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CS325 – Analysis of Algorithms
Project 3 – Linear Programming

Problem 1:
Part A:
Decision Variables:
p1w1 - No. of refrigerators shipped from P1 to W1
p1w2 - No. of refrigerators shipped from P1 to W2
p2w1 - No. of refrigerators shipped from P2 to W1
p2w2 - No. of refrigerators shipped from P2 to W2
p3w1 - No. of refrigerators shipped from P3 to W1
p3w2 - No. of refrigerators shipped from P3 to W2
p3w3 - No. of refrigerators shipped from P3 to W3
p4w2 - No. of refrigerators shipped from P4 to W2
p4w3 - No. of refrigerators shipped from P4 to W3
w1r1 - No. of refrigerators shipped from W1 to R1
w1r2 - No. of refrigerators shipped from W1 to R2
w1r3 - No. of refrigerators shipped from W1 to R3
w1r4 - No. of refrigerators shipped from W1 to R4
w2r3 - No. of refrigerators shipped from W2 to R3
w2r4 - No. of refrigerators shipped from W2 to R4
w2r5 - No. of refrigerators shipped from W2 to R5
w2r6 - No. of refrigerators shipped from W2 to R6
w3r4 - No. of refrigerators shipped from W3 to R4
w3r5 - No. of refrigerators shipped from W3 to R5
w3r6 - No. of refrigerators shipped from W3 to R6
w3r7 - No. of refrigerators shipped from W3 to R7

Objective Function:
Minimize the total cost of shipping from Plant to Retailer Through warehouse.
Minimize the following expression:
10 p1w1 + 15 p1w2 + 11 p2w1 + 8 p2w2 + 13 p3w1 + 8 p3w2 + 9 p3w3 + 14 p4w2 + 8 p4w3 + 5 w1r1 + 6w1r2 + 7 w1r3 + 10 w1r4 + 12 w2r3 + 8 w2r4 + 10 w2r5 + 14 w2r6 + 14 w3r4 + 12 w3r5 + 12 w3r6 + 6 w3r7

Constraints:
Constraint on Supply:
No. of refrigerators from each plant not exceeding their specified supply (Processing the supply of shipment)
p1w1 + p1w2 <= 150
p2w1 + p2w2 <= 450
p3w1 + p3w2 + p3w3 <= 250
p4w2 + p4w3 <= 150
Constraint on Demand:

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The demand of each retailer should be met. (Assuming '=', In class, solved with '>='. In this case, both giving same solution)

w1r1 >= 100
w1r2 >= 150
w1r3 + w2r3 >= 100
w1r4 + w2r4 + w3r4 >= 200
w2r5 + w3r5 >= 200
w2r6 + w3r6 >= 150
w3r7 >= 100

Constraint on Balancing Transshipment:

The Number of refrigerators received at each warehouse from plants should be equal to number of refrigerators sent from warehouse to retailers.

w1r1 + w1r2 + w1r3 + w1r4 - p1w1 - p2w1 - p3w1 = 0
w2r3 + w2r4 + w2r5 + w2r6 - p1w2 - p2w2 - p3w2 - p4w2 = 0
w3r4 + w3r5 + w3r6 + w3r7 - p3w3 - p4w3 = 0

Non-negativity constraints on all Decision variables:

p1w1 >= 0
p1w2 >= 0
p2w1 >= 0
p2w2 >= 0
p3w1 >= 0
p3w2 >= 0
p3w3 >= 0
p4w2 >= 0
p4w3 >= 0
w1r1 >= 0
w1r2 >= 0
w1r3 >= 0
w1r4 >= 0
w2r3 >= 0
w2r4 >= 0
w2r5 >= 0
w2r6 >= 0
w3r4 >= 0
w3r5 >= 0
w3r6 >= 0
w3r7 >= 0

Optimal Solution:

Refrigerators from Plant (p) to Warehouse (w):

p1 to w1 = 150
p2 to w1 = 200
p2 to w2 = 250

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p3 to w2 = 150
p3 to w3 = 100
p4 to w3 = 150

Refrigerators from Warehouse (w) to Retailer (r):
w1 to r1 = 100
w1 to r2 = 150
w1 to r3 = 100
w2 to r4 = 200
w2 to r5 = 200
w3 to r6 = 150
w3 to r7 = 100
The minimal cost for getting all refrigerators from their plants, to warehouses, to the retailers is **$17,100.00**

Part B:
We "closed" warehouse 2 by removing $p_i w_2$ and $w_2 r_k$ terms (can be also done by making those terms 0)

This prevented the warehouse from receiving any refrigerators from the plants, and subsequently being unable to ship fridges out to retailers. This resulted in an infeasibility error in Lindo. Looking at the transshipment graph, 'w3' is the only warehouse that can supply for 'r5', 'r6', and 'r7' once 'w2' is closed. The total demand of r5 + r6 + r7 = 200 + 150 + 100 = 450.
The total supply available to w3 = p3 + p4 = 250 + 150 = 400. So w3 will be 50 fridges short of being able to supply for the retailers for which it is the sole middle man.

Part C:
Limit shipments on warehouse 2 to 100 shipments per week.
We implemented this change by adding the following constraint to Part A:
p1w2 + p2w2+p3w2 +p4w2 <= 100
Since the old bottle-neck left us short by 50 units to accommodate the needs of r5 + r6 + r7, adding 100 additional units between r5 and r6 makes the model feasible again.
The new optimum cost is **$18,300.00**.
The new optimal shipping routes include:
Refrigerators from Plant (p) to Warehouse (w):
p1 to w1 = 150
p2 to w1 = 350
p2 to w2 = 100
p3 to w3 = 250
p4 to w3 = 150

Refrigerators from Warehouse (w) to Retailer (r):
w1 to r1 = 100

w1 to r2 = 150
w1 to r3 = 100
w1 to r4 = 150
w2 to r4 = 50
w2 to r5 = 50
w3 to r5 = 150
w3 to r6 = 150
w3 to r7 = 100

Part D:
Formulating Transshipment problem:
1. **Identify Decision Variables**: No. of refrigerators shipped from plant to warehouse and warehouse to retailer.
P_iW_j -> No. of refrigerators shipped from plant P_i to warehouse W_j . Where $i \in n$ and $j \in q$
W_jR_k -> No. of refrigerators shipped from warehouse W_i to retailer R_k . Where $j \in q$ & $k \in m$.

2. **Formulate the Objective Function**: The objective of the transshipment problem is to minimize the total cost of shipping refrigerators from Plant to Retailer.
The cost of shipping from plant P_i to warehouse W_j is given by cp(i,j) and the cost of shipping from warehouse W_i to retailer R_k is given by cw(j,k).
So, The total cost will be $\sum_{i \in n, j \in q} P_iW_j * cp(i,j) + \sum_{j \in q, k \in m} W_jR_k * cw(j,k)$
So, objective function will be

$$\mathbf{min} \left(\sum_{i \in n, j \in q} P_iW_j * cp(i,j) + \sum_{j \in q, k \in m} W_jR_k * cw(j,k)\right)$$

3. **Formulate the constraints**: The possible 3 constraints in any Transshipment problem are:
a. Constraint on Supply:
No. of refrigerators from each plant not exceeding their specified supply (Processing the supply of shipment)
$\sum_{i \in n, j \in q} P_iW_j <= S_i$, where S_i is the supply associated with each plant P_i.
b. Constraint on Demand:
The demand of each retailer should be met. (Assuming '=', In class, solved with '>='. In this case, both giving same solution)
$\sum_{j \in q, k \in m} W_jR_k = D_k$, where D_k is demand associated with each retailer R_k.
c. Constraint on Balancing Transshipment:
The Number of refrigerators received at each warehouse from plants should be equal to number of refrigerators sent from warehouse to retailers.
$\sum_{k \in m} W_1R_k = \sum_{i \in n} P_iW_1$
$\sum_{k \in m} W_2R_k = \sum_{i \in n} P_iW_2$
$\sum_{k \in m} W_3R_k = \sum_{i \in n} P_iW_3$
.
.
.
$\forall_{i \in n} (\sum_{k \in m} W_3R_j = \sum_{i \in n} P_iW_j)$

d. Non-negativity constraints on all Decision variables:
$\forall_{i \in n, j \in q}\ P_iW_j >= 0$
$\forall_{j \in q, k \in m}\ W_jR_k >= 0$

Problem 2:
Part A:
1. Minimize the following expression:
21 T + 16 L + 40 SP + 41 CR + 585 SS + 120 SMT + 164 CH + 884 O
Constraint on Protein
0.85 T + 1.62 L + 2.86 SP + 0.93 CR + 23.4 SS + 16 SMT + 9 CH > 15
Constraint on Fat
0.33 T + 0.2 L + 0.39 SP + 0.24 CR + 48.7 SS + 5 SMT + 2.6 CH + 100 O > 2
0.33 T + 0.2 L + 0.39 SP + 0.24 CR + 48.7 SS + 5 SMT + 2.6 CH + 100 O < 8
Constraint on Carbohydrates
4.64 T + 2.37 L + 3.63 SP + 9.58 CR + 15 SS + 3 SMT + 27 CH > 4
Constraint on Sodium
9 T + 28 L + 65 SP + 69 CR + 3.8 SS + 120 SMT + 78 CH < 200
40% leafy green
0.6 L + 0.6 SP - 0.4 T - 0.4 CR - 0.4 SS - 0.4 SMT - 0.4 CH - 0.4 O > 0
Non-negativity constraints
T > 0
L > 0
SP > 0
CR > 0
SS > 0
SMT > 0
CH > 0
O > 0

2. Lindo Report:

LP OPTIMUM FOUND AT STEP 12
OBJECTIVE FUNCTION VALUE

1) 114.7541

VARIABLE	VALUE	REDUCED COST
T	0.000000	16.901640
L	0.585480	0.000000
SP	0.000000	14.513662

CR	0.000000	36.289616
SS	0.000000	408.387970
SMT	0.878220	0.000000
CH	0.000000	97.551910
O	0.000000	886.404358

The minimum calories that the salad has is 114.75 calories

3)
Cost of Lettuce = (Serving/100gms)*cost = (58.5/100)* $0.75 = $0.43911
Cost of Smoked Tofu = (Serving/100gms)*cost = (87.822/100)* $2.15 = $1.888173
Total cost of salad = $0.43911 + $1.888173 = **$2.33**

PART B:

1. Minimize the following expression:
T + 0.75 L + 0.5 SP + 0.5 CR + 0.45 SS + 2.15 SMT + 0.95 CH + 2 O

Constraint on Protein
0.85 T + 1.62 L + 2.86 SP + 0.93 CR + 23.4 SS + 16 SMT + 9 CH > 15

Constraint on Fat
0.33 T + 0.2 L + 0.39 SP + 0.24 CR + 48.7 SS + 5 SMT + 2.6 CH + 100 O > 2
0.33 T + 0.2 L + 0.39 SP + 0.24 CR + 48.7 SS + 5 SMT + 2.6 CH + 100 O < 8

Constraint on Carbohydrates
4.64 T + 2.37 L + 3.63 SP + 9.58 CR + 15 SS + 3 SMT + 27 CH > 4

Constraint on Sodium
9 T + 28 L + 65 SP + 69 CR + 3.8 SS + 120 SMT + 78 CH < 200

40% leafy green
0.6 L + 0.6 SP - 0.4 T - 0.4 CR - 0.4 SS - 0.4 SMT - 0.4 CH - 0.4 O > 0

Non-negativity constraints
T > 0
L > 0
SP > 0
CR > 0
SS > 0

SMT > 0
CH > 0
O > 0

2) Lindo Report:

NO. ITERATIONS= 12
LP OPTIMUM FOUND AT STEP 10
OBJECTIVE FUNCTION VALUE
1) 1.554133

VARIABLE	VALUE	REDUCED COST
T	0.000000	1.002081
L	0.000000	0.402912
SP	0.832298	0.000000
CR	0.000000	0.486914
SS	0.096083	0.000000
SMT	0.000000	0.405609
CH	1.152364	0.000000
O	0.000000	7.281258

Minimum cost of the low calorie salad = $1.55

3)
Calories of Spinach =(Serving/100gms)*calories = (83.2298/100)* 40 = 33.2919
Calories of Sunflower Seeds = (Serving/100gms)*calories = (9.6083/100)* 585 = 56.2086
Calories of Chickpeas = =(Serving/100gms)*calories = (115.2364/100)* 164 = 188.9879
Total calories in salad = 33.2919 + 56.2086 + 188.9879 = **278.4882 calories**

PART C:
1)

- You can take the code from part 1 where we minimize the calories and add the constraint that the cost has to be less than $2.

- You can take the code from part 2 where we minimize the cost and add the constraint that the calories has to be less than 250.
- Another way to do this is to use the minimum weights of all the ingredients so this will in turn take care of minimum costs and minimum calories.

You can take the code from part 1 where we minimize the calories and add the constraint that the cost has to be less than $2.

Minimize the following expression:
21 T + 16 L + 40 SP + 41 CR + 585 SS + 120 SMT + 164 CH + 884 O

Constraint on Protein
0.85 T + 1.62 L + 2.86 SP + 0.93 CR + 23.4 SS + 16 SMT + 9 CH > 15

Constraint on Fat
0.33 T + 0.2 L + 0.39 SP + 0.24 CR + 48.7 SS + 5 SMT + 2.6 CH + 100 O > 2
0.33 T + 0.2 L + 0.39 SP + 0.24 CR + 48.7 SS + 5 SMT + 2.6 CH + 100 O < 8

Constraint on Carbohydrates
4.64 T + 2.37 L + 3.63 SP + 9.58 CR + 15 SS + 3 SMT + 27 CH > 4

Constraint on Sodium
9 T + 28 L + 65 SP + 69 CR + 3.8 SS + 120 SMT + 78 CH < 200

40% leafy green
0.6 L + 0.6 SP - 0.4 T - 0.4 CR - 0.4 SS - 0.4 SMT - 0.4 CH - 0.4 O > 0

Adding the cost constraint
1T + 0.75L + 0.50SP + 0.50CR + 0.45SS + 2.15SMT + 0.95CH + 2O < 2

Non-negativity constraints
T > 0
L > 0
SP > 0
CR > 0
SS > 0
SMT > 0
CH > 0
O > 0

Lindo Report:

LP OPTIMUM FOUND AT STEP 13

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OBJECTIVE FUNCTION VALUE
1) 134.7601

VARIABLE	VALUE	REDUCED COST
T	0.000000	165.106415
L	0.000000	48.747345
SP	0.550343	0.000000
CR	0.000000	109.689537
SS	0.029429	0.000000
SMT	0.796086	0.000000
CH	0.000000	67.332855
O	0.000000	1201.935669

Combination of Spinach, Sunflower seeds, Smoked Tofu.

Calories in the salad is **134.7601 calories**

Cost of Spinach = (Serving/100gms)*cost = (55.034/100)* $0.50 = $0.28
Cost of Sunflower seeds = (Serving/100gms)*cost = (2.9429/100)* $0.45 = $0.013
Cost of Smoked Tofu = (Serving/100gms)*cost = (79.6086/100)* $2.15 = $1.711
Cost is of salad = $2

You can take the code from part 2 where we minimize the cost and add the constraint that the calories has to be less than 250.

Minimize the following expression:
1T + 0.75L + 0.50SP + 0.50CR + 0.45SS + 2.15SMT + 0.95CH + 2O < 2

Constraint on Protein
0.85 T + 1.62 L + 2.86 SP + 0.93 CR + 23.4 SS + 16 SMT + 9 CH > 15

Constraint on Fat
0.33 T + 0.2 L + 0.39 SP + 0.24 CR + 48.7 SS + 5 SMT + 2.6 CH + 100 O > 2
0.33 T + 0.2 L + 0.39 SP + 0.24 CR + 48.7 SS + 5 SMT + 2.6 CH + 100 O < 8

Constraint on Carbohydrates

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4.64 T + 2.37 L + 3.63 SP + 9.58 CR + 15 SS + 3 SMT + 27 CH > 4

Constraint on Sodium
9 T + 28 L + 65 SP + 69 CR + 3.8 SS + 120 SMT + 78 CH < 200

40% leafy green
0.6 L + 0.6 SP - 0.4 T - 0.4 CR - 0.4 SS - 0.4 SMT - 0.4 CH - 0.4 O > 0

Adding the calorie constraint
21 T + 16 L + 40 SP + 41 CR + 585 SS + 120 SMT + 164 CH + 884 O

Non-negativity constraints
T > 0
L > 0
SP > 0
CR > 0
SS > 0
SMT > 0
CH > 0
O > 0

Lindo Report:

LP OPTIMUM FOUND AT STEP 13
OBJECTIVE FUNCTION VALUE
1) 1.62

VARIABLE	VALUE	REDUCED COST
T	0.000000	1.002098
L	0.000000	0.396025
SP	0.761996	0.000000
CR	0.000000	0.532741
SS	0.093830	0.000000
SMT	0.168941	0.000000
CH	0.880222	0.000000

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O	0.000000	8.431896

Minimum cost of the low calorie salad = $1.62

Calories of Spinach = (Serving/100gms)*calories = (76.1996/100)* 40 = 30.48
Calories of Sunflower Seeds = (Serving/100gms)*calories = (9.6083/100)* 585 = 54.89
Calories of Smoked Tofu= (Serving/100gms)*calories = (16.8941/100)* 120 = 20.27
Calories of Chickpeas= (Serving/100gms)*calories = (88.0222/100)* 164 = 144.36
Total calories in salad = 30.48 + 54.89 + 20.27 + 144.36= **250 calories**

Problem 3:
Part A:
We can find the shortest path from vertex A to other vertices by following objective and constraints:

Objective Function:
Maximize the sum of distances of each vertex in the graph.
Maximize the following expression:
a + b + c + d + e +f + g + h + i + j + k + l + m

Constraints:
Constraint on Distance:
The distance vertex V_i to V_j should be less than $w(V_j, V_i)$
b - a <= 2
c - a <= 3
d - a <= 8
h - a <= 9
a - b <= 4
c - b <= 5
e - b <= 7
f - b <= 4
d - c <= 10
b - c <= 5
g - c <= 9
i - c <= 11
f - c <= 4
a - d <= 8
g - d <= 2
j - d <= 5
f - d <= 1

h - e <= 5
c - e <= 4
i - e <= 10
i - f <= 2
g - f <= 2
d - g <= 2
j - g <= 8
k - g <= 12
i - h <= 5
k - h <= 10
a - i <= 20
k - i <= 6
j - i <= 2
m - i <= 12
i - j <= 2
k - j <= 4
l - j <= 5
h - k <= 10
m - k <= 10
m - l <= 2

Constraint on Start node:
The distance of start node should be equal to 0.
a = 0

Non-negativity constraints on all Decision variables:
b >= 0
c >= 0
d >= 0
e >= 0
f >= 0
g >= 0
h >= 0
i >= 0
j >= 0
k >= 0
l >= 0
m >= 0

Optimal Solution:
Shortest Path from vertex A to other vertices:
A to B = 2
A to C = 3
A to D = 8
A to E = 9
A to F = 6

A to G = 8
A to H = 9
A to I = 8
A to J = 10
A to K = 14
A to L = 15
A to M = 17

Part B:
If we add a new vertex 'Z' to the graph, which has no path from A to Z, The shortest path from vertex 'A' to all other vertices remains same. As vertex A has shortest path to all other vertices and has no shortest path to vertex 'Z', we can say that vertex Z is not connected to any of the vertices. Hence, the shortest path from vertex 'A' to other vertices remains same and there is no shortest path from vertex 'A' to 'Z'.
We did this by adding a constraint 'z - a <= 0' to the existing constraints in Part A.

Optimal Solution:
Shortest Path from vertex A to other vertices:
A to B = 2
A to C = 3
A to D = 8
A to E = 9
A to F = 6
A to G = 8
A to H = 9
A to I = 8
A to J = 10
A to K = 14
A to L = 15
A to M = 17
A to Z = 0

Part C:

The shortest path from each vertex to vertex 'M' can be solved by making vertex 'M' as start node and altering the directions of the edges.
By default, The original directions give the shortest path from other vertices to vertex 'M'. So, If we change the direction of the edge, then we can get the shortest path from vertex 'M' to other vertices.

The shortest distances from vertex 'M' to other vertices are as follows:
A to M = 17
B to M = 15

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C to M = 15
D to M = 12
E to M = 19
F to M = 11
G to M = 14
H to M = 14
I to M = 9
J to M = 7
K to M = 10
L to M = 2

Part D:

The shortest path from any vertex 'X' to vertex 'Y' that pass through vertex 'I' can be calculated by calculating the shortest path from all vertices in graph to vertex 'I' and then calculate shortest path from vertex 'I' to other vertices of the graph. The sum of both gives the shortest path from vertex 'X' to vertex 'Y' passing through vertex 'I'.
The shortest path from all other vertices to vertex 'I' can be calculated by using same procedure as in Part B, starting from vertex 'I' and change the direction of edges. Make sure to remove the vertices which are not reachable from vertex 'I'. The shortest path from vertex 'I' to other vertices can be calculated by using same procedure as in Part A, starting from vertex 'I' and find shortest path to other vertices. Make sure to remove the vertices which didn't satisfy in the first step.

We wrote 2 separate programs for each: one for calculating shortest path from all vertices to vertex 'I' and another for calculating shortest path from vertex 'I' to all other vertices.

The shortest path from other vertices to vertex 'I' are:
A to I = 8
B to I = 6
C to I = 6
D to I = 3
E to I = 10
F to I = 2
G to I = 5
H to I = 5
J to I = 2
K to I = 15
L to I = 0
M to I = 0
Note: vertex 'I' is not reachable from vertices 'L' and 'M'

The shortest path from vertex 'I' to other vertices are:

I to A = 20
I to B = 22
I to C = 23
I to D = 28
I to E = 29
I to F = 26
I to G = 28
I to H = 16
I to J = 2
I to K = 6
I to L = 0
I to M = 0

The Total shortest Distance from vertex 'X' to vertex 'Y' through vertex 'I' are:

A to I and I to A = 28
B to I and I to B = 28
C to I and I to C = 29
D to I and I to D = 31
E to I and I to E = 39
F to I and I to F = 28
G to I and I to G = 33
H to I and I to H = 21
I to I and I to I = 0
J to I and I to J = 8
K to I and I to K = 22
L to I and I to L = 0
M to I and I to M = 0

Several combinations can also be calculated by summing the two results.
Shortest distance vertex 'X' to vertex 'Y' through vertex 'I' is given by:
SUM (Shortest_Distance (X , I) , Shortest_Distance(I , Y))

Lindo Code: Lindo code and output for problem 1 and 3 were e-mailed to the professor.

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