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Portfolio models with return forecasting and transaction costs

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ABSTRACT

In this paper, we advance portfolio models by incorporating return projection and further analyze their *realized* performance. To ensure practicality, the transaction costs and the optimization of short-selling weights are taken into account in portfolio rebalancing. Using the daily returns of international ETFs over a period of 14 years, the empirical results show that including return forecasting improves the realized performance due to more efficient asset allocation but not a reduction in trading costs. The models that are based on trade-off between return and volatility, such as the mean-variance and Omega models, show higher increases in performance than those mainly focus on controlling loss, such as the linearized value-at-risk, the conditional value-at-risk, and the downside risk. The superiority of forecasting risky portfolios over the equally-weighted diversification varies intertemporarily across various portfolio models. The benefit of inclusion of prediction is larger when the market is less volatile.

1. Introduction

The quality of the input and the methods used to interpret the information affect the effectiveness of portfolio models. Markowitz (1952) suggests that the optimal asset allocation starts with "observation and experience," but his model merely takes the first two moments of historical returns into account the decision-making. The application of the mean-variance (MV) portfolio and its succeeding improvements is challenged mainly due to the approach to portraying the trade-off between risk and return. Given the poor predictability of expected return, does adding forecasting in the portfolio models improve the *realized* performance? If so, which portfolio models and what economic scenarios might be more suitable to incorporating the forecasting mechanism? This study responds to the call of Markowitz (1952) to better form "beliefs about the future performances of available securities" (pp. 77) by employing a conventional time-series method. Specifically, we develop the portfolio models by synthesizing return forecasting in constructing the optimal asset allocation.

The mean-variance framework suggested by Markowitz (1952) serves as the foundation for modern portfolio theory. Numerous approaches have been developed to address various issues that may skew the implementation of the authentic MV model. An inflating variance-covariance matrix yields computational complexity because of the natures of the quadratic programming. Thus, the use of the MV model may be limited as the number of assets grows. Konno and Yamazaki (1991) and Chang (2005) propose a mean-absolute deviation (MAD) model in which risk is defined as the mean of the absolute value of the difference in return. Vercher, Bermudez, and Segura (2007) show a down side risk (DSR) model is useful when the returns depart from a normal distribution. To capture the

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future potential downside risk, the value-at-risk (VaR) is to estimate the loss that the investor can tolerate given a certain confidence level (Jorion, 1997). To generate the global optimum, we apply the linearized VaR (LVaR) model by Yu, Chiou, and Mu (2015), which advances Benati and Rizzi (2007), to construct portfolios. The VaR measurement holds some limitations in managing portfolio volatility, such as generating multiple local minima and lacking sub-additive properties. The conditional value-at-risk (CVaR) model improves the above issues of the VaR model. Rockafellar and Uryasev (2000) and Low, Alcock, Faff, and Brailsford (2013) document that the CVaR can quantify downside risk more precisely than traditional MV models since the CVaR models asymmetry in asset return distribution. Kapsos, Christofides, and Rustem (2014) and Yu, Chiou, Lee, and Chuang (2019) suggest applying the Omega model to optimize the relative likelihood of return or loss over a threshold in portfolios under asymmetric distribution of returns.

We advance the feasibility of various portfolio models by incorporating forecasting asset return. Recent research such as Chao (2016), Neely, Rapach, Tu, and Zhou (2014), and Li and Tsiakas (2017) develop methods that include macroeconomic and technical analysis factors to improve predictability of equity premium. Another research line focuses on return forecast by using return distribution. Zhu (2013) develops a portfolio model that incorporates quantile regressions and copula to include the higher moments to better forecast returns. Gebka and Wohar (2019) document that the 75th quantile demonstrates the highest predictive power to the future returns than other quantiles but could not conclude the economic rationale of its predictability.

The objective of this paper is to maximize the expected returns as well as the realized returns with considering transaction costs and short-selling constraints. Since portfolio management focuses on risk-return efficiency, it is not the main objective of our paper to deal with the issues of the estimation error. The frequency of macroeconomic factors in previous papers to forecast equity premium do not fit the context of our paper as forecasting the future daily returns is needed to model the portfolios. Comparing the outcomes of various return-projection models, Panopoulou and Vrontos (2015) and Ustun and Kasimbeyli (2012) document that simple forecasting techniques perform better than the computationally intensive frameworks in managing portfolios. Avramov and Chordia (2006) find the models that are based on risk-loading factors demonstrate poor out-of-sample predictability. Our study applies a time-series model, the Autoregressive Integrated Moving Average (ARIMA), to project the returns as a response to the above critiques.

Our study contributes to the literature in two ways. First, we advance a wide range of risk-return portfolio models by explicitly integrating return forecasting and furthermore evaluate their *ex post* performance. One of the key issues for modern asset management is how to model future returns so that portfolio can yield out-of-sample performance while the investing objectives can be achieved. Panopoulou and Vrontos (2015) test the effectiveness of various combinations of pricing factors suggested by previous studies and information in modeling Markowitz (1952) optimal portfolios. Ustun and Kasimbeyli (2012) apply forecasting techniques to generate returns and return errors in modeling mean–variance–skewness portfolio with other 11 objective functions. Our study focuses on how to advance portfolio frameworks to generate higher realized performance. We examine the models that are extensively applied by professionals and academics, including mean-variance (MV), mean-absolute deviation (MAD), downside risk (DSR), value-at-risk (VaR), conditional value-at-risk (CVaR), and Omega ratio. Our study also explores whether the portfolio models with projecting returns are preferred over a naïve investing strategy. Many portfolio models are developed to yield higher *ex-ante*risk-return efficiency, however, DeMiguel, Garlappi, and Uppal (2009) have shown that none of the "sophisticated models" consistently yields a higher realized performance than an equally-weighted ($1/N$) asset allocation. We examine if the inclusion of return forecasting, even by applying a conventional time-series model, in constructing risky portfolios provides a possible method of outperforming diversified portfolios that do not use risk-return information. We rebalance these portfolios with and without including return projection over 14 years and compare their out-of- sample performance. The effectiveness of adding return predictions is evaluated by the Sharpe ratio of realized return, portfolio market value, transaction costs, and portfolio diversity.

Second, to ensure the feasibility of the strategies and the usefulness of our findings, our tests incorporate certain conditions that affect the application of the portfolio models, including the allowance of short sales and the incorporation of transaction costs when the portfolios are rebalanced. Angel, Christophe, and Ferri (2003) suggest that short selling provides investors a chance to arbitrage when market downturns are expected despite it can increase the volatility of portfolio returns. Yu and Lee (2011) optimize the proportion of short selling with considering portfolios' risks. Furthermore, modeling transaction costs improves the feasibility of portfolio construction. Atkinson and Mokkhavesa (2004) suggest that the portfolio rebalancing frequencies and scale are larger than what they should be if the transaction costs are not modeled. Our methodology integrates Woodside-Oriakhi, Lucas, and Beasley (2013) in rebalancing asset allocation with transaction costs in portfolio optimization. Specifically, both the weights allowing short selling and the transaction costs, including purchasing, selling, short selling, and repurchasing are modeled in making portfolio decisions.

The empirical results show that using the return forecasting increases the realized performance of the portfolio models. There are significant time-variation and cross-model difference in the economic value yielded from including the return forecasting to the portfolio models. The increase in realized return ranges from 16 (LVaR) to 716 (Omega) basis points per year. The benefit of incorporating the return predication is more consistent and significant for the Omega and MV models. The economic value of including the return projection in the portfolio models comes from efficient asset allocation rather than a reduction in transaction costs. The benefits over the naïve diversification fluctuate over the sample period and across portfolio models. Most specifically, they vanish during the financial crisis. This suggests that the inclusion of return forecasting improves performance especially when the market is stable.

This paper is organized as follows. Section 2 presents the portfolio models and their application in the study. Section 3 describes how to incorporate the ARIMA forecasting mechanism in the portfolio models. Section 4 reports the data, performance measures, and the design of the tests. The empirical findings are demonstrated in Section 5. Section 6 concludes.

2. Portfolio models and their empirical applications

This study evaluates the economic value of including the forecasting mechanism in the risky portfolio models that are widely applied. Specifically, we advanced the mean-variance (MV) model and its deviations, including the mean-absolute deviation (MAD) model, the downside risk (DSR) model, the linearized value-at-risk (LVaR) model, the conditional value-at-risk (CVaR) model, and the Omega model by incorporating projecting returns. These portfolios are rebalanced considering situations in which short sales are allowed and transaction costs are charged. We applied a simple weighted method for multiple objective programming due to multiple objectives can be in the optimization.

2.1. Mean-variance model (MV)

In our study, the objective function is modified to incorporate the short-selling position and to minimize portfolio variance. The weight of asset j is decomposed into weights of long position (+) and short-selling position (−), $w_j = w_j^+ - w_j^-$. The optimization is determined by solving the following equations:

$$\text{Min} \quad \sum_{j=1}^{n} w_j^2 \sigma_j^2 + \sum_{i=1}^{n} \sum_{j=1(j\neq i)}^{n} \sigma_{ij}\left(w_i^+ - w_i^-\right)\left(w_j^+ - w_j^-\right) \tag{1}$$

$$\text{Min} \quad \sum_{j=1}^{n} w_j^- \tag{2}$$

$$\text{Min} \quad \sum_{j=1}^{n}\left(p_1 l_j^+ + p_2 l_j^- + p_3 s_j^+ + p_4 s_j^-\right) \tag{3}$$

$$\text{s.t.} \quad \sum_{j=1}^{n} \overline{r}_j\left(w_j^+ - w_j^-\right) \geq E, \tag{4}$$

$$\sum_{j=1}^{n}\left(w_j^+ + k w_j^- + p_1 l_j^+ + p_2 l_j^- + p_3 s_j^+ + p_4 s_j^-\right) = 1, \tag{5}$$

$$w_j^+ = w_{j,0}^+ + l_j^+ - l_j^-, \tag{6}$$

$$w_j^- = w_{j,0}^- + s_j^+ - s_j^-, \tag{7}$$

$$0.01 \quad u_j \leq w_j^+ \leq u_j, \tag{8}$$

$$0.01 \quad v_j \leq w_j^- \leq v_j, \tag{9}$$

$$u_j + v_j \leq 1, \tag{10}$$

$$u_j, \quad v_j \in \{0, \ 1\}, \tag{11}$$

$$j \ = \ 1, \ 2, \ \ldots \text{n},$$

where $\overline{r}_j$ is the average historical return of security j; E is the required return; n is the number of available securities; w_j^+ is the proportion of security j invested at portfolio rebalancing; w_j^- is the proportion of security j being short sold by investors at portfolio rebalancing; $w_{j,\ 0}^+$ is the long position weight of security j prior to portfolio rebalancing; $w_{j,\ 0}^-$ is the short-selling position weight of security j prior to portfolio rebalancing; p_1, p_2, p_3, and p_4 are the transaction costs of buying, selling, short selling, and repurchasing, respectively; and k is the initial margin requirement for short selling. With each rebalancing, the weights are described in Eqs. (5) - (11): l_j^+ is the buying weight of security j; l_j^- is the selling weight in the long position of security j; s_j^+ is the short-selling weight of security j; and s_j^- is the repurchasing weight of security j. The binary variables u_j and v_j are used to indicate the long and short positions and to model the upper bounds of the weight.

Differing from the original MV model, we consider the portfolio short-selling positions and transaction costs: Eqs. (2) and (3) minimize the short-selling positions and trading costs; Eq. (4) requires the portfolio return not less than the required return after considering short selling; and Eq. (5) defines the budget constraint with taking into account trading costs. When modeling the rebalancing weights in Eqs. (6) and (7), we structure the weight of asset not less than one percent in Eqs. (8) – (11). This setting of the minimal weights is to avoid small asset positions (e.g., to hold $100 of a certain stock in a multi-billion-dollar portfolio) that are not very meaningful to portfolio management.

2.2. Mean absolute deviation (MAD) model

The MV model lacks efficiency as the number of assets increases due to the need to solve large-scale quadratic programming. Konno and Yamazaki (1991) thus propose the MAD model in which variances are replaced by measures of mean-absolute deviation. We follow Chang (2005) to split the weights of the long and short position, $\left|\sum_{j=1}^{n}(r_{jt} - E(R_j))w_j\right| = d_t = d_t^+ + d_t^-$, $d_t^+ \geq 0$, and $d_t^- \geq 0$, then $\sum_{j=1}^{n}(r_{jt} - E(R_j))w_j = d_t^+ - d_t^-$. The objective is as follows:

$$\text{Min} \ \sum_{j=1}^{n}\left(\left|\left(w_j^+ - w_j^-\right)r_{jt} - \left(w_j^+ - w_j^-\right)E\left(R_j\right)\right|\right),$$

The objective can be converted into a linear combination of the optimal portfolio variance and the short-sale weights:

$$\text{Min} \ \frac{1}{T}\sum_{t=1}^{T} d_t \tag{12}$$

$$\text{s.t.} \ \ d_t + \sum_{j=1}^{n}\left(w_j^+ - w_j^-\right)r_{jt} - \left(w_j^+ - w_j^-\right)E\left(R_j\right) \geq 0\,, \tag{13}$$

$$d_t - \sum_{j=1}^{n}\left(w_j^+ - w_j^-\right)r_{jt} - \left(w_j^+ - w_j^-\right)E\left(R_j\right) \geq 0, \ t = 1, \cdots, T, \tag{14}$$

constraints (4)–(11).

2.3. Downside risk (DSR) model

The variance of return can be questionable to measure risk since it includes the up-side profitability. But the risk that investors should care about is a possible loss in portfolio value. Vercher et al. (2007) consider the downside risk formulation for the portfolio selection and reformulate it as a linear model. From the above MAD model, we have $\left|\sum_{j=1}^{n}(r_{jt} - E(R_j))w_j\right| = d_t = d_t^+ + d_t^-$, where $d_t^- = \frac{1}{2}(d_t - \sum_{i=1}^{n}(r_{it} - R_i)w_i) \geq 0$, $d_t^+ \geq 0$, and $d_t^- \geq 0$. Through the linearizing process in Chang (2005), we have $\sum_{j=1}^{n}(r_{jt} - E(R_j))w_j = d_t^+ - d_t^-$.

$$\frac{\left|\sum_{j=1}^{n}\left(r_{jt} - E\left(R_j\right)\right)w_j\right| - \sum_{j=1}^{n}\left(r_{jt} - E\left(R_j\right)\right)w_j}{2} = \frac{d_t^+ + d_t^- - d_t^+ + d_t^-}{2} = d_t^- \tag{15}$$

Therefore, the DSR model can be expressed as the following linear model:

$$\text{Min} \ \frac{1}{T}\sum_{t=1}^{T} d_t^- \tag{16}$$

$$\text{s.t.} \ \ d_t^- + \sum_{j=1}^{n}\left(r_{jt} - E\left(R_j\right)\right)\left(w_j^+ - w_j^-\right) \geq 0, \ \ t = 1, \ \ \ldots, \ \ T, \quad \left(\text{for } d_t^+ \geq 0\right), \tag{17}$$

$$d_t^- \geq 0 \ \ \ t = 1, \ \ \ldots, \ \ T, \tag{18}$$

constraints (4)–(11).

2.4. Linearized value-at-risk (LVaR) model

The value-at-risk (VaR) model has been widely applied by financial institutions. Among various interpretations, Benati and Rizzi (2007) model VaR as a mixed integer linear programming in order to improve computation efficiency and to assure global optimization. We apply the advanced linearized model by Yu et al. (2015), which improves Benati and Rizzi (2007) and Lin (2009) to construct and rebalance VaR portfolios:

$$\text{Max} \ \ r^{VaR} \tag{19}$$

$$\text{s.t.}\quad x_t = \sum_{j=1}^{n}\left(w_j^+ - w_j^-\right) r_{jt},\quad t = 1,\quad \ldots,\quad T, \tag{20}$$

$$x_t \geq r^{Min} + (r^{VaR} - r^{Min})y_t,\quad t = 1,\quad \ldots,\quad T, \tag{21}$$

$$\sum_{t=1}^{T}\frac{1}{T}(1 - y_t) \leq \alpha^{VaR}, \tag{22}$$

$$\sum_{t=1}^{T}\frac{1}{T}x_t \geq E, \tag{23}$$

$$y_t \in \{\ 0,\ \ 1\ \},\quad t = 1,\quad \ldots,\quad T,\quad \text{and}\begin{cases} y_t = 0,\ x_t \geq r^{Min} \\ y_t = 1,\ x_t \geq r^{VaR} \end{cases} \tag{24}$$

constraints (4)–(11),where x_t is the portfolio return on day t, $x_t = \sum_{j=1}^{n} w_j r_{jt}$; $0 \leq \alpha \leq 1$; r^{Min} is the minimum return that can be observed in the market during a period of T days; r^{VaR} is the threshold return set by the investor; α^{VaR} is the confidence level of the investor between 0 and 1; and y_t indicates whether the return on the tth day is lower than the probability α^{VaR}. Constraint (20) sets x_t as the linear combination of r_{jt}, and Eqs. (21) and (22) prevent to select the portfolios that yield VaR lower than a given threshold. Eq. (23) set the mean of portfolio return over a period cannot be less than the required return. Eq. (24) shows the binary variable y_t indicating whether return on tth day is lower than r^{VaR} at the probability α^{VaR}.

2.5. Conditional value-at-risk (CVaR) model

The VaR bears some undesirable mathematical characteristics such as a lack of subadditivity and convexity, leading an increase in risk when a portfolio is more diversified. To improve the above issues, we consider the distribution-free condictional value-at-risk (CVaR) model in the study. The CVaR model extends the VaR while estimating extreme risk under specific confidence levels (Low, et al., 2013). We modify Rockafellar and Uryasev (2000) by transforming the objectives, the portfolio variance, and the short-selling weights into a linear function. Specifically,

$$\text{Min}\quad \xi + \frac{1}{(1-\alpha)T}\sum_{t=1}^{T}\eta_t \tag{25}$$

$$\text{s. t.}\ \eta_t \geq -\sum_{j=1}^{n} r_{jt}\left(w_j^+ - w_j^-\right) - \xi,\quad t = 1,\quad \ldots,\quad T, \tag{26}$$

$$\eta_t \geq 0,\quad t = 1,\quad \ldots,\quad T, \tag{27}$$

constraints (4)–(11),where ξ indicates the threshold of portfolio loss value; α is the confidence level; and η_t is the auxiliary variable transformed from the original objective of the loss function to a linear function. Eq. (25) is to establish the minimum of CVaR, and Eqs. (26) and (27) set the portfolio loss so that it remains positive and greater than the threshold.

2.6. Omega model

The Omega ratio is defined as following (Kapsos et al., 2014):

$$\omega \quad = \quad \frac{E\left(r_j\right) - \tau}{E\left[\tau - r_j\right]^+} + 1,$$

where τ is a threshold that partitions the return to the desirable (gain) and the undesirable (loss); r_j denotes the random return of asset j. The value of τ is determined by the investor. We applied the Omega ratio, a lower partial moment, to model the portfolio. The Omega ratio is free from the assumption of Gaussian distribution and can be suitable for modeling performances under various scenarios with a threshold return. We modify Kapsos et al. (2014) and Yu et al. (2019) as the following:

$$\text{Max}\ \theta \tag{28}$$

$$\text{s.t.}\ \delta\left(\sum_{j=1}^{n}\left(w_j^+ - w_j^-\right)\overline{r}_j^i - \tau\right) - (1-\delta)\frac{1}{T^i}\sum_{t=1}^{T^i}\eta_t^i \geq \theta, \tag{29}$$

$$\eta_t^i - \sum_{j=1}^{n} \overline{r}_j^i \left(w_j^+ - w_j^-\right) + \tau, \tag{30}$$

$$\eta_t \geq 0 \quad t = 1,\ 2,\ \ldots,\ \ T, \tag{31}$$

constraints (4)–(11), for $t = 1, 2, \ldots, T^i$, $i = 1, 2, \ldots, l$,where δ determines the risk-return preference of the model which is assigned by the user, T^i is the ending period of the investment horizon from the ith likelihood distribution for robust portfolio selection and τ is the threshold of portfolio return. In our empirical test, τ is set as one basis point in our empirical test. l is specified as 1 for dealing with certain return distribution like the previous models do.

3. Incorporating return forecasting in portfolio models

In this paper we use the Autoregressive Integrated Moving Average (ARIMA) to model the asset return forecasting. This approach has been widely applied to fit the time-series data and to predict values in the series. The generalized form, an ARIMA (*p, d, q*) process, can be expressed as follows:

$$\left(1 - \sum_{i=1}^{p} \phi_i L^i\right)\left(1 - L^i\right)^d r_t = \delta + \left(1 - \sum_{i=1}^{q} \theta_i L^i\right)\varepsilon_t \tag{32}$$

where L is the lag operator; ϕ_i is the autoregressive parameter; θ_i is the moving-average parameter; and ε_t is the error term. When using the ARIMA model, it is critical to determine the parameters, the number of autoregressive terms (p), the number of non-seasonal differences needed for stationarity (d), and the number of lagged forecast errors in the prediction equation (q). The model is more advanced than moving average (MA) model that is applied by Neely et al. (2014) and is suitable to fashion the returns of the diversified assets such as national market ETFs. Our study follows Ustun and Kasimbeyli (2012) and Panopoulou and Vrontos (2015) and models the dynamics of returns as a conventional ARIMA (1, 0, 1) process. The previous 60 daily returns are used to estimate the next 20 daily returns and to construct the various risky portfolios. Since pricing of the diversified assets are less volatile than equities, the returns of national market ETFs are suitable to be fashioned by an ARIMA model.

We demonstrate the MV with forecasting (MV_F) model as an example of how to incorporate the forecasting return. Specifically, the expected returns and the abnormal returns are modeled in the objective functions:

$$\text{Min} \quad \sum_{j=1}^{n} w_j^2 \sigma_j^2 + \sum_{i=1}^{n} \sum_{j=1(j\neq i)}^{n} \sigma_{ij}\left(w_i^+ - w_i^-\right)\left(w_j^+ - w_j^-\right)$$

$$\text{Max} \quad \sum_{j=1}^{n} \widehat{r}_j \left(w_j^+ - w_j^-\right) \tag{33}$$

$$\text{Max} \quad \sum_{j=1}^{n} \overline{\varepsilon}_j \left(w_j^+ - w_j^-\right) \tag{34}$$

$$\text{Min} \quad \sum_{j=1}^{n} w_j^-$$

$$\text{Min} \quad \sum_{j=1}^{n} \left(p_1 l_j^+ + p_2 l_j^- + p_3 s_j^+ + p_4 s_j^-\right)$$

s.t. (4)–(11),where $\widehat{r}_j$ is the predicted return of security j, and $\overline{\varepsilon}_j$ is the average residual of security j over the sample period. The error term at time t is $\varepsilon_t = r_t - \widehat{r}_t$, where r_t represents the actual value and $\widehat{r}_t$ is the forecasting value estimated by an ARIMA model. The two objectives are to maximize the expected portfolio return in the estimating period (Eq. (33)) and to maximize the abnormal return in the sample period (Eq. (34)). Both are desired by investors.

We apply the simple weighted method suggested by Yu and Lee (2011) to convert the multiple objectives model into a single objective. The MV_F model can be transformed as a linearized single-objective function:

$$\text{Min} \quad \sum_{j=1}^{n} w_j^2 \sigma_j^2 + \sum_{i=1}^{n} \sum_{j=1}^{n} \sigma_{ij}\left(w_i^+ - w_j^-\right)\left(w_j^+ - w_i^-\right) + \sum_{i=1}^{n}\left(p_1 l_i^+ + p_2 l_i^- + p_3 s_i^+ + p_4 s_i^-\right) + \sum_{i=1}^{n} w_j^-$$

$$- \sum_{i=1}^{n}\left(w_i^+ - w_i^-\right)\widehat{R}_i - \sum_{i=1}^{n}\left(w_i^+ - w_i^-\right)\overline{\varepsilon}_i, \tag{35}$$

Table 1
Summary statistics of the investments.

Asset	Symbol	Mean	SR	Skewness	Kurtosis
iShares MSCI Australia Index	EWA	0.0999	0.363	−0.239	11.072
iShares MSCI Austria Index	EWO	0.0936	0.331	−0.011	14.245
iShares MSCI Belgium Index	EWK	0.0627	0.231	−0.395	6.717
iShares MSCI Brazil Index	EWZ	0.1538	0.428	−0.055	8.903
iShares MSCI Canada Index	EWC	0.0985	0.434	−0.456	8.364
iShares MSCI France Index	EWQ	0.0615	0.221	−0.031	4.686
iShares MSCI Germany Index	EWG	0.0897	0.327	0.165	7.850
iShares MSCI Hong Kong Index	EWH	0.1028	0.418	0.303	6.868
iShares MSCI Italy Index	EWI	0.0326	0.092	−0.106	5.024
iShares MSCI Japan Index	EWJ	0.0641	0.255	0.092	5.765
iShares MSCI Malaysia Index	EWM	0.0929	0.467	−0.529	8.571
iShares MSCI Mexico Index	EWW	0.1387	0.535	0.132	13.490
iShares MSCI Netherlands Index	EWN	0.0631	0.232	−0.308	5.394
iShares MSCI Singapore Index	EWS	0.1038	0.414	0.161	7.153
iShares MSCI South Korea Index	EWY	0.1631	0.495	0.484	12.626
iShares MSCI Spain Index	EWP	0.0804	0.278	−0.010	5.493
iShares MSCI Sweden Index	EWD	0.1227	0.399	0.017	4.900
iShares MSCI Switzerland Index	EWL	0.0959	0.458	−0.232	5.960
iShares MSCI Taiwan Index	EWT	0.0965	0.323	0.152	6.084
iShares MSCI United Kingdom Index	EWU	0.0435	0.164	−0.307	7.818
SPDR S&P 500	SPY	0.0628	0.345	−0.020	10.648

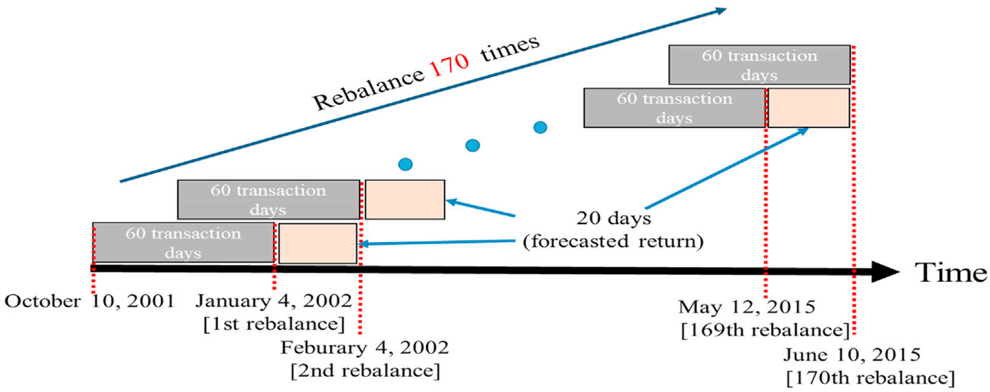


Fig. 1. Portfolio rebalancing mechanism.

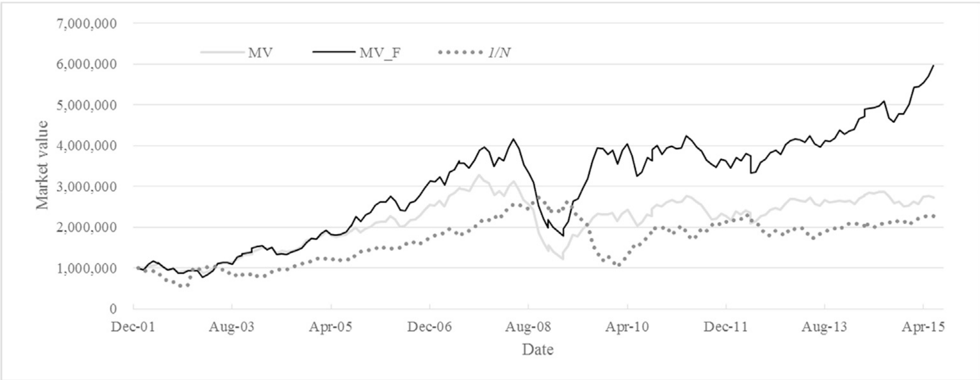
s.t. (4)–(11).Similar to the MV_F model, the two extra objectives, Eqs. (33) and (34), can be added to the other portfolio models. The multi-objective problem is transformed into a single-objective problem by employing the simple-weighted method. The linearized single-objective function, like Eq. (35), ensures the globally optimal solution. Our paper categorizes the models into two groups according to whether or not they incorporate return forecasting and analyzes their realized performance. We label the original models without return forecasting as MV, MAD, DSR, LVaR, CVaR, and Omega. The corresponding models that incorporate return forecasting are MV_F, MAD_F, DSR_F, LVaR_F, CVaR_F, and Omega_F.

4. Data and empirical test design

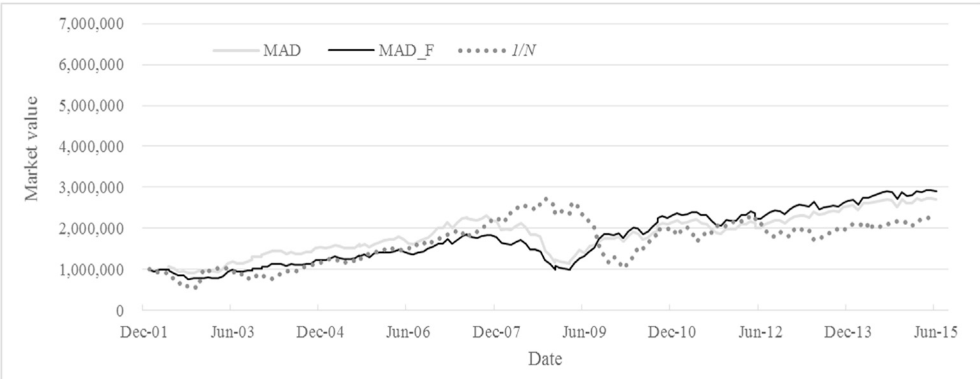
To verify the effectiveness of the proposed models, we collect 21 national Exchange Traded Funds (ETFs) from the period of October 10, 2001 to June 10, 2015 to perform the empirical tests. Table 1 presents the summary statistics of the assets, including their Sharpe ratio (SR), skewness, and kurtosis during the sample period. The portfolio strategies using the Morgan Stanley Capital International (MSCI) indices are feasible and are of high liquidity. The selected ETFs represent more than 90% of the world market capitalization during the period.

Fig. 1 shows the process of rebalancing the asset allocation over the 3,440 trading days. Similar to Yu and Lee (2011), the portfolio is formed by using the previous 60 daily returns to estimate the parameters and to forecast returns for the next 20 business days. The asset allocation is adjusted every 20 transaction days. The first of the 170 rebalances takes place on January 4, 2002, using the data from the preceding 60 days. Applying ARIMA (1, 0, 1), the same data set is used to forecast the next 20 daily returns of the assets. The last rebalance is on June 10, 2015.

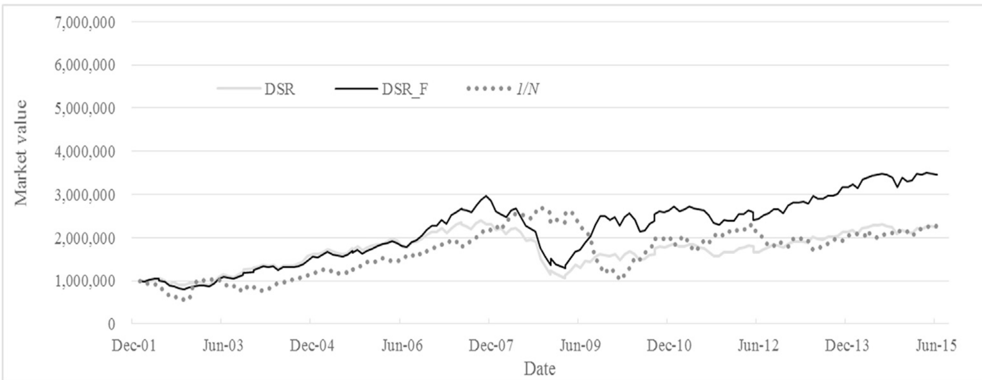
Our study focuses on the realized performance when investors follow the diversification strategies. Previous studies such as



A: Mean-Variance Models



B: Mean-Absolute Deviation Models

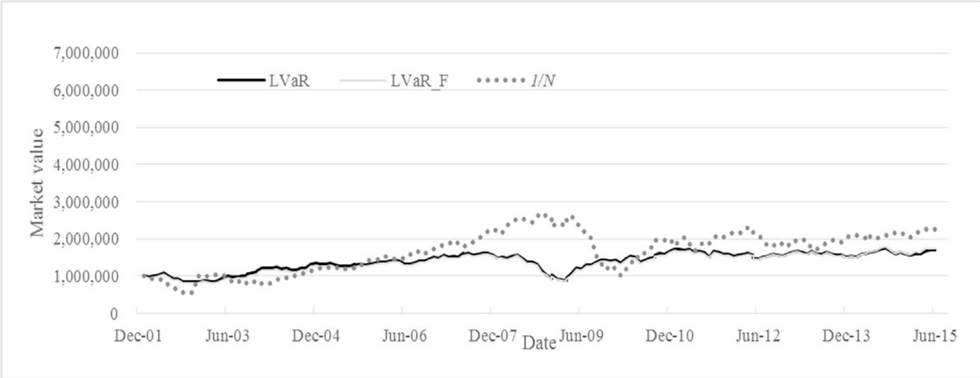


C: Downside Risk Models

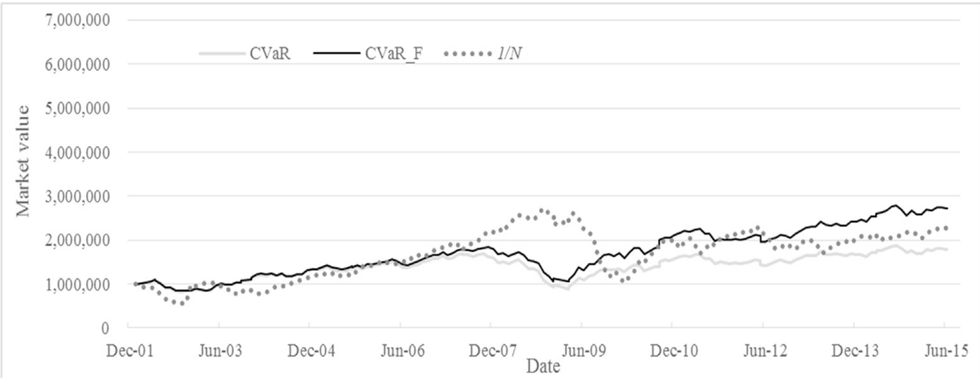
Fig. 2. The market value trend of the portfolio models.

DeMiguel et al. (2009) indicate that a poor estimation of asset returns challenges the application of risky portfolio models. They use a naïve diversification ($1/N$) as the benchmark portfolio. The realized portfolio market values are generated as the portfolios are rebalanced every period according to the optimization results.

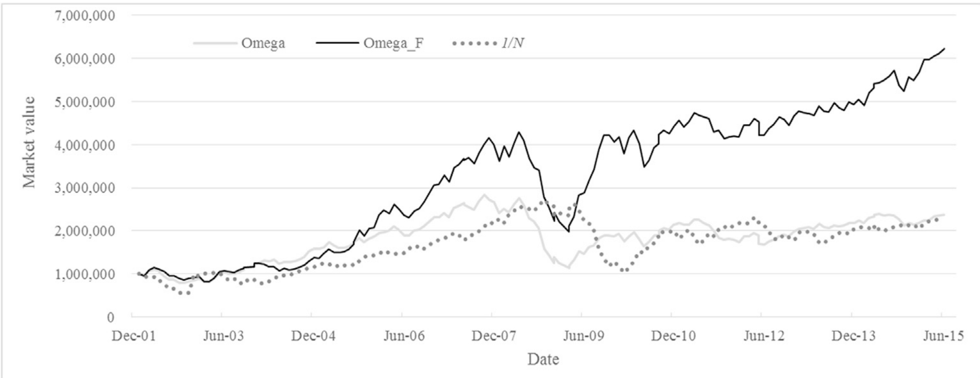
We use the realized portfolio return to measure the Sharpe and Omega ratios of the portfolios in order to evaluate the effectiveness of the above risk-return portfolio models. The Sharpe ratio (SR) is defined as



D. Linearized Value-at-Risk Models



E: Conditional Value-at-Risk Models



F: Omega Models

Fig. 2. . (*continued*).

$$SR = \frac{\overline{r}_P - r_f}{\sigma_P}, \tag{36}$$

where $\overline{r}_P$ is the expected return of the portfolio, r_f is the risk free rate, and σ_P is the standard deviation of the portfolio.

Considering the impact of higher moment risk on performance assessment and estimation errors, we also use the Omega ratio to measure portfolio performance. Unlike the Sharpe ratio that uses the first two moments, the lower partial moment is more feasible in estimating the performance of a return distribution that departs from normality.

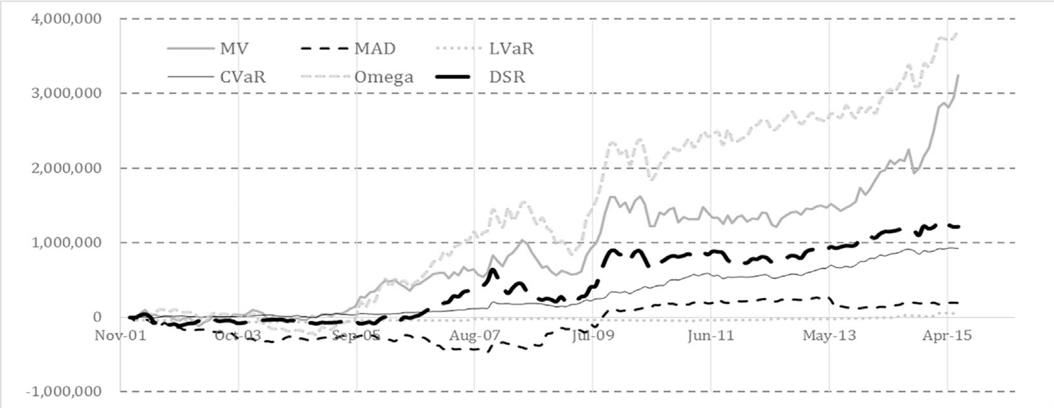


Fig. 3. Difference in Market Value: Models with Forecasting Return vs. without Forecasting Return.

Table 2
Summary statistics of Portfolio performance.

Model	Mean	St. Dev	SR	Omega	Max	Min	Return Distribution			
							$r<0$	$5\%>r>0$	$10\%>r>5\%$	$r>10\%$
1/N	0.0605	0.2664	0.17	1.42	0.5714	−0.2648	42.60	39.05	15.38	2.96
MV	0.0743	0.2040	0.29	1.15	0.1674	−0.2664	39.64	42.60	14.79	2.96
MV_F	0.1322	0.2275	0.52	1.43	0.2081	−0.1941	39.64	33.14	21.89	5.33
MAD	0.0736	0.1601	0.37	1.61	0.1172	−0.2164	39.64	46.75	11.24	2.37
MAD_F	0.0787	0.1586	0.40	1.65	0.1234	−0.1633	41.42	45.56	10.06	2.96
DSR	0.0599	0.1685	0.27	1.46	0.1100	−0.2329	42.60	43.79	12.43	1.18
DSR_F	0.0917	0.1741	0.44	1.44	0.1267	−0.1949	41.42	41.42	14.20	2.96
LVaR	0.0391	0.1544	0.16	1.21	0.1504	−0.1512	42.60	46.15	9.47	1.78
LVaR_F	0.0407	0.1556	0.17	1.27	0.1507	−0.1536	43.79	44.97	9.47	1.78
CVaR	0.0431	0.1478	0.19	1.40	0.1062	−0.1701	42.01	49.11	8.28	0.59
CVaR_F	0.0737	0.1448	0.41	1.73	0.1135	−0.1600	40.83	49.11	9.47	0.59
Omega	0.0637	0.1975	0.25	1.23	0.1343	−0.2703	40.83	41.42	14.79	2.96
Omega_F	0.1354	0.2098	0.57	1.49	0.1932	−0.2021	40.83	36.69	18.93	3.55

The transaction costs and the average number of assets are used to assess the effectiveness of these models. Though rebalancing portfolios enhances the efficiency of the investment, frequent asset replacement can lead to high transaction costs that erode the market value. Since the trading costs vary from broker to broker and from asset to asset, we use the fees that are generally accepted in the U.S. market and set all transaction costs (p_1, p_2, p_3, and p_4) at 25 basis points of the trading value. The required return is 0.01% for various models. The weights of each portfolio are obtained by using updated asset returns in each period and by assuming the initial investments to be \$1 million. Since asset short sales are allowed, the margin (k), set as 100% in the study, needs to be paid before the asset is short sold. The budget for the investment in the following period depends on the market value at the end of the previous period. For the LVaR and CVaR models, we present the results under the confidence level (α) of 90%. The threshold of portfolio return τ is set as one basis point in our empirical test.

5. Empirical results

The core question in this study is whether return forecasting improves the realized performance of the portfolio models. Previous finance literature has documented the poor out-of-sample performance of risky portfolio models due to the weak predictability of return. Using forecasting returns by an ARIMA model in the above portfolio models, we rebalance the asset allocation while considering the transaction costs and weight constraints. The empirical tests evaluate the *ex post* effectiveness in portfolio management by using the data of the international ETFs.

Fig. 2 demonstrates the realized portfolio value for various portfolios with and without the return forecasting. Most of the portfolios that include return forecasting yield higher ending market value than their corresponding models that do not include return forecasting and the naïve diversified (1/*N*) portfolio. The only exception is the linearized value-at-risk models (in Panel D). The outperformance of the models incorporating the return prediction primarily is outside the financial crisis period (before 2007 and after 2010). The market value of the 1/*N* portfolio does not necessarily underperform the risky portfolios, particularly those not including the forecasting mechanism. Because high uncertainty in the estimating parameters weakens the effectiveness of the error-sensitive risky portfolio models, the return forecasting mechanism causes a negative impact on the performance during the financial crisis period. Among them, the LVaR model performs worse than the other risky models since this framework focuses on control downside risk. The simple equally-

Table 3
Additional Realized Return: Portfolio Models with Return Forecasting vs. Portfolio Models without Return Forecasting.

	MV	DSR	MAD	LVaR	CVaR	Omega
Mean	0.0579	0.0318	0.0051	0.0017	0.0307	0.0716
Max	0.1016	0.0885	0.0713	0.0205	0.0491	0.1010
Min	−0.0671	−0.0800	−0.0543	−0.0469	−0.0138	−0.0768
Outperformance %	56.21	57.99	54.44	53.85	60.95	56.21

Table 4
Portfolio value and total transaction costs.

Model	Ending Portfolio Value ($K)	Average Number of Investing Assets	Total Transaction Costs ($ 1,000)
1/*N*	2,092.3	21.0	2.6
MV	2,729.8	11.3	28.6
MV_F	5,971.8	5.5	138.6
MAD	2,703.9	3.3	152.8
MAD_F	2,897.3	2.8	166.6
DSR	2,248.6	5.9	83.8
DSR_F	3,456.4	4.3	134.8
LVaR	1,695.9	11.6	441.7
LVaR_F	1,734.6	11.5	431.8
CVaR	1,790.0	5.0	219.3
CVaR_F	2,710.2	4.7	259.9
Omega	2,367.5	8.8	46.6
Omega_F	6,236.1	3.6	255.0

weighted diversification yields a higher market value than the original CVaR and DSR portfolios without including return forecasting.

Fig. 3 shows, among the portfolio models, a significant variation in the economic value yielded by the addition of the return forecasting method. We define the benefit as the difference in the market value between the forecasting-return model and its corresponding original model. The benefit of adding the return predication is more consistent and significant for the Omega and MV models. The other four models either are of volatile realized benefits over the sample period, such as the CVaR, the MAD, and the DSR models, or yield a low value, such as the LVaR. Though the patterns of the benefit differ among the models, the size of the benefit consistently shrinks during the financial crisis between 2007 and 2009. This suggests that incorporating the return forecasting is useful when the market is stable. Since the pricing of diversified assets is efficient in general, the projected returns based upon historical data provide limited information when the market experiences a significant instability.

Table 2 presents the performance of the portfolio models. The realized return of the models with return forecasting is higher than the corresponding models without return forecasting as well as the naïve diversification portfolio, excluding the LVaR model. Adding return forecasting in the risky portfolio models improves the risk-adjusted performance, as measured by the Sharpe and Omega ratios. The distribution of the realized performance also shifts to the right-hand side when the return forecasting is incorporated. Among the models, incorporating return forecasting brings a greater increase in return to the Omega-ratio and MV portfolios. These models differ from the other four portfolios in their objectives focusing on possible profitability by taking on a certain level of risk. The models that emphasize a control of the loss, such as the DSR, the LVaR, and the CVaR, in general yield lower returns with lower volatility. The installation of return forecasting enhances their performance without considerably increasing the standard deviation of the realized return.

Table 3 demonstrates the summary statistics of the outperformance of the models with the return forecasting mechanism. Adding the return projection from an ARIMA to the portfolio models enhances the realized portfolio return from 16 (391 of LVaR vs. 407 of LVaR_F) to 716 (637 of Omega vs. 1354 of Omega_F) basis points per year. The adding returns vary within each of the portfolios, compared to the mean of the excess return. We also compute the percentage to which the models with return forecasting outperform the original models (the periods of positive difference in return generated by the forecasting models over the total periods). The likelihood of outperformance ranging from 54% to 61% suggests the usefulness of including the return projection.

We further investigate the source of the higher performance generated by the return forecasting. The superior profitability generated by the mechanism of return projection in the portfolio models may come from (1) more efficient asset allocation, and/or (2) the saving of the transaction costs in the rebalancing. As shown in Table 4, all the models with the return projection are of higher transaction costs than their corresponding portfolios. For instance, the total trading expenses for the MV model are about $28,600 while MV_F are $138,600 over the 170 rebalances. Noticeably, the models of the greatest economic value, the MV and Omega-ratio portfolios, have the most incremental trading expenses associated with incorporating the return projection. The only exception, the LVaR model, shows a minor reduction in transaction costs by including the forecasting mechanism ($442K vs. $432K). The results show that transaction costs are not a factor in justifying the improvement in portfolio performance. Though the Omega_F yields the highest market value among the models with high transaction costs. The trading costs only count for a tiny proportion of the difference in market value between the LVaR and LVaR_F models. The asset allocations that incorporate the return forecasting seem demonstrate a higher efficiency than those without including return projection.

Including the return projection diminishes portfolio diversification. As demonstrated in Table 4, it is common that the average

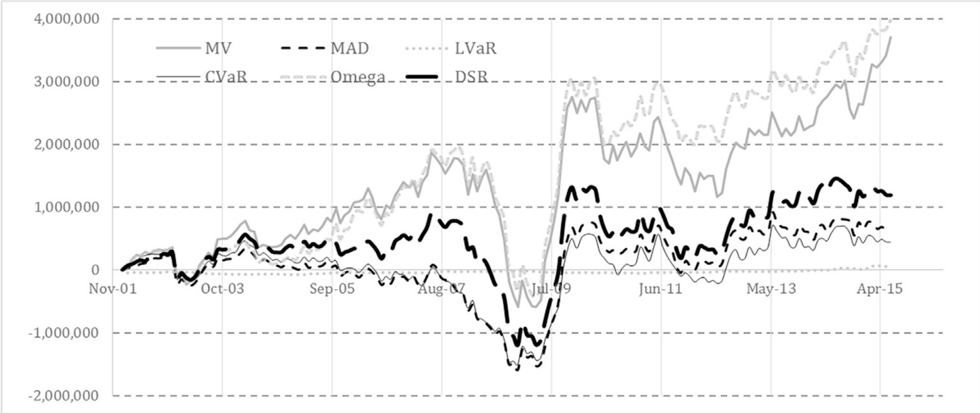


Fig. 4. Difference in Market Value: Models with Forecasting Returning vs. the 1/*N* Portfolio.

Table 5
Outperformance of the Models with the Return Forecasting Mechanism. Portfolio Models with Return Forecasting vs. the 1/*N* Portfolio.

	MV	DSR	MAD	LVaR	CVaR	Omega
Mean	0.0716	0.0312	0.0181	−0.0198	0.0132	0.0748
Max	0.3908	0.3908	0.3806	0.3296	0.3352	0.3908
Min	−0.5573	−0.5423	−0.5689	−0.5636	−0.5747	−0.5585
Outperformance %	56.21	48.52	47.93	47.93	46.75	52.07

number of assets invested decreases when the return forecasting is included in the portfolio models. Among them, the MV and Omega-ratio models show the greatest increase in portfolio concentration, but the level of asset diversity for the LVaR changes slightly when the return projection is incorporated. As observed by comparing the results of the portfolios with and without the forecasting return, the increase in portfolio value barely can be attributed to a cut in transaction costs but is instead associated with the change in asset allocation.

We further utilize an equally-weighted (1/*N*) portfolio applied by DeMiguel et al. (2009) as the benchmark for testing the performance of the new models. Though we show the usefulness of the return projection in constructing the risky portfolios, it is unclear whether the models with forecasting outperform a diversification without the knowledge of portfolio optimization techniques. Fig. 4 presents the difference in the market value between the forecasting-return model and the 1/*N* portfolio. Though the relation among the portfolio models is similar to the results of the economic value generated from the return projections shown in Fig. 3, the benefits over the naïve diversification are more unstable. Due to the limited predictability of return by its own time-series during market downturn, the excess value significantly drops becoming loss during the period from 2007 to 2009. This finding indicates that an inclusion of return forecasting using the time-series model only helps improve the performance of asset allocation when the market is stable.

Table 5 summarize the outperformance of the models with the return forecasting mechanism over the 1/*N* portfolio. The MV and Omega portfolios with the return prediction demonstrate higher performances than the other models and are the only two yielding higher likelihoods (56% and 52%) to outperform the naïve diversification. On the other hand, the equally-weighted diversification show higher return than LVaR model. There is a significant variation in the excess return over the 1/*N* portfolio within each of the portfolios.

Our empirical results show the usefulness of the inclusion of the return prediction in the risky portfolio models. The models that are based on both return and volatility, such as the MV and the Omega ratio, are more effective than those mainly focusing on controlling the loss, such as the LVaR, the CVaR, and the DSR. We show that incorporating the return projecting in the models improves realized portfolio value in the long term, especially when the economy is not expected to experience a significant recession. Our findings also verify DeMiguel et al. (2009), confirming that a naïve diversification does not essentially underperform sophisticated portfolio models. Investors may want to consider appropriate investing objectives and evaluate the market outlook when they determine whether the return forecasting should be incorporated.

6. Conclusions

We examine the effectiveness of adding return forecasting in various portfolio models. Specifically, we synthesize an Autoregressive Integrated Moving Average (ARIMA) model with various widely-applied approaches in managing investments, including mean-variance (MV), mean-absolute deviation (MAD), downside risk (DSR), linearized value-at-risk (LVaR), conditional value-at-risk (CVaR), and Omega ratio models. The naïve buy-and-hold market portfolio is used as the benchmark. To ensure the feasibility of the strategies, our empirical models incorporate the transaction costs and optimize short-sale weights according to market practices. The realized performance, including return, Sharpe ratio, Omega ratio, market value, transaction costs, and portfolio diversity, is presented.

The empirical results using 3,440 daily data of international Exchange Traded Funds show that adding the return forecasting in these models increases the realized performance even when the trading costs are considered. The additional returns yielded from the inclusion of the return forecasting vary significantly among the models, ranging from 16 (LVaR) to 716 (Omega) basis points per year. The benefit is more consistent and significant for the Omega and MV models. As incorporating the return projection leads to higher transaction costs and less portfolio diversity, the economic values of the return projection come from more efficient asset allocation but not from a reduction in trading expenses. The benefits over the naïve diversification deviate across portfolio models, are time-varying, and are particularly low during the period of financial crisis. Inclusion of return forecasting improves the *ex post* performance for the models that emphasize both return and risk in portfolio decisions, such as the MV and Omega, when the market is stable.

We advance a wide range of risk-return portfolio models by explicitly integrating them with return forecasting, transaction costs, and short-sales. Although the portfolios with the forecasting mechanism perform better than their corresponding original models, they do not consistently outperform a naïve diversification. The forecasting mechanism more likely yields outperformance when the market is relatively stable. Future research can compare the effectiveness of incorporating various return forecasting mechanisms in different portfolio models and asset classes.

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Appendix A. Supplementary data

Supplementary data to this article can be found online at https://doi.org/10.1016/j.iref.2019.11.002.

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