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NE 255

Homework 3

Problem 1: Attached in the end of this document.

Problem 2:
a) Attached in the end of this document.
b) Attached in the end of this document.
c) Quadrature are defined by directional cosines μ_l and their weight w. Order of the quadrature n specifies number of μ levels in [-1,1] interval. Because of quadrature being symmetric, finding values of the angles in half the range is sufficient. Hence number of μ levels is n/2. Also sum of all w in an octant is 1, since there are 8 octants in a sphere, w for the whole sphere is 8. Evaluated S_4, S_6, S_8, S_{10}, S_{12} and all of them came out to be 4π. This makes sense, as all the μ are unit vectors on the sphere.,

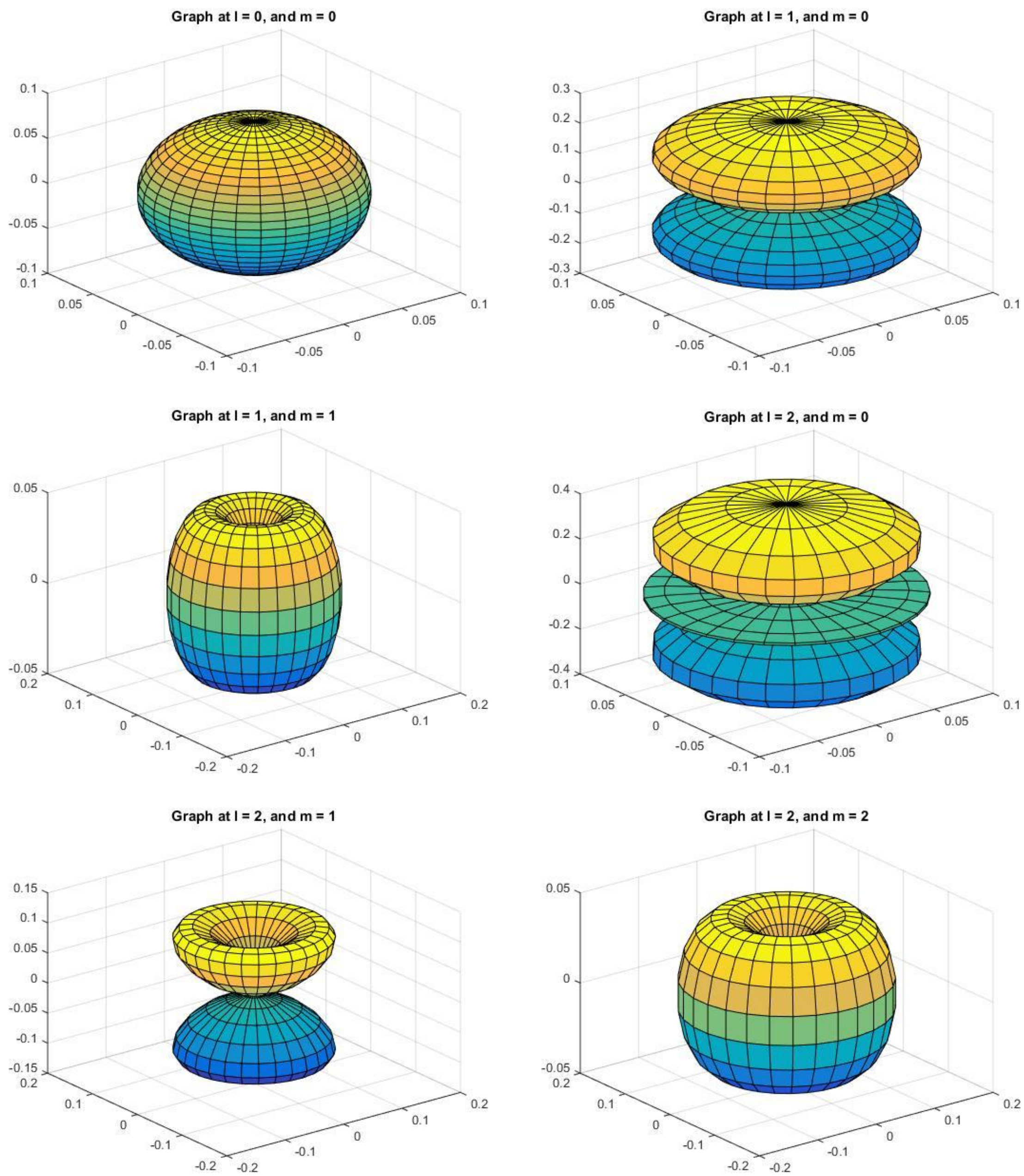
Problem 3:

a) Diffusion equation gives only ϕ as a solution and removes the angle dependence meaning the rates will only depend on scalar flux. When deriving the assumptions made that are made are: azimuthally symmetric, scattering is linearly anisotropic, angular flux is at most linearly anisotropic. To increase accuracy diffusion theory has discretized and homogenized space, discretized energy (few-group).
Deterministic method discretizes all independent variables, which obtain equation and then solve the using a numerical method. These solutions include truncation errors. Increase accuracy in deterministic methods requires discretized space, direction, and energy.
Monte Carlo methods require huge number of samples. In this method physics of every interaction of every particle is sampled until something statistically valid can be presented. In order to increase accuracy and runtime Monte Carlo methods need to have: continuous spatial resolution, continuous direction and energy representation.

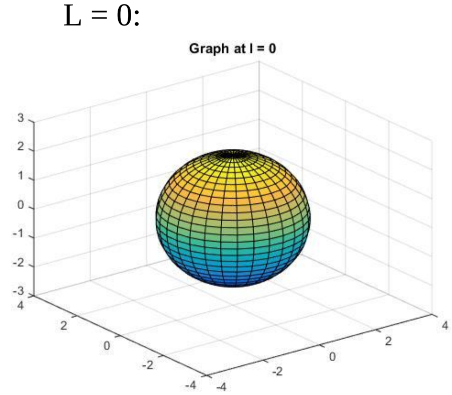
b) Deterministic methods are used by discretizing all variables, then solving the resulting equations which yield a solution with truncation errors. Deterministic methods require selecting grids for all independent variables. Commonly, this is easier than choosing parameters for Monte Carlo problems. Another advantage of deterministic method is that they are fast and solution is of same quality everywhere.
Disadvantages of deterministic method are that they are memory intensive, and difficult to parallelize on CPU. They are constrained by what you can mesh and as mentioned above solutions contain truncation errors. Lastly, solution qualities are governed by variable discretization.

Problem 4:

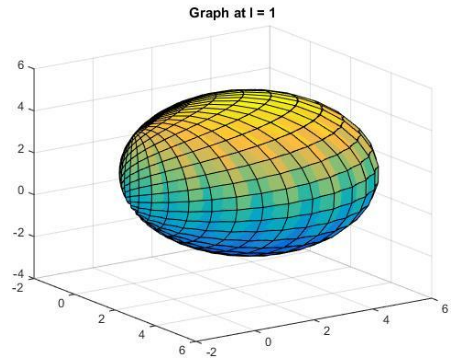
a) Plots:


Graph at l = 0, and m = 0
Graph at l = 1, and m = 0
Graph at l = 1, and m = 1
Graph at l = 2, and m = 0
Graph at l = 2, and m = 1
Graph at l = 2, and m = 2

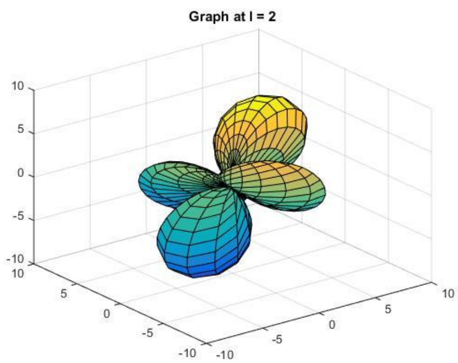
b) L = 0:


Graph at l = 0

L = 1:



L = 2:



Problem 5:
List of major data libraries and countries that manage them:

Acronym	*Name*	*Country/Agency*
ENDF	Evaluated Nuclear Data File	US and Canada
JEFF	Joint Evaluated Fission and Fusion File	Nuclear Energy Agency
JENDL	Japanese Evaluated Nuclear Data	Japan
CENDL	China Evaluated Nuclear Data Library	China
BROND	Library of Recommended Evaluated Neutron Data	Russia

PROBLEM 1:

Heuristic Equation of SP_N Equation:

$$\left(\frac{\ell'+1}{2\ell'+1}\right)\frac{d}{dx}\phi_{\ell'+1}(x)+\left(\frac{\ell'}{2\ell'+1}\right)\frac{d}{dx}\phi_{\ell'-1}(x)+\Sigma_t(x)\phi_{\ell'}=\Sigma_{s\ell'}(x)\phi_{\ell'}(x)+s_{\ell'}(x)$$

with $\phi_{-1}=0$; $\phi_{N+1}=0$ or $\frac{d}{dx}\phi_{N+1}=0$

2^{nd} order form of P_1 equations with Marshak boundary condition in diffusion equations:

$$-\frac{d}{dx}D\frac{d\phi_0}{dx}+\Sigma_a(x)\phi_0(x)=S_0(x) \qquad 0<x<X$$

$$\frac{1}{2}\phi_0(0)-D\frac{d\phi_0}{dx}(0)=2J^+(0) \qquad \text{where } D=\frac{1}{3[\Sigma_t(x)-\Sigma_{s\ell}(x)]}$$

$$\frac{1}{2}\phi_0(X)+D\frac{d\phi_0}{dx}(X)=2J^-(X)$$

For 3D: $\frac{d}{dx}D\frac{d}{dx}\longrightarrow\nabla\cdot D\nabla\equiv\frac{\partial}{\partial x}D\frac{\partial}{\partial x}+\frac{\partial}{\partial y}D\frac{\partial}{\partial y}+\frac{\partial}{\partial z}D\frac{\partial}{\partial z}$

$\pm\frac{d}{dx}\rightarrow\vec{n}\nabla$

3D Diffusion (P_1) equations

$$-\nabla\cdot D\nabla\phi_0(\vec{r})+\Sigma_a(\vec{r})\phi_0(\vec{r})=S_0(\vec{r}) \quad ,\ \vec{r}\in V$$

$$\frac{1}{2}\phi_0(\vec{r})+D\vec{n}\cdot\nabla\phi_0=2J^-(\vec{r}) \quad ,\ \vec{r}\in\partial V$$

$\phi_{\ell'}\rightarrow\vec{\phi}_{\ell'}\rightarrow(\phi_{\ell'}^x,\phi_{\ell'}^y,\phi_{\ell'}^z)^t$

Even ℓ', derivative replaced by divergence: $\frac{d}{dx}\rightarrow\nabla\cdot$

Odd ℓ', derivative changed to gradient: $\frac{d}{dx}\rightarrow\nabla$

First Order SP_N equations:

$$\nabla\cdot\vec{\phi}_1+\Sigma_a\phi_0=S_0$$

$$\left(\frac{\ell'+1}{2\ell'+1}\right)\nabla\phi_{\ell'+1}+\left(\frac{\ell'}{2\ell'+1}\right)\nabla\phi_{\ell'-1}+\Sigma_t\vec{\phi}_{\ell'}=\Sigma_{s\ell'}\vec{\phi}_{\ell'}+S_{\ell'} \quad ,\ \text{odd } \ell'$$

$$\left(\frac{\ell'+1}{2\ell'+1}\right)\nabla\cdot\vec{\phi}_{\ell'+1}+\left(\frac{\ell'}{2\ell'+1}\right)\nabla\cdot\vec{\phi}_{\ell'-1}+\Sigma_t\phi_{\ell'}=\Sigma_{s\ell'}\phi_{\ell'}+S_{\ell'} \quad ,\ \text{even } \ell'>0$$

<u>SP_5 Equations:</u>

$$\nabla\cdot\phi_1+\Sigma_a\phi_0=S_0$$

$$\frac{2}{3}\nabla\phi_2+\frac{1}{3}\nabla\phi_0+|\Sigma_t-\Sigma_{s1}|\vec{\phi}_1=0$$

$$\frac{3}{5}\nabla\vec{\phi}_3+\frac{2}{5}\nabla\vec{\phi}_1+|\Sigma_t-\Sigma_{s2}|\phi_2=0$$

$$\frac{4}{7}\nabla\phi_4+\frac{3}{7}\nabla\phi_2+|\Sigma_t-\Sigma_{s3}|\vec{\phi}_3=0$$

$$\frac{5}{9}\nabla\vec{\phi}_5+\frac{4}{9}\nabla\vec{\phi}_3+|\Sigma_t-\Sigma_{s4}|\phi_4=0$$

$$\frac{5}{11}\nabla\phi_4+|\Sigma_t-\Sigma_{s5}|\vec{\phi}_5=0$$

$$\vec{\phi}_5=-\frac{1}{\Sigma_t-\Sigma_{s5}}\left(\frac{5}{11}\nabla\phi_4\right)$$

$$\vec{\phi}_3=-\frac{1}{\Sigma_t-\Sigma_{s3}}\left(\frac{3}{7}\nabla\phi_2+\frac{4}{7}\nabla\phi_4\right)$$

$$\vec{\phi}_1=-\frac{1}{\Sigma_t-\Sigma_{s1}}\left(\frac{1}{3}\nabla\phi_0+\frac{2}{3}\nabla\phi_2\right)$$

PROBLEM 2:

From lecture notes:

$$\sum_{a=1}^{\frac{N(N+2)}{8}} w_a = 1 \Rightarrow \sum_{a=1}^{N(N+2)} w_a = 8$$

$$\int_{4\pi} d\hat{\Omega} = \frac{4\pi}{8} \sum_{a=1}^{N} w_a = 4\pi$$

$$\int_{4\pi} d\hat{\Omega}\, |\hat{\Omega}| = 4\pi \sum_{a=1}^{N/8} w_a |\Omega| = 4\pi$$

a) S_4 Quadrature:

(μ_1, η_1, ξ_2)

1

$1 \quad 1 \rightarrow (\mu_2, \eta_1, \xi_1)$

(μ_1, η_2, ξ_1)

$\mu = \eta = \xi$

$$4\pi \left[w_1 \left(\sqrt{\mu_1^2 + \mu_1^2 + \mu_2^2} \right) + w_1 \left(\sqrt{\mu_1^2 + \mu_2^2 + \mu_1^2} \right) + w_1 \left(\sqrt{\mu_2^2 + \mu_1^2 + \mu_1^2} \right) \right]$$

$$= 4\pi \left[3 w_2 \sqrt{2\mu_1^2 + \mu_2^2} \right]$$

$$= 4\pi \underbrace{\left[3(0.3333) \sqrt{2(.3500212)^2 + (.8688903)^2} \right]}_{1}$$

$\boxed{S_4 \text{ LQ}_N \text{ quadrature set gave } 4\pi}$

b) S_6 Quadrature:

$(\mu_2, \eta_1, \xi_2) \quad 1 \rightarrow (\mu_1, \eta_1, \xi_3)$

$2 \quad 2 \rightarrow (\mu_1, \eta_2, \xi_2)$

$1 \quad 2 \quad 1 \rightarrow (\mu_1, \eta_3, \xi_1)$

$(\mu_3, \eta_1, \xi_1) \quad (\mu_2, \eta_2, \xi_1)$

$\mu = \eta = \xi$

$$4\pi \left[w_1 \left(\sqrt{\mu_1^2 + \eta_1^2 + \xi_3^2} \right) + w_1 \left(\sqrt{\mu_1^2 + \eta_3^2 + \xi_1^2} \right) + w_1 \left(\sqrt{\mu_3^2 + \eta_1^2 + \xi_1^2} \right) + w_2 \left(\sqrt{\mu_1^2 + \eta_2^2 + \xi_2^2} \right) + w_2 \left(\sqrt{\mu_2^2 + \eta_2^2 + \xi_1^2} \right. + w_2 \left(\sqrt{\mu_2^2 + \eta_1^2 + \xi_2^2} \right) \right]$$

$$= 4\pi \left[3 w_1 \sqrt{2\mu_1^2 + \mu_3^2} + 3 w_2 \sqrt{\mu_1^2 + 2\mu_2^2} \right]$$

$$= 4\pi \underbrace{\left[3(.1761263) \sqrt{2(.2666355)^2 + (.9261808)^2} + 3(.1572071) \sqrt{(.2666355)^2 + 2(.6815076)^2} \right]}_{1}$$

$\boxed{S_6 \text{ LQ}_N \text{ quadrature set gave } 4\pi}$

c) Attached Code & Document