Closed Loop Control

Control input: D

Output: Vout

$$P = \frac{V_1 V_2' D(1-D)}{2 f_{sw} L}$$

↓

small D/d → small power

d = 0.5 → max power

d > 0.5 → decrease power again

Output Capacitor

Energy balance:

$$\boxed{P_{c\_in} - P_{c\_out} = P_{c\_stored}} \quad 1$$

$$\boxed{P_{c\_in} = \text{Power from DAB} = P}$$

$$P_{c\_out} = \text{Power consumed by power} = \boxed{\frac{V_{out}^2}{R_{load}}}$$

$$P_{c\_stored} = \text{Energy change in cap / second}$$

$$\boxed{E = \frac{1}{2} C V_{out}^2} \rightarrow \frac{dE}{dt} = \frac{d}{dt}\left(\frac{1}{2} C V_{out}^2\right)$$

$$\frac{dE}{dt} = \frac{1}{2} C \frac{d}{dt}\left(V_{out}(t)^2\right)$$

$$= \frac{1}{2} C \left[2\, V_{out}(t) \frac{dV_{out}}{dt} \right]$$

$$\boxed{\frac{dE}{dt} = C\, V_{out} \frac{dV_{out}}{dt}}$$

↳ Recall 1

$$P_{C\_in} - P_{C\_out} = P_{C\_stored}$$

$$\boxed{P - \frac{V_{out}^2}{R_{load}} = C\, V_{out} \frac{dV_{out}}{dt}}$$

Steady state → $\frac{dV_{out}}{dt} = 0$ 2

$$P - \frac{V_{out}^2}{R_{load}} = 0$$

$$P = \frac{V_{out}^2}{R_{load}} \rightarrow \frac{V_1 V_2' D(1-D)}{2 f_{sw} L} = \frac{V_{out}^2}{R_{load}}$$

$$P_{C\_in} = P_{C\_ou}$$

$$\boxed{P_{v\,(steady)} = \frac{V_1 V_2' D(1-D)}{2 f_{sw} L} = \frac{V_{out}^2}{R_{load}}}$$ 3

Add small perturbations

System is slightly disturbed from steady state:

$d = D + \hat{d}$

$V_{out} = V_0 + \hat{v}$

$\hat{d}$ = small change in phase shif

$\hat{v}$ = small change in output voltage

Substitute 2 with d vout

$$P_{C-in} - P_{C-out} = P_{C-stored}$$

$$\frac{V_1 V_2' d(1-d)}{2 f_{sw} L} - \frac{V_{out}^2}{R_{load}} = C\, V_{out} \frac{d V_{out}}{dt}$$

$$\frac{V_1 V_2'(D+\hat{d})(1-(D+\hat{d}))}{2 f_{sw} L} - \frac{(V_0+\hat{v})^2}{R_{load}}$$

$$= C\,(V_0+\hat{v})\frac{d(V_0+\hat{v})}{dt}$$

$$(D+\hat{d})(1-(D+\hat{d})) = (D+\hat{d})(1-D-\hat{d})$$

$$= (D+\hat{d}) - D(D+\hat{d}) - \hat{d}(D+\hat{d})$$

$$= D+\hat{d} - D^2 - D\hat{d} - D\hat{d} - \hat{d}^2$$

$$= -D^2 - \hat{d}^2 - 2D\hat{d} + D + \hat{d}$$

$$= D(1-D) + \hat{d}(1-2D) - \hat{d}^2$$

$\hat{d}$ = small → $\hat{d}^2$ = very small → drop $\hat{d}^2$

$$= D(1-D) + \hat{d}(1-2D)$$

Therefore,

$$P_{C\_in} = \frac{V_1 V_2'}{2 f_{sw} L}\left[D(1-D) + \hat{d}(1-2D)\right]$$

$$= \frac{V_1 V_2' D(1-D)}{2 f_{sw} L} + \frac{V_1 V_2'(1-2D)\hat{d}}{2 f_{sw} L} \quad (3)$$

$$\boxed{P_{C\_in} = P_{0(steady)} + Gain_p \times \hat{d}} \quad (4)$$

$$P_{C\_out} = \frac{(V_0+\hat{v})^2}{R_{load}};$$

$$(V_0+\hat{v})^2 = V_0^2 + 2V_0\hat{v} + \hat{v}^2$$

$\hat{v}^2$ = very small → drop $\hat{v}^2$

$$(V_0+\hat{v})^2 \approx V_0^2 + 2V_0\hat{v}$$

Therefore,

$$P_{C\_out} = \frac{V_0^2 + 2V_0\hat{v}}{R_{load}}$$

$$= \frac{V_0^2}{R_{load}} \; (3) + \frac{2V_0}{R_{load}}\hat{v}$$

$$\boxed{P_{C\_out} = P_{0\_steady} + \frac{2V_0}{R_{load}}\hat{v}} \quad (5)$$

$$C(V_0+\hat{v})\frac{d(V_0+\hat{v})}{dt}$$

V_0 constant → $\frac{dV_0}{dt} + \frac{d\hat{v}}{dt} = 0 + \frac{d\hat{v}}{dt}$

↳ $C(V_0+\hat{v})\frac{d\hat{v}}{dt}$

$$= CV_0\frac{d\hat{v}}{dt} + C\hat{v}\frac{d\hat{v}}{dt}$$

$\hat{v} \cdot \frac{d\hat{v}}{dt}$ → very small → drop it

$$\boxed{\approx C\, V_0 \frac{d\hat{v}}{dt}}$$ 6

Put 4 5 6 together:

$$P_{C\_in} - P_{C\_out} = C V_0 \frac{d\hat{v}}{dt}$$

$$\left(\cancel{P_{0\_steady}} + Gain_p \hat{d}\right) - \left(\cancel{P_{0\_steady}} + \frac{2V_0}{R_{load}}\hat{v}\right)$$

$$\parallel$$

$$C V_0 \frac{d\hat{v}}{dt}$$

$$Gain_p \hat{d} - \frac{2V_0}{R_{load}}\hat{v} = C V_0 \frac{d\hat{v}}{dt}$$

Simplify div by $\frac{2V_0}{R_{load}}$

$$\frac{Gain_p \hat{d} R_{load}}{2V_0} - \hat{v} = \frac{C \cancel{V_0} R_{load}}{2\cancel{V_0}} \frac{d\hat{v}}{dt}$$

$$\frac{R_{load}}{2V_0} Gain_p \hat{d} - \hat{v} = \frac{C R_{load}}{2} \frac{d\hat{v}}{dt}$$

$$\frac{C R_{load}}{2} \frac{d\hat{v}}{dt} + \hat{v} = \frac{R_{load}}{2V_0} Gain_p \hat{d}$$

Define $\tau = \frac{C R_{load}}{2}$

$$Gain_{DC} = \frac{R_{load}}{2V_0} Gain_p$$

$$\boxed{\tau \frac{d\hat{v}}{dt} + \hat{v} = Gain_{DC} \hat{d}} \quad (7)$$

Convert (7) to Laplace Domain

$$\tau \hat{v}(s) s + \hat{V}(s) = G_{DC}(s) \hat{D}(s)$$

$$\hat{v}(s) [\tau s + 1] = G_{DC}(s) \hat{D}(s)$$

$$\boxed{\frac{\hat{V}(s)}{\hat{D}(s)} = \frac{G_{DC}(s)}{\tau s + 1} = G_{VD}(s)} \quad (8)$$

Transfer function

with

$$G_{DC} = \frac{R_{load}}{2V_0} \frac{V_1 V_2' (1-2D)}{2 f_{sw} L}$$

↳ DC gain = if D changed by 1 unit, Vout changes by G_{DC} volts

$$\tau = \frac{C R_{load}}{2}$$

↳ Time constant: how fast does the system respond?

Pole

$$G_{vD} = \frac{G_{DC}}{\tau s + 1} \rightarrow 1 + \tau s = 0$$

$$\boxed{s = -\frac{1}{\tau}}$$

w domain

$s \rightarrow jw$

$$G_{vD}(jw) = \frac{G_{DC}}{1 + jw\tau}$$

Pole frequency (w_p) $\rightarrow$ when real part of the denominator equals the magnitude of the imaginary part.

Real part $= 1$

Imaginary part $= w\tau$

$$1 = w_p \tau$$

$$\boxed{w_p = \frac{1}{\tau} = \frac{2}{R_{load} C}}$$

PI Controller

Laplace Domain

$$G_c = K_p + \frac{K_i}{s}$$

$$= \frac{K_p s + K_i}{s}$$

Open loop

$$v_o' \longrightarrow \boxed{G_c} \xrightarrow{d} \boxed{G_{vd}} \longrightarrow v_o$$

$$L(s) = G_c \, G_{vd}$$

$$= \frac{K_p s + K_i}{s} \times \frac{G_{DC}}{\tau s + 1}$$

$$\boxed{L(s) = \frac{K_p (s + K_i/K_p)}{s} \times \frac{G_{DC}}{\tau (s + \frac{1}{\tau})}} \quad 9$$

Pole Zero Cancelation

$G_c \Rightarrow$ zero : $-K_i/K_p$

$G_{VD} \Rightarrow$ pole : $-\frac{1}{\tau}$

zero controller = plant pole

$$\frac{K_i}{K_p} = \frac{1}{\tau}$$

$$\boxed{K_i = \frac{K_p}{\tau}} \quad (10)$$

Substitute (10) into (9)

$$L(s) = \frac{K_p\left(s + (K_p/\tau)/K_p\right)}{s} \times \frac{G_{DC}}{\tau\left(s+\frac{1}{\tau}\right)}$$

$$= \frac{K_p\cancel{\left(s+\frac{1}{\tau}\right)}}{s} \times \frac{G_{DC}}{\tau\cancel{\left(s+\frac{1}{\tau}\right)}}$$

$$\boxed{L(s) = \frac{K_p G_{DC}}{s\tau}} \quad (11)$$

Bandwidth (Crossover Frequency, W_c)

↳ Frequency where the open loop gain equals 1

$|L(j\omega_c)| = 1$, recall 11

$$\left| \frac{K_p G_{DC}}{\tau (j\omega_c)} \right| = 1$$

$$\frac{K_p G_{DC}}{\tau (\omega_c)} = 1$$

$$K_p G_{DC} = \tau \omega_c$$

$$K_p = \frac{\tau \omega_c}{G_{DC}} \; ; \; K_i = \frac{K_p}{\tau}$$

with

$$G_{DC} = \frac{R_{load}}{2V_o} \frac{V_1 V_2' (1-2D)}{2 f_{sw} L}$$

$$\tau = \frac{C R_{load}}{2}$$

$$\omega_c = 2\pi f_{bw}$$