

ECHELON INSTITUTE OF TECHNOLOGY

Department of computer Applications

QUESTION BANK :(Probability & Statistics)

Course :BCA (DATA SCIENCE)

session :July -Dec 2023

Q1. Define Distribution function and its three properties with prove.

Q2.A random variable X has the following probability function:

| | | | | | | | | |
|----------------|---|---|----|----|----|-------|--------|----------|
| Values of X,x: | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| P(x) | 0 | k | 2k | 2k | 3k | k^2 | $2k^2$ | $7k^2+k$ |

(i) find k.

(ii) Evaluate $P(X < 6)$, $P(X \geq 6)$, and $p(0 < x < 5)$.

(iii) if $P(X \leq a) > \frac{1}{2}$, find the minimum value of a.

(iv) Determine the distribution function of X.

Q3. The diameter of an electric cable, say X, is assumed to be a continuous random variable with p.d.f :

$$F(x) = 6x(1-x), 0 \leq x \leq 1.$$

(i) check that $f(x)$ is p.d.f

(ii) Determine a number b such that $P(X < b) = P(X > b)$.

Q4. The probability distribution of a r.v .X is :

$$f(x) = k \sin \frac{1}{5} \pi x, 0 \leq x \leq 5.$$

Determine the constant k and obtain the median and quartiles of the distribution.

Q5. In a binomial distribution consisting of 5 independent trials, probabilities of 1 and 2 successes are 0.4096 and 0.2048 respectively. Find the parameter 'p' of the distribution.

Q6. The probability of a man hitting a target is $\frac{1}{4}$:

(i) If he fires 7 times what is the probability of his hitting the target at least twice?

(ii) How many times must he fire so that the probability of his hitting the target at least once is greater than $\frac{2}{3}$?

Q7. Find moment generating function of Binomial and Poisson distribution.

Q8. Find moments of Poisson distribution.

Q9. A car hire firm has two cars, which it hires out day by day. The number of demands for a car on each day is distributed as a Poisson distribution with mean 1.5 . calculate the proportion of days on which

(i) neither car is used

(ii) the proportion of days on which some demands is refused .

Q10. Define Normal Distribution and chief characteristics of normal distribution.

Q11. Find median and moment generating function of Normal distribution.

Q12. Let X is normally distributed and the mean of X is 12 and S.D. is 4 .

(a) (i) $X \geq 20$,

(ii) $X \leq 20$

(iii) $0 \leq x \leq 12$

(b) find x' , when $P(X > x') = 0.24$

Que13 . Define primary data, its method of collection and its merits and demerits.

Que14 . Define secondary data, its method of collection and its merits and demerits.

Que15 . Define classification of data and its types of classification with example.

Que16 . Define histogram and draw the histogram of the following data.

| | | | | | | |
|----------------|-----|-------|--------|--------|---------|---------|
| Age (in years) | 2-5 | 5- 11 | 11 -12 | 12 -13 | 13 - 14 | 14 - 15 |
| No. of boys | 150 | 306 | 497 | 477 | 496 | 143 |

Que17 . Define mean and its properties.

Que18 . a) Find the arithmetic mean of the following marks obtained by 10 students in statistics.

52,40,70,75,43,40,35,65,48,62

b) Given the following frequency distribution. Calculate the mean.

| | | | | | | | | | |
|----------------|---------|---------|---------|---------|---------|---------|---------|---------|---------|
| Monthly wages | 125-175 | 175-225 | 225-275 | 275-325 | 325-375 | 375-425 | 425-475 | 475-525 | 525-575 |
| No. of workers | 2 | 22 | 19 | 14 | 3 | 4 | 6 | 1 | 1 |

Que19. Calculate the mean and quartile deviation for the following frequency distribution of the no. of marks obtained by 49 students in a class.

| | | | | | | | | |
|-----------------|------|-------|-------|-------|-------|-------|-------|-------|
| Marks group | 5-10 | 10-15 | 15-20 | 20-25 | 25-30 | 30-35 | 35-40 | 40-45 |
| No. of students | 5 | 6 | 15 | | 10 | 5 | 4 | 2 |

Que20. Define mode and find the mode of the following data.

| | | | | | | | | |
|-------------------|------|-------|-------|-------|-------|-------|-------|-------|
| Marks obtained | 0-10 | 10-20 | 20-30 | 30-40 | 40-50 | 50-60 | 60-70 | 70-80 |
| No. of candidates | 33 | 108 | 221 | 439 | 529 | 495 | 322 | 153 |

Que21. Define harmonic mean, geometric mean and geometric mean for combined series.

Que22. Define exponential and uniform distribution and find its moments and moment generating function.

Que23. Define joint probability mass function and conditional probability function and for the joint probability of two random variables X and Y given below.

| | | | | |
|--------|------|------|------|------|
| Y X | 1 | 2 | 3 | 4 |
| 1 | 4/36 | 3/36 | 3/36 | 1/36 |
| 2 | 1/36 | 3/36 | 3/36 | 2/36 |
| 3 | 5/36 | 1/36 | 1/36 | 1/36 |
| 4 | 1/36 | 2/36 | 1/36 | 5/36 |

i) find marginal distribution of X & Y.

ii) find conditional distribution of X given the value Y = 1.

Que24. Define marginal probability density function of X and Y and conditional density function and find marginal density function of X and Y and conditional density function of Y when X=x and conditional density function of X given Y=y for the joint p.d.f of two dimensional random variable.

$$f(x,y) = \begin{cases} x^2 + y^2 & ; 0 < x < 1 ; 0 < y < 2 \\ 0 & ; \text{elsewhere} \end{cases}$$

Test whether X and Y are independent. For the above joint distribution find the conditional density of x given Y=y.

Que25. The average salary of Male employees in a firm was Rs. 5200 and that of Female was Rs.4200. The Mean salary of all employees was Rs.5000. Find the percentage of male & female employees.

Que26. The median and the mode of the following wage distribution are known to be Rs.3350 and Rs.3400 respectively. Find the values of f_3 and f_4 .

| | | | | | | | |
|------------------|--------|-----------|-----------|-----------|-----------|-----------|-----------|
| Wages in RS | 0-1000 | 1000-2000 | 2000-3000 | 3000-4000 | 4000-5000 | 5000-6000 | 6000-7000 |
| No. of employees | 4 | 16 | F_3 | F_4 | 40 | 6 | 4 |

Que27. The geometric mean of 10 observations on a certain variable was calculated as 16.2. It was later discovered that one of the observations was wrongly recorded as 12.9 in fact it was 21.9 apply appropriate corrections and calculate the correct geometric mean.

Que28. Define merits & demerits of all measure central tendencies.

Que29. Calculate:

i) Quartile Deviation (Q.D)

ii) Mean deviation (M.D) from mean for the following data.

| | | | | | | | |
|-----------------|------|-------|-------|-------|-------|-------|-------|
| Marks | 0-10 | 10-20 | 20-30 | 30-40 | 40-50 | 50-60 | 60-70 |
| No. of students | 6 | 5 | 8 | 15 | 7 | 6 | 8 |

Que30 . Find moments and β_1, β_2, Y_1 & Y_2 coefficients and find the nature of curve for the following data:

| | | | | | |
|-----------|-------|-------|-------|-------|--------|
| Score | 50-60 | 60-70 | 70-80 | 80-90 | 90-100 |
| frequency | 1 | 3 | 6 | 9 | 3 |

Que31. Define expectation and expected value of a Random variable for discrete & continuous.

Que32. Let x be R.V with probability distribution:

| | | | |
|----------|-------|-------|-------|
| X, x | -3 | 6 | 9 |
| $P(X=x)$ | $1/6$ | $1/2$ | $1/3$ |

Find $E(x)$ & $E(x^2)$ & $E(2x+1)^2$

Que33. Find the expectation of the No. on a die thrown.

Que34. Find the expectation of the sum of numbers obtained on two dies.

Que35. State and prove addition and multiplication theorem of expectation.

Que36. Define moment generating function and its properties.

Que37. Let the r.v x assume the value 'r' with the probability law: $p(x=r) = q^{r-1} p$: $r=1, 2, \dots$

Find M.g.f of x and hence its mean & variance.

Que38. The joint p.d.f of two random variables X and Y given by $f(x,y) = \frac{9(1+x+y)}{2(1+x)(1+y)}$; $0 < x < \infty, y >= 0$

Test whether X and Y are independent. For the above joint distribution find the conditional density of x given Y=y .

Ques 39. Define correlation coefficients and calculate the correlation coefficients for the following heights (in inches) of fathers(X)and their sons (Y):

| | | | | | | | | |
|---|----|----|----|----|----|----|----|----|
| X | 65 | 66 | 67 | 67 | 68 | 69 | 70 | 72 |
| Y | 67 | 68 | 65 | 68 | 72 | 72 | 69 | 71 |

Ques 40. A computer while calculating correlation coefficients between two variables X and Y from 25 pairs of observations obtained the following results:

$n=25$, $\sum x = 125$, $\sum y = 100$, $\sum xy = 508$, $\sum x^2=650$, $\sum y^2=460$. find correlation coefficients of the given data.

Ques 41. Obtain the equations of two lines of regression for the following data. Also obtain the estimate of X for Y =70.

| | | | | | | | | |
|---|----|----|----|----|----|----|----|----|
| X | 65 | 66 | 67 | 67 | 68 | 69 | 70 | 72 |
| Y | 67 | 68 | 65 | 68 | 72 | 72 | 69 | 71 |

Q42. For 10 randomly selected observations, the following data were recorded :

| | | | | | | | | | | |
|----------------------|---|---|---|----|---|----|----|----|----|----|
| Observation No. | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| Overtime hrs.(X) | 1 | 1 | 2 | 2 | 3 | 3 | 4 | 5 | 6 | 7 |
| Additional units (Y) | 2 | 7 | 7 | 10 | 8 | 12 | 10 | 14 | 11 | 14 |

Determine the coefficients of regression and regression equation using the non linear form:

$$Y = a+bx+cx^2.$$

Ques 43. Fit an exponential curve of the form $Y=ab^x$ to the following data :

| | | | | | | | | |
|---|-----|-----|-----|-----|-----|-----|-----|-----|
| X | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| Y | 1.0 | 1.2 | 1.8 | 2.5 | 3.6 | 4.7 | 6.6 | 9.1 |

Q44. Fit an power curve of the following data :

| | | | | | | |
|---|-----|-----|-----|-----|-----|-----|
| X | 1 | 2 | 3 | 4 | 5 | 6 |
| Y | 1.0 | 1.2 | 1.3 | 1.8 | 2.1 | 7.1 |

Q45. The Ranks of some 16 students in Math's and physics are as follows. Two no. in brackets denotes the ranks of the students in Math's and physics .

(1,1) ,(2,10) ,(3,3) ,(4,4) ,(5,5) ,(6,7) ,(7,2) ,(8,6) ,(9,8) ,(10,11) ,(11,15) ,(12,9) ,(13,14) ,(14,12) ,(15,16) ,(16,13).

Calculate the rank correlation coefficients for the proficiencies of this group in Math's and physics .

Q46. Fit a straight line to the following data :

| | | | | |
|---|-----|-----|----|----|
| X | 3 | 7 | 9 | 10 |
| Y | 168 | 120 | 72 | 63 |