matrices

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***Problem Statement* - Construct a trianlge from the tangents and a point on the circle .**

Solution

Consider a point **A** from which the tangents are drawn to the circle O with center **O** and radius $5cm$. The length of the tangents are $12cm$.
The tangents direction vectors are given by

$$\mathbf{m}\top\sum\mathbf{m}=0 \tag{1}$$

where,

$$\sum=(\mathbf{VA}+\mathbf{u})(\mathbf{VA}+\mathbf{u})^T-(\mathbf{A}^T\mathbf{VA}+2\mathbf{u^T}+f)\mathbf{V} \tag{2}$$

The normal vectors of the tangents can be derived from the eigen values and eigen vectors of the $\sum$ matrix.

$$n_i=\Gamma\begin{pmatrix}\sqrt{|\lambda_1|}\\ \pm\sqrt{|\lambda_2|}\end{pmatrix} \tag{3}$$

where Γ is the eigen vector matrix and λ_i are the eigen values of the $\sum$ matrix. If $\mathbf{V}$ is invertible, given the normal vector $\mathbf{n}$, the tangent points of contact are given by

$$\begin{aligned}\mathbf{q}_i&=\mathbf{V}^{-1}\left(\kappa_i\mathbf{n}-\mathbf{u}\right),i=1,2\\ \text{where }\kappa_i&=\pm\sqrt{\frac{f_0}{\mathbf{n}^\top\mathbf{V}^{-1}\mathbf{n}}}\end{aligned} \tag{4}$$

Let **B** is any point on the tangent **AP**.find the reflection of **B** using the formula

$$\mathbf{D}=\mathbf{B}+2\frac{c-\mathbf{n}^T\mathbf{B}}{\|\mathbf{n}\|^2} \tag{5}$$

where

$$\mathbf{m}=\mathbf{B}-\mathbf{O} \tag{6}$$

$$\mathbf{m}^T\mathbf{n}=0 \tag{7}$$

$$c=\mathbf{n}^T\mathbf{O} \tag{8}$$

Find the equations of the lines AQ and BD and use them to find the intersection **C**.