(B) \quad \text{(If Independent)}$$

$$P(A \text{ and } B) = P(A) \cdot P(B|A) \quad \text{(If dependent)}.$$

Ex 15 BCA A class of 80 students, 25 of them are Girls, 55 are boys, 10 of them are rich and remaining poor. 20 of them are fair complexioned. What is prob. of selecting a fair rich girls. $\frac{20}{80} \times \frac{10}{80} \times \frac{1}{2}$

Ex 17 A problem given to three students A, B & C. What is prob. prob. will be solved.

Taraet will be [illegible]

Addition theorem

This theorem is useful to find probability of union of two events. (combined prob). It is applied to find prob. of happening either one or other event.

$$P(A \text{ or } B) = P(A) + P(B) \quad \text{(If mutually Exclusive)}$$

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B).$$

(If Not mutually Exclusive)

Ex 14 BCA In single throw of two dice, find prob. of either doublet or 9 will appear.

$$P(A \text{ or } B) = \frac{6}{36} + \frac{4}{36} = \frac{10}{36} \quad \text{Ans.}$$

Ex 10 BCA Two dice are thrown, getting two number whose sum is divisible by 4 or 5.

$$P(4) + P(8) + P(12) + P(5) + P(10)$$
$$= \frac{3}{36} + \frac{5}{36} + \frac{1}{36} + \frac{4}{36} + \frac{3}{36} = \frac{16}{36}$$

Ex 13 BCA A drawer contains 50 bolts and 150 nuts. Half of bolts & nuts are rusted. If one item is chosen at random, what is prob. that it is rusted or a bolt.

$$P(A \text{ or } B) = P(A) + P(B) - P(A \cap B)$$
$$= \frac{100}{200} + \frac{50}{200} - \frac{25}{200} = \frac{125}{200} \quad \text{Ans.}$$