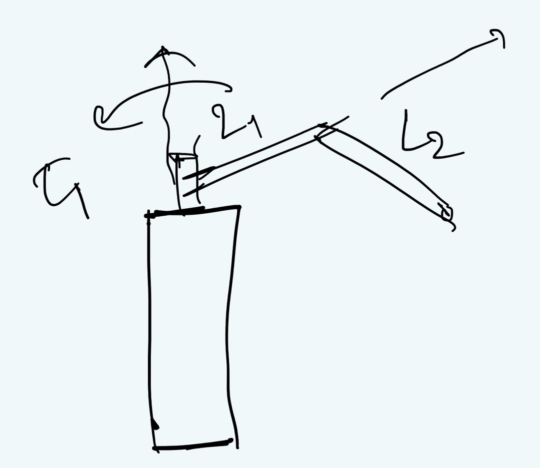
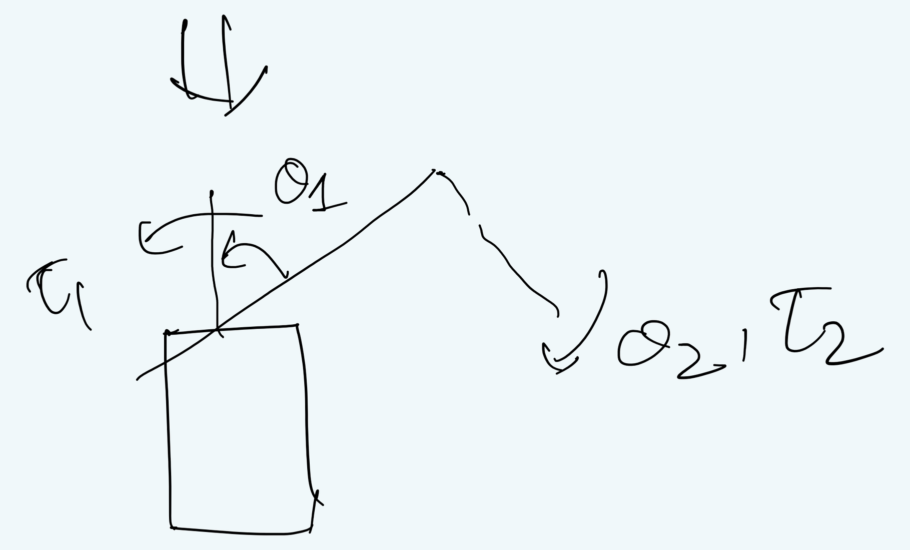
Tuesday, 30 May 2023 9:02 PM

FURUTA PENDULUM

↳ Similar to forward dynamics for Manipulators.

→ 

Assuming the Arm1, Arm2 Inertia tensors are along the principal axis



J_1, J_2 – inertia tensors of Arm 1,2 ↳ Along principal axes.

→ Rotation Matrices for Arm 1,2 are defined as,

$$R_1 = \begin{bmatrix} \cos\theta_1 & \sin\theta_1 & 0 \\ -\sin\theta_1 & \cos\theta_1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R_2 = R_1 \times \begin{bmatrix} 0 & 0 & -1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

$$R_2 = \begin{bmatrix} 0 & \sin\theta_2 & -\cos\theta_2 \\ 0 & \cos\theta_2 & \sin\theta_2 \\ 1 & 0 & 0 \end{bmatrix}$$

Deriving the dynamics of system by Newton-Lagrangian method.

→ Define Lagrangian –

$$L = E_k - E_p \quad [\text{Kinetic, potential energies}]$$

Evaluating E_k, E_p from the classical dynamics ↳ (frame of references---)

$$E_k = E_{k_1} + E_{k_2}$$
$$E_p = E_{p_1} + E_{p_2}$$

From Euler lagrange Equation –

$$\boxed{\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) + b_i \dot{q}_i - \frac{\partial L}{\partial q_i} = Q_i} \quad \text{—— (1)}$$

* $q_i = [\theta_1 \ \ \theta_2]^T$,

$b_i = \underbrace{[b_1 \ \ b_2]^T}_{\text{Considering viscous forces}}$, $Q_i = \underbrace{[\tau_1 \ \ \tau_2]^T}_{\text{Torques generated by motors.}}$

Term-wise evaluation of equation (1) yields the equations of motion of the system. As mentioned in the reference. (Linear and Angular quantities are computed)

$$\ddot{\theta}_1 = \begin{pmatrix} -\hat{J}_2 b_1 \\ m_2 L_1 l_1 \cos\theta_2 b_2 \\ -\hat{J}_2^2 \sin 2\theta_2 \\ -0.5\, \hat{J}_2 m_2 L_1 l_2 \cos\theta_2 \sin 2\theta_2 \\ \hat{J}_2 m_2 L_1 l_2 \sin\theta_2 \end{pmatrix}^T \begin{pmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \dot{\theta}_1 \dot{\theta}_2 \\ \dot{\theta}_1^2 \\ \dot{\theta}_2^2 \end{pmatrix} + \frac{\begin{pmatrix} \hat{J}_2 \\ -m_2 L_1 l_1 \cos\theta_2 \\ 0.5\, m_2^2 l_2^2 L_1 \sin 2\theta_2 \end{pmatrix}^T \begin{pmatrix} \tau_1 \\ \tau_2 \\ g \end{pmatrix}}{\hat{J}_0 \hat{J}_2 + \hat{J}_2^2 \sin^2\theta_2 - m_2^2 L_1^2 l_2^2 \cos^2\theta_2}$$

"Similarly for $\ddot{\theta}_2$"