FORMULA

Matrices

Symmetric matrix $\rightarrow$ $A^T = A$

Skew-Symmetric matrix $\rightarrow$ $A^T = -A$

* $A + A^T$ = Symmetric
* $A - A^T$ = Skew Symmetric

* $$A = \underbrace{\frac{1}{2}(A + A^T)}_{\text{Symmetric}} + \underbrace{\frac{1}{2}(A - A^T)}_{\text{Skew Symmetric}}$$

Elementory Operation

$$A = IA$$

- The interchange of any two rows or two columns $R_j \leftrightarrow R_i$
- The multiplication of the elements of any row or column by non zero number

$$R_i \rightarrow KR_i$$
$$C_i \rightarrow KC_i$$

- The addition to the elements of any row or column, the corresponding elements of any other row or column multiplied by any non zero number

$R_i \to R_i + R_j$

$C_i \to C_i + C_j$

Inverse of Matrix

$$A^{-1} = \frac{1}{|A|} \, Adj(A) \qquad \{|A| \neq 0\}$$

* if $|A|$ is 0 than A^{-1} DOES NOT EXIST

$Adj(A)$ = Transpons of Cofactors $= C^T$

Invertible Matrix

$AB = I$
$BA = I$

then, A is inverse of B
B is inverse of A

$B = A^{-1}$

Condition
1) Square matrix
2) $|A| \neq 0$

Adjoint of matrix

Transpose of Cofactors

$Adj\,A = C^T$

Minor & Cofactor

M_{ij} → Value computed from the determinant of a square matrix

C_{ij} → Cofactor is defined as the signed minor

$$A = (-1)^{i+j}$$

Singular matrix ⇒ $|A| = 0$

Non Singular matrix ⇒ $|A| \neq 0$

Linear Equations

$$a_1 x + b_1 y + c_1 z = d_1$$
$$a_2 x + b_2 y + c_2 z = d_2$$
$$a_3 x + b_3 y + c_3 z = d_3$$

$$A = \begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix} \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \quad B = \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix}$$

$$AX = B$$
$$X = A^{-1}B$$

Orthogonal matrix

$$A \times A^T = I$$

Conjugate matrix

Replacing its element by the corresponding conjugate complex number is called conjugate

$$A = \begin{bmatrix} 1+i & i \\ 2 & -3i \end{bmatrix}$$ Denoted by $\bar{A}$

$$\bar{A} = \begin{bmatrix} 1-i & -i \\ 2 & 3i \end{bmatrix}$$

Conjugate transpose

Denoted by A^θ

$$\bar{A} = \begin{bmatrix} 1-i & -i \\ 2 & 3i \end{bmatrix} \qquad A^\theta = \begin{bmatrix} 1-i & 2 \\ -i & 3i \end{bmatrix}$$

Rank of Matrix

Refers to the number of linearly independent rows and coloumn in matrix

A matrix rank is said to be ~~zero~~ of rank zero when all elements become zero

Hermition Matrix

$$A^{\theta} = A^{T}$$

$$A = \begin{bmatrix} 4 & 1-i & 7 \\ 1+i & 6 & -i \\ 7 & i & 5 \end{bmatrix} \qquad A^{\theta} = \begin{bmatrix} 4 & 1+i & 7 \\ 1-i & 6 & i \\ 7 & -i & 5 \end{bmatrix}$$

$$A^{T} = \begin{bmatrix} 4 & 1+i & 7 \\ 1-i & 6 & i \\ 7 & -i & 5 \end{bmatrix}$$

Skew Hermition Matrix

$$(A^{\theta})^{T} = -A$$

Characteastic Equation of Matrix

$$A = \begin{bmatrix} 1 & -1 \\ 0 & 0 \end{bmatrix}$$

$$|A - \lambda I| = 0$$

Subtract the diagonal with λ

$$\begin{vmatrix} 1-\lambda & -1 \\ 0 & 1-\lambda \end{vmatrix} = 0$$

Method 1: $(1-\lambda)(1-\lambda)+0=0$

$(1-\lambda)^2=0$

$\lambda^2-2\lambda+1=0$

Given matrix is a 2x2 matrix So quadatic equation appears if is it 3x3 then cubic equation

Method 2: $\boxed{\lambda^2 - P_1\lambda + |A| = 0}$

P_1 = Sum of main diagonal

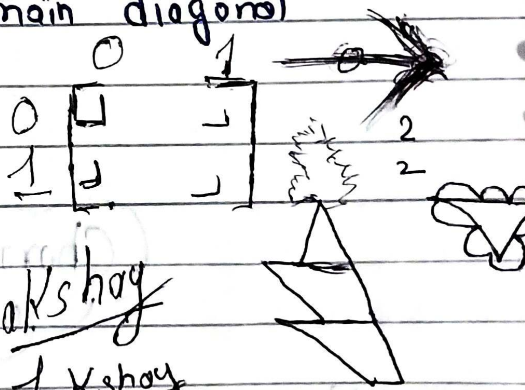
3x3 $\boxed{\lambda^3 - P_1\lambda^2 + P_2\lambda - |A| = 0}$ $|A| \neq 0$

Lakshay

P_1 = Sum of diagonal

P_2 = Sum of minors of main diagonal

Eigen Value and Vectors



There be minimum one

Out

Lakshay

Lakshay

Orthogonal Matrices

$\boxed{A \times A^T = I}$ if matrix is orthogonal then $\boxed{A^{-1} = A^T}$

$A^{-1} = A^T$

Q) Prove that following matrix is orthogonal and hence find A^{-1}

$$A = \frac{1}{3}\begin{bmatrix} -2 & 1 & 2 \\ 2 & 2 & 1 \\ 1 & -2 & 2 \end{bmatrix}$$ 1x3 3x3

Sol. $$A = \frac{1}{3}\begin{bmatrix} -2 & 1 & 2 \\ 2 & 2 & 1 \\ 1 & -2 & 2 \end{bmatrix}$$

$$A^T = \frac{1}{3}\begin{bmatrix} -2 & 2 & 1 \\ 1 & 2 & -2 \\ 2 & 1 & 2 \end{bmatrix} = A^{-1}$$

$$A \times A^T = \frac{1}{3}\begin{bmatrix} -2 & 1 & 2 \\ 2 & 2 & 1 \\ 1 & -2 & 2 \end{bmatrix} \times \frac{1}{3}\begin{bmatrix} -2 & 2 & -1 \\ 1 & 2 & -2 \\ 2 & 1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 4+1+4 & -4+2+2 & 2+(-2)+4 \\ -4+2+2 & 4+4+1 & -2+(-4)+2 \\ -2+(-2)+2 & 2+(-4)+2 & -1+4+4 \end{bmatrix} = \begin{bmatrix} 9 & 0 & 4 \\ 0 & 9 & -4 \\ 2 & 0 & 7 \end{bmatrix}$$

simple
data warehouse for movies, Sale data assume
Sale of marlies and making inventory in creating
this data warehouse all the sales +
R h

Rank of matrix

if determinat of a matrix is 0 (it means it has less than rank of its size

$A = \begin{bmatrix} - & - & - \\ - & - & - \\ - & - & - \end{bmatrix}_{3\times3}$ if $|A| = 0$ Rank will be less than 3

~~to~~ Extract any 2x2 matrix and check determinat
if it has non zero determinat means it rank is 2

$or_2\ A \times A^{T} = I$

$$= \frac{1}{3}\begin{bmatrix} 1+(1+i)(1-i) & 1(1+i)+(1+i)(-1) \\ (1-i)(1)+(-1)(1-i) & (1-i)(1+i)+1 \end{bmatrix}$$

$$= \frac{1}{3}\begin{bmatrix} 1+1+1 & 1+i-1-i \\ 1-i-1+i & 1+1+1 \end{bmatrix}$$

$$A \times A^{\theta} = \frac{1}{3}\begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Rank of Matrix

$$A = \begin{bmatrix} 1 & +1 & 1 \\ 1 & -1 & 0 \\ 1 & 1 & 1 \end{bmatrix}_{3\times3}$$

Take determinat of matrix A

$$|A| = 1(-1-0) - 1(1-0) + 1(1-(-1))$$

$$|A| = 0$$

Rank of the matrix is less than 3 (max is 3)

$$\begin{vmatrix} 1 & 1 \\ 1 & -1 \end{vmatrix} = \cancel{1(1\times1)}\ -1-1=0 \qquad \begin{vmatrix} 1 & 1 \\ -1 & 0 \end{vmatrix} = 0-(-1) = 1$$

a) Componets of DBMS enviroment
b) Explain the function of Data base admi

Unitary matrix

$$A \times A^{\theta} = A^{\theta} \times A = I$$

$$A \times A^T = \frac{1}{3}\begin{bmatrix} -2 & 1 & 2 \\ 2 & 2 & 1 \\ 1 & -2 & 2 \end{bmatrix} \times \frac{1}{3}\begin{bmatrix} -2 & 2 & 1 \\ 1 & 2 & -2 \\ 2 & 1 & 2 \end{bmatrix}$$

$$\cancel{-2\times(-2)+1+4} \quad = \frac{1}{9}\begin{bmatrix} 9 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 9 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Unitary Matrices

$\boxed{A \times A^\theta = I}$ ★ if matrix is unitary then $A^{-1} = A^\theta$

$A^\theta = A^T$ Complex Conjugate

$A^\theta = \bar{A}$

Q) Prove that $A = \begin{bmatrix} 1 & 1+i \\ 1-i & -1 \end{bmatrix}$ is unitary find A^{-1} also

Sol. $A^T = \frac{1}{\sqrt{3}}\begin{bmatrix} 1 & 1-i \\ 1+i & -1 \end{bmatrix}$

$A^{\theta'} = \frac{1}{\sqrt{3}}\begin{bmatrix} 1 & 1+i \\ 1-i & -1 \end{bmatrix}$

$A \times A^\theta = \frac{1}{\sqrt{3}}\begin{bmatrix} 1 & 1+i \\ 1-i & -1 \end{bmatrix} \times \frac{1}{\sqrt{3}}\begin{bmatrix} 1 & 1+i \\ 1-i & -1 \end{bmatrix}$

Charactestic equation

For 2x2 matrix

$$\boxed{\lambda^2 - \underbrace{(\text{Trace of } A)}_{\text{Sum of diagonals}}\lambda + \det A = 0}$$

For 3x3 matrix

$$\lambda^3 - (\text{Trace of } A)\lambda^2 + (\text{Sum of minor along diagonal})\lambda - \det A = 0$$

eigen values eigen vectors

1. $(A - \lambda I)x = 0$

$$A = \begin{bmatrix} 8 & -8 & -2 \\ 4 & -3 & -2 \\ 3 & -4 & 1 \end{bmatrix}$$

2. $|A - \lambda I| = 0$

$$= \begin{vmatrix} 8-\lambda & -8 & -2 \\ 4 & -3-\lambda & -2 \\ 3 & -4 & 1-\lambda \end{vmatrix}$$

3. Find character equation

$$\Rightarrow \lambda^3 - [\text{Sum Diagona}]\lambda^2 + (\text{Sum minor diagonal})\lambda - |A| = 0$$

$$= \lambda^3 - 6\lambda^2 + 11\lambda - 6 = 0$$

$\boxed{\lambda = 1, 2, 3}$ Eigen Values

Eigen vector

$$\begin{bmatrix} \cancel{8}\,7 & -8 & -2 \\ 4 & -4 & -2 \\ 3 & -4 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$7x_1 - 8x_2 - 2x_3 = 0 \quad - (1)$$
$$4x_1 - 4x_2 - 2x_3 = 0 \quad - (2)$$

$$\frac{x_1}{\begin{vmatrix} 8 & -2 \\ -4 & -2 \end{vmatrix}} = \frac{-x_2}{\begin{vmatrix} 7 & -2 \\ 4 & -2 \end{vmatrix}} = \frac{x_3}{\begin{vmatrix} 7 & -8 \\ 4 & -4 \end{vmatrix}}$$

$$\frac{x_1}{8} = \frac{x_2}{6} = \frac{x_3}{4}$$

$$X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 8 \\ 6 \\ 4 \end{bmatrix} = \begin{bmatrix} 4 \\ 3 \\ 2 \end{bmatrix}$$

Same as for $\lambda = 2$ & $\lambda = 3$

Characteristic Equation of a matrix

1) $A = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}_{2\times 2}$

$|A - \lambda I| = 0$ leading diagonal

$$\begin{bmatrix} 1-\lambda & -1 \\ 0 & 1-\lambda \end{bmatrix} = 0$$

$$(1-\lambda)^2 + 0 = 0$$

$$1 + \lambda^2 - 2\lambda = 0$$

$$\boxed{\lambda^2 - 2\lambda + 1 = 0}$$

2) $A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}_{3\times 3}$

Sol $|A - \lambda I| = 0$

$$\begin{bmatrix} -2-\lambda & 2 & -3 \\ 2 & 1-\lambda & -6 \\ -1 & -2 & 0-\lambda \end{bmatrix} = 0$$

$$-2-\lambda\left[(1-\lambda)(0-\lambda) - (-2)(-6)\right] - 2\left[2(0-\lambda) - (-6)(-1)\right] + 3\left[2(-2) - (-1)(1-\lambda)\right]$$

$$-2-\lambda\left[0-\lambda-0+\lambda^2 - 12\right] - 2\left[-2\lambda-6\right] + 3\left[-4 - (-1+\lambda)\right]$$

$$-2-\lambda\left[\lambda^2 - \lambda - 12\right] + 4\lambda + 12 + (-12) + 1 - \lambda]$$

Cayley hamilton theorem

Q) Verify CHT for

$$A = \begin{bmatrix} 1 & 2 & 0 \\ 2 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$ A^{-1} and A^{-4}

$(A - \lambda I)x = 0$

$\Rightarrow \lambda^3 - \lambda^2 - 5\lambda + 5 = 0$

$\boxed{\text{Put } \lambda = A}$

$A^3 - A^2 - 5A + 5I = 0$

A^{-1}, A^{-4}

① $A \times A^{-1} = I$

② $AI = A$

$A^3 - A^2 - 5A + 5I = 0$

multiply by A

$A^4 - A^3 - 5A^2 + 5AI = 0$

$A^4 = A^3 + 5A^2 + 5A$

$A^3 - A^2 - 5A + 5AI = 0$

multiply A^{-1}

$A^3 \cdot A^{-1} - A^2 \cdot A^{-1} - 5A^{-1} + 5A \cdot A^{-1} = 0$

$A^2 - AI - 5I + 5A^{-1} = 0$

$A^{-1} = \frac{1}{5}\left[-A^2 + A + 5I\right]$