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NE 255

HOMEWORK 4

* Problem 1: Sweeping process in one dimension along one angle using diamond diff scheme:

$$\mu_n \frac{d}{dx} \psi_n^{m+1}(x) + \sigma(x)\psi_n^{m+1}(x) = q_n^m(x)$$

Half integer meshpoints. Cell Centred points are:

$$x_i = \frac{1}{2}\left[x_{i+\frac{1}{2}} + x_{i-\frac{1}{2}}\right]$$

Define:

$$\sigma(x) = \sigma_i \,;\; x_{i-\frac{1}{2}} < x < x_{i+\frac{1}{2}}$$

Integrate over $x_{i-\frac{1}{2}}$ and $x_{i+\frac{1}{2}}$:

$$\mu_n\left[\psi_n(x_{i+\frac{1}{2}}) - \psi_n(x_{i-\frac{1}{2}})\right] + \sigma_i \Delta_i \psi_n(x_i) = \Delta_i q_n(x_i)$$

where: $\Delta_i = x_{i+\frac{1}{2}} - x_{i-\frac{1}{2}}$

& $\psi_{in} \equiv \psi_n(x_i)$

$$\mu_n\left[\psi_{i+\frac{1}{2},n} - \psi_{i-\frac{1}{2},n}\right] + \sigma_i \Delta_i \psi_{in} = \Delta_i q_{in}$$

∴ Diamond difference scheme has $\alpha = 0$, whereas step difference sets have $\alpha = \pm 1$.

$$\psi_{in} = \frac{1}{2}\left[\psi_{i+\frac{1}{2},n} - \psi_{i-\frac{1}{2},n}\right]$$

a) for $\mu > 0$:

$\psi_{i-\frac{1}{2}} \Rightarrow$ Incoming flux ; $\psi_{i+\frac{1}{2}} \Rightarrow$ Outgoing flux

Eliminate $\psi_{i+\frac{1}{2},n}$

$$\psi_{in} = \left(1 + \frac{\sigma_i \Delta_i}{2|\mu_n|}\right)^{-1}\left[\psi_{i-\frac{1}{2},n} + \frac{\Delta_i q_{in}}{2|\mu_n|}\right]$$

$$\boxed{\psi_{i+\frac{1}{2},n} = 2\psi_{in} - \psi_{i-\frac{1}{2},n}}$$

b) for $\mu < 0$:

$\psi_{i+\frac{1}{2}} \Rightarrow$ Incoming flux ; $\psi_{i-\frac{1}{2}} \Rightarrow$ Outgoing flux

Eliminate $\psi_{i-\frac{1}{2},n}$

$$\psi_{in} = \left(1 + \frac{\sigma_i \Delta_i}{2|\mu_n|}\right)^{-1}\left[\psi_{i+\frac{1}{2},n} + \frac{\Delta_i q_{in}}{2|\mu_n|}\right]$$

$$\boxed{\psi_{i-\frac{1}{2},n} = 2\psi_{in} - \psi_{i+\frac{1}{2},n}}$$

c) Reflecting boundary on right edge, have to transition from $\mu > 0$ to $\mu < 0$:

∴ Boundary values determine the relationship. We start there and sweep until reflecting boundary.

Initial Condition: $\psi_{I+\frac{1}{2},N+1-n} = \psi_{I+\frac{1}{2},n}$

d) What data to store in sweeping process?

When using angular flux to generate flux moments during solution process, we need to store $\boxed{\text{flux moments}}$ from one iteration to next.

* Problem 2:

$$\mu_a \frac{d\psi_a}{dx} + \Sigma_t \psi_a(x) = 0 \Rightarrow \mu_a \frac{d\psi_a}{dx} = -\Sigma_t \psi_a(x)$$

a) $\mu_a > 0$; Expression for flux at some location x' in terms of x:

$$\frac{1}{\psi_a} d\psi_a = \frac{-\Sigma_t}{\mu_a} dx$$

$$\int_x^{x'} \frac{1}{\psi_a} d\psi_a = \int_x^{x'} \frac{-\Sigma_t}{\mu_a} dx \Rightarrow \ln \psi_a(x') - \ln \psi_a(x) = \frac{-\Sigma_t}{\mu_a}(x'-x)$$

$$\boxed{\psi_a(x') = \psi_a(x) \exp\left[\frac{-\Sigma_t}{\mu_a}(x'-x)\right]}$$

b) Expression for $\psi_{a,i+\frac{1}{2}}$ in terms of $\psi_{a,i-\frac{1}{2}}$? $\Delta_i = x_{i+\frac{1}{2}} - x_{i-\frac{1}{2}}$ and $h \equiv \frac{\Sigma_t \Delta_i}{2|\mu_a|}$

$$\psi_{a,i+\frac{1}{2}} = \psi_{a,i-\frac{1}{2}} \exp\left[\frac{-\Sigma_t}{\mu_a}\Delta_i\right]$$

$$\boxed{\psi_{a,i+\frac{1}{2}} = \psi_{a,i-\frac{1}{2}} \exp[-2h]}$$

c) $\psi_{a,i+\frac{1}{2}}$ in terms of $\psi_{a,i-\frac{1}{2}}$ and h?

From notes and $\alpha = 0$, we get these two equations:

$$\frac{\mu}{\Delta_i}\left[\psi_{a,i+\frac{1}{2}} - \psi_{a,i-\frac{1}{2}}\right] + \Sigma_{ijk}\psi_{ijk} = 0$$

$\alpha = 0$:

$$\psi_{ijk} = \frac{1}{2}\left[\psi_{a,i+\frac{1}{2}} + \psi_{a,i-\frac{1}{2}}\right]$$

$$\frac{\mu}{\Delta_i}\left(\psi_{a,i+\frac{1}{2}} - \psi_{a,i-\frac{1}{2}}\right) = -\frac{\Sigma_{ijk}}{2}\left(\psi_{a,i+\frac{1}{2}} + \psi_{a,i-\frac{1}{2}}\right)$$

$$\frac{\mu}{\Delta_i}\psi_{a,i+\frac{1}{2}} + \frac{1}{2}\Sigma_{ijk}\psi_{a,i+\frac{1}{2}} = \frac{\mu}{\Delta_i}\psi_{a,i-\frac{1}{2}} - \frac{1}{2}\Sigma_{ijk}\psi_{a,i-\frac{1}{2}}$$

$$\psi_{a,i+\frac{1}{2}}\left[\frac{\mu}{\Delta_i} + \frac{1}{2}\Sigma_{ijk}\right] = \psi_{a,i-\frac{1}{2}}\left[\frac{\mu}{\Delta_i} - \frac{1}{2}\Sigma_{ijk}\right]$$

$$\psi_{a,i+\frac{1}{2}}\left[1 + \underbrace{\frac{\Sigma_i \Delta_i}{2\mu}}_{h}\right] = \psi_{a,i-\frac{1}{2}}\left[1 - \underbrace{\frac{\Sigma_i \Delta_i}{2\mu}}_{h}\right]$$

$$\boxed{\psi_{a,i+\frac{1}{2}} = \frac{1-h}{1+h}\psi_{a,i-\frac{1}{2}}}$$

d) Accuracy of the relationship:

Part b exp through power series: $\psi_{a,i+\frac{1}{2}} = \psi_{a,i-\frac{1}{2}} e^{-2h}$

$$= 1 + (-2h) + \frac{(-2h)^2}{2!} = \underline{1 - 2h + 2h^2 + \ldots}$$

Part c through power series: $\psi_{a,i+\frac{1}{2}} = \psi_{a,i-\frac{1}{2}}\left(\frac{1-h}{1+h}\right)$

$$= \underline{1 - 2h + 2h^2 + \ldots}$$

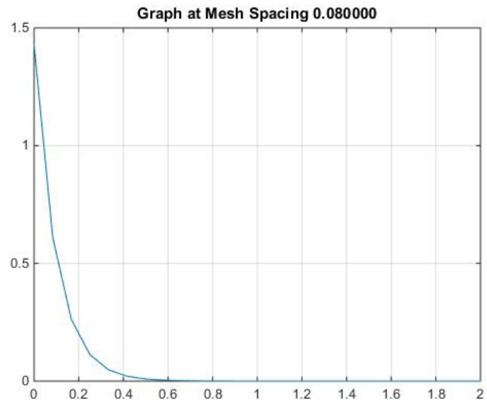
Same through the second order term meaning these relationship is second order accurate

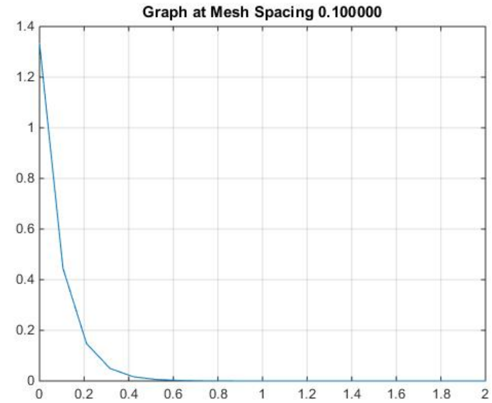
c) To avoid negative flux $h < 1$ meaning $\frac{\Sigma_t \Delta i}{2|\mu|} < 1 \Rightarrow \Delta i < \frac{2|\mu|}{\Sigma_t}$ is required.

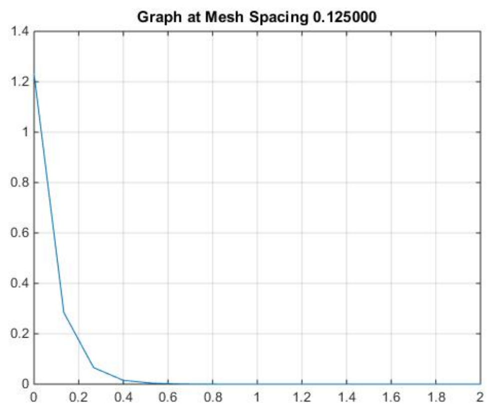
When Σ_t is large and μ_n is small, mesh spacing can become problematic and result in negative flux. More S_N order increases more mesh size should decrease.

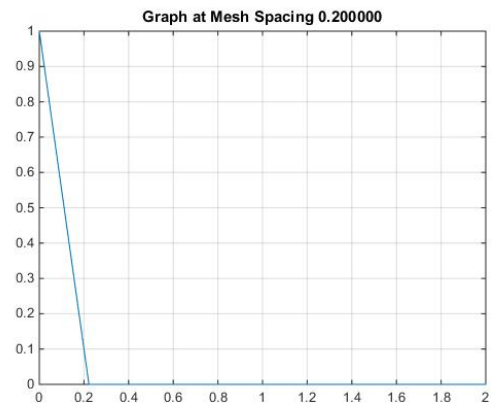
Problem 3:

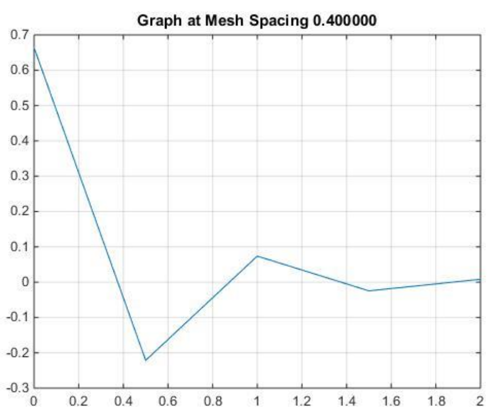
a) Cell-Centered flux:







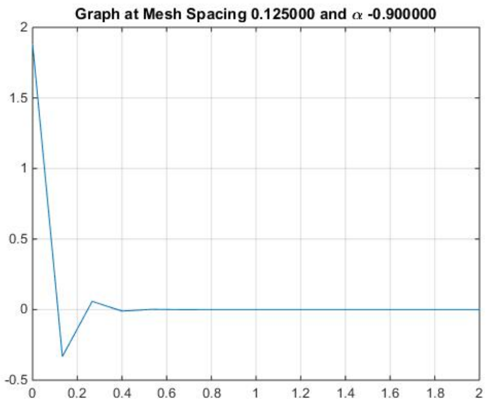


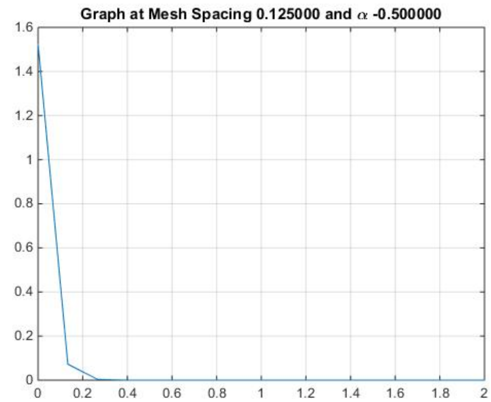


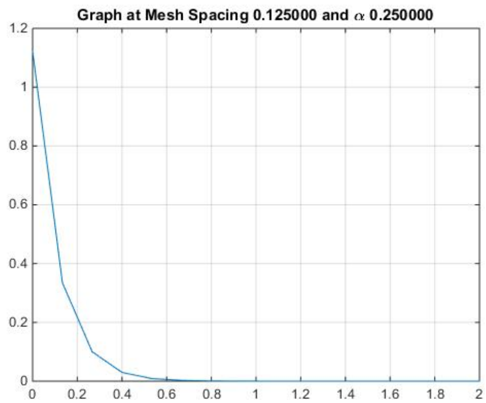
In this case, $\alpha = 0$ for all graphs. When $\Delta_i = 0.4$ we observe negative flux. This is the case as requirement stated in problem 2; mesh spacing (Δ_i) $< 2\mu/\Sigma_t$ doesn't hold true ($0.4 < 0.2$).

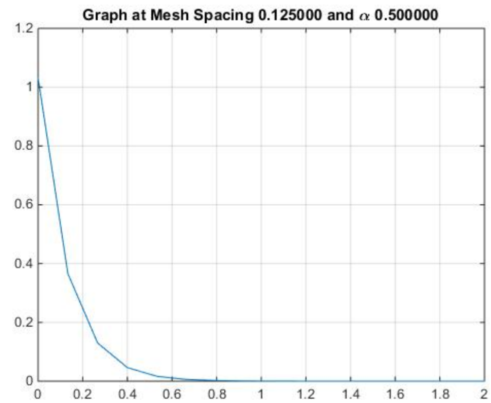
b) Impact of α when $\alpha = [-0.9, -0.5, 0.25, 0.5, 0.9]$.

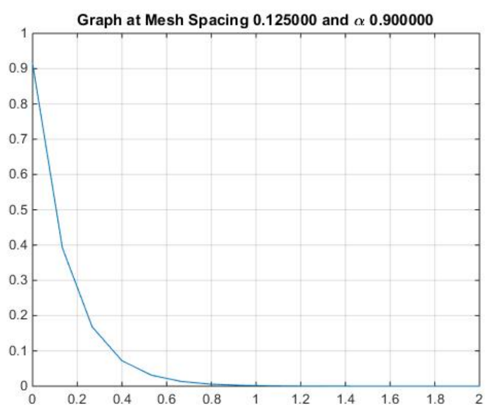
Let's look at different α's at $\Delta_i = 0.125$. Graphs at other Δ_i are present in the folder.







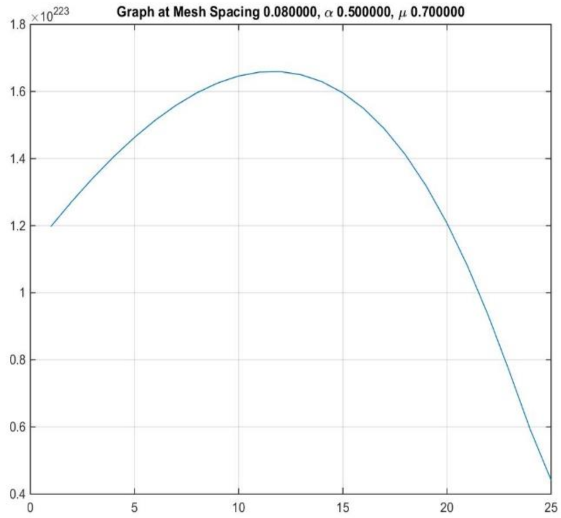


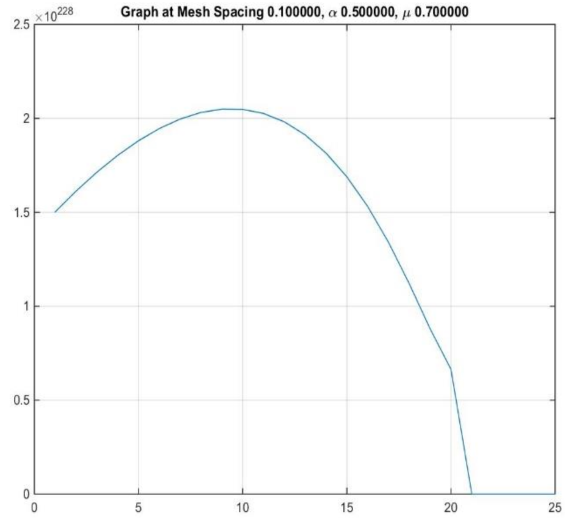


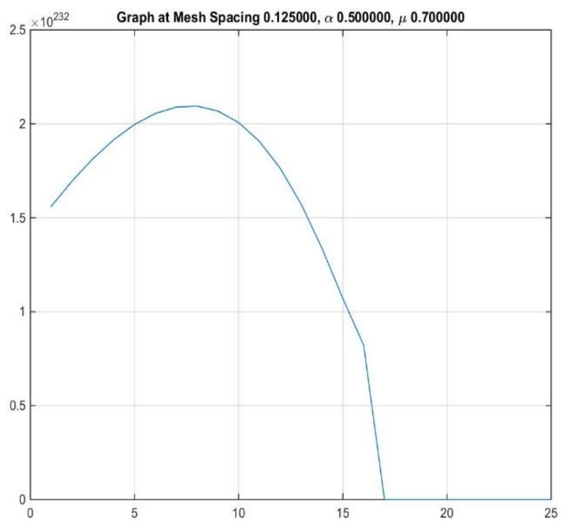
When α term is introduced, the requirement stated in problem 2 is changed by factor of $1+\alpha$ in this case because of going $x = 0$ to $x = 2$. $\Delta_i * (1+\alpha) < 2\mu/\Sigma_t$ should hold true to prevent negative flux.

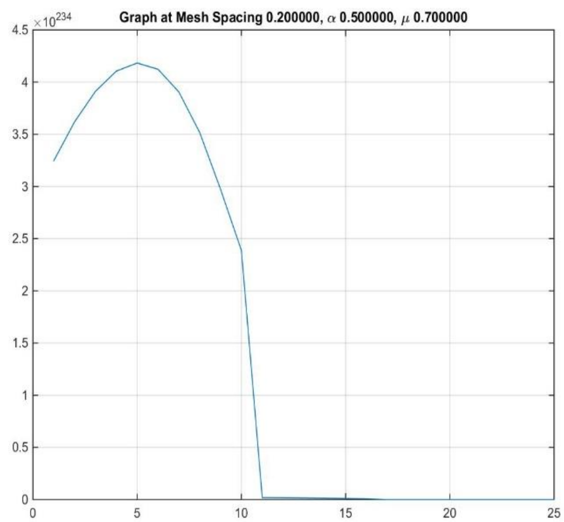
c) Adding source:

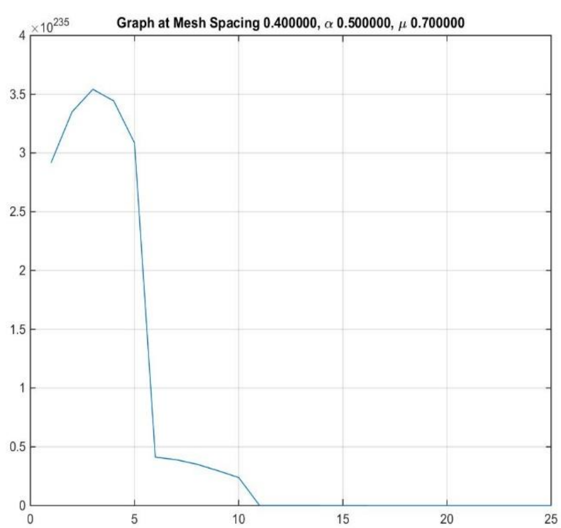
Graphs at $\mu = 0.7$, $\alpha = 0.5$, and for all mesh spaces here. Other graphs are in the folder.


Graph at Mesh Spacing 0.080000, α 0.500000, μ 0.700000
$\times 10^{223}$
1.8
1.6
1.4
1.2
1
0.8
0.6
0.4
0
5
10
15
20
25


Graph at Mesh Spacing 0.100000, α 0.500000, μ 0.700000
$\times 10^{228}$
2.5
2
1.5
1
0.5
0
0
5
10
15
20
25


Graph at Mesh Spacing 0.125000, α 0.500000, μ 0.700000
$\times 10^{232}$
2.5
2
1.5
1
0.5
0
0
5
10
15
20
25


Graph at Mesh Spacing 0.200000, α 0.500000, μ 0.700000
$\times 10^{234}$
4.5
4
3.5
3
2.5
2
1.5
1
0.5
0
0
5
10
15
20
25


Graph at Mesh Spacing 0.400000, α 0.500000, μ 0.700000
$\times 10^{235}$
4
3.5
3
2.5
2
1.5
1
0.5
0
0
5
10
15
20
25

d) $\alpha = 0$, $\Sigma_s = 0.9$:

As we increases Σ_s, the scalar flux increase and there is point of inflection on the graphs.

* Problem 4:

$$\frac{\mu_a}{h_i}\left(\psi^g_{a,i+\frac{1}{2}} - \psi^g_{a,i-\frac{1}{2}}\right) + \Sigma^g_{t,i}\,\psi^g_{a,i} = 2\pi \sum_{a=1}^{N} w_a \sum_{g'=1}^{G} \Sigma^{gg'}_{s,i}(a'\to a)\,\psi^{g'}_{a',i} + \frac{\chi_g}{2}\sum_{g'=1}^{G} \nu_{g'}\,\Sigma^{g'}_{f,i}\,\phi^{g'}_i + \frac{1}{2}Q^g_i$$

Neutrons only downscatter from fast group (1 and 2) to thermal group (3, 4, 5). Assume thermal groups can upscatter in other thermal groups and can downscatter. Assume there is an external source and fission source.

① $$\frac{\mu_a}{h_i}\left(\psi^1_{a,i+\frac{1}{2}} - \psi^1_{a,i-\frac{1}{2}}\right) + \Sigma^1_{t,i}\,\psi^1_{a,i} = \cancelto{0}{2\pi \sum_{a=1}^{N} w_a \sum_{g'=1}^{G} \Sigma^{1g'}_{s,i}(a'\to a)\,\psi^{g'}_{a',i}} + \frac{\chi_1}{2}\sum_{g'=1}^{G} \nu_{g'}\,\Sigma^{g'}_{f,i}\,\phi^{g'}_i + \frac{1}{2}Q^1_i$$

$$\boxed{\frac{\mu_a}{h_i}\left(\psi^1_{a,i+\frac{1}{2}} - \psi^1_{a,i-\frac{1}{2}}\right) + \Sigma^1_{t,i}\,\psi^1_{a,i} = \frac{\chi_1}{2}\sum_{g'=1}^{G} \nu_{g'}\,\Sigma^{g'}_{f,i}\,\phi^{g'}_i + \frac{1}{2}Q^1_i}$$

② $$\frac{\mu_a}{h_i}\left(\psi^2_{a,i+\frac{1}{2}} - \psi^2_{a,i-\frac{1}{2}}\right) + \Sigma^2_t\,\psi^2_{a,i} = \cancelto{0}{2\pi \sum_{a=1}^{N} w_a \sum_{g'=1}^{G} \Sigma^{2g'}_{s,i}(a'\to a)\,\psi^{g'}_{a',i}} + \frac{\chi_2}{2}\sum_{g'=1}^{G} \nu_{g'}\,\Sigma^{g'}_{f,i}\,\phi^{g'}_i + \frac{1}{2}Q^2_i$$

$$\boxed{\frac{\mu_a}{h_i}\left(\psi^2_{a,i+\frac{1}{2}} - \psi^2_{a,i-\frac{1}{2}}\right) + \Sigma^2_{t,i}\,\psi^2_{a,i} = \frac{\chi_2}{2}\sum_{g'=1}^{G} \nu_{g'}\,\Sigma^{g'}_{f,i}\,\phi^{g'}_i + \frac{1}{2}Q^2_i}$$

③ $$\frac{\mu_a}{h_i}\left(\psi^3_{a,i+\frac{1}{2}} - \psi^3_{a,i-\frac{1}{2}}\right) + \Sigma^2_{t,i}\,\psi^3_{a,i} = 2\pi \sum_{a=1}^{N} w_a \sum_{g'=1}^{G} \Sigma^{3g'}_{s,i}\,\psi^{g'}_{a',i} + \cancelto{0}{\frac{\chi_3}{2}\sum_{g'=1}^{G} \nu_{g'}\,\Sigma^{g'}_{f,i}\,\phi^{g'}_i} + \frac{1}{2}Q^3_i$$

$$\boxed{\frac{\mu_a}{h_i}\left(\psi^3_{a,i+\frac{1}{2}} - \psi^3_{a,i-\frac{1}{2}}\right) + \Sigma^3_{t,i}\,\psi^3_{a,i} = 2\pi \sum_{a=1}^{N} w_a \sum_{g'=1}^{G} \Sigma^{3g'}_{s,i}\,\psi^{g'}_{a',i} + \frac{1}{2}Q^3_i}$$

④ $$\frac{\mu_a}{h_i}\left(\psi^4_{a,i+\frac{1}{2}} - \psi^4_{a,i-\frac{1}{2}}\right) + \Sigma^4_{t,i}\,\psi^4_{a,i} = 2\pi \sum_{a=1}^{N} w_a \sum_{g'=1}^{G} \Sigma^{4g'}_{s,i}\,\psi^{g'}_{a',i} + \cancelto{0}{\frac{\chi_4}{2}\sum_{g'=1}^{G} \nu_{g'}\,\Sigma^{g'}_{f,i}\,\phi^{g'}_i} + \frac{1}{2}Q^4_i$$

$$\boxed{\frac{\mu_a}{h_i}\left(\psi^4_{a,i+\frac{1}{2}} - \psi^4_{a,i-\frac{1}{2}}\right) + \Sigma^4_{t,i}\,\psi^4_{a,i} = 2\pi \sum_{a=1}^{N} w_a \sum_{g'=1}^{G} \Sigma^{4g'}_{s,i}\,\psi^{g'}_{a',i} + \frac{1}{2}Q^4_i}$$

⑤ $$\frac{\mu_a}{h_i}\left(\psi^5_{a,i+\frac{1}{2}} - \psi^5_{a,i-\frac{1}{2}}\right) + \Sigma^5_{t,i}\,\psi^5_{a,i} = 2\pi \sum_{a=1}^{N} w_a \sum_{g'=1}^{G} \Sigma^{5g'}_{s,i}\,\psi^{g'}_{a',i} + \cancelto{0}{\frac{\chi_5}{2}\sum_{g'=1}^{G} \nu_{g'}\,\Sigma^{g'}_{f,i}\,\phi^{g'}_i} + \frac{1}{2}Q^5_i$$

$$\boxed{\frac{\mu_a}{h_i}\left(\psi^5_{a,i+\frac{1}{2}} - \psi^5_{a,i-\frac{1}{2}}\right) + \Sigma^5_{t,i}\,\psi^5_{a,i} = 2\pi \sum_{a=1}^{N} w_a \sum_{g'=1}^{G} \Sigma^{5g'}_{s,i}\,\psi^{g'}_{a',i} + \frac{1}{2}Q^5_i}$$