Finance

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1. Use of Various Probability Distributions in Finance

Variability is one of the most important concepts in finance. To model variability along with changes, we need the concept of a random variable. A random variable can denote :

- The return on an investment
- The price of an equity
- The sales volume of a store
- The turnover rate at your organization

The financial model can be divided into 2 categories :

- Discrete Time model
- Continuous Time model

1.1. Discrete Distributions

1.1.1 Bernoulli Distribution

It is one of the simplest distributions when it comes to Finance. A random variable X is said to follow a **Bernoulli distribution** with a parameter p if it takes values any two vales (say 1 and 0) with probability p and $1-p$.

The use of this distribution in finance is pedogogical. In one period model it is convinient to modelize variations of logarithms of a stock price by Bernoulli distribution. If the day-0 price is S_0 then the day-1 price (S_1) can be uS_0 or dS_0.

$$ln(S_1) = ln(S_0) + X$$

where X is a Bernpoulli random variable taking values ln(u) and ln(d), u and d being up and down levels respectively.

1.1.2 Binomial Distribution

Stock returns in one period were represented by a Bernoulli distribution. Now considering a multi-period model and successive returns to be independent (Binomial) Bernoulli random variables. If we assume the parameters u and d to be constant over time (constant volitality) the log price variations are driven by binomial distributions. Binomial distribution was also the foundation of famous option valuaton model developed by Cox-Ross-Rubinstein (1979). The model states that :

$$S_{t+1} = S_t \times X_{t+1}$$

where S_t is day-t stock price s_{t+1} is the price of stock on $(t+1)^t h$ day and X_{t+1} is the up/down level of the stock (i.e. u or d) with probabilities p and 1-p. As said earlier that the random variables $X_1, X_2, ...$ are independent. From the above equation S_t can also be written as

$$S_t = S_0 \times \Pi_{s=1}^{t} X_t$$

this can be rewritten as

$$ln(\frac{S_t}{S_0}) = \sum_{s=1}^{t} X_t$$

The probabilty of stock being $k \cdot u$ price up after t days is :

$$ln((S_t) = ln(S_0) + k \cdot u) = \binom{n}{k} p^k (1-p)^{t-k}$$

The above models also gives us some moments of log returns :

$$E[ln(\frac{S_t}{S_0})] = t(pln(u) + (1-p)ln(d))$$

$$\sigma^2[ln(\frac{S_t}{S_0})] = tp(1-p)ln(\frac{u}{d})^2$$

1.2. Continuous Distribution

1.2.1 Guassian Distribution

It is one of the most common and most used probability distribution. It is the best distribution with respect to value of stocks. The pdf of a gaussian random variable with expectation μ and deviation σ is :

$$\frac{1}{\sqrt{2\pi\sigma^2}} e^{\frac{-1}{2}(\frac{x-\mu}{\sigma})^2}$$

The curve of f_X of Gaussian Distribution is parallel to the line $x = \mu$. Many other mathematical and statistical processes have been derived using Gaussian distribution.

1.2.2 Log-Normal Distribution

It is a derived distribution from Gaussian. If the day-0 price is S_0 and day t price is S_t then

$$r = ln(\frac{S_t}{S_0})$$

It can be rewritten as $S_t = S_0 \cdot e^r$. The Log-Normal Distribution for parameters (μ and σ) [1] is given by :

$$f_X(x) = \begin{cases} \frac{1}{x \cdot \sqrt{2\pi\sigma^2}} e^{\frac{-1}{2}(\frac{ln(x)-\mu}{\sigma})^2} & \text{if } x > 0 \\ 0 & otherwise \end{cases}$$

It is also very useful while determining stock prices. It is base for a very famous option valuation model namely Black-Scholes.

$$E(X) = e^{(\mu + \frac{\sigma^2}{2})}$$

$$Var(X) = e^{(2\mu + \sigma^2)} \cdot (e^{\sigma^2} - 1)$$

[1]Here μ and σ are the mean and deviation of Gaussian random variable respectively