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A brief review of portfolio optimization techniques

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Abstract
Portfolio optimization has always been a challenging proposition in finance and management. Portfolio optimization facilitates in selection of portfolios in a volatile market situation. In this paper, different classical, statistical and intelligent approaches employed for portfolio optimization and management are reviewed. A brief study is performed to understand why portfolio is important for any organization and how recent advances in machine learning and artificial intelligence can help portfolio managers to take right decisions regarding allotment of portfolios. A comparative study of different techniques, first of its kind, is presented in this paper. An effort is also made to compile classical, intelligent, and quantum-inspired techniques that can be employed in portfolio optimization.

Keywords Portfolio optimization · Statistical measures · Machine learning · Deep learning · Reinforcement learning · Evolutionary techniques · Quantum computing

1 Introduction

In financial terms, a portfolio is a collection of assets/investments. Due to changing market dynamics, diversification of investment is a crucial mechanism used to reduce risk on investment, as proposed by Markowitz et al. Markowitz (1959). Based on degree of risk and return, portfolio can be further categorized as (i) Aggressive Portfolio (ii) Defensive Portfolio (iii) Income Portfolio (iv) Speculative Portfolio and (v) Hybrid Portfolio. Table 1 provides a brief description on the different types of portfolios.

During the buying and selling of the assets, a transaction cost is imposed by the brokers or financial institutes as commission or other expenses Mansini et al. (2015). Henceforth, an optimum number of asset selection is important to reduce the transaction cost. In order to handle this situation, a proximal approach involving time penalization is proposed in García-Galicia et al. (2019) which involves a three-step process viz., (i) a dynamic stock price process (ii) transformation of solution as a continuous-time discrete-state Markov

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decision processes Bäuerle and Rieder (2011) and (iii) a time penalization technique on transaction cost. Selection of limited number of assets makes the investment more manageable and explainable Babaei and Giudici (2021). A sparse and robust portfolio selection process is proposed in Lee et al. (2020) based on two step process viz., (i) develop a sparse mean-variance portfolio selection model using semi-definite relaxation and (ii) extend the model by using L_2-norm regularization and worst-case optimization to formulate two sparse and robust portfolio selection models.

Most of the methods used in portfolio selection proces suffers from estimation error and hence there is a need to regularize the portfolio process Bruder et al. (2013). A lagrangian regularization method is suitable for continuous time markov chain as proposed in Vazquez and Clempner (2020). Based on the complexity of the problem, it is quite evident that any portfolio management techniques requires a dynamic decision making process and hence a reinforcement based learning techniques Wu et al. (2021) are highly suitable to effectively manage the portfolios.

Portfolio management Cooper et al. (2001), hence involves dynamic decision process where the portfolios are evaluated, selected, and prioritized based on financial objective and risk tolerance setup by the organization or client or institute Jeffery and Leliveld (2004). Portfolio management requires careful SWOT (Strength-Weakness-Opportunity-Threat) analysis across all the investments. The choice of right stock is done based on different category of portfolio as described in Table 1. Some of the key elements of portfolio management are (i) Asset Allocation (ii) Diversification and (iii) Rebalancing. as mentioned in Table 3. Portfolio Management can be of two types viz., (i) Passive Portfolio Management and (ii) Active Portfolio Management as mentioned in Table 4.

Further, the types of portfolio can be extended to (i) Transaction Cost Portfolio (ii) Robust Portfolios (iii) Regularized Portfolios and (iv) Reinforcement learning based Portfolios as mentioned in Table 2.

Portfolio management also requires optimization techniques to select the best portfolio. Over the years, portfolio optimization techniques have also evolved from techniques like mean-variance (MV) Markowitz (1959), variance with skewness (VwS) Samuelson (1975), Value-at-Risk (VaR) Jorion (1997), Conditional Value-at-Risk Rockafellar and Uryasev (2000), Mean-absolute deviation (MAD) Konno and Yamazaki (1991) and Minimax (MM) Young (1998) to more advanced heuristic and meta-heuristic based methods. In recent years, evolutionary algorithms (EA) and Swarm Intelligence (SI) have become popular in the field of portfolio optimization. Some of the SI based techniques used are particle swarm optimization (PSO) Kennedy and Eberhart (1942), ant colony optimization (ACO) Coloni et al. (1996), bacterial foraging optimization (BFO) Passino (2002), artificial bee colony (ABC) Karaboga (2005), cat swarm optimization (CWO) Chu et al. (2006), firefly algorithm (FA) Yang (2009), invasive weed optimization (IWO) Karimkashi and Kishk (2010), bat algorithm (BA) Yang (2010) and fireworks algorithm (FA) Tan and Zhu (2010). Other than the above mentioned techniques, quantum inspired approaches have also become popular in the field of portfolio optimization.

The objective of this paper is to list different portfolio optimization techniques along with their advantages and limitations. This will help the audience to have a compiled view of the different state-of-art techniques and related advancements which has taken place in recent years.

The remainder of the paper is organized as follows. Section 2 briefly lists the motivation behind this article. Section 3 briefly lists different types of portfolios. Section 4 provides a brief definition and different types of measures used in portfolio management. Section 5 provides a brief introduction of portfolio optimization and its relevance. Section 6 lists out the classical approaches employed in portfolio optimization. Section 7 lists the intelligent approaches. The paper is concluded in Section 8. The organization of this paper is described in Fig. 1.

2 Motivation

In every time bond project management, success is determined by two factors viz., *"doing the right thing"* and *"doing things right"* Rämö (2002). The portfolio management techniques hence require better allocation of stock to maximize return while keeping the risk low. Portfolio management and optimization have always been a challenging problem and have aroused interest among the researchers in the technical as well as financial domains Haugh and Lo (2001). Despite multiple advancements done in new theories and computation power, portfolio optimization still remains a challenging problem to solve. Also, as per the "Bureau of Labor Statistics", the employment of financial analyst including portfolio managers is expected to increase by 5% from 2019 to 2029. Henceforth, there is a need for better tools and mechanisms to perform portfolio optimization. Some of the key factors which make it an area of interest includes

(i) dynamically changing market factors (ii) dynamically changing constraints like regularity constraints, liquidity, taxes, transaction costs, management fees, etc., and (iii) need for high computation power to determine the right distribution of selection in a timely bound manner. The main contribution of this paper is that it presents a brief review of classical and intelligent approaches that can be employed for portfolio optimization. A brief study of quantum-inspired based approaches are also touched upon, which opens new areas for efficient and faster computing in this direction.

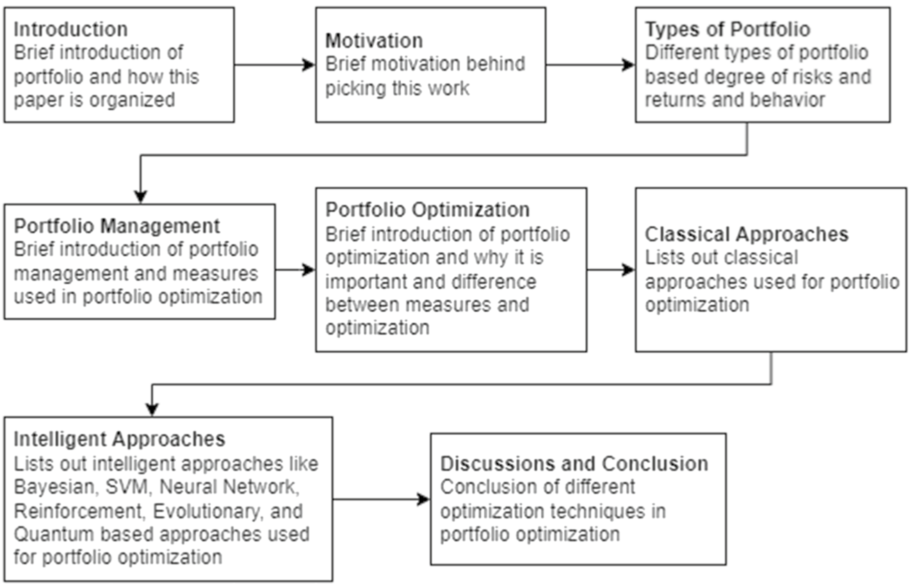


Fig. 1 Paper organization flow

3 Types of portfolios

Based on the degree of risk and return, the portfolio can be categorized into five categories viz., (i) Aggressive Portfolio (ii) Defensive Portfolio (iii) Income Portfolio (iv) Speculative Portfolio and (v) Hybrid Portfolio. Aggressive portfolio aims for higher returns and often undertake higher risks to achieve the objective. The preference is given to the companies that are in initial growth stages. On the contrary, defensive portfolio aims for minimal risk. The preference is given to the companies that provide daily need products to ensure the risk is minimum in adverse conditions. An income portfolio is somewhat similar to a defensive portfolio, but focuses on gain from dividends or other recurring benefits. Few examples of income portfolio includes real estates, FMCG, and other stable industries. The speculative portfolio aims for extremely high risk and is often termed as gambling. Few examples of speculative portfolio includes initial public offers (IPOs), investments into initial level of product research, and takeover targets. The hybrid portfolio aims to provide optimum return with optimum degree of risk. It uses an amalgamation of different types of assets based on the degree of risks and returns.

Furthermore, based on behavior the types of portfolio can be extended into four categories viz., (i) Transaction cost portfolio (ii) Robust portfolio (iii) Regularized portfolio and (iv) Reinforcement learning portfolio. During buying and selling of assets, there is transaction cost involved as transaction fee and charges. The transaction cost portfolio aim to reduce the transaction cost by employing time penalization techniques. Contrary to this, robust portfolio aims to reduce transaction cost in sparse and robust portfolio selection process. Typically, an optimization process often leads to estimation error that are handled in regularized portfolio. Furthermore, a changing market dynamics requires continuous learning of market conditions that are handled in reinforcement learning portfolio. A detail optimization technique to handle reinforcement learning portfolio is listed in Section 7.4. The types of portfolios are listed in Tables 1, 2

4 Portfolio management

Portfolio in literal terms means a collection of financial investments in the form of stock, bond, cash, assets, and other forms of commodities which tends to give some form of return in future. A portfolio can be managed by an individual or a group of financial institutes specialized in managing portfolio. Hence, building portfolio is one of the key parameters in growing business. Proper analysis of portfolio can help a company to analysis if the business would receive more or less investments. It also helps to analyze the overall strategy and focus of a company. In few cases, it has been observed that companies are opening multiple business models at once, despite their inability to execute them successfully Aversa et al. (2017). These factors must be analyzed by portfolio managers while selecting the right assets.

At any given time t, a portfolio can be represented as

$$P(t) = \sum_{j=1}^{n} w_j(t) s_j \tag{1}$$

where, $s_1, s_2, ..., s_n$ are the securities and $w_1, w_2, ..., w_n$ are the weights or quantities associated with the securities. The weight at time t is represented as

$$w_{tj} = \frac{\$ \text{ invested in securities } s_j}{\text{Total } \$ \text{ invested in portfolio } P_t}$$

Let us assume a list of securities as $GOOGL$, $AMZN$, $AAPL$ with optimum weights as $3,\ 2,\ 5$, then the corresponding portfolio at time t is represented as

$$P_t = 3 \times GOOGL + 2 \times AMZN + 5 \times AAPL$$

Based on the market dynamics there could be extended investment or withdrawal of existing stocks. The decision involves allocation or re-allocation of stock and requires dynamic opportunity, multiple goals and objective consideration. A good analyst should consider multiple factors before making any allocation and de-allocation decisions Cooper et al. (2001). An efficient portfolio management involves dealing with problems like (i) future events and opportunities (ii) dynamic decision making capabilities (iii) investments made could be in different stages making decision making more complex and (iv) limitations in resources which are allocated. A portfolio manager typically work with four goal viz., value maximization, balancing, strategic direction, and right number of projects. Further, portfolio optimization can be performed based on multiple parameters, as mentioned in Table 5.

The following subsections list out different measures used in portfolio management. Subsection 4.1 briefly describes mean variance risk measure while subsection 4.2 describes mean-absolute deviation. Further minimax is described in subsection 4.3 while lower partial moments are described in subsection 4.4.

4.1 Mean variance risk measure

A revolutionary technique for portfolio selection as proposed by Markowitz Markowitz (1959) is based on mean-variance. As per the proposal, mean-variance can be used for effective portfolio selection when the objective is to (i) minimize variance for a given expected return and (ii) maximize expected return for a given variance Mean-Variance (MV) Markowitz (1959) analysis is very useful and easy to implement when the distribution function is not quadratic in nature. Above all, mean-variance methodology is very fast. MV can also be employed while tracking errors Roll (1992); Feldstein (1969). One of the major drawbacks of mean-variance analysis is that it cannot be employed in high complex problems where the distribution function is polynomial in nature. A comprehensive guide on mean-variance is available in Markowitz and Todd (2000). The objective of the MV model is to find the weight of the assets that will minimize the variance at a given rate of return. The mathematical model is given by Markowitz and Todd (2000)

$$minimize \sum_{i=1}^{n} \sum_{j=1}^{n} \sigma_{i_j} x_i x_j \tag{2}$$

$$\text{subject to} \sum_{j=1}^{n} r_j x_j \geq \rho M_0, \tag{3}$$

Also, $0 \leq x_j \geq u_j, j = 1, 2, ..., n$ where, ρ_{i_j} is the covariance between assets i and j, x_j is the amount invested in an asset j, r_j is the expected return of asset j per period, ρ is a parameter representing the minimal rate of return required by an investor, M_0 is the total amount of fund and u_j is the maximum amount of money which can be invested in asset j. The challenge in the MV model arises when it tries to compute covariance and hence cannot be employed in large scale portfolio optimization problems.

Further, in large implementation, data is generally not normally distributed and there lies asymmetry in the probability distribution. A mean-variance-skewness based measure thus becomes more effective in such scenarios Samuelson (1975). In the traditional approaches, the trade-off between risk and return is not explicit leading to less intutive investment decisions. A stochastic based optimization algorithm is presented in Yin et al. (2002), which has been used for a class of stock liquidation problems that are based on hybrid geometric Brownian motion models. It allows regime changes by using continuous-time finite-state Markov chain. A controlled regime-switching system Yang et al. (2015) has been used where the observation is noisy. In such scenarios, the focus has been to minimizing the variance subject to a fixed terminal expectation, and thereby employ a Wonham filter Borisov (2011) to convert the partially observable system into a completely observable system. A continuous-time penalization García-Galicia et al. (2019) has been employed when the market is characterized as arbitrage-free that involves transaction cost and no short-selling of stocks. A lagrange Ito and Kunisch (2008) multiplier approach has been proposed in García-Galicia et al. (2019) to solve convex non-linear problems.

A sparse mean-variance portfolio selection model Lee et al. (2020) can be employed when the assets are sparse and robust at the same time. A suitable portfolio selection is performed by using two convex portfolio selection models to handle both sparsity and robustness in the assets with dynamically changing markets. The proposed technique in Lee et al. (2020) assumes that (i) the portfolio should not contain too many assets and (ii) the portfolio should be robust.

4.2 Mean-absolute deviation

The mean-absolute deviation (MAD) model as proposed in Konno and Yamazaki (1991) Konno and Koshizuka (2005) can be employed for large scale and highly diversified portfolio selection problems. A long term asset liability management (ALM) Bauer et al. (2006) model and mortgage backed security Hayre (2002) model can also be employed where the investments are done for a longer period and the investments are made in very diversified portfolios. A robust portfolio optimization using MAD Moon and Yao (2011) was applied on different stock market data NYSE (2003). It has been observed that the model takes advantages of reduction in computational complexity and provides optimum results. MAD is also observed to outperform the mean-variance Markowitz (1959) based models. This model suffers a drawback, as it penalizes both positive and negative deviations. Mathematically, the model can be represented as Konno and Yamazaki (1991)

$$\begin{aligned} &minimize\ w(x) = E \mid \sum_{j=1}^{n} R_j x_j - E \sum_{j=1}^{n} R_j x_j \mid \\ &\text{subject to } \sum_{j=1}^{n} E \mid R_j \mid x_j \geq \rho M_0, \text{ and} \\ &\sum_{j=1}^{n} x_j = M_0, \end{aligned} \tag{4}$$

$0 \leq x_j \leq u_j, j = 1, 2, ..., n$ where R_j is the return of asset j, x_j is the amount invested in asset j, ρ is a parameter representing the minimal rate of return required by an investor, M_0 is the total amount of fund and u_j is the maximum amount of money which can be invested in an asset j.

4.3 Minimax

Minimax (MM) Cai et al. (2004) Li et al. (2019) uses the minimum return as a measure of risk. In scenarios where the assets returns are multivariate and normally distributed, then both Minimax Cai et al. (2004) and mean-variance Markowitz (1959) leads to the same result. Minimax Cai et al. (2004)Li et al. (2019) has certain advantages when the returns are not normally distributed. Minimax Cai et al. (2004) is fast due to its property of linear programming and can be employed for more complex models and constraints. One of the disadvantages of Minimax Cai et al. (2004) is its sensitivity to outliers, and hence it cannot be employed when the historical data is missing. Mathematically, the model can be represented as Cai et al. (2004)

$$\begin{aligned} &max\ M_p \\ &\text{subject to } \sum_{j=1}^{N} w_j y_j - M_p \geq 0, t = 1, 2, ..., T, \\ &\sum_{j=1}^{N} w_j \overline{y_j} \geq G, \\ &\sum_{j=1}^{N} w_j \leq W, \\ &w_j \geq 0, j = 1, 2, ..., N \end{aligned} \tag{5}$$

where, y_{i_j} is the return on one dollar invested in a security j in a time period t, $\overline{y_j}$ is the average return on security j, w_j is the portfolio allocation to security j, M_p is the minimum return on portfolio, G is the minimum level of return and W is the total allocation.

4.4 Lower partial moment

Lower Partial Moment (LPM) Nawrocki (1992) Brogan and Stidham (2008) uses a set of moments to estimate downside risk in a portfolio. Based on continuous study, there could be multiple such kinds of moments. Hence, as a part of LPM Nawrocki (1992), multiple

N such kinds of lower moments are analyzed. It is further used to calculate sortino ratio, omega ratio, and kappa ratio Chen (2016). The model can be defined as Nawrocki (1992)

$$LPM_{\alpha}(\tau, Ri) = \int_{-\infty}^{\tau} (\tau - R)^{\alpha} \, \partial F(R) \tag{6}$$

where, α is the degree of LPM, τ is the target return, R is the return, and $\partial F(R)$ is the cumulative distribution function of the asset return R.

5 Portfolio optimization

Portfolio optimization is the process of selecting the best combination of assets based on pre-defined objectives. The objectives could be maximization of return or minimization of risk, or both. Each optimization technique uses different portfolio measures as fitness functions. In fact, due to large number of assets to choose from, manual selection of right set of assets is difficult and hence requires advanced techniques as mentioned in Sections 6 and 7. Further, each optimization technique uses different portfolio measures as objective functions to come up with an optimum combination of assets as mentioned in Table 5. The optimization techniques are based on heuristics and do not require historical data for optimization. This paper presents several optimization techniques using different portfolio measures as listed in Table 5. Several optimization techniques can be used based on their complexities, limitations, and advantages as highlighted in Tables 6, 7, 8, 9.

6 Classical approaches to portfolio optimization

Portfolio optimization is inevitably an important process to minimize risk and maximize return on investments. Henceforth, it has always been a challenging task to have a proper mix of risks associated along with expected returns with limited number of assets to be considered. Almost all the classical approaches like mean-variance Markowitz and Todd (2000), variance with skewness Samuelson (1975), mean absolute deviation Konno and Yamazaki (1991); Konno and Koshizuka (2005), minimax Cai et al. (2004) Li et al. (2019), and lower partial moments Nawrocki (1992) mentioned in Table 5 are based on asset and cardinality constraints Cesarone et al. (2011). There are numerous approaches mentioned in the literature McNeil et al. (2015)Galai et al. (2001), which mainly incorporate a combination of portfolio states Ray and Bhattacharyya (2015). Value-at-Risk (VaR) Dowd (2007)Holton (2003)Jorion (2007) is a measure of, how the market value of an asset is likely to reduce over a period of time under certain market conditions. VaR Jorion (2007) requires determination of three parameters viz., (i) (ii) time horizon (iii) confidence level and unit of VaR. While VaR Jorion (2007) is fairly a simplistic technique, it does not consider risk measures. As mentioned in Ray and Bhattacharyya (2017), a conditional Value-at-Risk (CVaR) Rockafellar and Uryasev (2002) based technique can be employed to consider risk measures. The condition associated with value-at-risk is the weighted average of VaR and losses exceeding value-at-risk Krokhmal et al. (2002). VaR Jorion (2007) and CVaR Rockafellar and Uryasev (2002) focus mainly on maximizing the expected return. Both VaR Jorion (2007) and CVaR Rockafellar and Uryasev (2002) are efficient when the underlying risk factors are normally distributed, but face challenges for discrete distribution, making it non-convex

Rockafellar (1970). Another challenge while using VaR Jorion (2007) is that it lacks an analytical functional form and is generally a nonlinear, non-convex and non-smooth function.

There are multiple smoothing techniques presented in Xu et al. (2007)Xu (2003)Xu and Ng (2006) which can be employed while using VaR Jorion (2007) and CVaR Rockafellar and Uryasev (2002) to get optimum results. While working with long-term investments, there is some amount of uncertainty involved, which can be handled by using Monte Carlo simulation Eckhardt (1987). The scenario based analysis can be further done using Mulvey (2001) and Hibiki (2001). Furthermore, an uncertainty set can be employed with random interval Chen et al. (2011) as an optimization technique. VaR Jorion (2007) requires a perfect knowledge of data, but in reality it is prone to errors. Hence, an estimation based on wrong data points may be misleading and inaccurate. The technique proposed in Chen et al. (2011) can be employed in such scenarios having high performance and low cost. Another hybrid technique which combines mean-variance portfolio optimization technique and Shefrin Statman's behavioral portfolio technique is proposed in Das et al. (2010). The proposed mental accounting (MA) Das et al. (2010) framework includes structure of portfolios, definition of risk, and attitude towards risks. The proposed methods in Das et al. (2010) result in better connection between portfolio production and consumption goals.

Value at Risk (VaR) Dowd (2007)Holton (2003)Jorion (2007) is one of the primitive statistical approaches used for portfolio optimization. Both risk and return are correlated with each other, and at times inversely related. High risks may lead to high returns and vise versa. VaR is easily interpretable and at times self explanatory. VaR requires estimation of three parameters viz., (i) time period (ii) confidence level and (iii) unit of value at risk.

The time period denotes the time horizon that is considered for analysis. Typical buying and selling can be intraday, weekly, monthly, or yearly. Also, the period can be taken as 1 day, 10 days, or any other arbitrary value. The confidence level is the interval in which VaR Jorion (2007) would not exceed the maximum loss. Typically, the confidence value is taken as 95% or 99% depending on the objective set for portfolio optimization. The unit of VaR Jorion (2007) refer to the quantity of stock, bond, other types of assets to be considered while building the portfolio. A mathematical representation of portfolio is given in equation 1.

VaR Jorion (2007) could be misleading at times as it does not consider the worst case loss and cannot be applied in large portfolios. VaR Jorion (2007) is discrete in nature and difficult to simulate. Hence, there is a need to look at scenario based approaches like the Conditional Value at Risk (CVaR) Lim et al. (2011). Suppose $f(x, y)$ represents the percentile of loss or reward based on decision vectors $x = (x_1, x_2, ..., x_n)$ and random vectors $y = (y_1, y_2, ..., y_n$, then VaR can be represented as α percentile. VaR calculated considering loss distributed α is termed as the conditional value at risk. (CVaR) can be of two types, viz., "upper CVaR" ($CVaR^+$) or "lower CVaR" ($CVaR^-$). In $CVaR^+$, the expected loss exceeds VaR. Thus, (CVaR) can be represented as Lim et al. (2011)

$$CVaR = \lambda VaR + \ (1 - \lambda)\ CVaR^+,\ where\ 0 \leq \lambda \geq 1 \tag{7}$$

As denoted by the expression, CVaR Rockafellar and Uryasev (2002) can handle extreme losses by using dynamic weight α derived from $f(x, y)$ based on the decision and random vectors. Also, CVaR Rockafellar and Uryasev (2002) can be employed in volatile scenarios and helps in managing risk more efficiently as compared to VaR Jorion (2007). CVaR Rockafellar and Uryasev (2002) suffers a major disadvantage that it cannot indicate the maximum loss that can be incurred.

Some of the traditional techniques like hidden markov model Rabiner and Juang (1986), efficient frontier Best and Hlouskova (2000), and monte carlo simulation Pedersen (2014) are other popular classical approaches used in portfolio optimization.

The mean-variance Markowitz (1959) portfolio optimization is proposed as a parametric quadratic programming (PQP) in Zl et al. (2008). A bagging along with boosting can also be employed to construct an ensemble of classifiers Derbeko et al. (2002). Furthermore, bagging helps in reducing the variance Bauer and Kohavi (1999)Bühlmann and Yu (2002)Friedman and Hall (2007) and hence is an ideal choice by portfolio managers.

7 Intelligent approaches to portfolio optimization

Machine Learning (ML) Alpaydin (2020) based techniques are now very popularly used due to their ability to learn from historical data. A huge volume of dataset can be quickly churned and analyzed using these intelligent techniques. This section contains a list of intelligent approaches that can be applied for portfolio selection and optimization.

The following subsections mention different intelligent approaches used in portfolio optimization.

7.1 Bayesian approaches

During the 1770s, Thomas Bayes introduced the "Bayes Theorem" which is still relevant and used in solving multiple complex problems using its inferential technique. Bayes' theorem describes the probability of the occurrence of an event based on prior knowledge of conditions which might be related to the event. In the context of portfolio management, it represents the probability of risk of buying or selling based on prior knowledge of the market conditions.

According to Bayes' theorem, the conditional probability is represented as Alpaydin (2020)

$$p(C_k|x) = \frac{p(C_k)p(x|C_k)}{p(x)} \tag{8}$$

where, $p(C_K|x)$ is the posterior probability or the updated probability after taking into consideration other parameters; $p(C_k)$ is the prior probability or the probability that is assigned before any relevant evidence is taken into account; $p(x|C_k)$ is the likelihood; and $p(x)$ is the evidence from historical observations.

Portfolio optimization as proposed in Shenoy and Shenoy (2000) uses a Bayesian network to model risk and return. It has been employed to visualize the relationship between different variables in the model. Similar to financial analysts, it combines historical data with qualitative information to draw a relationship between different portfolios. The technique proposed in Shenoy and Shenoy (2000) is well suited in situation where qualitative and quantitative information are combined. Further Bayesian approaches can also be applied in new product development, as proposed in Yang and Xu (2017). It uses a three step process viz., (i) (ii) identify portfolio management factors and determine performance criteria (iii) model relationship among the factors within a similar time frame and (iv) develop a Bayesian network model. A fuzzy based Bayesian approach as proposed in Bai et al. (2020) can be employed to assess risk and devise risk-reduction strategies. The proposed technique takes project interdependency into consideration to determine the probability of all the risks by employing fuzzy based techniques. The error caused due to subjectivity of expert knowledge is also considered in the proposed approach Bai et al. (2020).

Bayesian network can also be used to identify risk transfer Guan et al. (2014)Guan and Guo (2014) and helps in constructing an independent network of risks. This particularly overcomes the drawback of greedy algorithms, which require a starting structure and other risk parameters. The results as proposed in Guan et al. (2014)Guan and Guo (2014) shows that the interdependent network of risks is an effective measure to determine the risk transfer in projects and helps in computing the value of portfolio risks.

7.2 Support vector machine based approaches

Support vector regression (SVR) Drucker et al. (1997) has been used to determine the quantity to buy and sell. SVR is a machine learning based approach where historical data is fed to the SVR Drucker et al. (1997) to learn the pattern from the historical path. In modern times, machine learning based techniques have become more popular due to their ability to run faster on huge datasets. They also have the ability to learn changing market positions of the stocks and can be employed for dynamic selection and rejection problems.

Given a training set of instance-label pairs $(x_i, y_i), i = 1, 2, ..., m$ where $x_i \epsilon R^n$ and $y_i \epsilon {-1}, 1$, for a linearly separable case, the data points can be correctly classified by

$$\langle w.x_i \rangle \ + b \geq \ +1, \ for\ y_i = +1 \tag{9}$$

$$\langle w.x_i \rangle \ + b \leq \ -1, \ for\ y_i = -1 \tag{10}$$

Combining the above two equations gives

$$y_i(\langle w.x_i \rangle \ + b) \ - 1 \geq 0, \ \forall i \tag{11}$$

where, w is the normal vector of the hyperplane and b is the bias value. The objective of SVM Cortes and Vapnik (1995) is to find the most optimum hyperplane. SVM Cortes and Vapnik (1995) has certain advantages over statistical based techniques like (i) (ii) ability to learn from historical data (iii) can be used to build feedback to continuously learn (iv) very effective in high dimensional space (vi) memory efficiency. SVM Cortes and Vapnik (1995) works very well when the data points are linearly separable, but in the scenario when there is overlap, it underperforms.

Further, SVM Cortes and Vapnik (1995) is also known as regularization formulation where the loss in prediction and penalty is traded off by one or more regularization parameters. Henceforth, it can be used to trace the solution paths for ϵ-support vector regression, uniclass SVM, and quadratic regression as mentioned in Zl et al. (2008).

7.3 Neural network based approaches

A neural network or a network of neurons, also referred to as the artificial neural network Haykin (2010), is used for solving complex computational and learning problems. A typical neural network has input layers, hidden layers, and output layers as shown in Fig. 2. The hidden layers help to refine learning from historical data.

In a neural network, the neuron is an information processing unit which forms the basis of any neural network model Haykin (2010). A nonlinear model of a neural network is

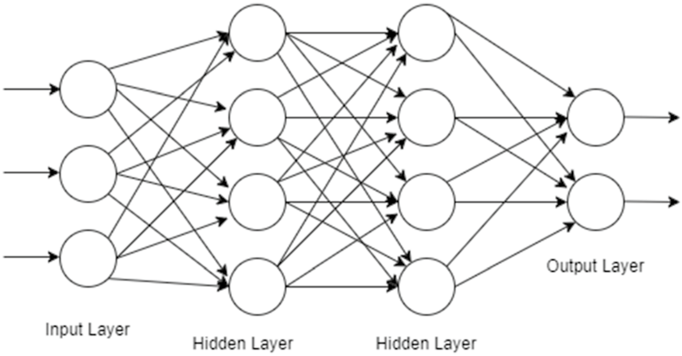


Fig. 2 Neural network with two hidden layers

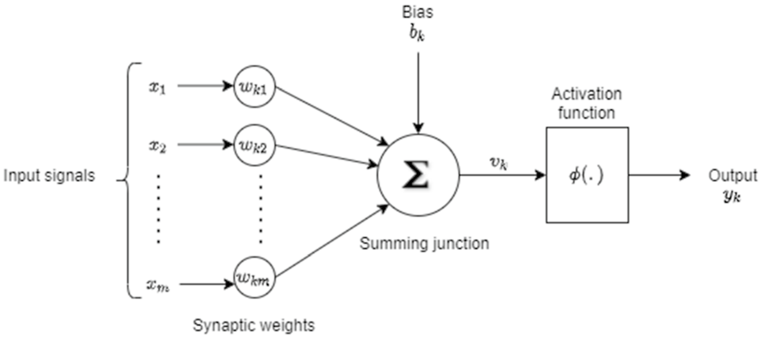


Fig. 3 Neural network with nonlinear model

shown in Fig. 3. As shown in Fig. 3, there are three basic elements of any neural network model viz., (i) (ii) synapses or connecting links with weights (iii) an adder for adding the input signals and (iv) an activation function for limiting the amplitude of the output. Bias b_k is also introduced to reduce or increase the net input to the activation function based on amplitude. A neuron k described in Fig. 3 is represented as Haykin (2010)

$$u_k = \sum_{j=1}^{m} w_{kj}x_j \tag{12}$$

and

$$y_k = \phi(u_k + b_k) \tag{13}$$

where, $x_1, x_2, ..., x_m$ are the input signals; $w_1, w_2, ...w_m$ are the synaptic weights of the neuron k. u_k is the linear combiner output; b_k is the bias; $\phi(.)$ is the activation function and y_k is the output signal. The activation potential is represented as

$$v_k = u_k + b_k \tag{14}$$

Portfolio optimization as proposed in Ban et al. (2018) uses a performance-based regularization (PBR) function, which helps in reducing the estimation error. PBR is considered for both mean-variance and mean-CVaR problems. A quadratic polynomial constraint is introduced in Ban et al. (2018) for two convex approximation. Further, the result was tested using k-fold based validation. Other than $L1$ and $L2$ regularization, there is also a need to have some mechanism to penalize incorrect predictions. A comprehensive penalization method is demonstrated in Eckstein and Kupper (2019) which can be used as a coherent measure for risk Artzner et al. (1999). It has been found that recurrent neural networks (RNN) are very efficient in understanding temporal dependencies. A comprehensive study with a hybrid model using long-short term memory (LSTM) as proposed by Schmidhuber et al. Schmidhuber and Hochreiter (1997), and Wiener et al. Saboia (1977), is presented in Choi (2018). In the proposed approach, the ARIMA model has been used to filter linear tendencies in data and passed the residual to the LSTM model. The proposed technique in Choi (2018) was tested on 150 S &P500 stocks and was compared with walk-forward optimization method Ładyżyński et al. (2013). Further, in the series of hybrid techniques, there has been works performed using ensemble techniques using deep learning and reinforcement learning. The objective is to generate financial-model-free reinforcement learning framework with deep learning techniques to come up with better portfolio management. The proposed technique uses (i) ensemble of identical independent evaluators (EIIE) topology (ii) online stochastic batch learning (OSBL) and (iii) reward function Jiang et al. (2017). The proposed technique used cryptocurrency and is examined in three back-test experiments with trading period of 30 minutes. In all the three back-tests, the proposed techniques seem to perform better than the integrated CNN (iCNN) (Jian and Liang, 2017). An optimal reward using reinforcement learning based techniques can be achieved in portfolio optimization, as proposed in Chaouki et al. (2020). A comparison of Sharpe ratio of major neural network based techniques are mentioned in figure 4.

Further, a risk-sensitive multi-agent network (RSMAN) Park et al. (2022) has shown better performance over deep neural networks (DNNs). The proposed framework in Park et al. (2022) uses (i) (ii) risk-sensitive agents (RSAs) and (iii) risk adaptive portfolio generator (RAPG) and hence can understand the market risk better than DNNs.

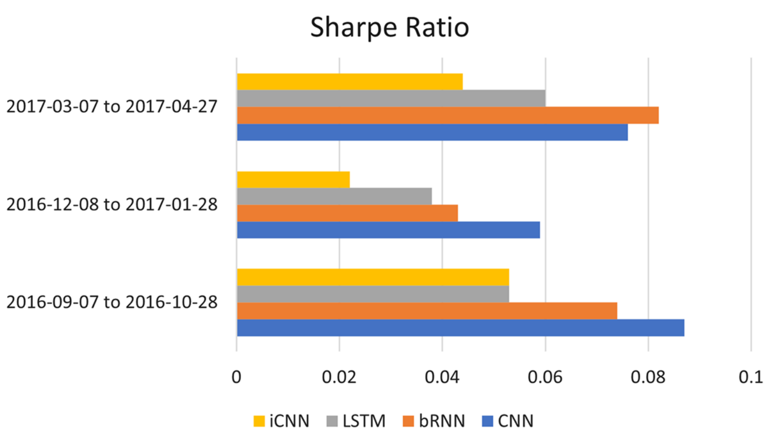


Fig. 4 Sharpe Ratio Comparison of Neural Network based appraoches Jiang et al. (2017)

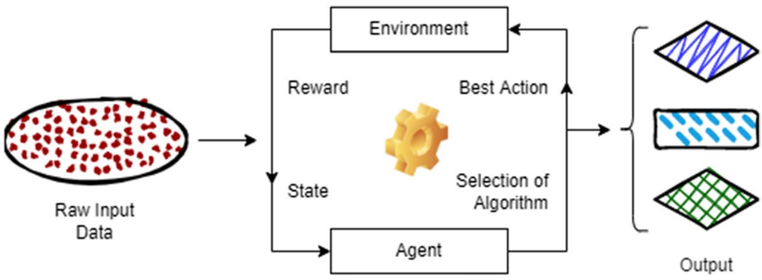


Fig. 5 Reinforcement Learning

7.4 Reinforcement approaches

Reinforcement follows a mechanism of dynamic learning or weight based on action taken by the agents. There is a continuous observation based on user actions and the learning weight is further defined as shown in Fig. 5. Reinforcement based learning is very popularly used in computer gaming and can be extended to portfolio optimization problems where the weights can be learned dynamically.

There are four major elements in reinforcement learning viz., (i) Policy (ii) Reward (iii) Value and (iv) Model. A policy defines the way the agents behaves in a certain scenario. It requires mapping of the current state with the actions taken by the agent. Further, based on the objective set and the agent's reaction, positive and negative rewards are given. The objective is to maximize the total reward. The distinction of good and bad rewards is defined by the value function. The environment behaviour is modelled by using appropriate model.

Reinforcement learning involves learning procedure through an agent. The agent interacts with the environment and takes appropriate action which modifies the state. The action taken by the agent leads to either reward or penalty. At any given point of time t (where t=0, 1, 2, ...), s_t represents the set of all possible states (where $s_t \in S$ represents the state of the agent). Also $a_t \in A(s_t)$ represents the action taken by the agent at t where $A(s_t)$ is the set of possible actions in state s_t. On every action a_t, reward $r_{t+1} \in R$ is generated, and the agent moves to the next state s_{t+1} Alpaydin (2020).

A state function is given by Alpaydin (2020)

$$v_\pi(s) = E_\pi[G_t | S_t = s] \tag{15}$$

where, v is the value function with state s of policy π. G_t is the expected return from the given state S_t.

The reinforcement based technique Sutton and Barto (2018) works on maximizing award to the agent. This technique can be certainly applied in portfolio optimization, where the weight can be dynamically computed against static weights used in traditional approaches. Portfolio Management System (PMS) using a convolutional neural network (CNN) and recurrent neural network (RNN) as proposed in Wu et al. (2021) uses a novel reward function involving Sharpe ratio and evaluation technique to measure the performance of the model. The proposed reward function further enhances the performance and supports resource allocation for empirical stock trading. With reinforcement learning (RL) there are two major branches which are developed viz., (i) Q-learning and (ii) policy gradient. While Q-learning is used to learn the optimal reaction and establish Q-table using Markovian circumstances, policy gradient is used to model optimal reward.

An adaptive technique, as proposed in Almahdi and Yang (2017), extends the work using recurrent reinforcement learning (RRL) Moody and Saffell (2001) and builds a variable weight allocation scheme to maximize expected drawdown. The proposed method fetches buy and sell signal and asset allocation by employing a risk adjusted objective function and calmar ratio Young (1991). Further, a deep reinforcement technique where investment decisions and actions are made periodically based on the present global objective, is proposed in Yu et al. (2019). An interactive system in real-time is proposed in Yu et al. (2019) along with other list of novel modules viz., (i) infused prediction module (IPM) (ii) data augmentation module (DAM) and (iii) behaviour cloning module (BCM) which can be implemented for both on-policy and off-policy reinforcement learning (RL). Further, there are different optimization techniques viz., (i) deep deterministic policy gradient (DDPG) (ii) proximal policy optimization (PPO) and (iii) policy gradient (PG) explored in Liang et al. (2018) that have been used on datasets of China stock market. The proposed technique in Liang et al. (2018) considers at optimum variable weight based portfolio allocation under expected maximum drawdown Almahdi and Yang (2017) and risk-adjusted profile as proposed in Galai et al. (2001). A comprehensive feature engineering technique can be found in Li (2017) with a comprehensive guide on deep Q learning (DQL) Achiam et al. (2019) with changing structure and hyperparameters in Mnih et al. (2015). In recent times, continuous time García-Galicia et al. (2019) and continuous control Aboussalah and Lee (2020) based techniques have also become popular within reinforcement based approaches. The approach mentioned in García-Galicia et al. (2019) can be applied in the context of continuous time reinforcement learning. It assumes that the process is a continuous-time discrete-state Markov chain with simple and dynamic constraints. Based on the assumptions, a memory-less model i.e., a continuous-time Markov process (CTMP) is proposed which changes state over time. Further, a multidimensional state space can be addressed by Stacked Deep Dynamic Recurrent Reinforcement Learning (SDDRRL) as proposed in Aboussalah and Lee (2020). It captures the latest market conditions and then tweaks the portfolio accordingly. In addition to that, it uses (i) (ii) Gaussian Process (*GP*) and (iii) Expectation Improvement (*EI*) as acquisition functions which make it more dynamic and enable it to learn continuously changing market parameters. The problem of vanishing gradient caused due to presence of memory gate is also handled in the proposed technique Aboussalah and Lee (2020).

In recent times, crypto-currencies have become very popular and are faced with different challenge due to erratic changes in price. Further, crypto-currencies are decentralized and are affected by government policies and actions Betancourt and Chen (2021). In addition to that, crypto-currencies show higher volatility and henceforth require a better and advanced techniques to be applied. Such kind of market space requires rapid incorporation of new assets, adapt to changing state, and take action Betancourt and Chen (2021). A proximal policy optimization (PPO) Schulman et al. (2017) has been shown to perform well while dealing with crypto-currencies and has been validated in three different setups with variable episode lengths Betancourt and Chen (2021). In scenarios where the transaction costs are dynamic, there are different novel modules proposed in Betancourt and Chen (2021) viz., (i) formulation of trading system with variable number of assets (ii) optimum management of transaction cost (iii) implementation and validation of proposed technique using dataset of crypto-currencies.

Further, the erratic and complex behavior of market is determined by endogenous (like law of demand and supply, interest rates, income, etc.) and exogenous factors (like social media, change in government policies, etc.). A combinatory approach using a restricted stacked autoencoder and convolutional neural network (CNN) is proposed in Soleymani and Paquet (2020) based on (i) high level feature extraction containing eleven correlated features and (ii) feature reduction using restricted stacked autoencoder to reduce training time. Due to reduction in the training time, the proposed technique Soleymani and Paquet (2020) can be employed for designing online learning and online portfolio management.

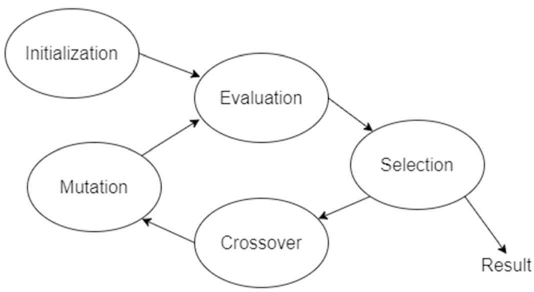
An extension of the work from Soleymani and Paquet (2020) is proposed in Soleymani and Paquet (2021) which is based on a graph convolutional reinforcement learning called DeepPocket. DeepPocket uses a restricted stacked autoencoder for feature extraction and focuses on exploiting the time-varying interrelations between the financial instruments. The interrelation is represented by a graph where the nodes are financial instruments and the edges represent the correlation between financial instruments. Recently, algorithmic-based hedging has become popular and requires a multi-period portfolio selection model. A dedicated multi-agent-based deep reinforcement learning technique is proposed in Lin et al. (2022) using a two-level nested agent. The multi-agent-based deep reinforcement learning (MABDRL) framework Lin et al. (2022) is employed in online portfolio selection problem where each agent is equipped with multiscale convolutional neural networks (MCNNs) Shi et al. (2019) and the ensemble of identical independent evaluators (EIIE) topology Jiang et al. (2017) to learn risk shift behaviour. Further, a graph convolutional based neural network combined with reinforcement learning can provide better explainability to the relationship between assets and their corresponding companies Shi et al. (2022). The relationship between the companies is represented as a relational graph convolutional network (R-GCN) Schlichtkrull et al. (2018), designed for heterogenous graph and is efficient in handling multi-relational data (Fig. 6).

7.5 Evolutionary approaches

Evolutionary computing Eiben et al. (2003) is a computational intelligence technique inspired from the biological evolution process. Similar to natural selection, the fitness of an individual in any given population is determined by how well they succeed in adapting and achieving their goals. In the context of portfolio selection problem, an individual candidate is selected based on its ability to efficiently seed as a future candidate in the evolution process.

There are two major forces that form the fundamentals of evolutionary systems viz., (i) variation operators (recombination and mutation) and (ii) selection procedure Eiben et al. (2003). The variation operators involve the process of recombining and mutation leading to an improvement in the fitness parameters also denoted as the adaptation process. This

Fig. 6 Genetic technique



leads to an increase in the viability of an individual, which is further reflected in the number of offspring. This process leads to better adaptability to the dynamic environment. It should be noted that during the selection procedure, at times even weak individuals can be selected. In other words, both recombination and mutation processes are stochastic in nature.

Combinatorial Optimization Problems (COPs) Ausiello et al. (2012) have been identified as one of the NP-hard problems and remain an active area of research Wolsey and Nemhauser (1999). Broadly there are two main approaches for solving COPs Cappart et al. (2020) viz., (i) extract scenario based techniques and (ii) heuristic based techniques. As mentioned in Section 7.4, reinforcement based techniques are best fit to handle such scenarios, but come with constraints while dealing with large datasets. This led to the exploratory works using evolutionary based approaches. An evolutionary based technique like the Genetic Algorithm (GA) Whitley (1994) can be employed in optimization problems. Typically, any GA Whitley (1994) based technique works on the five states viz., (i) initialization (ii) evaluation (iii) selection (iv) crossover, and (v) mutation Mirjalili (2019).

Particle swarm optimization (PSO) Kennedy and Eberhart (1942)Kennedy and Eberhart (1995) first intended to simulate social behaviour as a representation of the movement of organisms in a bird flock or fish school, can also be employed in volatile market condition. PSO Kennedy and Eberhart (1995)El-Shorbagy and Hassanien (2018) is a heuristic based technique that considers each member in the population as a particle. PSO Kennedy and Eberhart (1995)El-Shorbagy and Hassanien (2018) uses the technique of "gbest neighborhood topology" as proposed by Kennedy et al. Cura (2009). The underlying principle is that each particle remembers its best previous position and the best previous position visited by any particle in the whole swarm of particles. In short, a particle moves towards the best particle and the best previous position. Let us suppose that there are N dimensions, where each dimension represents an asset for each portfolio. Then each particle includes proportion variables x_{p_i} and decision variables z_{p_i} such that the proportion variables, $x_{p_i}(p = 1, 2, ..., P)$ where, P is the number of particles in the swarm. Also, the total number of dimensions that a particle owns will be $2 \times N$. PSO Kennedy and Eberhart (1995)El-Shorbagy and Hassanien (2018) is robust and can be applied for determining non-smooth global optimization problems and seems to perform better than other heuristic based techniques. It is also found to give high quality results and is less computationally intensive. One of the challenges with PSO is that it lacks mathematical foundation and is difficult to apply in real life scenarios.

The "cardinality constrained mean-variance (CCMV) Li et al. (2006) model" as proposed by Chang et al. Chang et al. (2000) and Fernandex and Gomez Fernández and Gómez (2007) has been employed for portfolio optimization by using PSO Kennedy and Eberhart (1942). The results have been compared with genetic algorithm (GA) Chang et al. (2000)Oh et al. (2005)Yang (2006), tabu search (TS) Chang et al. (2000), simulated annealing (SA) Chang et al. (2000)Crama and Schyns (2003)Derigs and Nickel (2004), neural networks Fernández and Gómez (2007), and other techniques like heuristic Mansini and Speranza (1999), meta-heuristic Derigs and Nickel (2003), and hybrid heuristic based approaches Schlottmann and Seese (2004). For comparison and benchmarking, the weekly prices from March 1992 to September 1997 have been used from Seng (1992). Based on the experiments conducted, it is observed that PSO Kennedy and Eberhart (1995)El-Shorbagy and Hassanien (2018) performs better than other heuristic based approaches. A further study on the cardinality constrained based portfolio optimization can be found in Salahi et al. (2014) where the quadratic programming (QP) Frank and Wolfe (1956) problem Anagnostopoulos and Mamanis (2011a) is formulated as a mixed-integer quadratic problem (MIQP) Lazimy (1982) by adding cardinality and quadratic constraints. On comparing integer and categorical particle swarm optimization (ICPSO) Goodman et al. (2016) and improved harmony search (IHS) algorithm Mahdavi et al. (2007), is observed to perform much faster than ICPSO Goodman et al. (2016) on large data sets. For comparison and benchmarking, the weekly prices from March 1992 to September 1997 have been used from Seng (1992). A detailed experiment for constrained portfolio optimization using PSO Kennedy and Eberhart (1995)El-Shorbagy and Hassanien (2018) is conducted in Zhu et al. (2011) on the Shanghai Stock Exchange 50 Index (SSE 50 Index) and it is found that PSO Kennedy and Eberhart (1995)El-Shorbagy and Hassanien (2018) outperforms GA and VBA (Visual Basic Application). A detailed heterogeneous multiple population particle swarm optimization (HMPPSO) algorithm is proposed in Yin et al. (2015) where the whole population is divided into smaller sub-populations and different variants of PSO Kennedy and Eberhart (1995)El-Shorbagy and Hassanien (2018) are applied on each sub-population. For comparison and benchmarking, the weekly prices from March 1992 to September 1997 Seng (1992) have been used. On comparing with other PSO Kennedy and Eberhart (1995)El-Shorbagy and Hassanien (2018) variants, HMPPSO Yin et al. (2015) seems to be more effective and robust and could be employed on high dimensional problems.

On the other hand, ant colony optimization (ACO) Forqandoost Haqiqi and Kazemi (2011) is a probabilistic model designed to select the best path designed based on pheromone-based communication of biological ants. ACO Forqandoost Haqiqi and Kazemi (2011) can be employed to solve the minimum cost problem, a situation which involves multiple nodes and non-directed arcs. The objective is to establish a minimum cost path between the source and destination. ACO Forqandoost Haqiqi and Kazemi (2011) works on both forward and backward modes and simulates forward movement of ants when they move from nest to food and backward movement when they move from food to nest. The forward movement typically employs probabilistic based selection while backward movement is performed based on the deposited memory, which further helps to avoid or reduce loops. The selection made in the forward movement is also done based on historical knowledge deposited with multiple to and fro ant movements. Typically, the selection of fragment *I* from *K* possible choices is done based on Forqandoost Haqiqi and Kazemi (2011)

$$Prob_i = \frac{\tau_i}{\sum_1^k \tau_k} \tag{16}$$

where, τ_k is the amount of pheromone and k is the k^{th} fragment. The evaluation is performed based on the evaporation and pheromone deposit phases given by

$$(t+1) = \tau_i(t)(1-\gamma) + \delta_i \tag{17}$$

where, γ is the evaporation rate and τ_i is the amount of pheromone in the fragment i.

ACO Forqandoost Haqiqi and Kazemi (2011) is employed where there is a dynamic change in path, distance, and other parameters. Since ACO is based on probabilistic distribution, hence a significant change in topology can make the selection process slow.

An extension of ACO Forqandoost Haqiqi and Kazemi (2011) is proposed in Deng and Lin (2010) to solve the cardinality constrained Markowitz mean-variance portfolio optimization (CCMPO) problem. For comparison and benchmarking, the weekly prices from March 1992 to September 1997 have been used from Seng (1992). The results show that ACO Forqandoost Haqiqi and Kazemi (2011) is more robust and effective than the standard PSO Kennedy and Eberhart (1995)El-Shorbagy and Hassanien (2018) for low risk investment scenarios. The proposed technique uses three major components viz., (i) solution construction (ii) constraint satisfaction and (iii) pheromone update Deng and Lin (2010). Further, a clustering based technique along with ACO Forqandoost Haqiqi and Kazemi (2011) is used to perform portfolio optimization Rezani et al. (2020). Clustering using *k*-means++ Bahmani et al. (2012) is used to diversify the portfolio and suitable stocks and their weights are chosen ACO Forqandoost Haqiqi and Kazemi (2011). The three stages viz., (i) coefficient allocation (ii) quality assessment and (iii) updating weight help in dynamically determining the right stock. Experiments conducted using one year data from SP500 (2016) is found to give the best results on cluster sizes of 8 where the proposed technique Rezani et al. (2020) gave the highest average return and Sharpe ratio. An average entropy based ACO Forqandoost Haqiqi and Kazemi (2011) is proposed in Li et al. (2012) where the parameters of the algorithm have been adjusted adaptively. The adaptive behaviour tends to avoid stagnation behaviour. ACO Forqandoost Haqiqi and Kazemi (2011) is further extended to solve multi-criteria optimization problem with population-based ACO (PACO) Guntsch and Middendorf (2003) based on the multi-colony ACO approach as proposed by Mariano and Morales Mariano and Morales (1999). However, most of the above techniques work on a single objective of either maximizing the return or minimizing the risk. Thus, these techniques are unable to deal with conflicting objectives of both maximization of returns and minimization of risks. Due to the above limitation, multi-objective evolutionary techniques Van Veldhuizen and Lamont (1998) have gained popularity in the recent past due to their ability to find multiple pareto-optimal solutions Lin (1976) out of conflicting objectives.

A comprehensive study on Multi-objective Evolutionary Algorithms (MOEAs) Zhou et al. (2011) viz., Non-dominated Sorting Genetic Algorithm II (NSGA-II) Yusoff et al. (2011), Pareto Envelop-based Selection Algorithm (PESA) Corne et al. (2000) and Strength Pareto Evolutionary Algorithm 2 (SPEA2) Zitzler et al. (2001), is available in Anagnostopoulos and Mamanis (2011b). It is observed that the performance using these techniques is independent of the risk function used in design. Both return and risk can be simultaneously optimized using a multi-objective algorithm, which makes is it more efficient than other techniques. A classification and genetic based technique is presented in Guennoun and Hamza (2012). The proposed technique uses minimum value-at-risk and maximum Value (MinVaRMaxVaL) using a two stage process using (i) k-means Alpaydin (2020) and (ii) dynamic optimization algorithm. Further, two more variants of the multi-objective particle swam optimization (MOPSO) Reyes-Sierra and Coello (2006) algorithm for portfolio optimization are reported in Babaei et al. (2015). A comparative study between MOPSOs Reyes-Sierra and Coello (2006), NSGA-II Yusoff et al. (2011), and SPEA2 Zitzler et al. (2001) is presented in Babaei et al. (2015). A novel mean-VaR optimization technique using a univariate Generalized Auto Regressive Conditional Heteroscedasticity (GARCH) model is proposed in Ranković et al. (2016). The proposed work in Ranković et al. (2016) considers both portfolios with fixed weight and portfolios with fixed holdings of assets. An efficient learning-guided hybrid multi-objective evolutionary algorithm (MODE-GL) is proposed in Lwin et al. (2017), which can be used in constraints like cardinality, quantity, reassignment, round-lot and class constraints Lwin et al. (2017). A mutation based technique, using multi-objective evolutionary algorithms (MOEAs) proposed in He and Aranha (2020), can be employed to find more than one interesting solution in a single run. A comparison of mean return error of different evolutionary techniques are mentioned in Fig. 7.

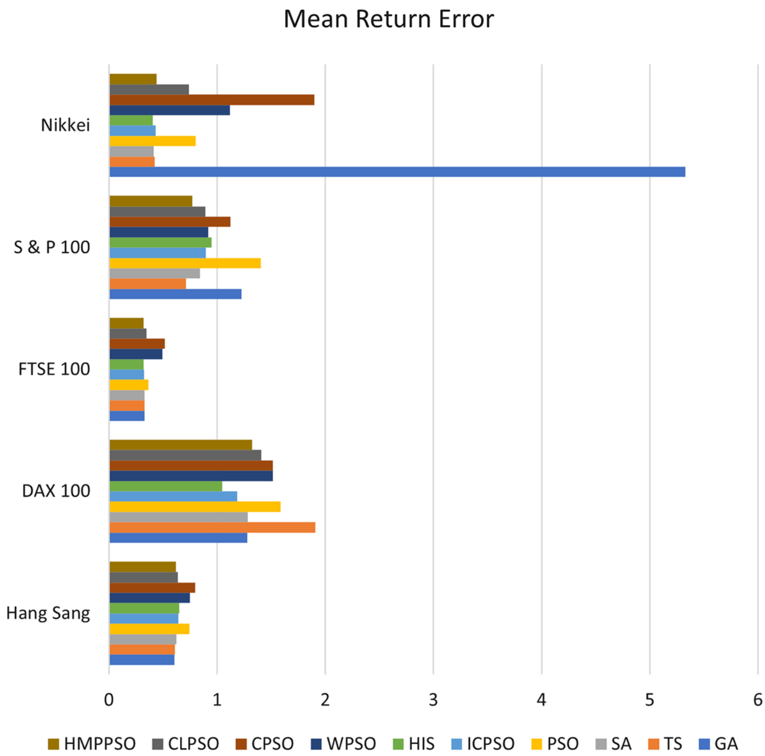


Fig. 7 Mean Return Error of different Evolutionary Techniques Cura (2009)Salahi et al. (2014)Yin et al. (2015)

Table 1 Types of portfolios

Sl. No.	Portfolio	Description
1.	Aggressive Portfolio	Aims for higher returns and undertakes high risks
2.	Defensive Portfolio	Aims for minimum risk and gives minimum return
3.	Income Portfolio	Very similar to defensive portfolio, but focuses on gains from dividends or other types of recurring benefits
4.	Speculative Portfolio	Aims for extremely high risk. Somewhat similar to gambling
5.	Hybrid Portfolio	Aims to provide optimum return with optimum degree of risk

7.6 Approaches based on quantum computing

In recent years, lots of effort have been put on trying hybrid based approaches to make portfolio optimization more realistic. It has been found that quantum and quantum-inspired computing techniques can help solve difficult optimization problems Orus et al. (2019). During the early twentieth century, Newton and Maxwell dominated the laws of physics and were assumed to be correct. Later it was however noted that there are certain flaws in these techniques which led to the formulation of a new mathematical framework like quantum computing Kaye et al. (2007).

Table 2 Extended types of portfolio management

Sl. No.	Portfolio	Description
1.	Transaction Cost Portfolio García-Galicia et al. (2019)	Aims to reduce the transaction cost by using time penalization techniques.
2.	Robust Portfolio Lee et al. (2020)	Aims to reduce transaction cost in sparse and robust portfolio selection process.
3.	Regularized Portfolio Vazquez and Clempner (2020)	Aims to reduce estimation error
4.	Reinforcement Learning Portfolio	Aims to continuously learn the changing market condition by applying several reinforcement learning approaches.

Table 3 Key elements of portfolio management

Sl. No.	Key element	Description
1.	Asset Allocation	An effective portfolio management involves proper asset allocation, keeping short term and long term objectives in mind. This requires proper understanding of asset, as all the assets do not move similarly. This also requires proper mix of asset selection.
2.	Diversification	Distributing risk and return within an asset is an important factor. Due to changing market dynamics, it is difficult to know exactly about any subset of an asset. Thus, diversification becomes an important key element in portfolio management.
3.	Rebalancing	Rebalancing is the process of bringing back the allocated assets to the desired asset mix. The process of rebalancing involves selling high-priced assets and putting that money in buying less-priced assets.

Table 4 Types of portfolio management

Sl. No.	Type of portfolio management	Description
1.	Passive Management	This involves long term strategy where the assets are allocated for long term.
2.	Active Management	This involves active participation where the assets are bought and sold regularly to gain maximum return.

Quantum computing (QC) is a part of quantum physics, where its inherent dynamics are governed by the Schrödinger equation (SE) Talbi et al. (2006)McMahon (2007). On multiple experiments, QC is shown to give better performance on complex and NP-hard problems which require large solution space. Quantum machines are represented by a wave function in the Hilbert space $\mathcal{H}$. The Hilbert space, $\mathcal{H}$ can also be imagined as an extension of a two-dimensional or a three-dimensional space to spaces with finite or infinite number of dimensions.

7.6.1 Qubit

Qubit is the smallest unit of information in quantum computing McMahon (2007) represented as $|0\rangle$ and $|1\rangle$, also known as two-state qubit vectors in quantum computing. The qubit vectors are also represented as McMahon (2007)

$$|0\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix} and |1\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

Every notation in the above equation is referred to as the complex conjugate transpose of each other.

7.6.2 Quantum superposition principle

The quantum superposition principle can be understood as the linear combination of quantum vectors represented in two states as McMahon (2007)

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$$

where, α and β are complex numbers which must satisfy the condition

$$| \alpha |^2 + | \beta |^2 = 1$$

where, α^2 and β^2 are the probabilities of measuring the basis states $|0\rangle$ and $|1\rangle$, respectively. Thus, a qubit can be represented as a superposition of two basis states viz., $|0\rangle$ and $|1\rangle$ implemented using quantum gates (Q-gates). Thus, these basis states exist simultaneously until a quantum measurement destroys the superposition. Hence, for n number of qubits, the number of states in a quantum machine is 2^nDiVincenzo (1998)Han and Kim (2002).

7.6.3 Quantum-inspired metaheuristic algorithms

Metaheuristic-based techniques are widely perceived optimization techniques to solve complex and large computational problems. Quantum-inspired metaheuristic techniques

Table 5 Portfolio optimization analysis models

Sl. No.	Model	Advantages	Limitations
1.	Mean-Variance Markowitz and Todd (2000)	Should be used when the risk parameters are small. Used when the return distribution is compact and portfolio decisions are made frequently.	Cannot be used in case of high risk parameters due to computational problems caused due to higher order polynomial functions. Can be applied when portfolio returns are normally distributed.
2.	Variance with skewness Samuelson (1975)	Can be employed where the portfolio returns are not normally distributed	Optimization can ensure only local optimal solution.
3.	Value-at-Risk Jorion (1997)	Easy to understand the level of risk. Applicable to large number of assets.	Could be misleading. Does not measure worst case loss. Difficult to employ in large portfolios.
4.	Conditional Value-at-Risk Rockafellar and Uryasev (2000)	Can handle extreme loss by using weighted average. Can be used in volatile scenario. Risk management can be performed efficiently.	CVaR does not indicate the maximum loss that can be incurred.
5.	Mean-absolute deviation (MAD) Konno and Yamazaki (1991)	MAD is a linear program and hence relatively fast, unlike mean-variance which is quadratic. Used when asset returns are multivariate and normally distributed.	Can lead to calculation risks caused due to skipping of covariance matrix. Penalizes both negative and positive deviations.
6.	Minimax Young (1998)	Has logical advantages when the returns are not normally distributed. Similar to MAD, it is a linear program and hence faster. Can be used for more complex models and constraints.	Sensitive to outliers. Cannot be used when historical data is missing.
7.	Lower Partial Moment Nawrocki (1992)Brogan and Stidham (2008)	Suits investors' perception about risk. Undesirable downside and desirable upside deviations are separated. Penalizes only downside deviations.	Computation of portfolio risk is tedious. Sensitive to outliers.

Table 6 Comparative study of classical and intelligent approaches

Sl. No.	Methods	Advantages	Limitations
1.	Statistical Based Approaches	Easy to implement. Good for smaller data sets and time periods. Easily interpretable	Does not consider multiple features into account.
2.	Regression Based Approaches	Simple, interpretable, and easy to implement. can learn iteratively from historical data.	Efficiency is highly dependent on the learning rate chosen.
3.	Bayesian Based Approaches	Can be easily extended, unlike other machine learning based techniques. Can be employed for reasoning based problems.	No universal method to construct a network from data. While training, it is not assured that all the training patterns are used while training.
4.	Neural Network Based Approaches	Better learning model as compared to other traditional techniques. Fault tolerance, distributed memory and at times can work with inadequate knowledge.	Should be used when dealing with large datasets. Not self-explanatory and absence of specific rules to determine the structure of network.
5.	Reinforcement Based Approaches	Can solve complex optimization problems. Learning model is very similar to learning patterns of humans. Can be used to adaptively learn based on changing states.	Should not be used for simple problem and on smaller data set. Too much reinforcement can adversely affect learning. The curse of dimensionality can limit the learning.
6.	Evolutionary Based Approaches	Conceptually simple, and outperforms classical techniques by imposing non-linear constraints and non-stationary conditions. Ability to parallelism and robust to dynamic changes.	Higher computation required. Results at times are not self-explanatory and reproducible.
7.	Quantum-based Approaches	Adding qubits can increase the storage exponentially and are useful to solve very complex compute extensive problems. Faster as compared to any other methods.	The energy required by quantum computer is much larger than traditional computers. Still there is a lot of unknowns as this is an ongoing area of research.

Table 7 Comparative study of different state-of-art approaches

Sl. No.	Methods	Advantages	Limitations
1.	Value-at-Risk (VaR) Jorion (1997)	Simple and easy to implement. Works well when the return is normally distributed and there is the presence of historical data.	Difficult to use in real-life scenario as the return is not always normally distributed.
2.	Conditional Value-at-Risk (CVaR) Rockafellar and Uryasev (2002)	Risk management with CVaR can be done better than VaR. CVaR also accounts for loss exceeding VaR.	CVaR mainly focus on risk and does not represent maximum loss that can be incurred.
3.	Particle swarm optimization (PSO) Kennedy and Eberhart (1942)	Simple to implement, robust, less computation power, can be used to run in parallel computing.	Can be difficult to specify initial design parameters. Can converge prematurely, especially in case of complex problem. Cannot handle discrete problem.
4.	Integer and categorical particle swarm optimization (ICPSO) Goodman et al. (2016)	Can be used for both integer and categorical values. Can be used when there is no natural ordering to solution value. Can be employed to handle a discrete problem.	Assumes that the variable are independent of each other. More sensitive to bias.
5.	Heterogeneous multiple population particle swarm optimization (HMPPSO) Yin et al. (2015)	Can be employed with problem having high dimensions. Efficient over PSO and multiple variants of PSO can be employed in each sub-population.	Requires large dataset to be applied. Can be applied when large population can be split as mutually exclusive partitions.
6.	Ant Colony Optimization (ACO) Deng and Lin (2010)	Can adapt to dynamic changes. Can search optimum solution in parallel. Convergence is guaranteed.	Time of convergence is not guaranteed. Probabilistic distribution can change in very iteration.
7.	Population-Based ACO (PACO) Guntsch and Middendorf (2003)	Combines the power of population and probabilistic based approach. Better convergence as compared to traditional ACO. Can be used to solve multi-criteria problem.	Requires selection of prerequisite parameters. More efficient when population is mutual exclusive.
8.	Support Vector Machine (SVM) Alpaydin (2020)	Easy to understand and implement. Effective in high dimensional space. Requires less computation power.	Not suitable for large dataset. Works well when there is a clear margin of separation. Performance degrades when there is noise in data.
9.	Bayesian-Based approach Karatzas and Zhao (2001)	Good at combining prior information in building strong decision making framework. Inference drawn are conditional based on circumstances.	No correct way to choose prior. The result at times are not self-explanatory.
10.	Long-Short term memory (LSTM) Choi (2018)	Can be employed to learn lengthy time period dependencies.	Hard to implement. Suffers from gradient exploding and vanishing problem.

Table 7 (continued)

Sl. No.	Methods	Advantages	Limitations
11.	Integrated CNN (iCNN) Chaouki et al. (2020)	Model free approach. Can be employed in absence of historical knowledge.	Initial allocation could lead into loss before yielding profit.

Table 8 Comparative study of different state-of-art approaches

Sl. No.	Methods	Advantages	Limitations
12.	Recurrent reinforcement learning (RRL) Moody and Saffell (2001)	Used while optimizing performance criteria like differential Sharpe ratio and differential downside deviation ratio. Discover investment policies dynamically. Avoids Bellman's curse of dimensionality. Used in discrete action space.	Cannot be used in immediate estimation scenario. Initial allocation could lead into loss before yielding profit.
13.	Stacked Deep Dynamic Recurrent Reinforcement Learning (SDDRRL) Aboussalah and Lee (2020)	Employed in continuous action is required in multi-dimensional state space. Requires updated market information to rebalance portfolio. Does not require any time series predictions.	A detailed and comprehensive study with neural based technique is missing. Computationally large. Requires current market state for better prediction.
14.	Multi-objective evolutionary algorithms (MOEAs) He and Aranha (2020)	Could be used to solve a complex multi-objective problem. Used in high dimensionality search space. Used with search space is non-linear, non-differential, non-continuous, and non-convex.	Lack of reproducibility. Parameter tuning and controlling is difficult.
15.	Non-Dominated Sorting Genetic Algorithm II (NSGA-II) Yusoff et al. (2011)	Characteristics of fast non-dominated sorting approach, fast crowded distance estimation procedure and simple crowded comparison operator.	Lack of reproducibility. Parameter tuning and controlling is difficult. Requires careful selection of parameters.
16.	Multi-objective particle swam optimization (MOPSO) Reyes-Sierra and Coello (2006)	Can be employed for multiple-objective scenario. Uses less computation power as compared to evolutionary techniques.	Can be difficult to specify initial design parameters. Can converge prematurely, especially in case of complex problem. Cannot handle discrete problem.
17.	Generalized auto regressive conditional heteroscedasticity (GARCH) Ranković et al. (2016)	Used when volatility of return of groups of stocks with large number of observation need to be evaluated.	More sensitive to the frequency of data used. Should be used while dealing with very large time series data.
18.	Quantum-inspired tabu search (QTS) Chou et al. (2011)Chou et al. (2014)	Performs better than other heuristic algorithms in optimization problem without premature convergence. Can be used to solve knapsack problem in shorter time. Can avoid the problem of over-fitting. Flexible, profitable and stable. Performs well in upward trends.	Not efficient with increase in quantity of items.

Table 9 Comparative study of different state-of-art approaches

Sl. No.	Methods	Advantages	Limitations
19.	Multi-objective quantum-inspired tabu search (MOQTS) Chou et al. (2014)	Can be employed while dealing with multiple objectives. Flexible and profitable. Can run more simulation with different parameters.	Need to be further evaluated with multiple different financial parameters.
20.	Quantum-inspired Firefly Algorithm with Particle Swarm Optimization (QIFAPSO) Zouache et al. (2016)	Converges faster than PSO and other firefly based techniques.	No experimental finding in portfolio optimization space.
21.	Quantum-inspired acromyrmex evolutionary algorithm (QIAEA) Montiel et al. (2019)	Can be employed to find an efficient global optimization method for complex systems. Performs better than quantum-inspired evolutionary algorithms. Has higher average accuracy and lower standard deviation.	Cannot be used for multiple objective scenarios.

combine the power of quantum computing and metaheuristics and have been shown to perform better than the classical counterparts Karmakar et al. (2017) in terms of exhibiting faster convergence and having better exploration and exploitation capabilities. The quantum-inspired metaheuristic algorithms emulate the principles of quantum mechanics. These algorithms resort to qubit based encoding Nielsen and Chuang (2001)DiVincenzo (1998) of the solution space and are characterized by quantum-inspired versions of the underlying operators. Due to their ability to solve complex and large computational problems, these quantum-inspired metaheuristics are widely used in both constrained and unconstrained problems Rebentrost and Lloyd (2018)Orus et al. (2019).

Of late, several incarnations of these quantum-inspired metaheuristics have been evolved to yield quantum-inspired versions of the classical metaheuristics including the genetic algorithm Davis (1991), tabu search Glover and Laguna (1998), PSO Kennedy and Eberhart (1995), and ACO Dorigo et al. (2006) to work in quantum space. Typical examples include the quantum-inspired genetic algorithm Narayanan and Moore (1996)Han et al. (2001)Saad et al. (2021), quantum-inspired tabu search Chiang et al. (2014)Chou et al. (2014)Kuo and Chou (2017), quantum-inspired PSO Alvarez-Alvarado et al. (2021) Agrawal et al. (2021), quantum-inspired ACO Dey et al. (2018)Mohsin et al. (2021) to name a few.

7.6.4 Quantum-inspired metaheuristic-based approaches

Portfolio optimization employs several techniques to maximize the profit and reduce risks. Henceforth, trading rules can be designed which can be employed to take decisions on when to sell and buy based on rules identified. A combination of trading rules can be termed as a trading strategy, which can be formulated as an optimization problem. A novel method of optimization is proposed in Kuo et al. (2013) where a quantum-inspired Tabu Search (QTS) Chou et al. (2011)Chou et al. (2014) algorithm is used to find the optimum combination of trading rules along with a sliding window to avoid over-fitting. The proposed method is evaluated with the Buy & Hold method and is found to give better results. Further, an extension of QTS Chou et al. (2011)Chou et al. (2014) called the multi-objective quantum-inspired tabu search (MOQTS) Chou et al. (2014) is proposed. While QTS Chou et al. (2011)Chou et al. (2014) works on fewer rules, MOQTS can handle more number of rules and is observed to outperform QTS Chou et al. (2011)Chou et al. (2014) in terms of profit and successful transaction rates.

A brief study on "Quantum-inspired Firefly Algorithm with Particle Swarm Optimization (QIFAPSO)" is available in Zouache et al. (2016) applied on some discrete optimization problems. The results from QIFAPSO Zouache et al. (2016) are compared with two classes of algorithms viz., quantum inspired algorithms and PSO inspired algorithms on high dimensional knapsack instance and is found to outperform by a big margin. Further, a dynamic portfolio optimization technique as proposed in Mugel et al. (2020) implemented (i) D-Wave Hybrid quantum annealing (ii) Variational Quantum Eigensolver (VQE) (iii) VQE Constrained on IBM-Q with the quantum inspired Tensor Network (TN) algorithm. Based on the tests conducted in Mugel et al. (2020), it is found that the D-Wave hybrid quantum annealing, and tensor network are able to handle large systems of approximately, 1272 fully-connected qubits without hitting computational limitations. It is also observed that D-Wave is very fast, whereas tensor network provides better portfolio optimization at a cost of higher computation time. Further, quantum-inspired acromyrmex evolutionary algorithm (QIAEA) as proposed in Montiel et al. (2019) is inspired from the acromyrmex ant species also known as leaf-cutter ants. On several experiments, it is shown as QIAEA gives the best performance in terms of precision and recall as compared to the classical GA Whitley (1994) or quantum-inspired GA Narayanan and Moore (1996).

A bird's eye view of the different classical and intelligent approaches to portfolio management is provided in Table 6. A detailed comparative study of different state-of-art approaches is mentioned in Tables 7, 8, 9.Further experimental results of different approaches are mentioned in Tables 10, 11, 12, 13, 14, 15.

Table 10 Comparison of different state of art techniques from literature

Index	Assets	Errors	GA	TS	SA	PSO	ICPSO
Hang Sang	31	Mean Euclidean Distance	0.0040000	0.0040000	0.0040000	0.0049000	0.0000824
		Variance of return error (%)	1.6441000	1.6578000	1.6628000	2.2421000	1.9003836
		Mean return error (%)	0.6072000	0.6107000	0.6238000	0.7427000	0.6409340
		Time (s)	18	9	10	34	57
DAX 100	85	Mean Euclidean Distance	0.0076000	0.0082000	0.0078000	0.0090000	0.0001298
		Variance of return error (%)	7.2180000	9.0309000	8.5485000	6.8588000	7.2061918
		Mean return error (%)	1.2791000	1.9078000	1.2817000	1.5885000	1.1876764
		Time (s)	99	42	52	179	254
FTSE 100	89	Mean Euclidean Distance	0.0020000	0.0021000	0.0021000	0.0022000	0.0000408
		Variance of return error (%)	2.8660000	4.0123000	3.8205000	3.0596000	3.3812060
		Mean return error (%)	0.3277000	0.3298000	0.3304000	0.3640000	0.3239293
		Time (s)	106	42	55	190	269
S & P 100	98	Mean Euclidean Distance	0.0041000	0.0041000	0.0041000	0.0052000	0.0000756
		Variance of return error (%)	3.4802000	5.7139000	5.4247000	3.9136000	4.5894404
		Mean return error (%)	1.2258000	0.7125000	0.8416000	1.4040000	0.8963859
		Time (s)	126	51	66	214	323
Nikkei	225	Mean Euclidean Distance	0.0093000	0.0010000	0.0010000	0.0019000	0.0000220
		Variance of return error (%)	1.2056000	1.2431000	1.2017000	2.4274000	1.8414523
		Mean return error (%)	5.3266000	0.4207000	0.4126000	0.7997000	0.4328426
		Time (s)	742	234	286	919	2676

Table 11 Comparison of different state of art techniques from literature

Index	Assets	Errors	HIS	WPSO	CPSO	CLPSO	HMPPSO
Hang Sang	31	Mean Euclidean Distance	0.0000820	0.0000951	0.0001018	0.0001220	0.0000790
		Variance of return error (%)	1.8044041	2.0118800	2.2931800	3.5785200	1.6758400
		Mean return error (%)	0.6483910	0.7484910	0.7959690	0.6352230	0.6212350
		Time (s)	55	119	162	152	136
DAX 100	85	Mean Euclidean Distance	0.0001296	0.0001746	0.0001863	0.0001912	0.0001547
		Variance of return error (%)	7.3849655	8.0357600	11.0700000	8.5494500	7.0019500
		Mean return error (%)	1.0492997	1.5159100	1.5134200	1.4103400	1.3258100
		Time (s)	159	459	502	548	463
FTSE 100	89	Mean Euclidean Distance	0.0000400	0.0000591	0.0000653	0.0000414	0.0000390
		Variance of return error (%)	3.2479355	4.0458500	5.3473600	2.9917200	2.8747300
		Mean return error (%)	0.3202570	0.4934480	0.5143340	0.3465540	0.3216470
		Time (s)	168	475	511	586	499
S & P 100	98	Mean Euclidean Distance	0.0000746	0.0000956	0.0001222	0.0000799	0.0000782
		Variance of return error (%)	3.9023695	3.8469800	7.1698400	3.4338900	2.8476000
		Mean return error (%)	0.9480060	0.9200940	1.1214000	0.8896880	0.7726480
		Time (s)	186	568	634	676	585
Nikkei	225	Mean Euclidean Distance	0.0000206	0.0000518	0.0000978	0.0000261	0.0000205
		Variance of return error (%)	1.6020636	3.2030600	7.5329400	1.3008500	1.1965100
		Mean return error (%)	0.4036726	1.1182400	1.8971600	0.7386130	0.4399000
		Time (s)	659	2242	2479	2376	2190

8 Discussions and conclusion

In this study, different types of portfolios and portfolio measures used in portfolio management and optimization have been discussed. Different portfolio measures are used depending on the different types of portfolios. A comparative study of different classical and intelligent approaches for portfolio optimization has been presented in this paper.

It is observed that from the comparative studies that the evolutionary optimization methods stand in good stead when it comes to optimized allocation of portfolios. Furthermore, the evolving quantum-inspired evolutionary techniques also promise to be better viable alternatives.

However, evolution of a better portfolio model is a challenge for the overall performance improvement of these portfolio optimization techniques. Moreover, robust and efficient optimization techniques remain to be investigated for yielding better optimization scenarios, including the optimization of the risk-return paradigm involving conflicting objectives. Researchers are constantly investing their efforts in this direction.

Table 12 Comparison of different state of art techniques from literature

	2016-09-07 to 2016-10-28			2016-12-08 to 2017-01-28			2017-03-07 to 2017-04-27		
Algorithm	MDD	fAPV	SR	MDD	fAPV	SR	MDD	fAPV	SR
CNN	0.224	29.695	0.087	0.216	8.026	0.059	0.406	31.747	0.076
bRNN	0.241	13.348	0.074	0.262	4.623	0.043	0.393	47.148	0.082
LSTM	0.280	6.692	0.053	0.319	4.073	0.038	0.487	21.173	0.060
iCNN	0.221	4.542	0.053	0.265	1.573	0.022	0.204	3.958	0.044
Best Stock	0.654	1.223	0.012	0.236	1.401	0.018	0.668	4.594	0.033
UCRP	0.265	0.867	-0.014	0.185	1.101	0.01	0.162	2.412	0.049
UBAH	0.324	0.821	-0.015	0.224	1.029	0.004	0.274	2.230	0.036
Anticor	0.265	0.867	-0.014	0.185	1.101	0.010	0.162	2.412	0.049
OLMAR	0.913	0.142	-0.039	0.897	0.123	-0.038	0.733	4.582	0.034
PAMR	0.997	0.003	-0.137	0.998	0.003	-0.121	0.981	0.021	-0.055
WMAMR	0.682	0.742	-0.0008	0.519	0.895	0.005	0.673	6.692	0.042
CWMR	0.999	0.001	-0.148	0.999	0.002	-0.127	0.987	0.013	-0.061
RMR	0.900	0.127	-0.043	0.929	0.090	-0.045	0.698	7.008	0.041
ONS	0.233	0.923	-0.006	0.295	1.188	0.012	0.170	1.609	0.027
UP	0.269	0.864	-0.014	0.188	1.094	0.009	0.165	2.407	0.049
EG	0.268	0.865	-0.014	0.187	1.097	0.010	0.163	2.412	0.049
B^k	0.436	0.758	-0.013	0.336	0.770	-0.012	0.390	2.070	0.027
CORN	0.999	0.001	-0.129	1.000	0.0001	-0.179	0.999	0.001	-0.125
M_0	0.335	0.933	-0.001	0.308	1.106	-0.008	0.180	2.729	0.044

**MDD* Maximum Drawdown, *fAPV* final Accumulated Portfolio Value, *SR* Sharpe Ratio

Table 13 Comparison of different state of art techniques from literature

Function	GA	ACO	PSO	QGA	AQGA	QIAEA
1	87.50	100.00	67.50	70.00	92.50	100.00
2	35.00	0.00	10.00	20.00	10.00	92.50
3	57.50	20.00	12.50	57.50	60.00	90.00
4	5.00	7.50	20.00	25.00	25.00	52.50
5	57.00	100.00	40.00	35.00	55.00	100.00
6	45.00	87.50	45.00	57.50	80.00	92.50
7	17.50	0.00	2.50	30.00	17.50	52.50
8	40.00	0.00	5.00	52.00	60.00	57.50
9	22.50	10.00	25.00	22.50	52.50	40.00
10	100.00	92.50	45.00	97.50	97.50	100.00
11	92.50	97.50	52.50	52.50	67.50	82.50
12	70.00	100.00	100.00	50.00	40.00	100.00
13	35.00	100.00	50.00	20.00	10.00	35.00
14	95.00	100.00	100.00	57.50	77.00	100.00
15	47.50	0.00	25.00	35.00	25.00	52.50

*Hit accuracy, Population size=40, No. of generations=50, No. of runs=40

Table 14 Comparison of different state of art techniques from literature

Method	Yearly Profit Return (%)
Buy and Hold	-7.20
RSI in normal operating	-12.00
RSI in abnormal operating	5.40
Genetic Logic Rule	2.02
Single Genetic Login Rule	-0.05
Genetic Neural Networks	10.27
QTS trading system	15.96

*Experiment result performed with stock index of TAIEX on data from 28-Jun-2000 to 2-Jul-2004

Table 15 Comparison of different state of art techniques from literature

Method / stock	Nasdaq (%)	DJI (%)	NYSE (%)	S &P 500 (%)	Average (%)
Buy and Hold	55.18	42.03	29.29	45.33	42.96
QTS	34.13	32.20	26.70	47.32	31.78
Best QTS	58.39	38.15	38.45	54.00	47.25
	23.39	10.49	11.85	10.71	14.11
Rates of QTS	70.00	100.00	86.67	100.00	89.17
MOQTS	70.15	14.54	23.93	30.75	34.44
Best MOQTS	77.12	38.45	54.00	31.11	50.17
of MOQTS	13.95	9.60	8.01	12.20	10.94
Rates of QTS	100.00	100.00	93.33	65.00	89.58

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