Theory of equation

n degree equation

$$f(x) = a_0x^n + a_1x^{n-1} + a_2x^{n-2} + a_3x^{n-3} + \cdots + a_n$$

$$f(x) = 0$$

$$a_0x^n + a_1x^{n-1} \cdots + a_n = 0$$

Complete equation	Incomplete equation
$x^3 + 4x^2 - x + 5 = 0$	$x^3 + 4x - 5 = 0$
	$= x^3 + 0x^2 + 4x - 5 = 0$

Date ..../..../........

Degree of equation

① A equation of first degree is called linear eq

ex- $ax + b = 0$

② Second degree → Quadratic equation $(ax^2 + bx + c = 0)$

Third degree → Cubic equation $(ax^3 + bx^2 + cx + d = 0)$

Roots of equation

Statisfies the $f(x) = 0$ is called roots of equation or zero of polynomial

① An equation of the n^{th} degree has n roots

② Imaginary roots in an equation occurs in conjugate pair

③ If a & b are two real numbers & $f(x)$ is a polynomial $f(a)$ & $f(b)$ are of opposite sign That is are is positive other is negative then there exists at least one real root of equation $f(x) = 0$ between a & b

④ Every equation of an odd degree has at least one real root whose sign is opposite to that of absolute term

Date ..../..../........

⑤ Every equation of even degree whose absolute term is negative has at least two real roots, one is negative & other is postive

Sythetic division

To find quotient & remainder when polynomial is divided by a bionomial

Example:- find the quotient & the reminder when $x^5 - 3x^4 + x^3 - 8x - 135$ is divided by $x - 4$

$x - 4 = 0$

$x = 4$

4	1	-3	1	0	-8	-135
+	–	4	4	20	80	288
	1	1	5	20	72	153

$Q(x) = x^4 + x^3 + 5x^2 + 20x + 172$

$R(x) = 153$

1	1	-16	+86	-176	105
	–	1	15	101	-75
	1	15	101	-75	30

$Q(x) = x^3 + 15x^2 + 101x - 75$

$R(x) = 30$

Date ..../..../........

Relation between the roots & the coefficient

let $f(x) = a_0x^n + a_1x^{n-1} + a_2x^{n-2} + \cdots\cdots + a_n = 0$ be algebraic equation of area n let $\alpha_1, \alpha_2, \cdots \alpha_n$ be its roots then

$$S_1 = \Sigma\alpha_1 = \text{Sum of all roots} = \frac{-a_1}{a_0} = -\frac{\text{Cofficient } x^{n-1}}{\text{Cofficient } x^n}$$

$x^2 - 4x - 5 = 0$ $\quad a_0 = 1$

$(x+5)(x+1)$ $\quad a_1 = -4$

$x = 5, -1$ $\quad a_2 = -5$

$\alpha = 5$

$\beta = -1$ $\quad \alpha + \beta = 5 - 1 = 4$

$$\alpha + \beta = \frac{a_1}{a_0} = \frac{4}{1}$$

$S_2 = \Sigma\alpha_1\alpha_2$ = Sum of the product of the roots taken two at a time

$$= (-1)^2 \frac{a_2}{a_0}$$

$$= (-1)^2 \frac{\text{Cofficient } x^{n-2}}{\text{Cofficient } x^n}$$

$\alpha\beta = -5 \times 1 = -5$

$$= (-1)^2 \frac{a_2}{a_0} = 1 \times \frac{-5}{1} = -5$$

Date ..../..../........

$S_n = \alpha_1 \alpha_2 \ldots \alpha_n =$ Product of all roots

$= (-1)^n \frac{a_n}{a_0} = (-1)^n \frac{\text{Constant term}}{\text{Cofficient of } x^n}$

Roots and cofficient of quadartic

$ax^2 + bx + c = 0$

roots $= \alpha, \beta$

$\boxed{\alpha + \beta = -\frac{b}{a}}$ $\boxed{\alpha\beta = \frac{c}{a}}$

Roots and cofficient of cubic

$ax^3 + bx^2 + cx + d = 0$

roots $= \alpha, \beta, \gamma$

$\boxed{\alpha + \beta + \gamma = -\frac{b}{a}}$ $\boxed{\alpha\beta + \beta\gamma + \gamma\alpha = \frac{c}{a}}$

$\boxed{\alpha\beta\gamma = -\frac{d}{a}}$

Roots and cofficient of bi-quadartic

$ax^4 + bx^3 + cx^2 + dx + e = 0$

roots $= \alpha, \beta, \gamma, \delta$

$\boxed{\alpha + \beta + C + \delta = -\frac{b}{a}}$ $\boxed{\Sigma\alpha\beta = \frac{c}{a}}$ $\boxed{\Sigma\alpha\beta\gamma = -\frac{d}{a}}$ $\boxed{\alpha\beta\gamma\delta = -\frac{e}{a}}$

Spiral

Date ..../..../........

Transformation of equation

① Roots with sign changed

Change the sign of every alternate term of given equation beginning with second term after making ~~two~~ the equation complete

ex → $x^2 - 4x + 3 = 0 \xrightarrow{\text{Change sign}} x^2 + 4x + 3 = 0$

$(x-1)(x-3)$ $\qquad$ $(x+1)(x+3) = 0$

$x = 1, 3$ $\qquad$ $x = -1, -3$

Chang the sign of every secondth term

② Roots multiplied by a constant K

To transform equation into another whose roots are the roots of the given equation multiplied by K

ex → $x^2 - 4x + 3 = 0$ $\qquad$ 4 time $y = 4x$ $\quad x = y/4$

$(x-1)(x-3) = 0$

$x = 1, 3$

$$(y/4)^2 - 4(y/4) + 3 = 0$$

$$y^2/16 - y + 3 = 0$$

$$(y-12)(y-4)$$

$$y = 4, 12$$

Spiral

Date ..../..../........

Reciprocal of the roots of given equation

Replacing $y = 1/x$ or $x = \frac{1}{y}$ in the given equation $f(x) = 0$ the transformed equation $f(1/y)$ or $f\left(\frac{1}{x}\right) = 0$ is required equation

To obtain an equation where roots are ~~re~~ reciprocal of the roots of given are simply put $\frac{1}{x}$ for x

$x^4 - 4x + 5 = 0$

$(x+1)(x-5)$

$x = -1, 5$

$f\left(\frac{1}{x}\right) = \left(\frac{1}{x}\right)^2 - 4\left(\frac{1}{x}\right) + 5$

$= 5x^2 + 4x - 1 = 0$

$(x+1)(5x-1) = 0$

$x = -1, \; x = \frac{1}{5}$

Reciprocal equation

an equation which remain unaltered by changing x into $\frac{1}{x}$ is called reciprocal equation

Example :- $x^5 - 5x^4 + 9x^3 - 9x^2 + 5x - 1 = 0$

$6x^6 - 25x^5 + 31x^4 - 31x^3 + 25x - 6 = 0$

Date ..../..../........

Properties

① A reciprocal equation of first kind of an odd degree has a root $= -1$

$$6x^5 - x^4 - 43x^3 + 43x^2 + x + 6 = 0$$

② A reciprocal equation of second kind of an odd degree has a root $= 1$

$$x^5 - 5x^4 + 9x^3 - 9x^2 + 5x - 1 = 0$$

③ A reciprocal equation of second kind of an even degree has root $= 1$ & -1

Roots diminished by h

Equation whose roots are the roots of given equation diminished by a constant h

ⓐ Division by synthetic method if convenient

ⓑ If we are to increase the root by h we are to take h negative

ex - $x^2 - 4x - 5 = 0$

$(x+1)(x-5) = 0$

$x = -1, 5$ show $x = -3, 3$ (2)

$(x - 2 + 2)^2 - 4(x - 2 + 2) - 5 = 0$

$y^2 - 9 = 0$

Shind