matrices

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Problem Statement - Construct a triangle from the B using the formula tangents and a point on the circle.

(5)

 $\mathbf{D} = \mathbf{B} + 2 \frac{c - \mathbf{n}^T \mathbf{B}}{\|\mathbf{n}\|^2}$

Solution

Consider a point **A** from which the tangents are drawn to the circle O with center \mathbf{O} and radius 5cm. The length of the tangents are 12cm.

The tangents direction vectors are given by

$$\mathbf{m} \top \sum \mathbf{m} = 0 \tag{1}$$

where,

$$\sum = (\mathbf{V}\mathbf{A} + \mathbf{u})(\mathbf{V}\mathbf{A} + \mathbf{u})^T - (\mathbf{A}^T\mathbf{V}\mathbf{A} + 2\mathbf{u}^T + f)\mathbf{V}$$
 (2)

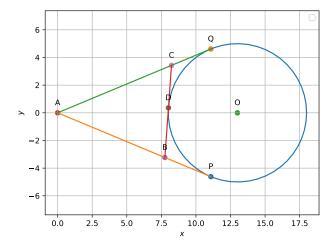
The normal vectors of the tangents can be derived from the eigen values and eigen vectors of the \sum matrix.

$$n_i = \Gamma \begin{pmatrix} \sqrt{|\lambda_1|} \\ \pm \sqrt{|\lambda_2|} \end{pmatrix} \tag{3}$$

where Γ is the eigen vector matrix and λ_i are the eigen values of the \sum matrix. If **V** is invertible, given the normal vector n, the tangent points of contact are given by

$$\mathbf{q}_{i} = \mathbf{V}^{-1} \left(\kappa_{i} \mathbf{n} - \mathbf{u} \right), i = 1, 2$$
where $\kappa_{i} = \pm \sqrt{\frac{f_{0}}{\mathbf{n}^{\top} \mathbf{V}^{-1} \mathbf{n}}}$
(4)

Let B is any point on the tangent AP.find the reflection of



where

$$\mathbf{m} = \mathbf{B} - \mathbf{O} \tag{6}$$

$$\mathbf{m}^T \mathbf{n} = 0 \tag{7}$$

$$c = \mathbf{n}^T \mathbf{O} \tag{8}$$

Find the equations of the lines AQ and BD and use them to find the intersection \mathbf{C} .