Lakshay Seth
NE 255

Homework 6

Problem 1: See attached below

Problem 2:

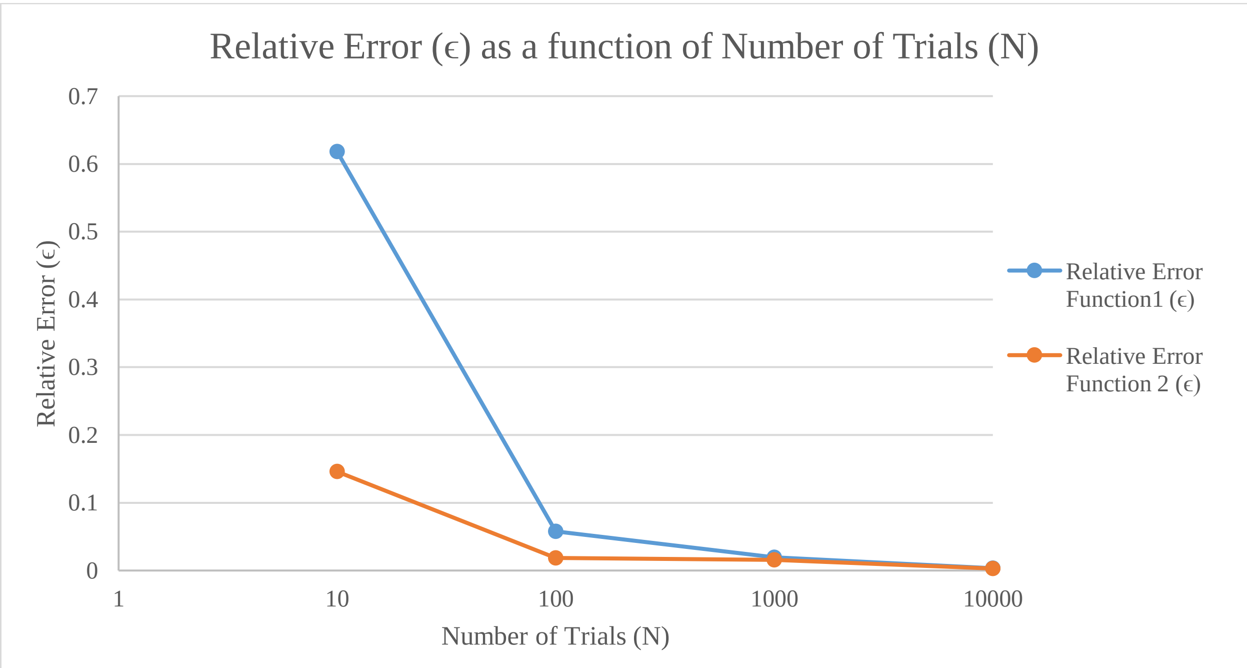
a) Relative Error using first function for the different samples (N) are:

Number of Samples (N)	Relative Error (ϵ)
10	0.6180
100	0.0578
1000	0.0196
10000	0.0034

b) Relative Error using second function for different samples (N) are:

Number of Samples (N)	Relative Error (ϵ)
10	0.1459
100	0.0186
1000	0.0158
10000	0.0029

Relative error decreases as a function of number of trials. Below is the graph of a sample run. It's important to note than when values are generating at random, it could be possible that small sample size (N) can converge faster than larger sample size (N). But the results from larger sample size will always be more consistent.



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HW6

Problem 1:

a) Neutron direction in 3D if neutron source is isotropic:

$\Omega = \Omega(\theta, \phi) = (u, v, w)$ $\qquad d\Omega = \sin\theta \, d\theta \, d\phi$ $\qquad f(x,y) = f_1(x) f_2(y)$

$$\frac{d\Omega}{4\pi} = \frac{\sin\theta \, d\theta \, d\phi}{4\pi} = \frac{-\partial(\cos\theta)\, d\phi}{4\pi} \qquad \mu = \cos\theta$$

$$= \frac{-\partial\mu \, d\phi}{4\pi}$$

$$f(\Omega) = f_1(\mu) f_2(\phi)$$

$$= \frac{1}{2} \frac{1}{2\pi} = \frac{1}{4\pi}$$

$$F_1(\mu) = \int_{-1}^{\mu} f_1(\mu') \, d\mu'$$

$$= \frac{1}{2}(\mu + 1) = \xi_1 \implies \boxed{\mu = 2\xi_1 - 1}$$

$$F_2(\phi) = \int_0^{\phi} f_2(\phi') \, d\phi'$$

$$= \frac{\phi}{2\pi} = \xi_2 \implies \boxed{\phi = 2\pi \xi_2}$$

b) Three material with radius R_1, R_2, R_3 and total cross section $\Sigma_{t1}, \Sigma_{t2}, \Sigma_{t3}$.

$$P_c \, ds = \Sigma_t \exp(-\Sigma_t s) \, ds$$

$$P_c(s) = \int_0^s \Sigma_t \exp(-\Sigma_t s') \, ds' \implies 1 - e^{-\Sigma_t(s) s}$$

Σ_t function of s, piecewise const but changing.

$n = \Sigma_t s$

$dn = \Sigma_t \, ds$

$$P_c(n) = \int_0^n e^{-n'} dn' = 1 - e^{-n}$$

$$g(n_c) \, dn_c = e^{-n_c} dn_c$$

$$\underbrace{G(n_c) \, dn_c}_{\xi} = 1 - e^{-n_c} \implies \boxed{\begin{aligned} n_c &= -\ln(1-\xi) \\ &= -\ln(\xi) \end{aligned}}$$

$$n_1 = S_{R_1} \Sigma_{t1} \; ; \; n_2 = S_{R_2} \Sigma_{t2} \; ; \; n_3 = S_{R_3} \Sigma_{t3}$$

n_c (Mean free path until next collision)

n_b (Mean free path until next boundary)

$n_b > n_c$: Boundary further from collision. Collision occurs.
Solve for distance.
Sample new n_c and calculate new n_b

$n_b < n_c$: Boundary closer than collision. Cross into next boundary
Store distance traveled. ($n_{dist} = n_{dist} + (n_c - n_b)$)
Update $n_c = n_c - n_b$.
Calculate new n_b with new boundaries and Σ_t.

c) Collision can be elastic, inelastic, absorption, capture interaction.

$$\Sigma_{tot} = \Sigma_{elast} + \Sigma_{inelast} + \Sigma_{abs} + \Sigma_{capt}$$

$$P_1 = \frac{\Sigma_{elast}}{\Sigma_{tot}} \qquad P_2 = \frac{\Sigma_{inelast}}{\Sigma_{tot}} \qquad P_3 = \frac{\Sigma_{abs}}{\Sigma_{tot}} \qquad P_4 = \frac{\Sigma_{capt}}{\Sigma_{tot}}$$

$$P_i = \frac{\Sigma_i}{\Sigma_t}$$

$$1 = \sum_{i=1}^{4} P_i$$

Elastic:

$$0 < \xi < P_1$$

Inelastic:

$$P_1 < \xi < P_1 + P_2$$

Absorbed:

$$P_1 + P_2 < \xi < P_1 + P_2 + P_3$$

Capture:

$$P_1 + P_2 + P_3 < \xi < P_1 + P_2 + P_3 + P_4$$