Sr. No. 60413

YMCA UNIVERSITY OF SCIENCE & TECHNOLOGY, FARIDABAD

B.C.A., 1st SEMESTER

Mathematics (BCA-17-103)

Time: 3 Hours | Max. Marks: 75

Instructions: *1. It is compulsory to answer all the questions (1.5 marks each) of Part -A in short.*

2. Answer any four questions from Part -B in detail.

3. Different sub-parts of a question are to be attempted adjacent to each other.

PART -A

Q1 (a) If A and B are any two sets, then show that $A - B = A \cap B'$. (1.5)

(b) If A and B are symmetric matrices, then show that AB is symmetric iff $AB = BA$ i.e. A and B commute. (1.5)

(c) Without expanding evaluate the determinant $\begin{vmatrix} 41 & 1 & 5 \\ 79 & 7 & 9 \\ 29 & 5 & 3 \end{vmatrix}$ (1.5)

(d) What is an equivalence relation? Explain. (1.5)

(e) Define injection and surjection. (1.5)

(f) Explain the removable discontinuity of a function. (1.5)

(g) If $y = x + e^x$; then find d^2x/dy^2 (1.5)

(h) If $f(x) = x - 2$ and $g(x) = f(f(x))$, then find $g'(x)$. (1.5)

(i) Evaluate the integral

$$\int \frac{x + \sin x}{1 + \cos x} dx$$

(1.5)

(j) Let $f(x) = x - [x]$, for every real number x, where $[x]$ is the integral part of x. Then evaluate $\int_{-1}^{1} f(x)dx$. (1.5)

PART -B

Q2 (a) Show that the homogeneous system of equations $x - 2y + z = 0$, $x + y - z = 0$, $3x + 6y - 5z = 0$, has a non trivial solution. Also, find the solution. (10)

(b) In a battle 70% combatants lost one eye, 80% an ear, 75% an arm, 85% a leg. x% lost all the four limbs. Then, find the minimum value of x. (5)

Q3 (a) If ω is an imaginary cube root of unity, then evaluate the determinant:

$$\begin{vmatrix} 1+\omega & \omega^2 & -\omega \\ 1+\omega^2 & \omega & -\omega^2 \\ \omega^2+\omega & \omega & -\omega^2 \end{vmatrix}$$

(5)

(b) Prove that the relation R on the set $N \times N$ defined by $(a,b)\ R\ (c,d) \Leftrightarrow a+d=b+c$ for all $(a,b),\ (c,d) \in N \times N$ is an equivalence relation.

(10)

Q4 Evaluate:

(i) $\lim_{x\to\infty}\left(1+\frac{2}{x}\right)^x$ (ii) $\begin{vmatrix} b+c & a-b & a \\ c+a & b-c & b \\ a+b & c-a & c \end{vmatrix}$ (iii) $\lim_{x\to 0}\left(\frac{a^x+b^x+c^x}{3}\right)^{1/x}$

(15)

Q5 (a) If $u=e^x\sin x;\ v=e^x\cos x$, then prove that:

$$\frac{d^2v}{dx^2}+2u=0.$$

(5)

(b) Evaluate the integral:

$$\int\frac{1}{(x-1)\sqrt{x^2+4}}dx$$

(10)

Q6 (a) If $R=\{(x,y): x, y\in Z,\ x^2+y^2\le 4\}$ is a relation in Z, then find the domain of R.

(5)

(b) If $f: R\to R$ is a function defined by $f(x)=10x-7$, then find $f^{-1}(x)$. (5)

(c) Find the set of points where the function $f(x)=x|x|$ is differentiable. (5)

Q7 (a) Evaluate the integral:

$$\int_0^{\pi/4}\frac{\sin x+\cos x}{3+\sin 2x}dx$$

(5)

(b) If $y=\sin^{-1}\left(\frac{\sin\alpha\sin x}{1-\cos\alpha\sin x}\right)$, then find the value of $y'(0)$.

(5)

(c) Prove that every square matrix can be uniquely expressed as the sum of a symmetric matrix and a skew-symmetric matrix.

(5)

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