# **Automatic Robot Taping with Force Feedback**

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Abstract—In surface treatment processes like plasma spraying and spray painting of workpieces, protecting the uninvolved surface by applying masking tape is necessary. Due to the operation complexity for different geometries, such taping tasks depend on a lot of manual works, which is tedious and tiring. This paper introduces an automatic agile robotic system and the corresponding algorithm to do the surface taping. The automatic taping system consists of a 3D scanner for workpiece 3D model reconstruction, a taping end-effector which is mounted on a robot manipulator to handle the taping task, and a rotating platform that is used to hold the workpiece. The surface covering method and the taping path planning algorithms using the scanned model are introduced. With the implementation of the compliance mechanism, the force feedback and the tape cutting mechanism, the system is able to handle the taping tasks for flat, cylindrical, freeform, and grooved surfaces. Experiments conducted on taping an engine inner liner shows that the surface can be covered with uniform taping overlap and only little wrinkle. The proposed system is a useful taping package for industrial applications such as workpiece repairing and surface protection, where surface treatments are involved.

#### I. INTRODUCTION

In industrial applications such as mechanical part repair, surface protection, and crack repair, surface treatments such as plasma-spraying, spray painting, and shot peening are critical to ensure the required characteristic of the mechanical workpieces. In such processes, only selected parts of the workpiece need to be treated, and the rest of the parts needs to be covered to protect the surface. To cope with this requirement, covering the workpiece surface using masking tapes is a general solution. Unlike simple geometries like tubes or flat surfaces which can be taped easily using a simple taping devices [1, 2], mechanical parts are usually quite complex in geometry, which is more difficult in the surface covering. Therefore, this taping process depends on a lot of manual work.

One example is in the aeroplane engine overhaul, where the damaged engine parts need to be repaired. The repair

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processes include plasma-spraying, where parts of the engine are coated at high deposition rate. In such application, covering only a single piece of engine part can cost a worker a few hours.

In order to make this process easier, the authors aim to come up with an automatic robotic taping solution.

Different from other manipulation tasks, taping has geometric constraints in the orientation of the masking tape. This constrain has to be satisfied if we want to prevent any wrinkle on the tapes, and to make sure that the tape is going towards the correct direction. In the manual taping, the worker observes the geometry of the part and uses his hand to control the orientation of the tape. At the same time, the worker uses his finger to press and push the tape to make sure that the tape is nicely attached to the surface of the part. In the overall process, the taping path should be properly planned in order to let the tape cover the entire area of interest.

Therefore, to automate this process using a robotic system, the following problems need to be solved.

- 1. To reconstruct the 3D model of the workpiece (precise CAD model is not available as the part may be deformed or broken).
- 2. To design the taping end-effector to apply the tape onto the surface, to cut the tape and to hold it.
- 3. To have force feedback solution to ensure proper tape attachment.
- 4. To come up with an efficient surface covering method and path planning method for the robot taping.
- 5. A platform to carry the workpiece and to collaborate with the taping robot arm.

In order to realize the robot taping, a robot motion planning strategy based on the 3D model of the part to be taped is crucial [3, 4]. Since for repair works the CAD model of the broken part is not directly available (because of deformation, cracks, losing of material etc.), we need to use the 3D scanning device to get the digital model.

Robot manipulations with 3D scanning model reconstructions have been investigated in surface treatment applications like spray painting [5-8] and laser coating removal [9]. These applications are scenarios that do not have any contact requirement and there is no restriction other than the surface geometry. In painting, the spray painting nozzle do not need to be in contact with the workpiece, but the evenness of the paint on the surface is the major concern [8, 9]. While for taping, to make an efficient tape attachment, the taping end-effector should orient the tape correctly, and then press the tape onto the surface with the required attachment force. It is tricky to ensure a tape segment attached nicely without any wrinkle. Therefore, a proper end-effector to realize the taping motion is important. Meanwhile, the efficient robot paths need to be generated based on the path planning method using the 3D scanning model. The force feedback is also critical to guarantee that the tape is attached onto the surface firmly.

Previous works [10, 11] [12, 13] have been done to tape regular shapes such as cylindrical shapes and freeform surfaces, and without any force feedback. In this paper, an improved taping end-effector with force feedback capability is added. The inner liner (a part of airplane engine) is used to demonstrate the capability of the system. Experimental results show that the automatic system can do faster taping, and the taping quality is good with more evenly distributed overlaps and less wrinkles.

The remaining parts of the paper are organized in the following manner. Section II introduces the taping system design. Section III describes the path planning method. Section IV introduces the robot execution. Section V discusses the further improvement required and concludes the paper.

### II. TAPING SYSTEM

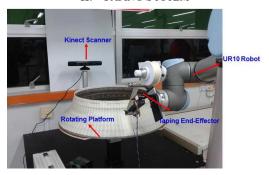


Figure 1: An automatic taping system

Figure 1 shows the robot taping system. The system includes a 3D scanner (Kinect sensor is used) for the 3D model reconstruction, a rotating platform, a taping robot, and a robot taping tool. The platform is used to mount the parts for taping and it can rotate to collaborate with the robot manipulator for the taping task. The taping end-effector is specially designed to meet the proper taping requirement.

#### A. Taping End-Effector Design

The taping end-effector is used to hold the tapes and to conduct the actual taping process. As shown in Figure 2, the "tape holder" is used to hold the masking tapes. The "tape guiding roller" is used to control the tape direction. The "tape roller" is used to press and roll the tape onto the surface, so that it is attached firmly.

Meanwhile for the force feedback, the "compliance spring" mechanism, together with a potentiometer based distance sensor is used. Based on the stiffness of the spring  $(K_s)$  and its deformation (dx) during the operation, the applying force along the normal spring direction can be calculated. In current cases, the compression direction (the main direction of the spring) is oriented to be collinear with the surface normal. Therefore, the pressing force is  $F = -K_s dx$ .

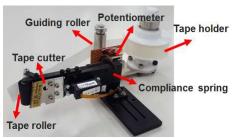


Figure 2: Taping tool design

In addition, a "tape cutter" which is driven by a linear motor is used to conduct the tape cutting action. The requirements for this tape cutter are firstly to be able to cut the tape easily. And secondly, to keep the tape in the location that is ready for the next taping step. A thin roller is located beside the tape roller. It can stop the tape from going away from the tape roller.

The overall taping process is illustrated below.

- 1. Reconstructing the 3D model of the workpiece.
- 2. Calibrate the relative pose between the robot and the platform, and the workpiece position.
- 3. Define the taping method and the parameters (speed, overlaps ratio and initial pose etc.)
- 4. Generate the taping paths for the taping area, and plan the robot path.
- 5. Robot execution with force feedback.

#### B. 3D Scanner

In this setup, the Kinect sensor is used (its random error in depth measurement is around a 2-3 mm at 0.5m range). For the taping application, the required manipulation accuracy is not as high as applications like polishing and machining. The model errors from the scanner can be compensated by the force feedback.

The scanners will provide the user with the 3D point cloud data, from where the x, y, z coordinates of the points can be calculated. The surface normal vectors can be calculated. Post-processing of the model, such as object segmentation to get the workpiece model, noise point filtering, and surface normal smoothing need to be done to generate the final digital model, which will be used in the subsequent path planning.

#### III. TAPING PATH PLANNING STRATEGY

When the skilled workers do the manual taping, they firstly plan the strategy to cover the area based on the geometry of the area. For example, they will decide whether to tape vertically, horizontally, or to wrap around the surface. After the strategy is decided, each tape segment to cover the surface is a point to point taping across the surface. In this case, the masking tape needs to be nicely attached to the surface with proper orientation to make sure that the tape goes through the path as expected. The authors' previous work talks about segmentation strategies and taping methods for different geometries [12, 13] in details. A brief introduction of the path planning for taping is described below for readers to better understand the contents.

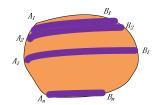


Figure 3: Area of interest for taping

## A. Surface area taping strategy

The general idea of the taping method is illustrated in Figure 3. The orange color is the area to be taped. The points  $A_1, A_2, ... A_n$  are the starting points,  $B_1, B_2, ... B_n$  are the corresponding ending points. So  $A_i, B_i$  forms a point pair to define a tape segment.

Firstly, for an area of interest as illustrated in Figure 3, we need to determine the main taping direction based on the geometry of the area. Program to check the adjacent tape distances are needed in order to check the minimal/ maximal overlapping. The points will need to be modified when adjacent tapes is not overlapping enough or overlaps too much. A general taping strategy for any complex geometry can be difficult. However, the good thing is that for one type of the workpiece, the taping strategy is always the same. Therefore, taping strategy database can be recorded for cope with different class of workpieces, making the taping point selection directly available.

#### B. Point-to-point taping

In the point-to-point taping (A to B as shown in Figure 4 for example), the tape path must follow the underlying surface in such a way that the tape does not rotate around an axis, denoted as Z, perpendicular to the taping plane. This restriction comes from the fact that the tape will start to wrinkle significantly if the tape is rotated around the perpendicular axis Z. When applying tape to a flat surface, it is trivial to guide the tape in a straight line without any wrinkle. However, when the surface is curved it becomes a difficult task to guide the tape in a wrinkle-free path.

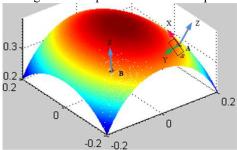


Figure 4: 3D surface model for taping

The concept of *geodesics curve* [14] can provide a valid path for taping as described in [15] as *geodesics* corresponds to a "straight line" on a freeform surface [16].

Computationally, a more efficient path generation method can be used based on the geometrics while taping. We start at a taping point p = (x, y, z), and the taping frame is defined according to the orientation of the tape. The Z axis is defined by the surface normal at p. The Y axis is along the taping direction. The X axis is then  $(Y \times Z)$ . The path planning is

numerically done based on the projection of the tape towards the Y direction and flip to the surface along the flipping axis defined by the surface geometry.

The taping element is defined to be a small trapezoid form by the edge of the tape and the adjacent flipping axis--the line around which the tape flip around, as shown in Figure 5. Starting from point A, the small tape element with very small length dl is attached on the surface.

For the small element, the taping starting point  $p_i$  is defined to be the middle point of the front edge of the minitape element as shown in Figure 5. The Z axis is defined to be along the normal vector direction.

$$z_i = n_i \tag{1}$$

Therefore, the next taping point  $p_{i+1}$  can be estimated as follow:

$$\hat{p}_{i+1} = p_i + y_i dl \tag{2}$$

Note that the taping point should lie on the surface. Therefore, a point  $\hat{p}_{i+1}$  is numerically projected to the surface to get the next taping point  $p_{i+1}$ .

Now the critical problem comes when determining the taping orientation of the small taping element. For taping on a flat surface, the X axis (and Y axis) of the tape do not change while continuously taping on the surface. However, for taping general surfaces, the tape elements need to be properly directed in order to satisfy the geometric constraint of the surface.

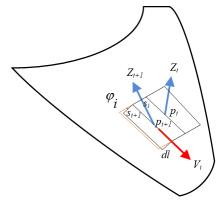


Figure 5: Flipping of the tape element while taping.

As shown in Figure 5, while the tape attaches to the surface, it is understandable that the tape actually flip to the surface along a certain screw axis which depends on the surface geometrics.

In the actual taping, as shown in Figure 5, the small taping element flip to the surface along a certain axis  $V_i$ . It is noted that this axis  $V_i$  is the intersection between the small taping element surface (flat)  $S_i$  and its next small taping element flat surface  $S_{i+1}$ . As a result this twist axis  $V_i$  is perpendicular to both  $z_i$  (the surface normal of  $S_i$ ) and  $z_{i+1}$  (the surface normal of  $S_{i+1}$ ).

Therefore, we have,

$$V_i = \frac{z_i \times z_{i+1}}{|z_i \times z_{i+1}|} \,. \tag{3}$$

Geometrically, the body frames of the two taping elements are identical (in orientation) when the tape are straight, the difference of them actually comes from the twist motion along this twist axis  $V_i$  for an angle  $\varphi_i$ . This angle is therefore the angle between  $z_i$  and  $z_{i+1}$ . Then, we have,

$$\varphi_i = \cos^{-1}(z_i \cdot z_{i+1}).$$
 (4)

Through this formulation, it is shown that the relationship between the orientation of the element frame of  $S_i$  (let's call this  $R_i$ ), and that of element frame of  $S_{i+1}$  (let's call this  $R_{i+1}$ ), are connected by the twisting motion,

$$R_{i+1} = e^{\varphi_i \hat{V}_i} R_i . \tag{5}$$

where  $\hat{V_i}$  is the skew-symmetric matrix of vector  $V_i$ . Its corresponding rotation matrix is calculated by the SO(3) matrix  $e^{\varphi_i\hat{V_i}}$ ,

$$e^{\varphi_i \hat{V}_i} = I_{3 \times 3} + \sin(\varphi_i) \hat{V}_i + (1 - \cos(\varphi_i)) \hat{V}_i^2$$
.

Details can be found in [17].

Therefore, the taping orientation will be

$$y_{i+1} = e^{\varphi_i \hat{V}_i} y_i . \tag{6}$$

In this way, the taping process can be conducted by following the geometry of the surface. Once we choose a particular initial orientation, the subsequent points along the surface can be determined by this method.

## **Initial taping orientation**

In the taping of a segment, we start from a starting point  $A_i$ , and we want to let the tape attach to the surface nicely without wrinkle and to end at  $B_i$ . To achieve this, the correct initial taping orientation needs to be determined. Details are provided in earlier work [12].

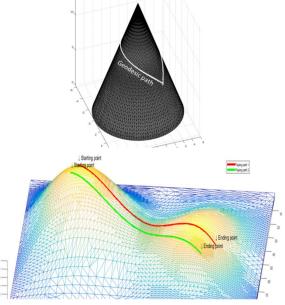


Figure 6: Point - to - point taping path examples. Upper: Ring taping path on a cone. Bottom: Point- to- point path on a freeform shape.

Examples of generating the point- to - point taping paths on cone surfaces and freeform surfaces are shown in Figure 6. It is noticed that the path generated from this algorithm actually corresponds to the geodesic path on the surface, when we compare the current path with the results generated from the

geodesic algorithm software toolbox. This proposed method is much faster in computation as compared with the geodesic path toolbox.

For a special case of the cylindrical shapes taping, the taping process can be conducted continuously by wrapping the tapes around the objects. Here the taping path will be only one continuous path. The initial taping angle can be calculated based on the tape overlapping requirement. Details can be found in [13], where path planning with the collision check is also introduced.

Based on this estimated initial orientation, the taping process is then conducted by following the method introduced in the last section.

### IV. SCANNING, PLANNING AND EXECUTION

The actual taping process consists of three main parts: Model scanning and post processing, Path planning, and Robot Execution. In this section, the taping of the inner liner workpiece is used as a demonstration example.

# A. 3D model reconstruction (scanning and post processing).

Kinect sensor can be used as a handheld device to scan an object. The 3D model of the object can be generated by holding the scanner and scan around the object. An easier way is that we can fix the scanner at a fixed location and fix the part on a rotating platform within the efficient scanning range. The scanning can be done while the rotating platform rotates for at least one cycle. The workpiece model is available after removing the background, filtering the noises and smoothen the surface normal vectors.

# B. Robot and platform motion generation.

In this inner liner taping, the workpiece is a rotationally symmetrical part. The taping strategy is to tape the surface patch by patch along the vertical section of the surface as shown in Figure 7. The markers shown in the figure are the taping paths for the robot to follow. After each taping patch, the rotating platform rotates some certain angle so that the next taping patch will be in the same relative position with respect to the robot as the previous taping patch. By this method, the robot motion is kept at minimum.

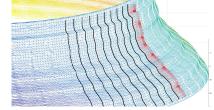


Figure 7: Taping path planning

After having the path for the patches, the instruction for cutting the tape after each patch is also added. With this, the execution of the taping task can be started. However, due to the inaccuracies from the object model, robot kinematics, and calibration error, the force feedback is needed to compensate for the error.

## C. Force feedback

, As can be seen in figure 8 (a) and 9 (a), the taping results could become unsuccessful with no force feedback is applied

due to some additional errors deliberately introduced to the executed paths. Therefore, the force feedback from the compliant spring is used to cope with this problem. Figure 8 (b) and 9 (b) shows the taping result with the force feedback.



Figure 8: Box vertical taping. (a) No feedback. (b) Force feedback



Figure 9: Inner linear taping. (a) No feedback. (b) Force feedback

Let  $d_n$  denotes the distance reading of the distance sensor. This distance corresponds to the current length of the spring on the end effector. If the force is exerted to the taping tool on its normal direction, this distance will decrease, and we can calculate the estimated force by considering the stiffness of the spring. The force feedback algorithm is simply designed to maintain the force within a defined force range, as illustrated in the following block.

```
Loop:

Move Robot to Pose

Measure F

If F_{min} < F < F_{max}
Go to Next

If F < F_{min}
d_n = d_n + 0.05(mm)
Else if F > F_{max}
d_n = d_n - 0.05(mm)
Pose = pose + d_n \times \mathbf{n}

Go to Loop

Next:
Pose = Next Pose
Go to Loop
```

Each distance adjustment loop only cost 20ms for UR 10 robot. Therefore, the control system responds efficiently without lagging.

# D. Execution of the taping process

The example of taping the inner liner surface is shown in Figure 10. The sample is fixed on the rotating platform. The tape attachment is conducted patch by patch. For each patch taping, the robot moves the end-effector to the desired initial attaching location, and applies the tape onto the surface. Then, the robot moves following the planned taping path with force feedback adjustment. After one patch is finished, the cutter is activated to cut the tape. Then, the rotating platform rotates for the desired angle to prepare for the next taping patch.

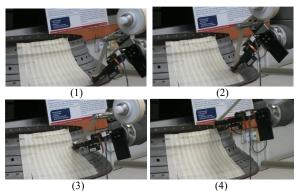


Figure 10: Taping of inner linear surface

The parameters for the workpiece taping are shown in Table I.

Table I: Taping parameters

Items	Quantity
Overlapping of tape	10%
Total tape length	19 meter
Number of segments	101
Total area	3500(cm <sup>2</sup> )
Total time	11 mins
Taping speed:	1.72m/min

More than ten trails have been tested, and results shown that the repeatability of the taping is good - no obvious wrinkle, even tape overlap and no failure in the attachment. This is important to make sure the system is stable in doing the surface covering tasks. An example of the completed taping result is shown in Figure 11.

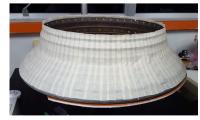


Figure 11: Taping results for the inner linear surface

In order to be aware of the key technique in manual taping and to better understand the process and to make improvement to the robotic taping solution, we recruited four volunteers (unskilled subjects are tested in current study) to do a manual taping. The task is defined to be: Taping the inner linear for 10 patches with proper overlapping as fast as possible while maintaining the good taping quality. Selected taping results of the four subjects and the robot are shown in Figure 12.

While doing the manual taping, volunteer subjects feel difficult in taping, uneasy to control the overlap size, and it is difficult to avoid wrinkle on curvatures. But works are much more dexterous in dealing with complex curvatures. Current robot taping solution can handle the general surface covering for the majority of the mechanical parts in the engine repairing. For tiny features like thin grooves and corners for example, the current taping tool cannot handle directly due to the size of the taping rollers and surface geometry constraints.

In such scenarios, human is more flexible for taping such tiny features. An efficient collaborative solution would be to let the robot do the main parts and live the minor difficult features for human. Otherwise, additional taping accessary tools will be needed.

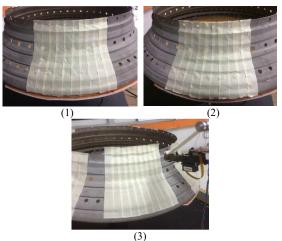


Figure 12: Taping comparison. (1) Result of subject No.1. (2) Result of subject No.2. (3) Result of the robot.

## V. CONCLUSION AND DISCUSSION

This paper introduces an automatic robot taping system and the corresponding methods to do surface covering using masking tapes. A new specially designed taping end-effector is applied to handle the taping task. The motions of the robotic system are planned based on the geometric information of the reconstructed workpiece model using a 3D scanner. In the robot execution, the force feedback is added to ensure the efficient contact force for the tape attachment.

Experiment of taping an engine part is conducted to show the function of the system and methods. Results show that the system can complete the taping task nicely with good repeatability and taping quality. The current average taping speed is 1.6m/min.

Such taping processes are common in industrial applications such as part joining, surface covering before coating of mechanical part for repair, crack repair, surface protection, etc. Such taping solutions is a useful taping package in such applications.

For different applications, the task conditions and requirements are different. The proposed solution is intended for general taping cases. However, due to the restriction of the taping tool size, taping tiny features may not be applicable because of the collision and the reachability of the tool. This will require smaller taping tools to handle the task, and at the same time, the scanner to model the workpiece will also need to be more accurate in order to be able to detect the tiny features. Since the number of workpiece types is finite in an industrial application, it is reasonable to design the suitable taping tools with proper size for the taping tasks. Meanwhile, accessories may be needed to deal with special features like corners, narrow grooves etc. Eventually, the taping methods and the processing steps for each workpiece type can be predefined so that once a part comes in for treatment, the

robotic system can finish the entire process using the correct tools and following the correct working flows.

#### ACKNOWLEDGMENT

This work is supported in part by the "Start-up research Grant for Young Faculty in Foshan University", and the Agency for Science, Technology and Research, Singapore, under SERC Grant 12251 00005.

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