

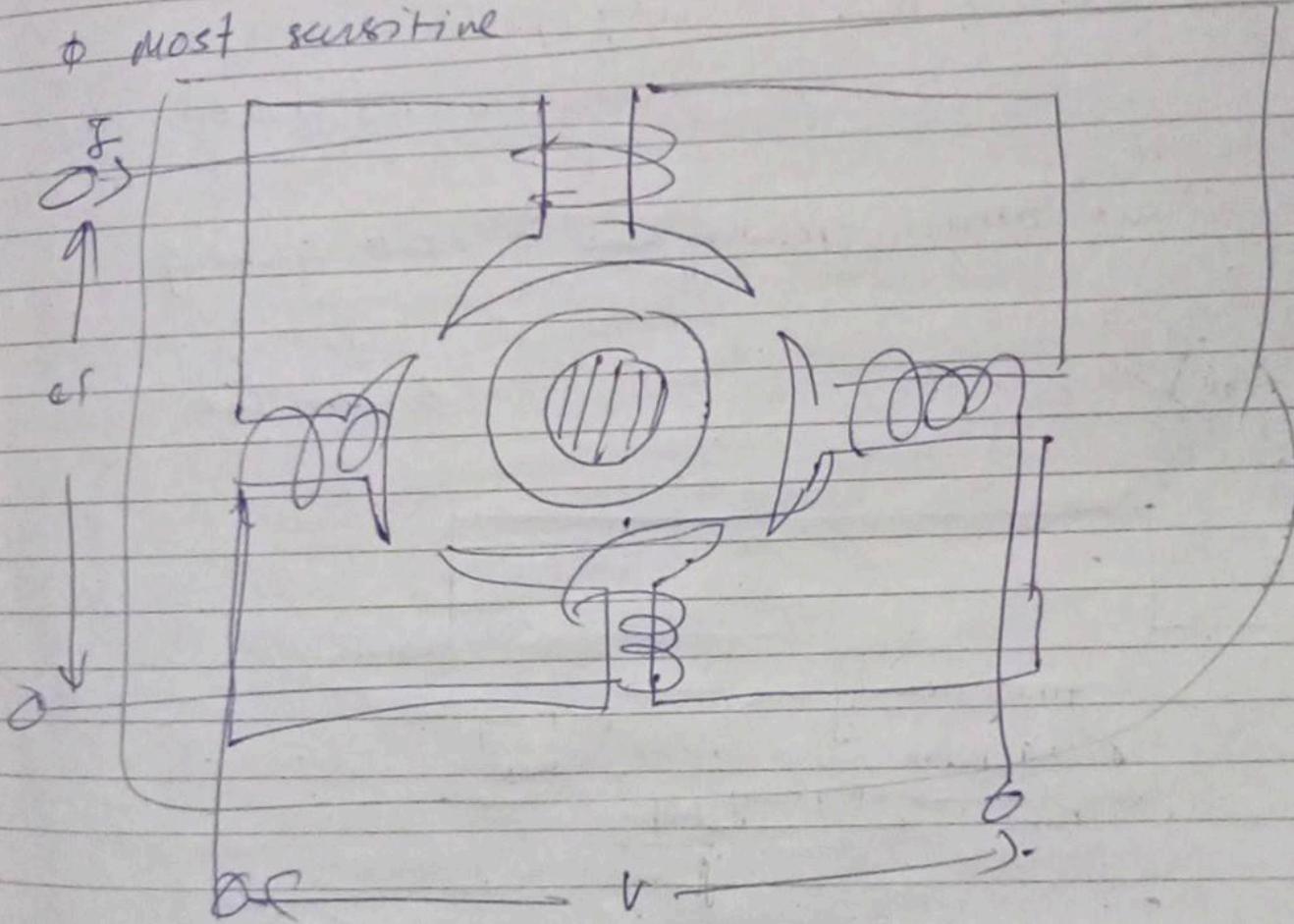
PSPS

Page No.	10
Date	2023

10 Jam

2ndⁿ cup type construction:

♦ most sensitive



L, give max torque per VA ratio

$$\left(\frac{\text{Magnet}}{\text{Pickup}} \right) \approx 1$$

L, 0.95 - 1 for induction principle

0.6 - 0.9 electromagnetic attraction
type ratio

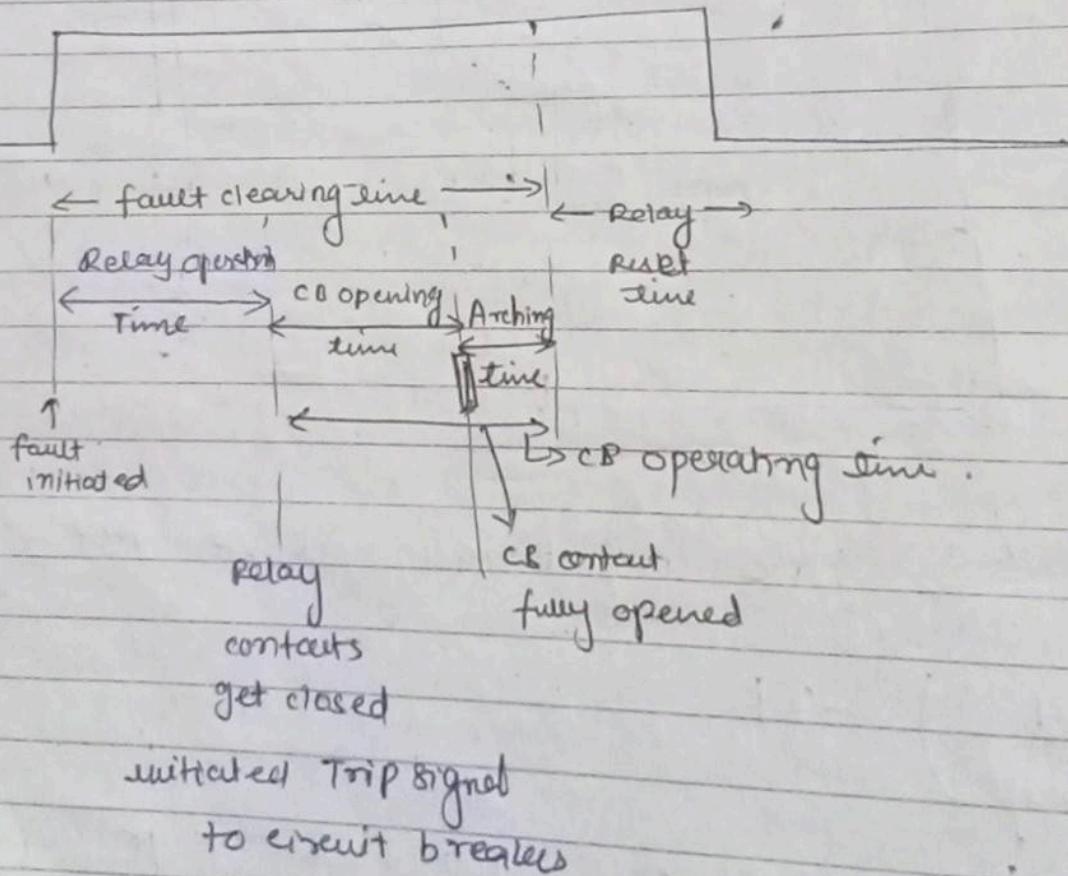
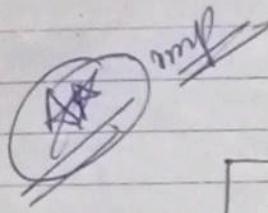


Scan with Fast Scan

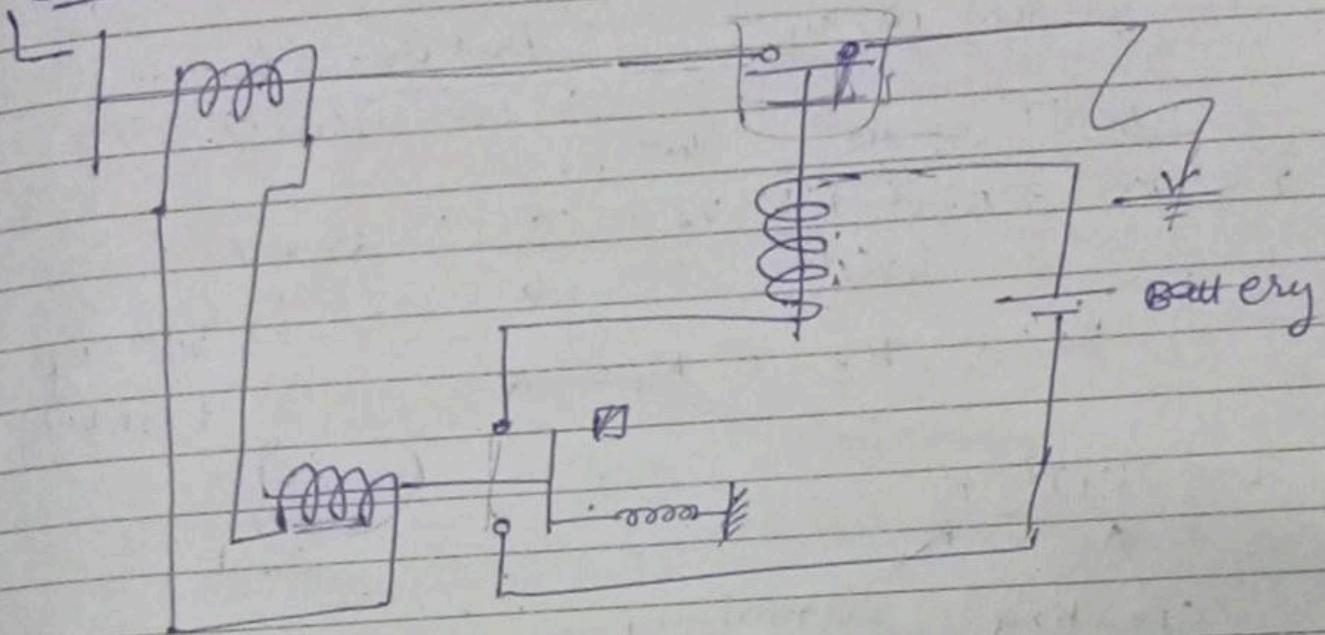
operating times :

Fault clearing time = Relay operating time +
circuit breakers operating time

Circuit breakers operating time = C-B. opening
time
+
Arching time



control



CB open hone ke time arch form hoti hau
thermo ionic emission 14 vajreh se

~~**~~ fund

Universal torque equation:

$$T = K_1 I^2 + K_2 V^2 + K_3 VI \cos(\theta - \epsilon) + K_4$$

\downarrow \downarrow \downarrow
current unit voltage unit direction unit
or
power unit

spring

↳ Derive any kind of relay from
this equati



Scan with Fast Scan

→ Not a directional relay

Page No. _____
Date _____

Over current relay : \therefore don't know dirn.

$$\therefore K_3 V I \cos(\alpha - \gamma) = 0$$

$$T = \frac{K_1 I^2}{L_{SO}} + \frac{K_2 V^2}{L_{SD}} + \frac{K_3 V I \cos(\alpha - \gamma)}{L_{SP}} + K_4$$

$$K_2 = 0, K_3 = 0, K_4 = -K$$

L spring
torque.

$$T = K_1 I^2 - K$$

↑ ↓
Operating torque Restraining torque

$$T_{op} = T_{rest} \rightarrow \text{over current relay}$$

operate

$$(I_{pick-up} = \sqrt{R(I)^2 + j\omega(I)^2})^2 = C$$

$$K_1 I^2 - K > 0$$

$$I = \sqrt{\frac{K}{K_1}} = I_{pick-up} \Rightarrow I_{pick-up} = C$$

$I_{pick-up} = \text{constant}$

$\left. \begin{array}{l} \text{pickup current } I_0 \\ \text{in magnet coil} \\ \text{we have} \end{array} \right\}$ magnet current above which over relay ~~current~~ will operate generally called "pickup".

$$K_1 I^2 > K$$

$$I > \sqrt{\frac{K}{K_1}}$$

$\left. \begin{array}{l} I > I_{pick-up} \end{array} \right\} \rightarrow \text{relay operate}$



transient fault: this fault includes the momentary loss of connectivity to components & services, the temporary unavailability of a service for time that causes data
a service is busy.

This faults often self correcting.

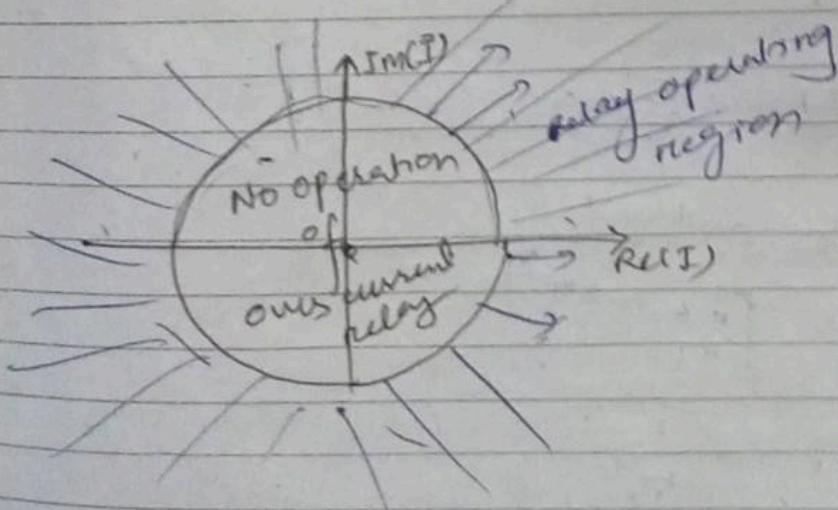
Restoration process of returning to its earlier good condition

Thermal stress?

57 spicu. $\sqrt{3}$

$$\Rightarrow \text{Re}(I)^2 + \text{Im}(I)^2 = c^2$$

6 this is circle
center at $(0,0)$.



Scan with Fast Scan

Date
12-1-23

$$\frac{I}{S} = 1.4$$

Page No.
Date

current setting (CSL) / ~~plug setting~~ / threshold value

$$\text{current setting (in %)} = \frac{I_{\text{pick-up}} \times 100}{I_{\text{relay}}}$$

ex: ① $I_{\text{relay}} = 5A$

$$I_{\text{pick-up}} = 7A$$

$$\text{CS(%)} = \frac{I_{\text{pick-up}} \times 100}{I_{\text{relay}}}$$

$$= \frac{7}{5} \times 100 = 140\%$$

② if $I_{\text{pick-up}} = 5A$

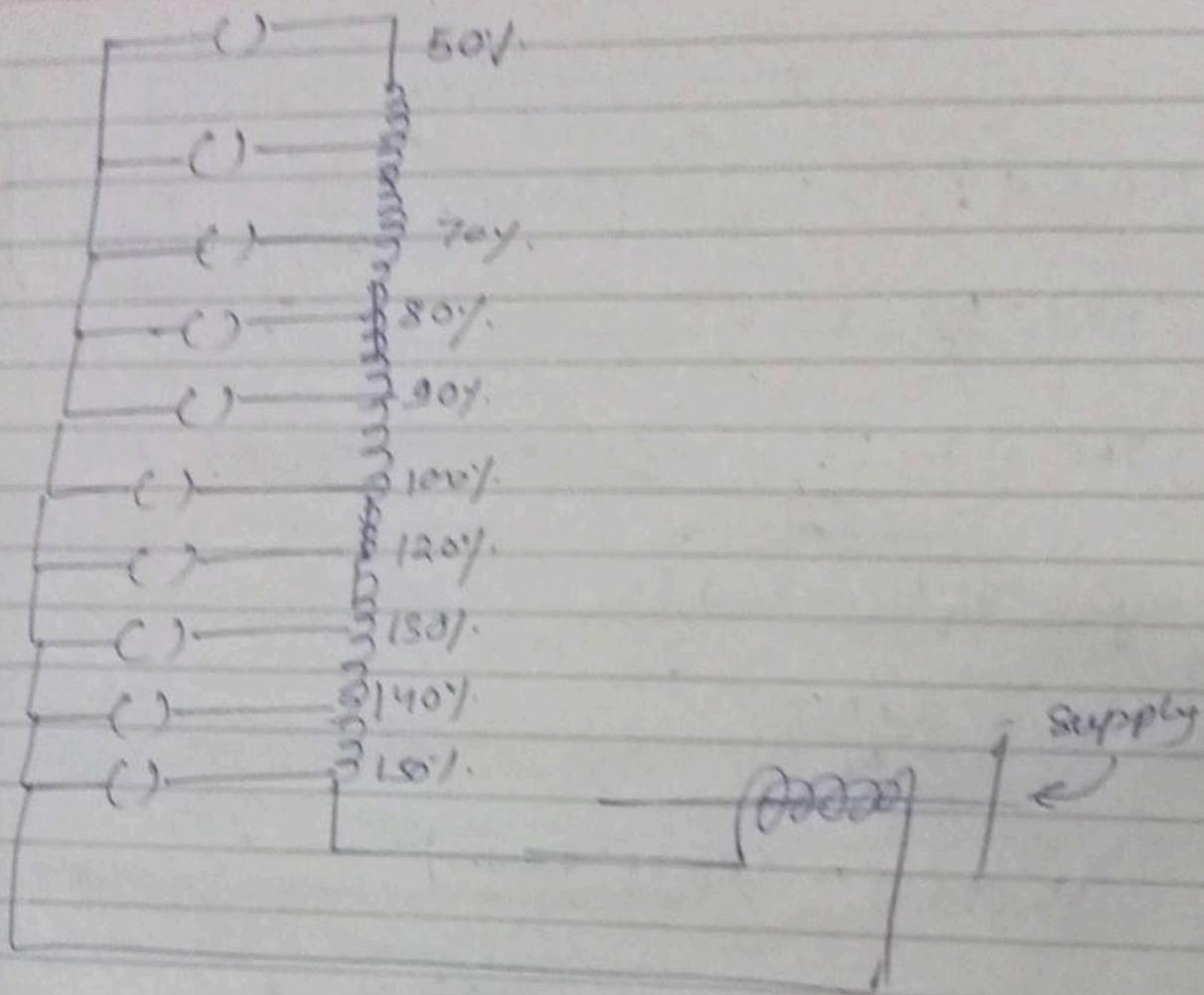
$$\text{min (CS(%))} = \frac{5}{5} \times 100$$

$$= 100\%$$

$$I_{\text{pick-up}} = \sqrt{\frac{K_C C}{R I}} \text{ constant}$$

$$I_{\text{pick-up}} \propto \frac{1}{\sqrt{R I}}$$





current setting of I_{Pick-up}

by merely

* Current setting we decide the pick-up current.

$T_{OP} \propto K_1 I^2$ —①

$$T_{OP} \propto \Phi I \quad [\because \Phi = \frac{NI}{R_e} \leftarrow \text{Resistance}]$$

$$T_{OP} \propto \frac{NI}{R_e} \times I$$

$$\underline{T_{OP} = K_0 \frac{NI^2}{R_e}} \quad -\textcircled{2}$$

compare eqⁿ ① and ②

$$K_1 = \frac{K_0 N}{R_e}$$

$$\Rightarrow K_1 \propto N \text{ (No. of turns)} \quad -\textcircled{3}$$

$CS(Y) \propto I_{\text{pick-up}}$ —④

$$I_{\text{pick-up}} = \sqrt{\frac{K_0}{K_1}}$$

$$\therefore I_{\text{pick-up}} \propto \frac{1}{\sqrt{K_1}} \quad -\textcircled{5}$$

~~④~~ by \otimes ④, ⑤

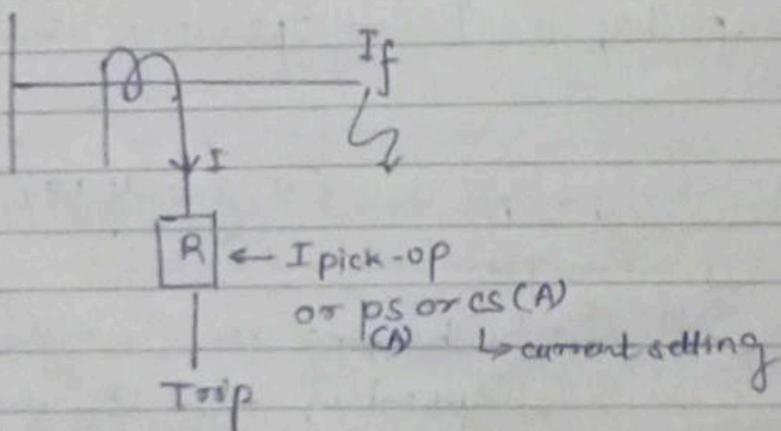
$$CS(Y) \propto I_{\text{pick-up}} \propto \frac{1}{\sqrt{N}}$$



Plug setting Multiplier (PSM).

$PSM = \frac{\text{secondary current of CT}}{\text{current setting (in ampere)}}$

$PSM = \frac{\text{Primary current}}{\text{CT ratio} \times \text{current setting (A)}}$.



$$PSM = \frac{I_f}{\text{CT ratio} \times \text{CS(A)}}$$

if $PSM > 1 \rightarrow$ relay operates

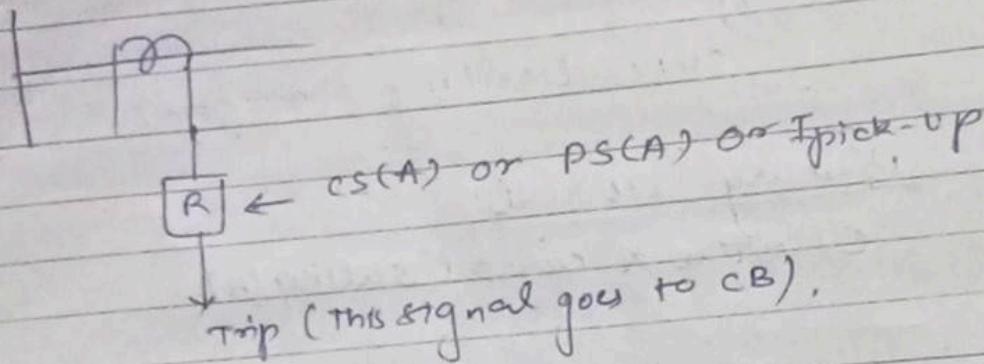
$PSM < 1 \rightarrow$ relay NOT operate

$PSM = 1 \rightarrow$ relay at verge of operation

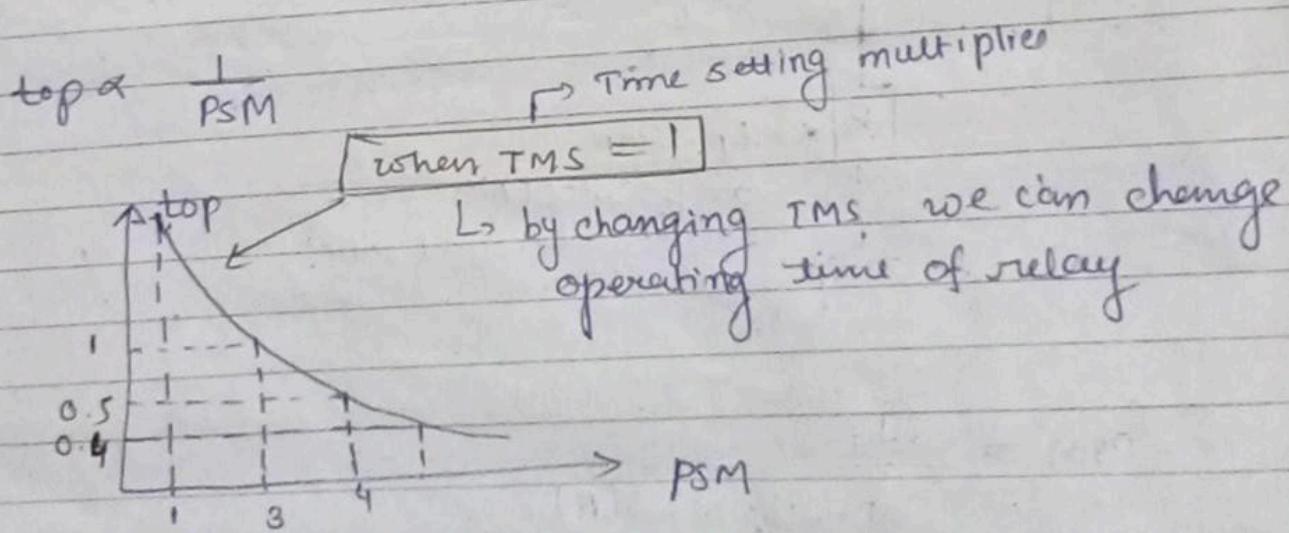
operating time
of relay is
infinite.



PSM vs operating Time



If $\uparrow \rightarrow \text{PSM} \uparrow \rightarrow$ speed of disc $\uparrow \rightarrow$ operating time \downarrow

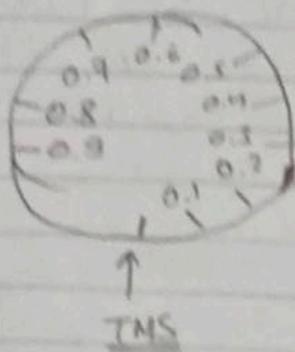


$$\text{top} = \text{TMS} \times \text{tripping}$$



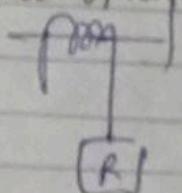
Ques

$$TMS = 1$$

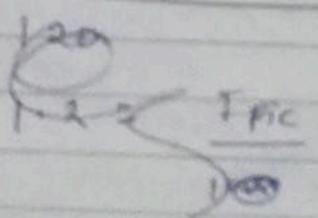


Que an over current relay is connected to the supply circuit through a 1000/1A current transformer, the relay is set at 120%. calculate $I_{\text{pick-up}}$.

given $CT = 1000$
 $CS = 120\%$



$$CS = \frac{I_{\text{pick-up}} \times 100}{I_{\text{inv}}}$$



$$120 = \frac{I_{\text{pick-up}} \times 100}{I_{\text{relay}}}$$

$$I_{\text{relay}} = 1.20$$

$$I_{\text{pick-up}} = \frac{120}{100} \times 1$$

$$I_{\text{pick-up}} = 1.2 \text{ A}$$



* question always give relay current.

✓ relay current

Ques A 5A over current relay is set at 100% .
and its relay operating coil has 144 turns.
if pick-up is adjusted to $6A$ calculate.
No. of amp. turns required in its relay
operating coil

$$I_{\text{relay}} = S \text{ or } I_{\text{pick-up}}$$

$$S = 100\%.$$

$$\frac{100}{6} \times \frac{I_{\text{pick-up}}}{I_{\text{relay}}} = 100$$

$$\boxed{I_{\text{pick-up}} = I_{\text{relay}}}$$

$$N = 144.$$

$$PSM = \frac{1}{2R}$$

Solⁿ

$$I_{\text{pick-up}} \propto \frac{1}{\sqrt{N}}$$

$$5 \propto \frac{1}{\sqrt{N_1}} \Rightarrow$$

$$\frac{5}{6} \propto \frac{\sqrt{N_2}}{\sqrt{N_1}}$$

$$6 \propto \frac{1}{\sqrt{N_2}}$$

$$\frac{5}{6} = \frac{\sqrt{N_2}}{\sqrt{N_1}}$$

$$\boxed{N_2 = 100}$$

Ans



Scan with Fast Scan

(Methods)

Classification of protective schemes:

1. Over current protection.
2. Distance protection.
3. Carrier current protection.
4. Differential protection.

Types of overcurrent relays.

9) Instantaneous over current relay: does not depend on mag of If.

- i) Definite time over current relay.
- ii) Inverse definite time over current relay, (IDMT).
- iii) Very inverse over current relay.
- iv) Extremely inverse over current relay

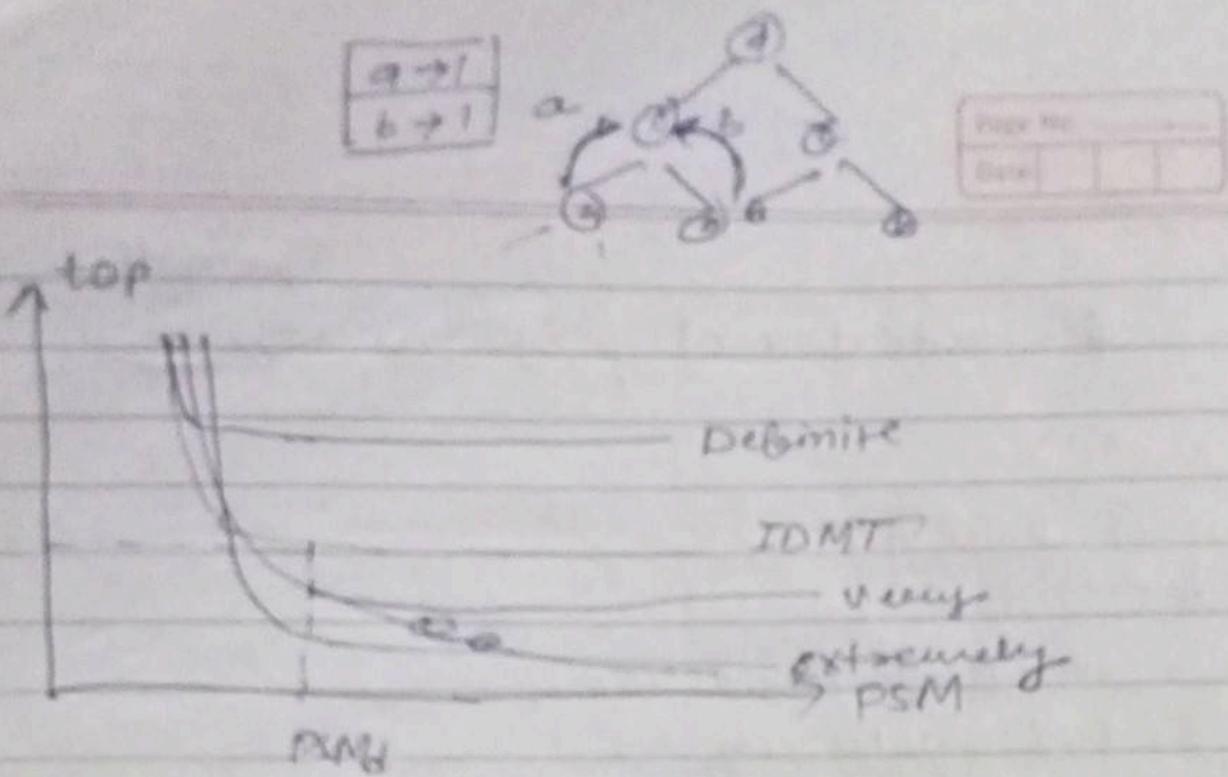
$$PSM_E = \frac{I_{fd}}{CT_{radio} \times CS(A)}$$

$$I \leq PSM < PSM_d$$

\hookrightarrow Inverse definite time.

$PSM > PSM_d \rightarrow$ Definite time





volume of core

$EI > VI > IDMT > Debnite$

#

~~Very imp~~

Distance Relays [3 types]

1. Impedance relay
2. Reactance relay
3. Mho relay

Impedance relay / Voltage restraint over current relay

$$T = K_1 I^2 + K_2 V^2 + K_3 V I \cos(\theta - \Gamma) + K_4$$

↓

I will provide operating torque

V will provide restraining torque.

For Impedance relay

$$K_1 = +ve, K_2 = -ve, K_3 = 0 = K_4$$

Here we are obtaining restraining force with the help of voltage.

$$T = K_1 I^2 - K_2 V^2$$

I , provides
Torque

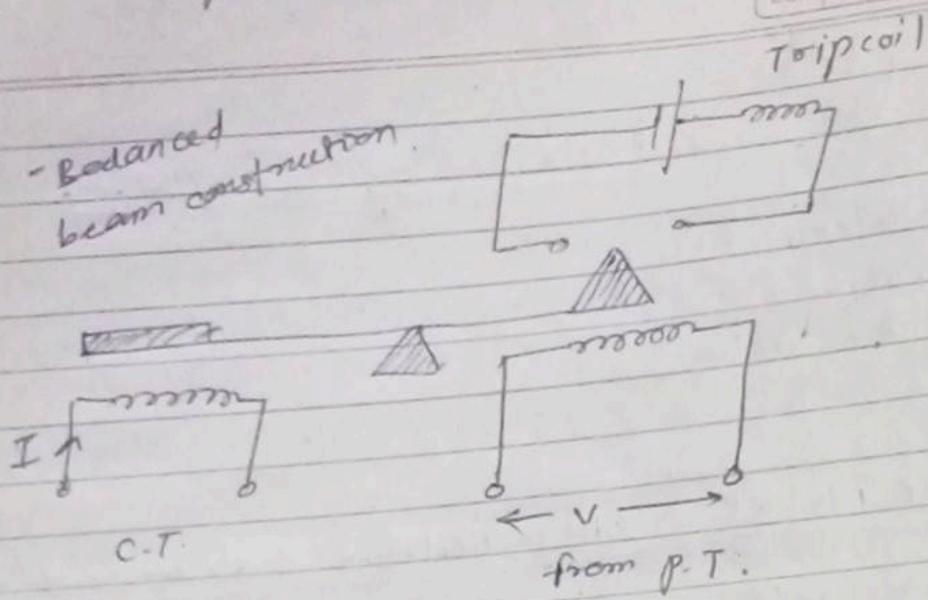
V , provides
restraining
torque



$$V = IR$$

Page No.	
Date	

- Balanced beam construction.



Under threshold condⁿ

$$T = 0 \Rightarrow \text{Top}$$

Zerowring = Treffpunkt

$$K_1 I^2 > K_2 V^2$$

$$\left(\frac{V}{I}\right)^2 = \frac{K_1}{K_2}$$

$$\frac{V}{I} = \sqrt{\frac{K_1}{K_2}} = z_{\text{set}} \quad \text{--- (1)}$$

$\hookrightarrow z_{\text{seen}}$

Top > Trag

$$K_1 I^2 > K_2 V^2$$

$$\left(\frac{V}{I}\right)^2 < \sqrt{\frac{K_1}{K_2}} \Rightarrow \frac{V}{I} < \sqrt{\frac{K_1}{K_2}}$$

$\hookrightarrow z_{\text{seen}} \quad \hookrightarrow z_{\text{set}}$



~~Relay~~

②

$$z_{seen} < z_{set}$$

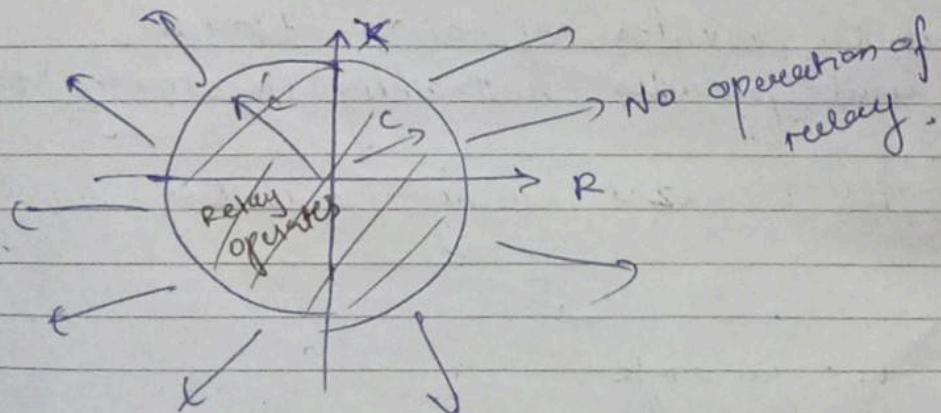
[relay will operate.]

from ①

$$\begin{aligned} z_{set} &= \sqrt{r_{set}^2 + x_{set}^2} = \sqrt{\frac{K_1}{K_2}} \\ &= \sqrt{(R_{set})^2 + (X_{set})^2} = c \end{aligned}$$

$$(R_{set})^2 + (X_{set})^2 = c^2$$

→ equation of circle center as origin. and radius is c .



Reactance Relay

$$\tau = k_1 I^2 + k_2 V^2 + k_3 VI \cos(\theta - \tau) + k_4$$

For Impedance relay ($Z = R + jX$)

$$k_1 = +\text{ve}, \quad k_2 = 0, \quad k_4 = 0, \quad k_3 = -VR$$

$$\tau = k_1 I^2 - k_3 VI \cos(\theta - \tau) \quad \text{--- (1)} \quad (\text{and "disk")}$$

\uparrow torque angle
 \uparrow power unit
current unit

(or directional restrained)

- Reactance relay is power restrained over current relay.

- current unit provides operating torque
- power unit provides restraining (restoring) torque

this relay is realized using induction type of relay.

$\tau \rightarrow$ torque angle

$\tau = 90^\circ$ (maximum torque angle).



Scan with Fast Scan

by eqn ①

$$T = K_1 I^2 - K_3 V I \cos(\theta - 90^\circ)$$

$$T = K_1 I^2 - K_3 V I \sin \theta$$

$$V = IR$$

under threshold cond'n

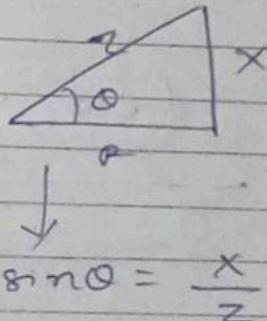
$$K_1 I^2 = K_3 V f \sin \theta$$

$$I = \frac{K_3 V \sin \theta \times f}{K_1}$$

$$\frac{V}{I} = \frac{K_1}{K_3 \sin \theta}$$

$$Z = \frac{K_1}{K_3 \sin \theta}$$

$$Z \sin \theta = \frac{K_1}{K_3}$$



$$\sin \theta = \frac{X}{Z}$$

$$\Rightarrow \boxed{X = \frac{K_1}{K_3}} \leftarrow \text{This is set value of resistance}$$

$$\boxed{X_{set} = \frac{K_1}{K_3}} \quad \text{--- (2)}$$

when $X < X_{set}$ then our relay will operate.



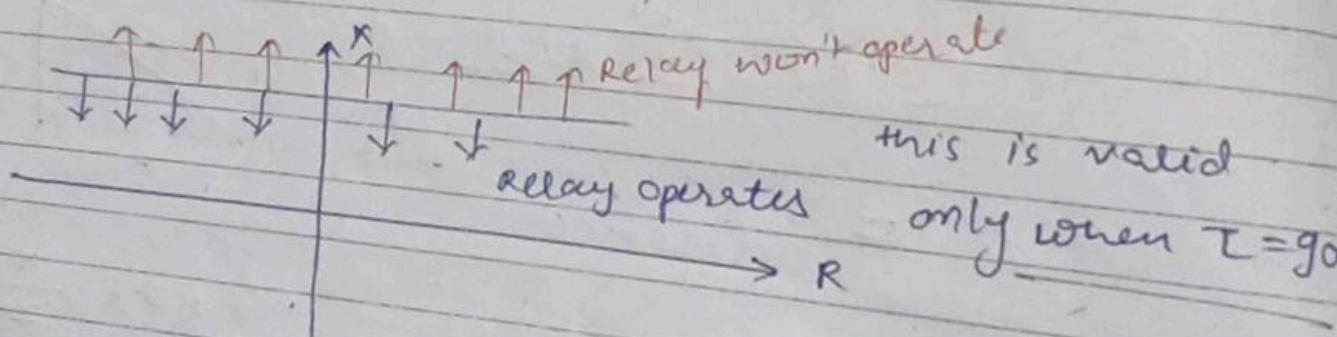
when relay operates.

$$\tau > 0 \quad T_{op} > T_{res}$$

$$K_1 I^h > K_3 v \sin \theta$$

$$\textcircled{2} \quad \frac{v}{I} \sin \theta < \frac{K_1}{K_3}$$

$$\boxed{x_{seen} < x_{set}} \rightarrow \textcircled{3}$$



When $\tau = 0$

$$T = K_1 I^2 - K_3 V I \cos(\theta - \tau)$$

$$T = K_1 I^2 - K_3 V I \cos \theta$$

under threshold cond'n

$$T = 0 = K_1 I^2 - K_3 V I \cos \theta$$

$$K_1 I = K_3 V \cos \theta$$

$$\frac{V}{I} \cos \theta = \frac{K_1}{K_3}$$

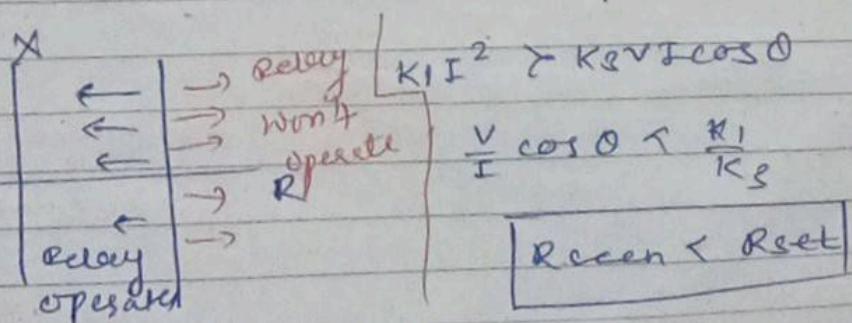
$$Z \cos \theta = \frac{K_1}{K_3}$$

$$R_{set} = \frac{K_1}{K_3}$$

set value of
resistance.

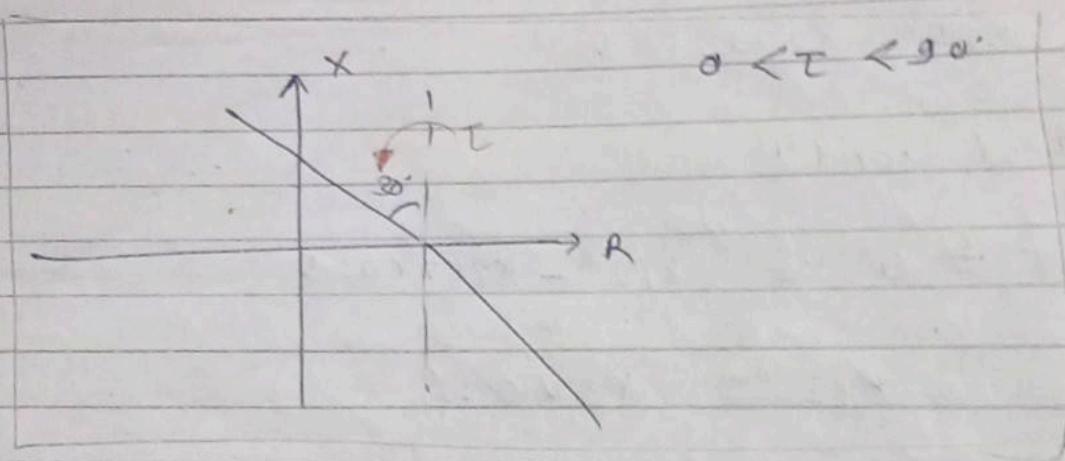
when relay operates

$$\tau > 0 \Rightarrow T_{op} > T_{res}$$



when $I=30$

$$T = K_1 I^2 - K_2 V I \cos(\theta - 30^\circ)$$



- This relay don't have diurnal feature

(distance relay)

Mho relay / Voltage restraint directional relay

- This relay has directional feature.
- used for protection of long transmission line.

$$T = K_1 I^2 + K_2 V^2 - K_3 V I \cos(\theta - \tau) + K_4$$

↑ to restoring force provide.

for mho relay

$$K_1 = 0, K_2 = -ve, K_3 = +ve, K_4 = 0$$

$$T = K_3 V I \cos(\theta - \tau) - K_2 V^2$$

under "normal cond"

$$\tau = 0 \Rightarrow$$

$$K_3 V I \cos(\theta - \tau) \leq K_2 V^2$$

$$\frac{V}{I} = \frac{K_3 \cos(\theta - \tau)}{K_2}$$

$$Z_{set} = \frac{K_3}{K_2} \frac{V}{I} = \frac{K_3}{K_2} \cos(\theta - \tau) \quad \leftarrow ①$$

Relay will operate when

$$\tau > 0 \Rightarrow K_3 V \cos(\theta - \tau) > K_2 V^2$$

operating torque < restoring torque



$$\frac{V}{I} \leftarrow \frac{k_3}{k_2} \cos(\theta - \tau)$$

$$[z_{seen} < z_{set}]$$

In this cond'n our relay
will operate

- (2)

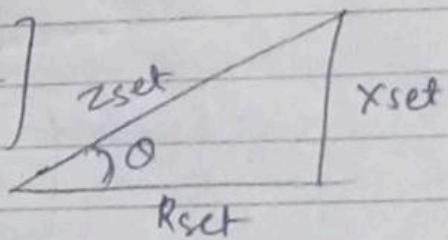
'plotting curve'

$$z_{set} = \frac{k_3}{k_2} \cos(\theta - \tau) \quad k_0 = \frac{k_3}{k_2}$$

$$\sqrt{r_{set}^2 + x_{set}^2} = k_0 \cos(\theta - \tau)$$

$$= k_0 [\cos \theta \cdot \cos \tau + \sin \theta \cdot \sin \tau]$$

$$z_{set} = k_0 \left[\frac{r_{set}}{z_{set}} \cdot \cos \tau + \frac{x_{set}}{z_{set}} \cdot \sin \tau \right]$$



$$z_{set}^2 = k_0 [r_{set} \cdot \cos \tau + x_{set} \sin \tau]$$

$$z_{set} \cos \theta = r_{set}$$

$$\cos \theta = \frac{r_{set}}{z_{set}}$$

$$\sin \theta = \frac{x_{set}}{z_{set}}$$



Scan with **Fast Scan**

$$R_{set}^2 + X_{set}^2 = K_0 R_{set} \cos \tau + K_0 X_{set} \sin \tau$$

$$\boxed{R_{set}^2 + X_{set}^2 - K_0 R_{set} \cos \tau - K_0 X_{set} \sin \tau = 0}$$

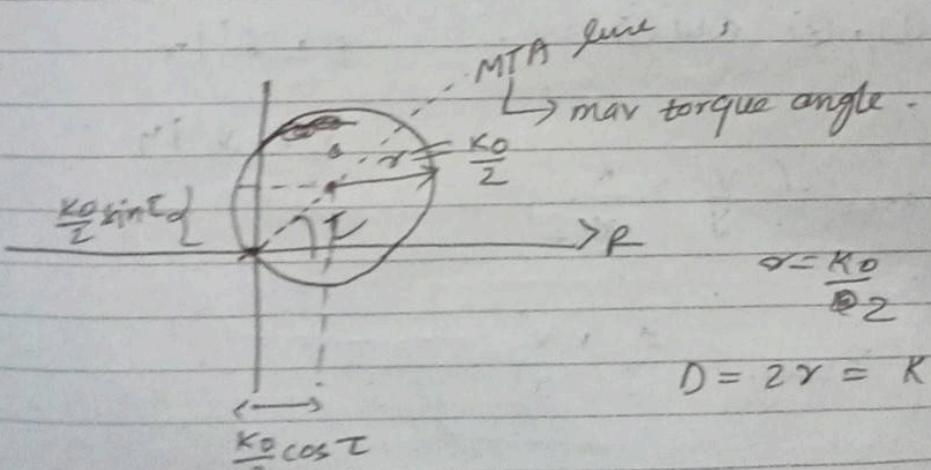
\Rightarrow This is circle passing through origin.

$$x^2 + y^2 + 2ax + 2by + c = 0$$

$$\text{Centre} = \left(\frac{K_0}{2} \cos \tau, \frac{K_0}{2} \sin \tau \right)$$

$$\text{radius} = \sqrt{a^2 + b^2 - c} = \frac{K_0}{2}$$

$$\text{Diameter} = K_0$$



$$\theta = \frac{K_0}{R/2}$$

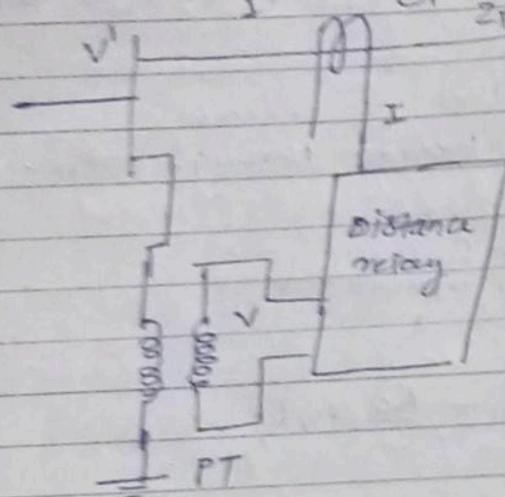
$$D = 2r = K_0$$

$$\text{Max torque angle} = 65^\circ - 85^\circ$$

17-1-23

Page No. _____
Date _____

Impedance seen by Relay:



$$(current\ transform\ \chi) \quad \chi = CT\ ratio = \frac{I'}{I} \quad \textcircled{1}$$

y = Potential-transformer ratio.

$$= \frac{V'}{V} \quad \textcircled{2}$$

$$V_L = I' Z_{L'}$$

$$Z_{seen} = \frac{V}{I} \quad \textcircled{3}$$

$$\chi = \frac{I'}{I}$$

from \textcircled{1}, \textcircled{2} ~~and~~

$$I = I'/\chi$$

$$V = V'/y$$

$$Z_{seen} = \frac{V'}{y} \times \frac{\chi}{I'} \quad \textcircled{4}$$

$$Z_{seen} = \frac{\chi}{y} \times \frac{V'}{I'} \quad \boxed{\textcircled{4}}$$

PT

CT =



Scan with **Fast Scan**

Ques 132 KV TL is protected by distance relay
 the relay is connected to the supply circuit
 through 132 KV / 110 V PT and 1000 / 5A CT
 Impedance of TL 120Ω , calculate

① impedance seen by the relay for a fault at middle of TL.

② calculate impedance seen by a relay for a fault at a middle of TL, with error in CT is $\pm 5\%$ and error in PT is $\pm 5\%$.

132 KV

$$PT \ y = \frac{132 \text{ KV}}{110 \text{ V}} = \frac{1^2}{110} = 1200$$

$$CT = x = \frac{1000}{5} \text{ A} = 200 \text{ A}$$

$$Z_{\text{seen}} = \frac{1000 \times 110}{5} \times 120$$

$$x = 200$$

$$y = \cancel{1200} \quad 1200$$

$$Z_{\text{TL}} = 120 \Omega$$

$$\text{Middle of T.L. } \cdot (Z_{\text{TL}})_{\text{middle}} = 60 \Omega = \frac{120 \Omega}{2}$$

$$Z_{\text{seen}} = \frac{x \times (Z_{\text{TL}})_{\text{middle}}}{y} = \frac{200}{1200} \times 60 = 10 \Omega$$



(Q) with error :

$$CT \text{ error} = \pm 5\%$$

$$PT \text{ error} = \pm 5\%$$

$$\text{Total error} = 10\%$$

$$z_{\text{seen}} = 10 \pm \frac{10 \times 0.1}{\sqrt{2}} \quad (?)$$

$$\boxed{z_{\text{seen}} = 9 \pm 1.1 \Omega}$$

~~10~~



20-1-23

- page (i) 52

Page No. _____
Date _____

1 mc

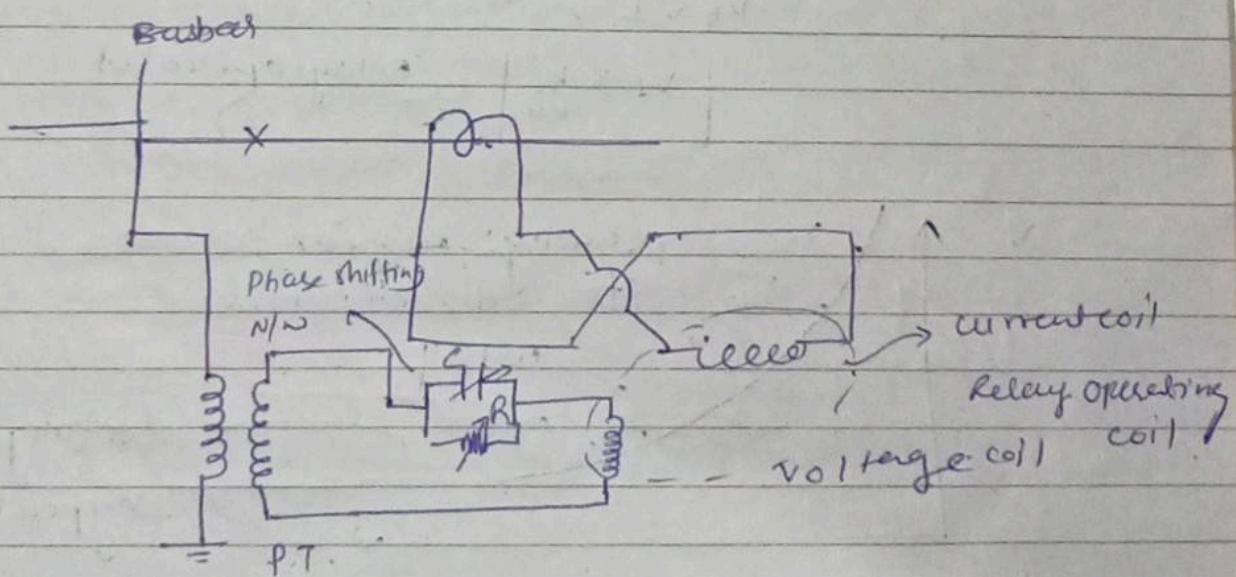
Directional relay:

- o whenever dirn of current has reversed, then also relay should operate. so we use directional relay.

$$T = K_1 I^2 + K_2 V^2 + K_3 VI \cos(\theta - \tau) + K_4$$

$$K_1 = K_2 = 0, K_3 = +ve, K_4 = -K$$

$$T = K_3 VI \cos(\theta - \tau) - K \downarrow \text{spring} \quad \text{Restraining torque.}$$



$$\text{Let } K_4 = K_3 \cos(\theta - \tau)$$

$$T = K_4 VI - K$$



Scan with Fast Scan

Under threshold condition,

$$R_o \cdot T = 0 \Rightarrow T_{op} = T_{res}$$

$$K_4 V I = K$$

$$VI = \frac{K}{K_4}$$

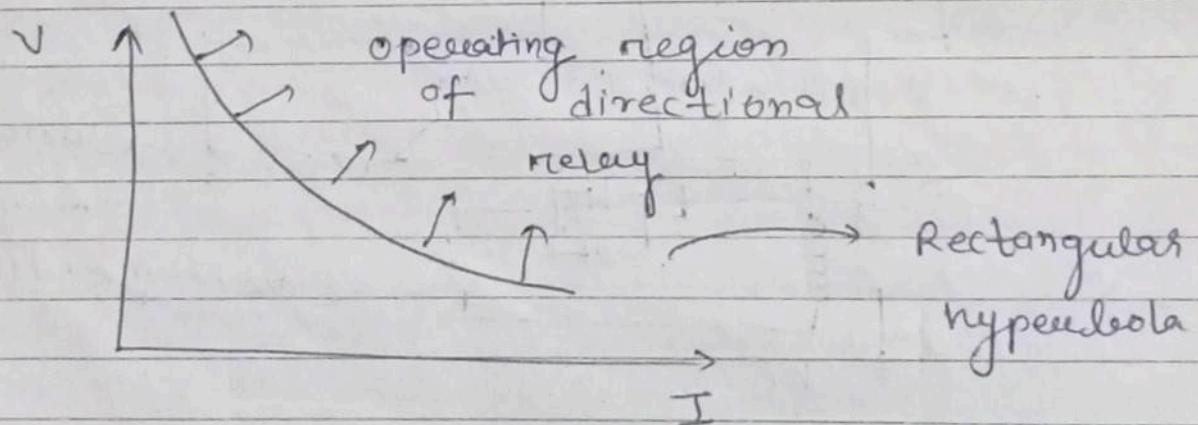
Relay will operate when

$$T_{op} > T_{res}$$

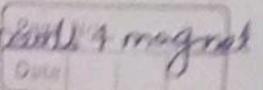
$$K_4 V I > K$$

$$VI > \frac{K}{K_4}$$

relay operates.



electromechanical relay;

→ moving  & magnet

Static relay:

→ No moving parts inside relay.

<1> electronics relays:

<2> Transductor (magnetic amplifier)

<3> rectifier bridge type  imp

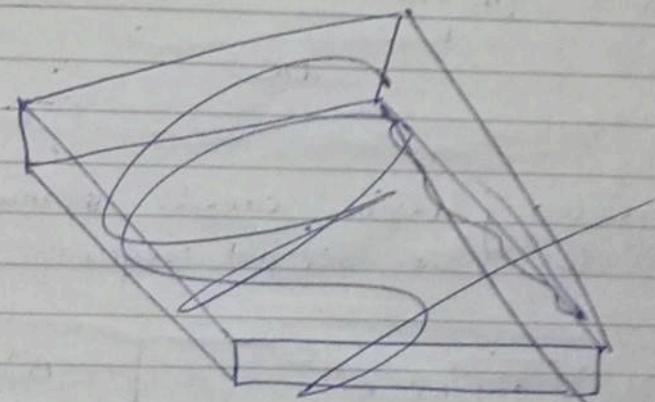
<4> Transistor relays

<5> Hall effect relays

<6> Gauss effect relays

 (imp)

Hall effect relays

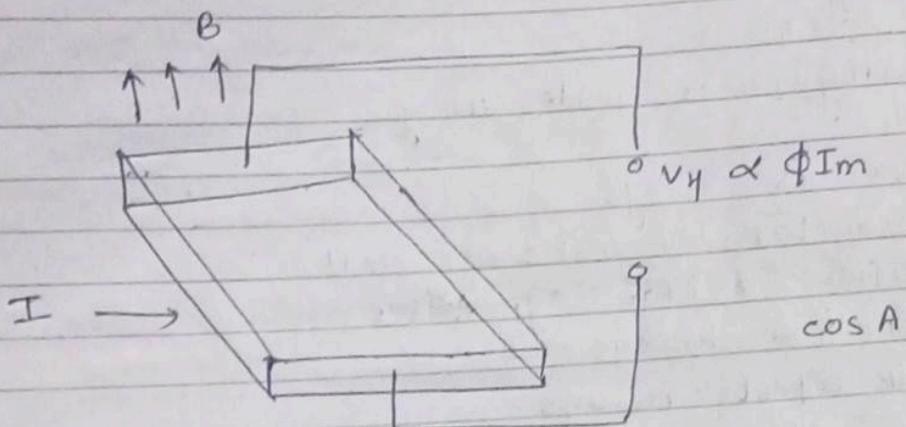


$$\phi = \phi_m \sin \omega t$$

$$i = I_m \sin(\omega t + \alpha)$$

$$\sin A \sin B = \frac{\cos(A-B) - \cos(A+B)}{2}$$

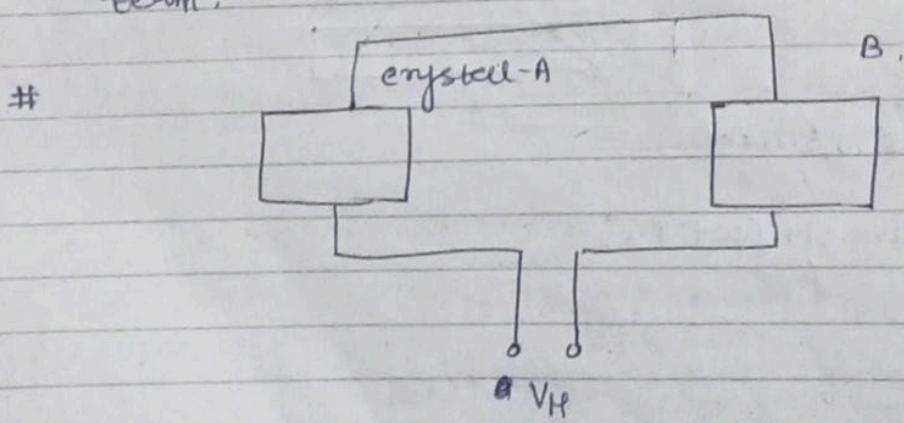
Page No. _____
Date _____



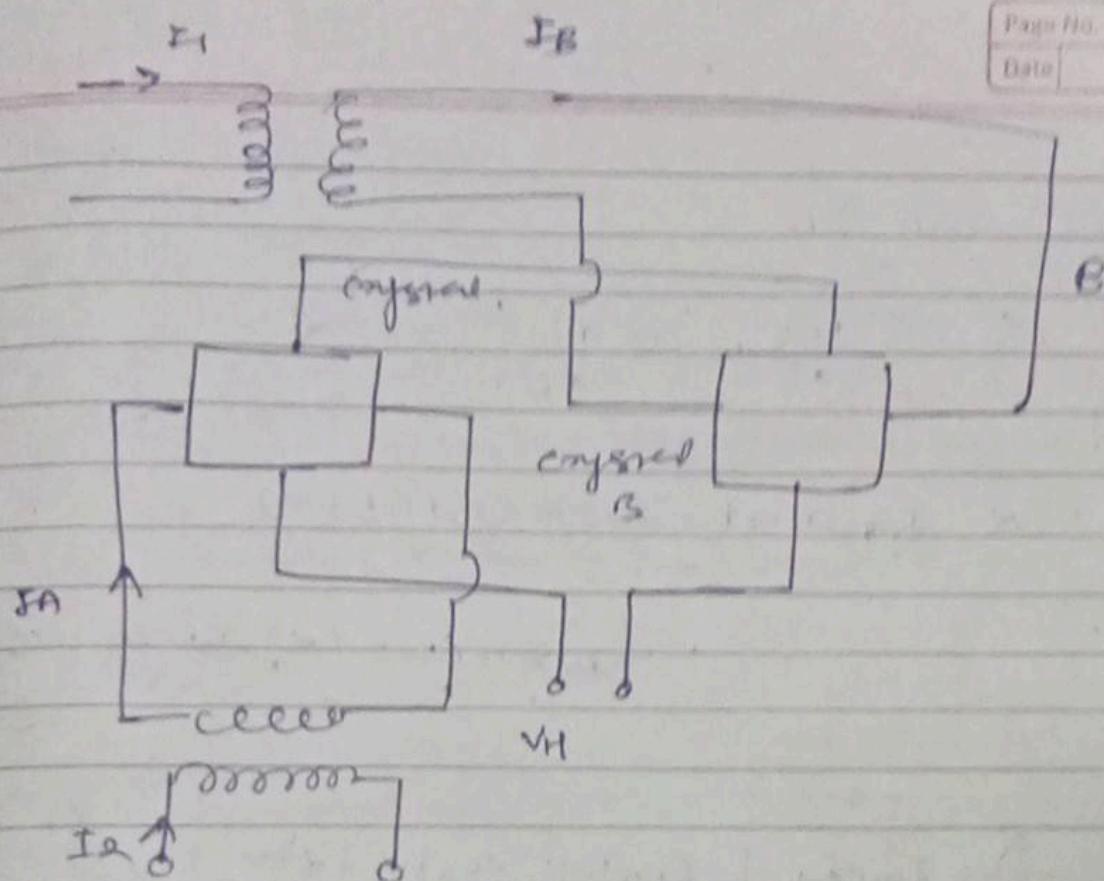
$$V_H \propto \phi_m \sin \omega t \cdot I_m \sin(\omega t + \alpha)$$

$$V_H \propto \phi_m \cdot I_m \cdot \underbrace{\cos \alpha}_{DC} \left[\cos \omega t - \cos(\omega t + \alpha) \right] \underbrace{\Delta \phi}_{AC}$$

→ double frequency term create maloperation in relay. so we doesn't require this ~~relay~~ term.



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Page No. _____
Date _____

doing cross connection to avoid double frequency term.

$$I_1 = I_m \sin(\omega t)$$

$$I_2 = I_m \sin(\omega t + \alpha)$$

flux through the crystal A & B,

$$\phi_A \propto I_1$$

$$\phi_B \propto I_2$$

and current through the crystal A & B.

$$I_A \propto \frac{dI_2}{dt}, I_B \propto \frac{dI_1}{dt}$$



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$$V_H \propto V_A - V_B$$

$$V_H \propto I_1 \frac{dI_2}{dt} - I_2 \frac{dI_1}{dt}$$

$$\propto I_m \sin \omega t - I_m \omega \cos(\omega t + \alpha)$$

$$= I_m \sin(\omega t + \alpha) \cdot \omega \cdot I_m \cos(\omega t)$$

$$\propto I_m^2 [\sin \omega t \cdot \cos(\omega t + \alpha) \\ - \sin(\omega t + \alpha) \cdot \cos \omega t]$$

$$\propto I_m^2 \sin(\omega t - (\omega t + \alpha))$$

$$\propto = I_m^2 \sin \alpha$$

$V_H \propto I_m^2 \sin \alpha$

→ USSR ने use करते हैं.

→ element is Indium Antimonite] rare crystal
Indium.

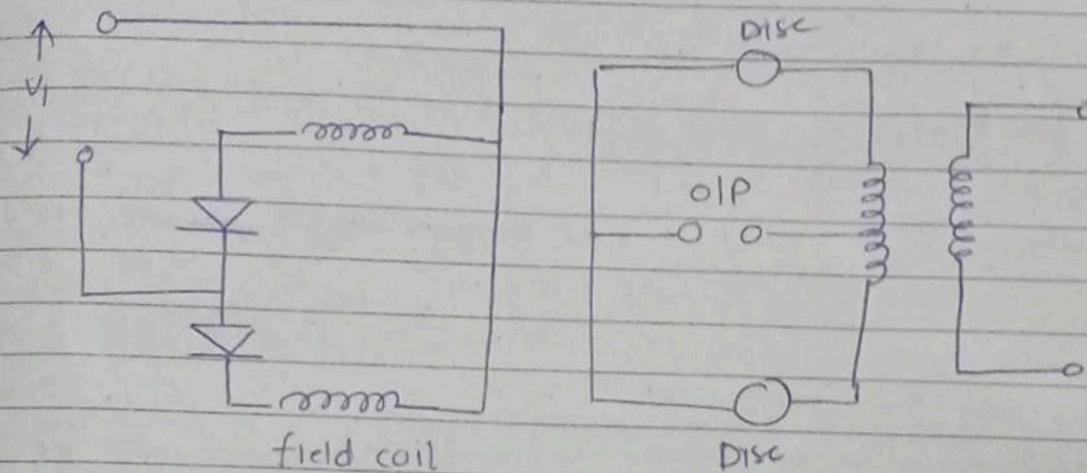
crystal used for
hole generation.



31-01-23

Page No. _____
Date _____

Gauss electric relay:



$$O.P \text{ voltage} \propto V_1 V_2 \cos \theta$$

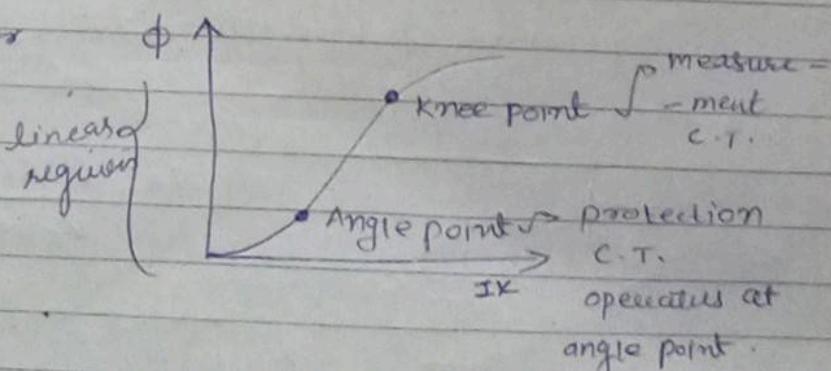
maximum voltage ($\theta = 0^\circ$)

minimum voltage ($\theta = 90^\circ$)

C.T/P.R

↳ Phase angle error

↳ Ratio error



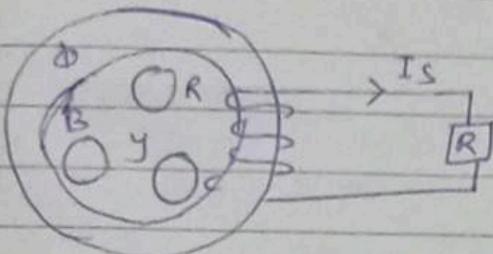
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* based on the insulation we decide voltage level of any line.

(CBCT)

Core Balanced CT:

→ leakage / earth current protection.



$$I_s \propto \phi$$

$$\phi \propto \phi_R, \phi \propto \phi_Y, \phi \propto \phi_B$$

$$\phi \propto (\phi_R + \phi_Y + \phi_B)$$

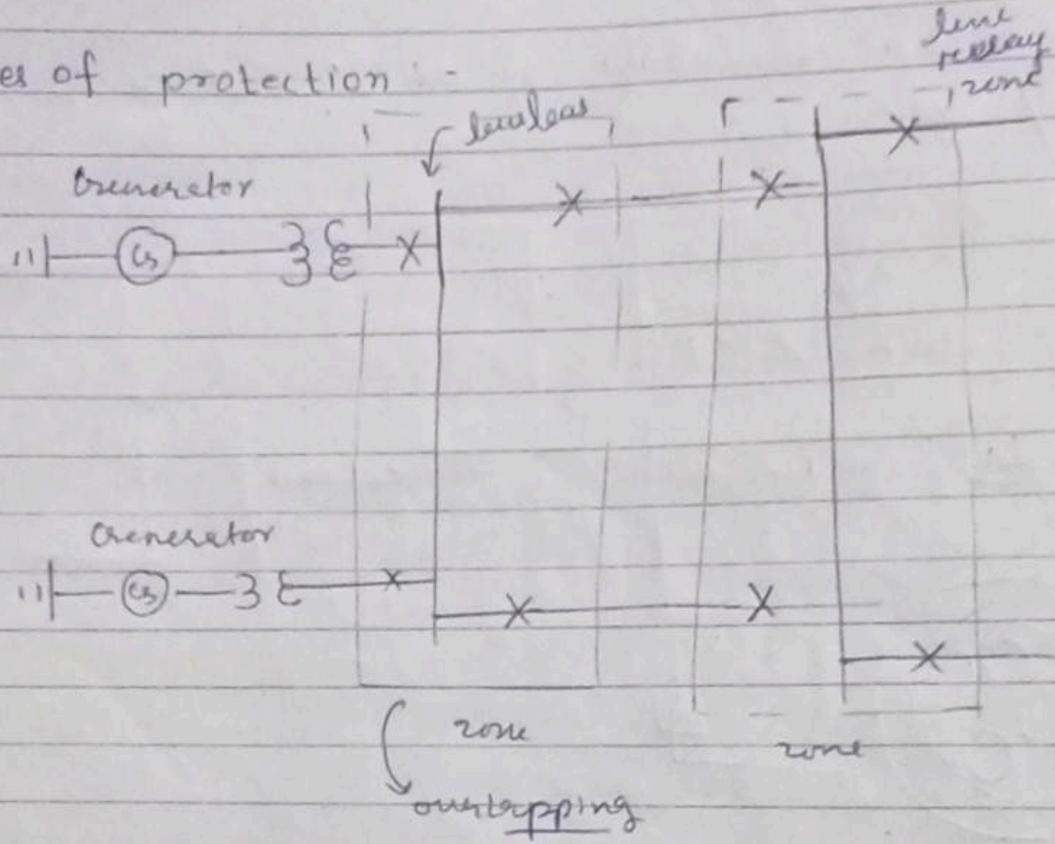
$$I_s \propto (\phi_R + \phi_Y + \phi_B)$$

$$I_n = \frac{1}{3} (I_R + I_Y + I_B) \quad (\text{neutral current?})$$

$$I_s \propto (I_R + I_Y + I_B)$$

$$I_s \propto 3I_o \quad \Rightarrow \text{core balance CT.}$$

zones of protection :-

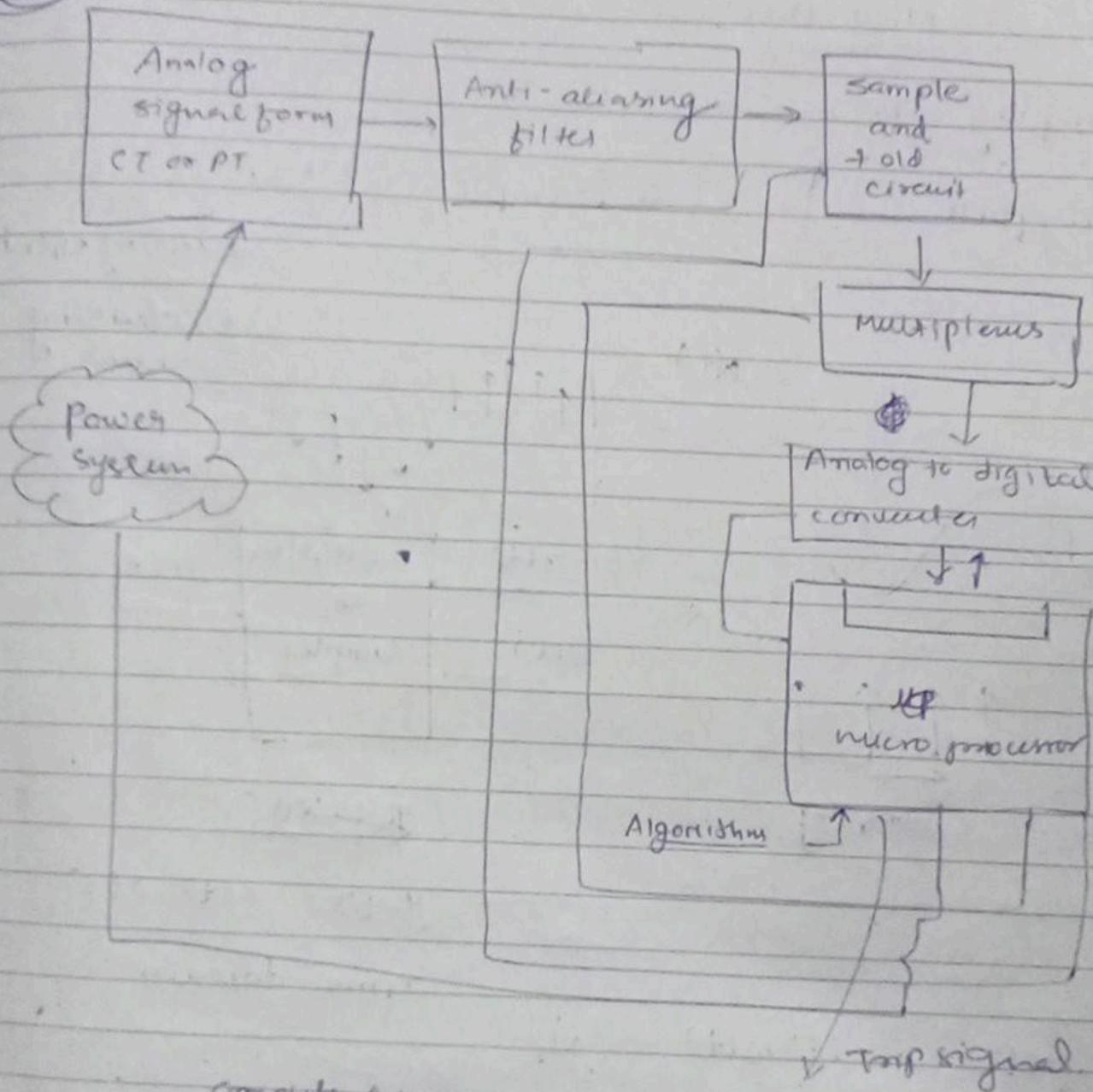


- * Protection means continuous monitoring of the system
- * overlap the system to avoid the island spot

Imp

Numerical Relay

Page No. _____
Date _____



complete block
diagram

→ watchdog timer ()
in microprocessor

Inside microprocessor, we
designed everything
overcurrent relay, underfreq
relay

→ Read



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~~dtf imp~~

sampling theorem:

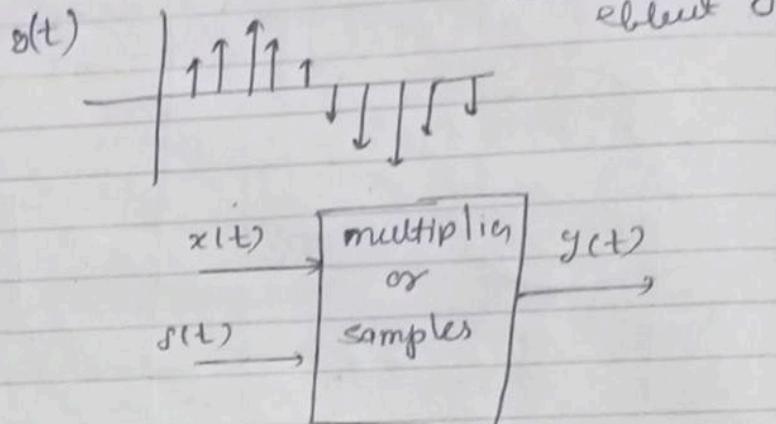
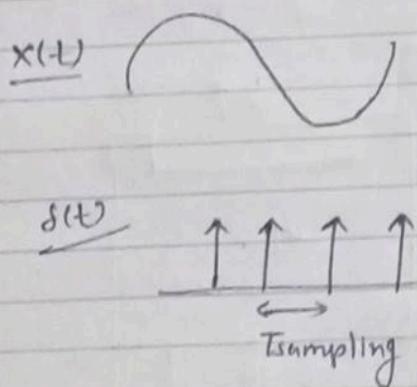
sampling theorem def:

Nyquist

$\infty f_{\text{sample}} = 2 \text{ mess}$

aliasing effect

anti aliasing effect



~~y(t) * delta~~

$$y(t) = x(t) \cdot \delta(t)$$

Time-domain.

$$\underline{g(t) = x(t) \cdot \delta(t)} \rightarrow (1)$$

$$\underline{s_0(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos n\omega t + b_n \sin n\omega t.} \rightarrow (2)$$

$$a_0 = \frac{1}{T_s} \left[\int_{-T_s/2}^{T_s/2} s_0(t) dt \right] \rightarrow (3)$$

$$t=0 \quad \downarrow \quad s_0(0) \quad \downarrow$$



$$a_0 = \frac{1}{T_s}$$

$$a_n = \frac{2}{T_s} \int_{-T/2}^{T/2} f(t) \cos n\omega t dt$$

$$a_n = \frac{2}{T_s} \int_{-T/2}^{T/2} f(0) \cos n\omega(0) dt$$

$$a_n = \frac{2}{T_s}$$

$$b_n = \frac{2}{T_s} \int_{-T/2}^{T/2} f(t) \sin n\omega t dt$$

$$b_n = 0$$

$$f(t) = \frac{1}{T_s} + \sum_{n=1}^{\infty} \frac{2}{T_s} \cos n\omega t + 0$$

$$f(t) = \frac{1}{T_s} (1 + \textcircled{2} \cos \omega t + \dots) \rightarrow \textcircled{3}$$

Substitute equation ③ in ①

$$y(t) = x(t) \frac{1}{T_s} (1 + 2\cos \omega t + 2\cos 2\omega t + 2\cos 3\omega t)$$

$\delta \rightarrow$ Sample Signal .

Page No.	
Date	

$$y(t) = \frac{1}{T_s} (x(t) + 2x(t)\cos\omega t + 2x(t)\cos 2\omega t + \dots)$$

$$x(t) \xleftarrow{\text{FT}} X(\omega)$$

sampling freq

$$e^{j\omega t} x(t) \longleftrightarrow X(\omega - \omega_s)$$

$$X(j\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt$$

$$e^{j\omega} X(j\omega) = \int_{-\infty}^{\infty} x(t) e^{-j(\omega - \omega_s)t} dt$$

$$e^{-j\omega t} x(t) \longleftrightarrow X(\omega + \omega_s)$$

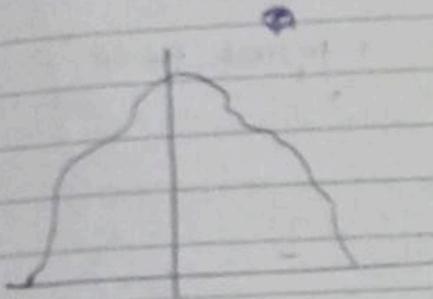
$$2\cos\omega t = 2 \cdot \left(\frac{e^{j\omega t} + e^{-j\omega t}}{2} \right)$$

$$y(\omega) = c(X(\omega)) + X(\omega - \omega_s) + X(\omega + \omega_s) \\ + X(\omega - 2\omega_s) + X(\omega + 2\omega_s) \\ + \dots$$

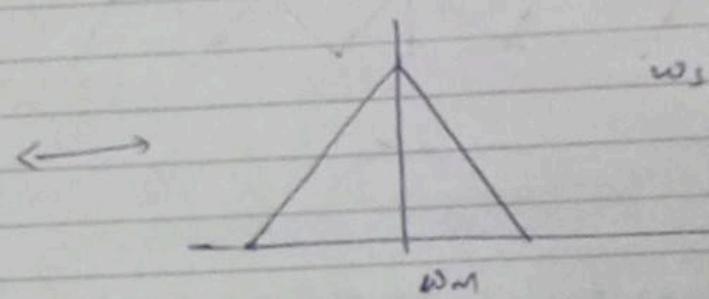


$$y(\omega) = \sum_{n=-\infty}^{\infty} x(\omega - n\omega_s)$$

Time-domain

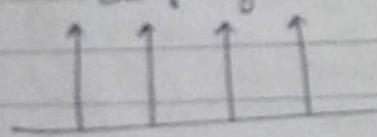


spectrum of
signal
frequency domain

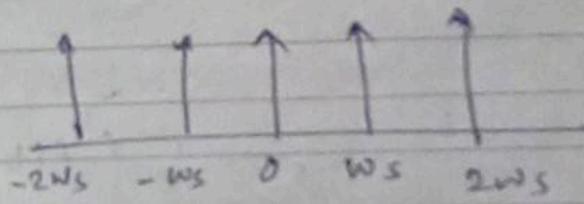


$$\omega_s > 2\omega_m$$

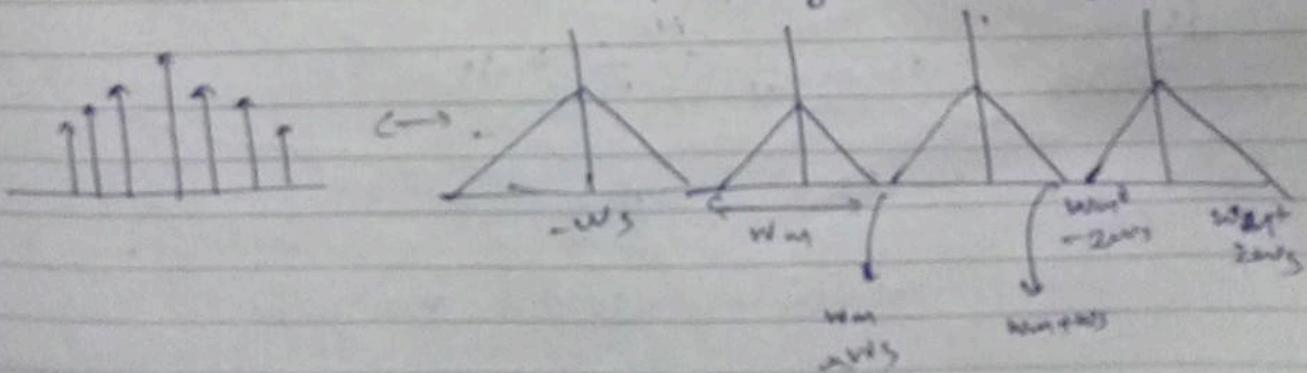
sampling



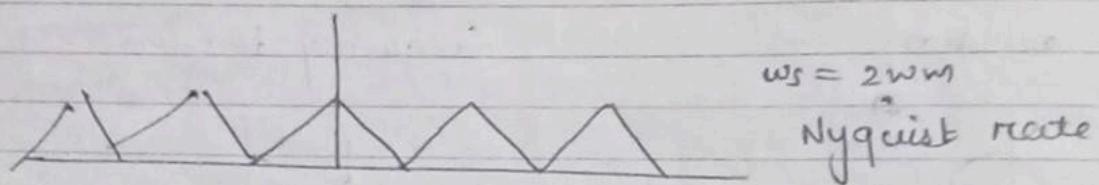
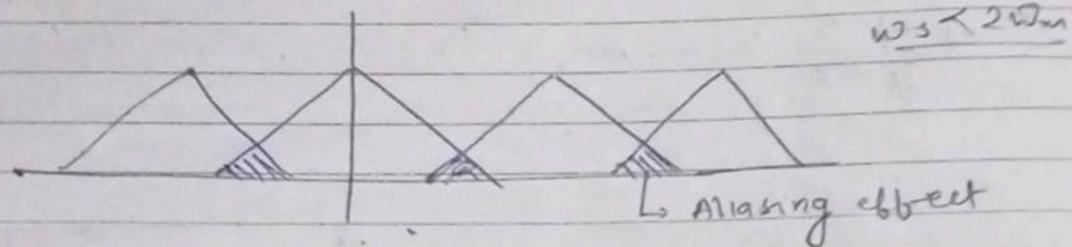
spectrum of sampled signal



spectrum of sampled signal $\omega_s \geq 2\omega_m$



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Fourier series - Numerical relay

$$f(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos n\omega t + b_n \sin n\omega t$$

$$a_0 = \frac{1}{T} \int_0^T f(t) dt \quad \text{--- (1)}$$

$$a_0 = \frac{1}{N} \sum_{k=1}^N f(k \cdot \Delta t)$$

$$\Delta t = T_{sampling}$$

N = no of sample per cycle

Fourier Transform
Discrete Fourier Transform

Page No. _____
Date _____

Discrete Fourier transform

$$a_0 = \frac{1}{N} \sum_{k=1}^N f(k \cdot \Delta t)$$

$$a_n = \frac{2}{N} \sum_{k=1}^N f(k \cdot \Delta t) \cdot \cos(n\omega_k \Delta t),$$

$$b_n = \frac{2}{T} \int_0^T f(t) \cdot \sin n\omega t dt.$$

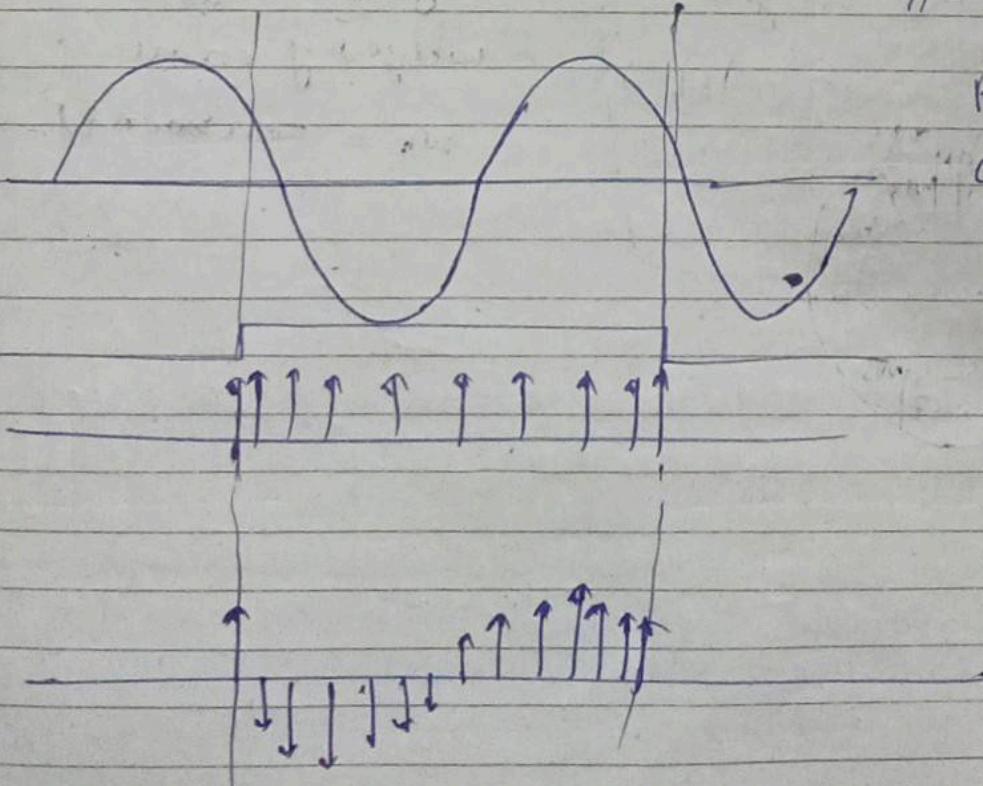
$$\Delta t = T_{\text{sampling}} = \frac{1}{f_{\text{sampling}}}$$

amplitude

$$f_i = \sqrt{a_i^2 + b_i^2}$$

phase angle

$$\theta = \tan^{-1} \left(\frac{b_i}{a_i} \right)$$

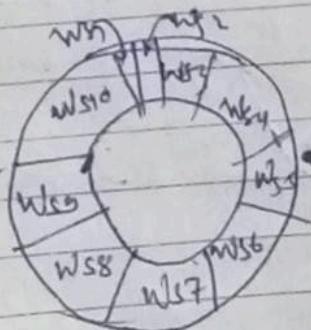


$f(10 \cdot \Delta t)$
 $f(9 \cdot \Delta t)$
 $f(8 \cdot \Delta t)$
 $f(6 \cdot \Delta t)$
 $f(5 \cdot \Delta t)$
 $f(4 \cdot \Delta t)$
 $f(3 \cdot \Delta t)$
 $f(2 \cdot \Delta t)$

pre computed
values stored
in memory

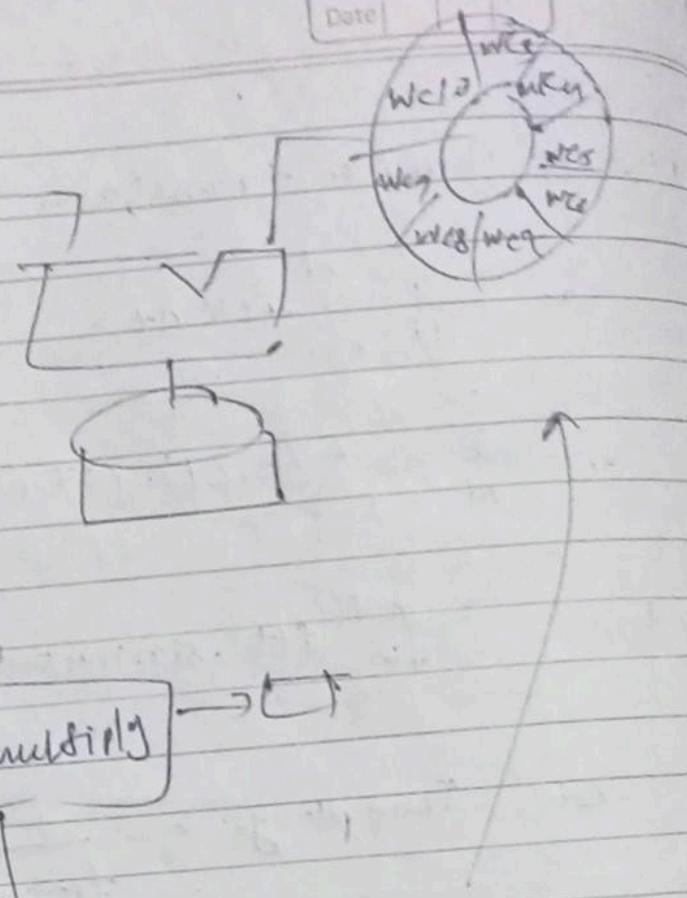
fundamental weight,

$$w_{sn} = \sin(n\pi x)$$



fundamental
weight of cosine

$$w_n = \cos(n\pi x)$$



Least error squared technique

$$\hat{i}(t) = K_1 e^{-t/\tau} + \sum_{n=1}^{\infty} K_n \sin(n\omega t + \phi_n)$$

↑ \curvearrowleft
 dc signal harmonics + fundamental

$$e^{-t/\tau} = \left(1 - \frac{t}{\tau} + \frac{t^2}{2\tau^2} - \frac{t^3}{3\tau^3} + \dots \right)$$

$$K_1 e^{-t/\tau} = K_1 - \frac{K_1 t}{\tau} + \frac{K_1 t^2}{2\tau^2}$$

$$\begin{aligned} \hat{i}(t) = & K_1 - \frac{K_1 t}{\tau} + \frac{t^2}{2\tau^2} + K_{21} \sin(\omega t + \phi_1) \\ & + K_{23} \sin(3\omega t + \phi_3) \end{aligned}$$

$$\begin{aligned} i(t) = & K_1 - \frac{K_1 t}{\tau} + \frac{t^2}{2\tau^2} + K_{21} \sin \omega t \cdot \cos \phi_1 + \\ & K_{21} \cdot \sin \phi_1 \cdot \cos \omega t \\ & + K_{23} \sin(3\omega t) \cos(\phi_3) \\ & + K_{23} (\cos(3\omega t) \sin \phi_3) + \dots \end{aligned}$$



Let us denote unknown by x and known by a

$$f(t) = a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + a_{14}x_4 + a_{15}x_5 \\ + a_{16}x_6 + a_{17}x_7 \\ = Ax + b$$

where unknown

known

$$x_1 = x_1$$

$$a_{11} = 1$$

$$x_2 = k_{21} \cos \theta_1$$

$$a_{12} = \sin \omega t$$

$$x_3 = k_{23} \cos \theta_2 \sin \theta_1$$

$$a_{13} = \cos \omega t$$

$$x_4 = k_{23} \cos \theta_3$$

$$a_{14} = \sin \omega t$$

$$x_5 = k_{23} \sin \theta_3$$

$$a_{15} = \cos \omega t$$

$$\alpha_6 = -k_1 / \tau$$

$$a_{16} = -t$$

$$x_7 = \frac{\omega}{2\tau^2}$$

$$a_{17} = t^2$$

$$i(t) = a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + a_{14}x_4 + a_{15}x_5 + \\ a_{16}x_6 + a_{17}x_7$$

$$i(t_2) = a_{21}x_1 + a_{22}x_2 + a_{23}x_3 + a_{24}x_4 + a_{25}x_5 + a_{26}x_6 \\ + a_{27}x_7$$





$$\dot{c}(t_7) = a_{71}x_1 + a_{72}x_2 + a_{73}x_3 + a_{74}x_4 + a_{75}x_5 + a_{76}x_6 + a_{77}x_7$$

$$\begin{bmatrix} \dot{c}(t_1) \\ \vdots \\ \dot{c}(t_7) \end{bmatrix} = \begin{bmatrix} a_{11} & \dots & a_{17} \\ \vdots & & \vdots \\ a_{71} & \dots & a_{77} \end{bmatrix} \begin{bmatrix} x_1 \\ \vdots \\ x_7 \end{bmatrix}$$

$$x_{7x1}(t) = A_{7x7}^{-1} I(t) 7x1,$$

$$F_1 = \sqrt{x_2^2 + x_3^2}$$

$$\phi_1 = \tan^{-1} \left(\frac{x_3}{x_2} \right)$$

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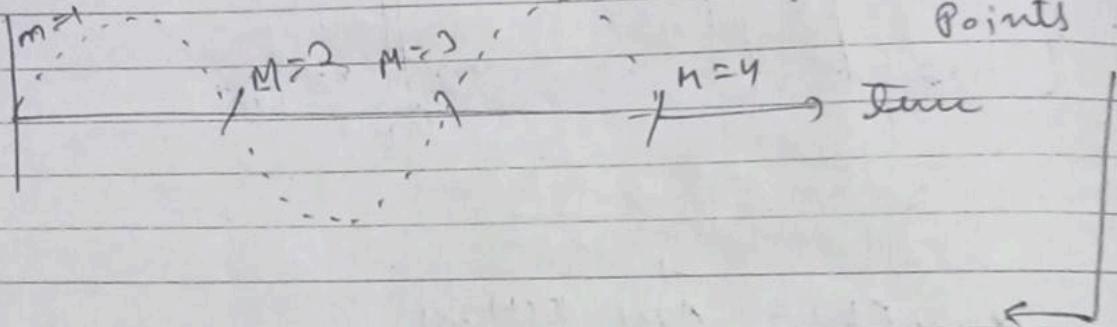
Page No. _____
Date _____

Frequency estimation:

1) zero crossing method.

2) Phasor based method.

$m = \text{No. of zero crossing points}$



$$f(tm) = \frac{m-1}{2} \cdot \frac{1}{(tm - t_1)}$$

t_1 : time period of first zero crossing

tm : time of m^{th} zero crossing

f : frequency estimate

* This method require no of prefilter. bandpass filters

low pass filter
postfilter?

↳ because harmonics also creates zero crossing point.

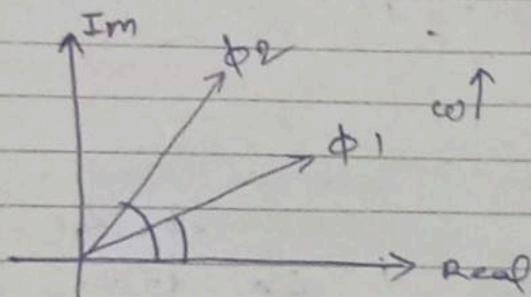


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* because of harmonics mutual capacity of motor decreases.

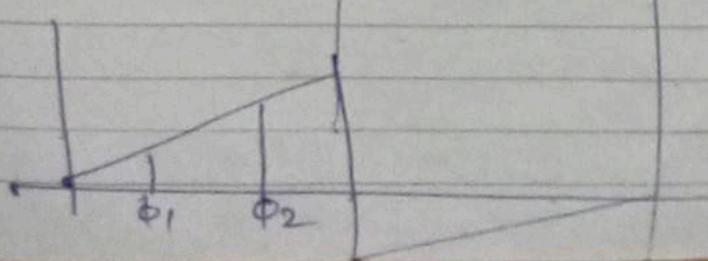
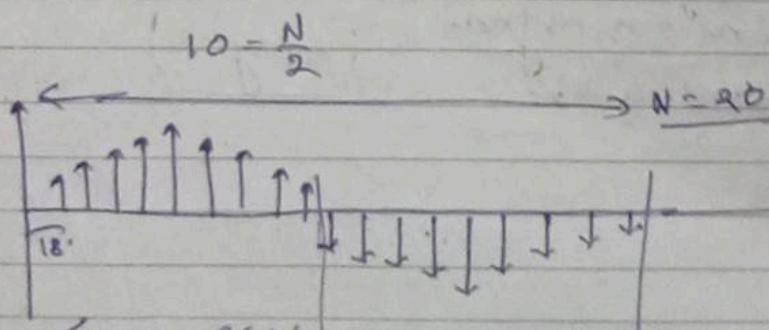
zero crossing method:

→ To integrate PV system with grid, we use this method.



$$2\pi f = \omega = \frac{d\phi}{dt}, \quad f = \frac{1}{2\pi} \frac{d\phi}{dt}$$

∴ f is rate of change of phasor value



$$\frac{360}{20} = 18^\circ$$



$$\underline{\phi} =$$

\downarrow angle \downarrow angle

$$\underline{\phi_2 - \phi_1} = 18^\circ$$

$$\underline{2\pi f K_2 T_s - 2\pi f K_1 T_s = 18^\circ}$$

K : integers no. of samples

T_s : sampling freq.

18°

$$t = \frac{1}{2n}$$

$$\begin{cases} \phi_2 - \phi_1 \\ \phi_2 + \phi_1 \end{cases}$$

$$f = \frac{18}{2\pi(K_2 - K_1)T_s}$$

$$f = \frac{1}{2n} \frac{\phi_2 - \phi_1}{t_2 - t_1}$$

- Asg:
- 1) phsor estimation by exponential form fourier series (before exam) that this be in discrete form
 - 2) half cycle DFT
 - 3) cosine transform
 - 3) Recursive DFT
- (spages)