Om Sai Ram

Jacobians & Hessians for the New functions:

1. Matyas Function

$$f(x) = 0.26(x_1^2 + x_2^2) - 0.48\, x_1 x_2 .$$

Jacobian (Gradient):

$$\nabla f = \begin{bmatrix} 0.52\,x_1 - 0.48\,x_2 \\ 0.52\,x_2 - 0.48\,x_1 \end{bmatrix}$$

Hessian:

$$H = \begin{bmatrix} 0.52 & -0.48 \\ -0.48 & 0.52 \end{bmatrix}$$

Minima : $\nabla f = 0$

$$0.52x_1 - 0.48x_2 = 0$$
$$0.52x_2 - 0.48x_1 = 0$$

Solving above equations gives $x_1 = x_2 = 0$.

Eigen values of H are 0.04 & 1 (both positive $\Rightarrow$ strictly convex)

Thus (0,0) is the global minima.

2. Rotated Hyper-Ellipsoid Function

$$f(x) = \sum_{i=1}^{d} \sum_{j=1}^{i} x_j^2$$

where $x = [x_1, \dots x_n]^T \in R^n$.

Gradient:

$$\nabla f = \frac{\partial f}{\partial x_k} = 2(n-k+1)\, x_k = \begin{array}{c} \begin{bmatrix} 2n x_1 \\ 2(n-1)x_2 \\ \vdots \\ 2x_n \end{bmatrix} \end{array} \begin{array}{l} \leftarrow k=1 \\ \vdots \\ \\ \leftarrow k=n \end{array}$$

Hessian:

$$\frac{\partial^2 f}{\partial x_k^2} = 2(n-k+1) = \begin{bmatrix} 2n & 0 & \dots & 0 \\ 0 & 2(n-1)x_2 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & 2 \end{bmatrix}$$

Critical Point (Global Minima)

$$\nabla f = 0$$

$$2n x_1 = 0 \Rightarrow x_1 = 0$$

$$2(n-1) x_2 = 0 \rightarrow x_2 = 0$$

$$\vdots \qquad \vdots$$

$$2 x_n = 0 \Rightarrow x_n = 0.$$

Thus, critical point is $x^* = 0$.

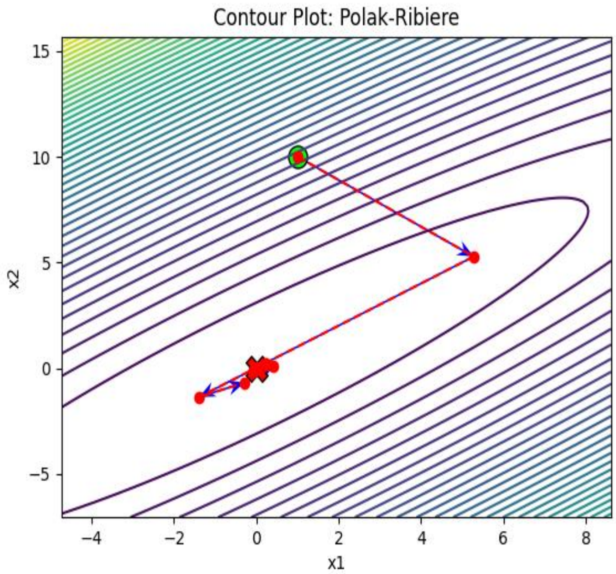
The Hessian is diagonal & positive definite, since all diagonal entries $2n, 2(n-1), \dots, 2$, are positive.

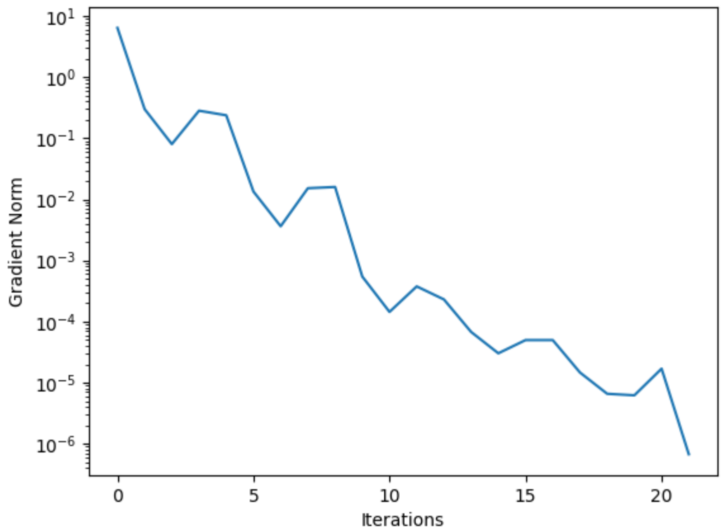
Therefore $x^* = 0$ is global minimum & $f(0) = 0$.

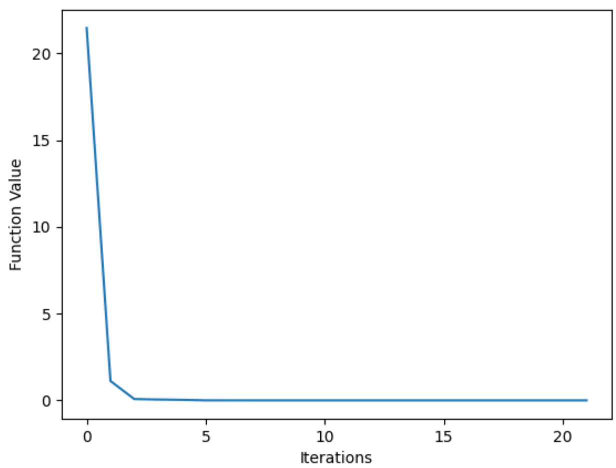
Algorithm convergence Analysis :-

1. Conjugate Gradient Methods.
   - Work well for quadratic function like Matayas & Hyper Ellipsoid
   - Struggle with non-quadratic functions like Rosenbrock and styblinski-Tang
   - Fletcher-Reeves tends to be more stable than Polak-Ribiere & Henstenes-Stiefel.
2. Quasi-Newton Methods:
   - BFGS performs best across all functions
   - ~~SR~~ DFP is less robust than BFGS but better than SR1
   - SR1 fails when denominator in the update becomes too small.
3. Specific failure:
   - SR1 fails on Rosenbrock function with poor initial points due to its positive definiteness
   - Conjugate gradient methods may converge slowly on styblinski-Tang function.
   - All methods converge for Matayas & Hyper-Ellipsoid function due to their convexity.

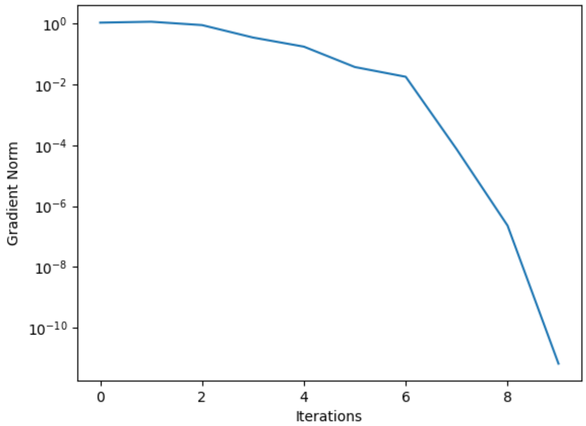
matyas_function_[1. 10.]_Polak-Ribiere

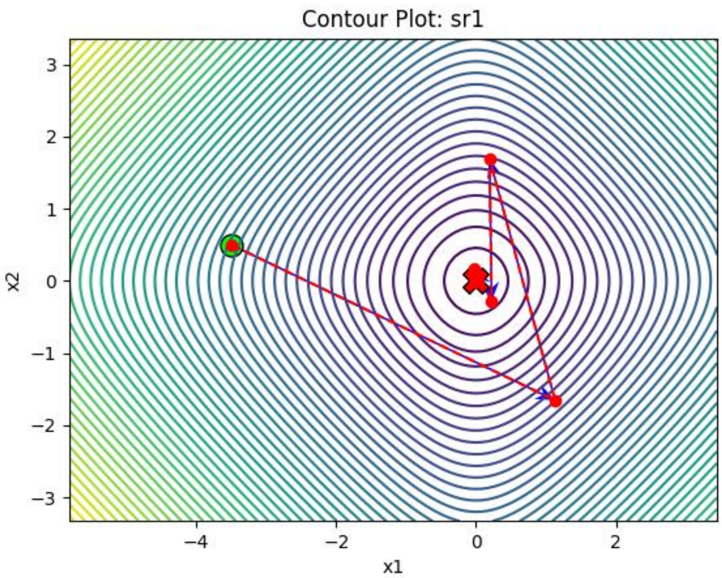


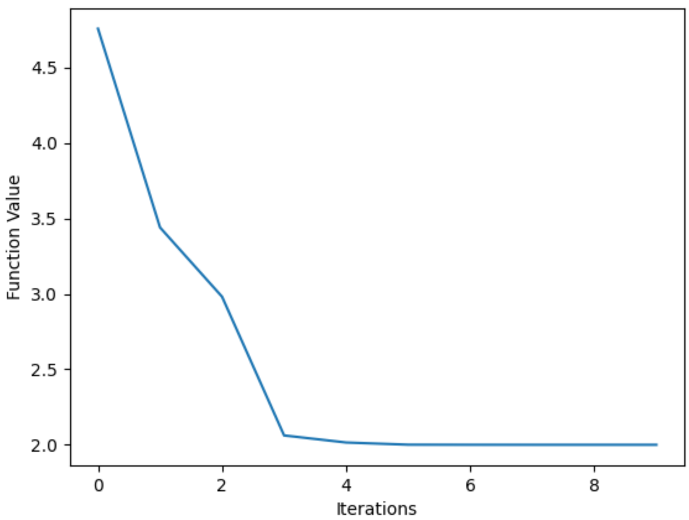




func_1_[-3.5 0.5]_sr1







three_hump_camel_function_[-2. 1.]_Fletcher-Reeves