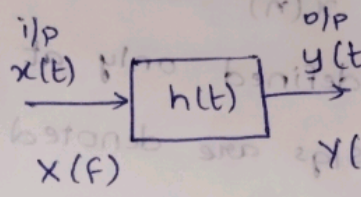
Signals & Systems

→ **Signal** :- A signal is defined as a function of one or more variables which conveys information on the nature of physical phenomena.

→ **System** :- A system is commonly defined as an entity that manipulates one or more signals to accomplish a function thereby yielding new signals.



$F[x(t)] = X(f)$

$F[y(t)] = Y(f)$

$F[h(t)] = H(f)$

$y(t) = x(t) * h(t)$ → impulse response

$Y(f) = X(f) \cdot H(f) \Rightarrow H(f) = \frac{Y(f)}{X(f)}$

→ **Classification of signals** :-

Based on their nature and characteristics in time domain, the s/gs can be broadly classified as a) continuous - time signals
b) discrete - time signals.

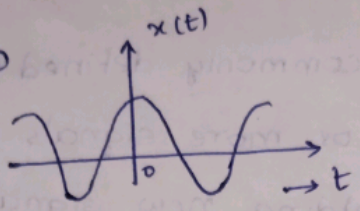
Both continuous - time and discrete - time s/gs may be further classified as

① Deterministic and random signals
② Periodic and non-periodic signals (aperiodic s/g)
③ Energy and Power signals
④ Causal and non-Causal signals
⑤ Even and odd signals.

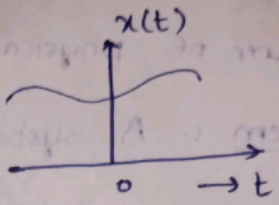
a) Continuous-time s/gs :- $x(t)$

A continuous time s/g is one which is defined for every value of time t. These type of signals are continuous both in amplitude and time.

eg :- ①



eg ②

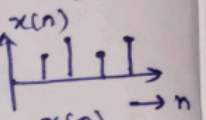


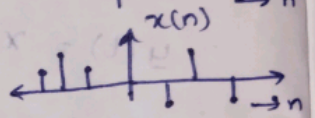
b) Discrete-time signals :- $x(n)$

These s/gs are continuous in amp but discrete in time

These signals are defined only at discrete instants of time. These s/gs are denoted by $x(n)$ where n is an integer ; $n = 0, \pm 1, \pm 2, \pm 3 \ldots$

eg :-



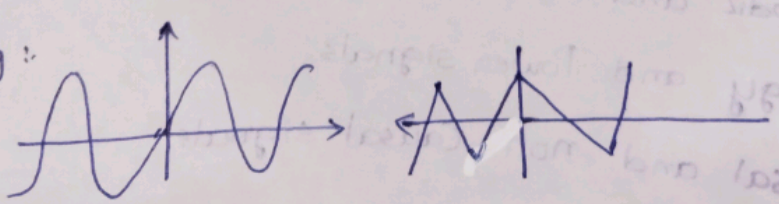


① Deterministic and random signals :-

Deterministic s/g :- It is a signal in which each value (magnitude & phase) of the signal is fixed & can be determined by a mathematical expression or rule.

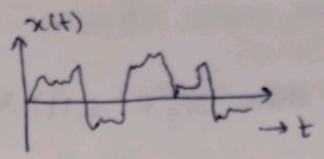
These type of signals have a regular pattern and can be represented by mathematical equation at any time, so that there is no uncertainty of its magnitude & phase at any given instant of time & the future values of the s/g can be predicted in advance with complete confidence.

eg :-



Random signal :- It has lot of uncertainity about its behaviour. The future values of this random s/g cannot be accurately predicted. The pattern of random signal is quite irregular.

eg :-



② Periodic and Non-periodic s/gs :-

A signal which has a definite pattern & which repeats itself at regular intervals of time is called periodic signal, and a s/g which does not repeat at regular intervals of time is called non-periodic (or) aperiodic signal.

Mathematically, a continuous-time signal $x(t)$ is said to be periodic if and only if

$$\boxed{x(t) = x(t+T)}$$ for all t i.e, for $-\infty < t < \infty$

where T is fundamental period ; ω is angular frequency

$$\omega = 2\pi f \Rightarrow f = \frac{\omega}{2\pi} = \frac{1}{T}$$

$$\Rightarrow T = \frac{2\pi}{\omega}$$

For DT periodic s/g

Fundamental freq = $\frac{2\pi}{N} = \frac{\omega}{K}$

angular freq = ω

Similarly, a discrete s/g $x(n)$ is said to be periodic, if it satisfies the condition

$$\boxed{x(n) = x(n+N)}$$ for all integers n.

$$\omega = 2\pi f = 2\pi \frac{K}{N}$$ where N is period

$$\Rightarrow f = \frac{K}{N} \quad ; \quad \boxed{N = \frac{2\pi}{\omega} K}$$

if $\boxed{f = \frac{K}{N} \text{ is ratio of integers}}$, then it is periodic, if not satisfied above conditions, then $x(t)$ or $x(n)$ is non-periodic

Note :

1) The sum of two periodic signals is periodic only if the ratio of their respective periods $\frac{T_1}{T_2}$ is rational no. or ratio of integers

2) The fundamental period is L.C.M of T_1 & T_2

3) If ratio $\frac{T_1}{T_2}$ is irrational no. then s/g's $x_1(t)$ & $x_2(t)$ do not have common period & $x(t)$ cannot be periodic

$x_1(t)$ T_1 $\quad x(t) = x_1(t) + x_2(t)$

$x_2(t)$ T_2 $\quad$ if $\frac{T_1}{T_2} = \frac{K}{M}$ = rational no.

Period of $x(t)$ is T : L.C.M of T_1 & T_2

$$T = mT_1 = kT_2$$

→ π is not a rational no. $\{\pi = 3.1416\}$

④ Energy & Power s/gs :

The total energy or normalized energy of continuous-time s/g $x(t)$ is given by

$$E = \lim_{T\to\infty} \int_{-T}^{T} |x(t)|^2 \, dt \text{ Joules}$$

For complex valued s/gs →

$$E = \int_{-\infty}^{\infty} |x(t)|^2 \, dt$$

The avg power or normalized avg power of C.T s/g is given by

$$P = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 \, dt \text{ watts}$$

and $\sqrt{P}$ is rms value of s/g $x(t)$

For discrete-time s/gs :

$$E = \sum_{n=-\infty}^{\infty} |x(n)|^2$$

$$P = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x(n)|^2$$

Note :-

1) A s/g is said to be an energy s/g if & only if its total energy E is finite i.e., E = finite $\{0 < E < \infty\}$

For an energy signal, avg power P = 0

Non-periodic s/gs are eg's of energy s/gs.

2) A s/g is said to be power s/g if its average power P is finite i.e., $0 < P < \infty$

For a power s/g, total energy $E = \infty$

Periodic s/gs are examples of power s/g's

3) No s/g can be both energy s/g & power s/g.

4) The s/gs that satisfy above properties are neither energy s/gs nor power s/gs.

5) For the s/g to be energy s/g is as $|t| \to \infty$ then s/g amplitude tends to zero i.e., $|x(t)| \to 0$

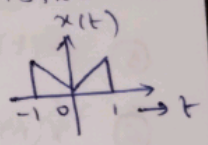
→ Prove the following :- (P.T.O)

a) The power of energy s/g is '0' over infinite time.

b) The energy of power s/g is '∞' over infinite time.

(4) Even and odd signals :-

A C.T s/g is said to be even s/g if it satisfies the condition $\boxed{x(t) = x(-t)}$ for all t

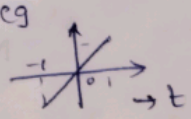


Similarly for D.T s/g $x(n) = x(-n)$ for all n.

- Even s/gs are symmetrical about vertical axis. Also called as symmetrical s/gs.

Odd s/g :- (antisymmetric s/g)

C.T s/g $\boxed{x(t) = -x(-t)}$ for all t



D.T s/g $x(n) = -x(-n)$ for all n

$\boxed{x_{even}(t) = \frac{x(t) + x(-t)}{2}}$; $\boxed{x_{odd}(t) = \frac{x(t) - x(-t)}{2}}$

↑ Even component of s/g ↑ odd part of s/g

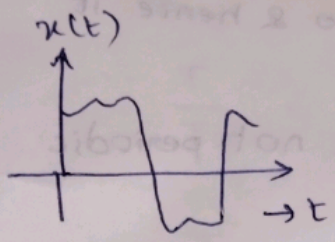
⑤ Causal & Non-Causal signals :-

A C.T s/g is said to be causal if $x(t) = 0$ for $t < 0$ otherwise the s/g is non-causal. A C.T s/g $x(t)$ is said to be anti-causal if $x(t) = 0$ for $t > 0$.

ie., causal s/gs does not exist for -ve time

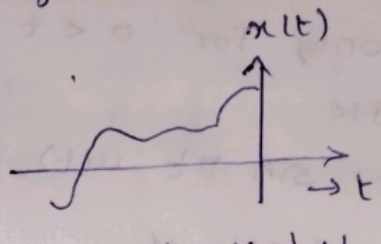
anti causal s/gs does not exist for +ve time.

Non-causal s/gs exists both in +ve & -ve time.



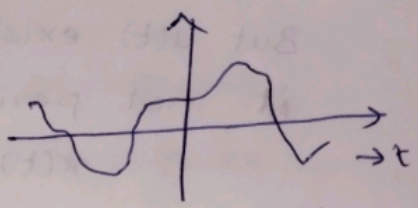
causal s/g

$x(t) = 0$ for $t < 0$



Anticausal s/g

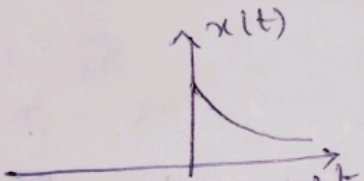
$x(t) = 0$ for $t > 0$



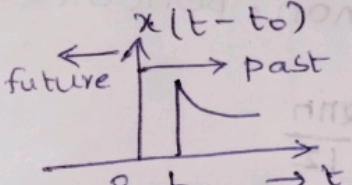
Non-causal s/g.

Note :-

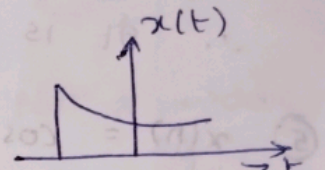
① causal system :- A system is said to be causal if the present value of o/p s/g depends only on present and/or past values of i/p s/g.



causal



causal



Non-causal s/g

→ Check whether the following s/gs are periodic or not ? If periodic, find the fundamental period.

① $\sin 12\pi t = x(t)$

$\omega = 12\pi = 2\pi(6)$ $\therefore f = 6 \Rightarrow T = \frac{1}{6}$.

periodic s/g ✓

② $e^{j4\pi t} = x(t)$

$\omega = 4\pi = 2(2\pi) \Rightarrow f = 2 \Rightarrow T = 1/2$

periodic