Zoned Pooling: A Novel Alternative Ride Share Model

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***Abstract*— We present an alternative model for ride share services termed the Zoned Pooling model, which optimizes fairness and overall efficiency by exploiting spacial locality. A Mutli-Agent simulation is constructed and modeled using some basic assumptions about passenger frequency and vehicle behavior to illustrate reasonable operation of the model, while at the same time highlighting its potential in actual systems. An analysis of some live analytics of the simulation is then conducted and presented following the simulation, from which conclusions regarding its application to actual implementation in real world scenarios are drawn.**

I. INTRODUCTION

Ride share and carpool modeling have emerged as a considerable area of research interest within the past few years as a result of explosive interest in commercial ride sharing companies; however, the principle issues of optimal groupings and routings are at their core mathematical in nature. For the grouping problem, the goal is to assign a number of riders to vehicles such that a utility function is maximized. A more formal definition, however, is presented by Gidofalvi et. al[7]:

DEFINITION 1. *For a given **maximum vehicle–share size** K, and **minimum savings** min_savings $\in [0,1]$, the vehicle–sharing problem is to find a disjoint partitioning $S = \{s_1 \uplus s_2 \uplus \ldots\}$ of R, such that $\forall s_j \in S$, s_j is* valid, $|s_j| \leq K$, *and the expression $\sum_{s_j \in S} \sum_{r_i \in s_j} p(r_i, s_j)$ is maximized under the condition that $\forall r_i \in s_j$ $p(r_i, s_j) \geq$ min_savings or $\{r_i\} = s_j$.*

Conversely in the route optimization problem, sometimes also referred to as the Vehicle Routing Problem, the issue instead becomes locating a path along a connected graph such that the route is minimized with respect to a set of costs. Laporte[12] formalizes this problem concretely, though in future works the end location is generalized to not necessarily be the same as the start location:

Let $G = (V, A)$ be a graph where $V = \{1, \ldots, n\}$ is a set of vertices representing *cities* with the *depot* located at vertex 1, and A is the set of arcs. With every arc (i, j) $i \neq j$ is associated a non-negative *distance* matrix $C = (c_{ij})$. In some contexts, c_{ij} can be interpreted as a *travel cost* or as a *travel time*. When C is symmetrical, it is often convenient to replace A by a set E of undirected edges. In addition, assume there are m available vehicles based at the depot, where $m_L \leq m \leq m_U$. When $m_L = m_U$, m is said to be *fixed*. When $m_L = 1$ and $m_U = n - 1$, m is said to be *free*. When m is not fixed, it often makes sense to associate a fixed cost f on the use of a vehicle. For the sake of simplicity, we will ignore these costs and unless otherwise specified, we assume that all vehicles are identical and have the same capacity D. The VRP consists of designing a set of least-cost vehicle routes in such a way that

(i) each city in $V \setminus \{1\}$ is visited exactly once by exactly one vehicle;
(ii) all vehicle routes start and end at the depot;
(iii) some side constraints are satisfied.

Both of these problems are NP-hard, and have thus spawned a considerable number of related works. The need for solving these problems is obvious - after all, being able to optimally or even semi-optimally solve either of these problems can mean the difference between a pooled ride share being completed in a timely manner and leaving one or multiple riders monumentally dissatisfied.

But at the same time, it is difficult to solve large numbers of these problems dynamically in the short periods of time often required by real world application. A number of works, which we elaborate on in the proceeding section, do this exclusively offline and thus skirt around the realistic problem of viable commercial application. This is understandable since the search space grows exponentially with longer distances and additional passengers.

We believe, however, that the complexity of the problem is largely due to an implicit yet pervasive assumption: that riders must be taken from a start point to an end point in a single trip. We believe that the difficulties associated with both grouping and route optimization can be largely mitigated by breaking the problem down into smaller trips. If the trips are sufficiently small, even polynomial approximations or even less complex decision schemes can be employed as each only contributes a small piece to the total trip - globally, however, we believe reasonable performance can still be achieved via this form of operation. In the context of ride sharing, we achieve this by partitioning a city into a set of smaller zones and allow drivers to only service riders within a particular zone. If a rider's final destination lies outside a particular zone boundary, he or she must make a series of transfers to get to the final zone which contains the desired destination.

The remainder of the paper is organized is follows. In Section II we present the related work, including those relating to the problems mentioned in this section as well as some other problems. Section III describes an architectural overview of the zoned pooling model as well as some preface for the experiments conducted. The specific experiments conducted are then outlined in Section IV, with the results following the setup. Finally, Section V and Section VI state the conclusions gained from this research and plans for future work.

II. RELATED WORK

A considerable number of ride sharing models have arisen in the operations research literature after the massive growth and explosion of interest generated by popular ride sharing companies such as Uber and Lyft, though a number existed even prior to the rise of these two giants. A number of these papers, in order to test their various models, make use of the NYC taxi dataset first published by Zhu et. al[2]. This dataset contains numerous entries which contain rider pickup and drop off locations, pick up and drop off times, and other various metrics of interest - for a much more comprehensive analysis of the included content, see the work of Swoboda[8]. For this reason, the authors of this paper have selected this dataset to be used for validation and testing with regards to further work done on the model proposed; however, its integration currently resides outside of the scope of this work in the interest of time. Research groups have approached this dataset from a variety of angles, including optimizing groupings and routes as mentioned in the introduction, as well as alternatives such as optimizing trips that span several forms of transport and even by evaluating fairness. In all cases, however, the problem is viewed as an optimization problem. As such, several optimization techniques, including Lagrangian relaxation and column generation[9], heuristic evaluation[9], and numerical gradient based approaches[9] have been exercised in tackling an overall minimization and / or maximization of an objective function.

Gidofalvi et. al[7] attempt to solve the grouping problem by treating it as a set partitioning problem and subsequently applying numerical approximation and heuristic techniques to arrive at good, but not necessarily optimal, solutions that can be scaled to higher input ranges. Alonso-Mora et al[5], meanwhile, reformulated the grouping problem as a dynamic vehicle assignment problem and attempt to optimize said assignments in an effort to reduce the number of unused seats as much as possible. Both papers employ some version of a look-ahead approach in which a number of routes are considered and the one which results in a minimal value (either solely distance, time, or a weighted sum of these two plus a variety of other metrics) is selected as the optimal grouping. The problem, however, is that this entire search process is done offline. While this is acceptable to do for the sake of analysis and testing proof of concept, they will fail to really gain attention from a practical standpoint due to the considerable amount of time needed to perform this minimization (which is an NP-hard problem to solve). Huang et. al[3] address this shortcoming by modeling the problem as a real time search, and by doing so attempt to refocus the attention on finding as good a solution as possible within a given deadline; however, this often requires a reasonable tradeoff in solution quality.

This route optimization problem, on the other hand, bears a striking amount of similarity to the Traveling Salesman problem, with the only caveat being that, rather that searching for a complete tour, an algorithm can simply designate the end destination as the final drop off point. Ma et. al[10] address this problem in the proposal of their T-Share model, which uses a combination of bi-directional search, schedule correction, and occasionally greedy approximations using only some basic assumptions about car quantities and rider behavior. A similar approach is taken by Santos et. al[11], albeit with a larger emphasis on greedy approximations and an additional emphasis on earnings per driver. As with the groupings problem, this is also a difficult problem to solve and thus both these papers also do so offline.

Next, we examine another class of papers that address either mixed-form (meaning that the trip could be comprised of several forms of transportation, including walking, car, plane, ferry, etc.) trips. The literature in these categories significantly more sparse, albeit slightly more interesting since these approaches generalize the trips to a larger degree than prior works. The exploration conducted by Ma and Wolfson[6], for example, analyzes combination trips of both car and slugging (a term they use for walking) and attempts to optimize trips by analyzing graphs in which dynamically generated walking edges can be added to result in new potential groupings. This is interesting in the sense that, in many regards, it serves as a viable model for new services like Uber Express pool; however, the scope of this paper only addressed groupings and not dynamic selection of pickup and drop off locations or route optimization. This did not occur until the work of Lin et. al[4], which expanded upon the prior paper by generating a generalized graph after groupings were determined, after which a variety of optimization techniques were used.

On the topic of fairness, however, there is very little literature. All of the prior works, and a variety of others that were omitted from this paper, focused exclusively on optimality without even considering this metric, which understandably is harder to define. Wolfson et. al[1] attempt to introduce a basic means of addressing the issue by adding in rider preferences as an additional parameter when considering optimal groupings or routes. A subsequent framework is also introduced to evaluate the slightly more complex problem. The principal motivator here is that riders may have an upper bound on the amount of delay or other inconveniences (walking to a destination, for example) they may be willing to suffer in exchange for a potentially lower price. However, the analysis here only considers the preferences of the rider and not fairness to the driver in terms of earnings distributions.

We mention the works across a variety of areas within the ride share domain because the model proposed in this work simultaneously tackles almost all of them at once by simply doing away with some assumptions that are implicitly made across almost all the works of literature. Firstly, this work does not assume that a rider necessarily need to be taken from a start to and end location in a single ride; instead, it addresses the issue by breaking the trip into smaller trips. By reducing the radius of travel for all of the sub-trips, computing optimized routes or grouping becomes highly simplified to the point of being almost unnecessary if the sub-trips are sufficiently small. Additionally, this work also tackles the issue of fairness among riders by eliminating the walking requirement and asymptotically bounding the wait time, as well as the issue of fairness to the riders by distributing earnings as much as possible within each of the sub-trips.

III. MODEL OVERVIEW AND ARCHITECTURE

Our proposed model involves several components that act semi-independently, but collectively operate to successfully simulate a variety of scenarios. We discuss each of these components in detail in the proceeding subsections.

A. *Transfer Points*

Transfer Points straddle the boundaries of up to 4 zones (assuming a grid pattern). Riders only move between zones by being hopping off at these particular points, which semi-centralizes the pickup locations of all inter-zone travelers and therefore reduces the complexity of pickup logic for the vehicles. A passenger at a transfer point is still considered in transit and as a result any wait time at a transfer point is considered as part of the whole trip. Additionally, each transfer point is designated with an orientation with regards to each of the zones they straddle (northwest, southwest, northeast, or southeast), which ultimately is used by the car to determine whether a rider needs to pass through a car's designated zone in order to reach his or her final destination.

B. *Riders*

Every rider, randomly or systematically, is given a start location and end destination in order for the process to begin. These coordinates are broadcasted to the Zone Controller monitoring the rider's current location in order for a car to be designated for pickup. Each rider incurs an initiation fee when the first car of the complete trip picks up the rider from the start location. The remaining fees are incurred on a per-mile (in the simulation a per-unit distance) basis for the complete length of the trip. If zone transfers are required, the rider must hop off at an inter-zone transfer point until a car in the next zone on the journey (may or may not be the final zone) arrives with enough capacity to pick up the rider.

C. Zone Controllers

Zone controllers manage all the traffic within or at the borders of a particular zone, and is responsible for ensuring that any rider that is passing through or terminating the trip via the current zone is promptly serviced. Each zone controller contains a reference to every car, rider, and transfer point on or within its designated zone boundaries, but has no responsibility for managing any agent outside its zone borders. The controller serves as the central entity for all cars in a particular zone - in this regard, it is the entity that matches cars to riders and vice versa. Candidate drivers for rider pickup are determined based on current direction of movement and current earnings. Under no circumstance, however, will a car be assigned to a rider which would require backtracking with respect to the current movement direction. Note that neither proximity to a rider nor complex grouping algorithms were used since the high spacial locality of all cars in a particular zone generally allowed for reasonably good assignments even if evaluated using this basic scheme. Additionally, managing vehicles via this locally centralized controller allows for fairness in rider allocation - in particular, if the earnings of riders within the zone are unequally distributed, then the zone controller will give preference to a driver with less earnings. This results in scenarios where true losers (drivers who continually fail to receive business) are virtually eliminated.

D. Cars and Drivers

Drivers serve as the main agents in the simulation, and as a result required the most complex behavioral logic. Vehicles all operate independently within a zone, following only minimal coordination from the zone controller to ensure that collisions between separate vehicles do not occur in the event of rider pickup. Roughly speaking, every vehicle follows the proceeding task hierarchy:

1) If a rider assignment is available and there is an open spot, head towards and pick up the assigned rider
2) If the car is at max capacity, drop off the rider or riders whose zone destination (which could be a final destination or a transfer point) is closest in proximity. Note here the lack of a complex route optimization scheme.
3) If no rider has been assigned (in other words there are no riders left in the zone) but there is at least one rider in the car, simply perform task 2 for that rider or riders
4) If there are no passengers AND no currently assigned rider, poll the zone controller to figure out if there are any transfer points which have unserviced riders headed in the direction of the car's zone and head there
5) If none of the above scenarios applies, then there is no one to service. In this case simply don't move and await further directions (to save gas).

As will be expressed in the experiments and results section, this simple set of primitive directions resulted in surprisingly well coordinated autonomous behavior. In addition, this evaluation scheme is not fixed as priorities could be inverted depending on the situation (i.e. supersaturation of riders at a particular transfer point may warrant an increase in priority of task 4 in order to service the increased demand and by doing so subvert the need for surge pricing); however, the general order of priority as outlined in the hierarchy is typically adhered to. Surprisingly, this scheme allowed for extremely consistent servicing of riders.

Cars collect payments from riders via the following scheme:

1) If the car picks up a rider from a starting location, a flat fare is collected as the trip initiation fee
2) When a car either drops a rider off at a transfer point or a final destination, a per unit distance fee is assessed based on the shortest DIRECT path between the zone pickup and drop off location. If a rider's trip spans multiple zones, each driver involved in the trip collects his or her portion of the trip fare when the rider's passage through the zone is

complete.

Because the total fare of a trip could now be distributed among several vehicles and because the zone controllers factored in driver fairness when determining rider assignments, the emergent behavior of this pricing scheme was that, probabilistically speaking, it was unlikely that a car did not get any business during a simulation. In this regard, this scheme can be considered more fair than a traditional situation in which a single car gets all of a rider's business in a winner-take-all manner.

IV. EXPERIMENTS AND RESULTS

Several experiments were performed in which the only parameters that varied were the inputs to the simulation. A total of six possible scenarios were examined, each of which consisted of a permutation of the following two input parameters:

- Rider spawn frequency: low (20% per time step), medium (50% per time step), high (80% per time step)
- Pickup and drop-off location probability: uniform across all zones, concentrated in particular zones

Each permutation was intended to serve as a depiction of a realistic scenario, though still idealized in the sense that they were conducted with mock data as opposed to real data. For example, a low rider frequency with uniform pickup and drop off location may reflect the situation in a rural community or a suburban community during off-peak hours. A high and concentrated scenario, however, would represent peak hours in an urban setting where a considerable number of riders may be headed towards or away from points which naturally generate a high amount of requests (i.e. airports, train stations, ferry terminals, etc.).

Each simulation time step corresponds to about one-third of a second, and each simulation was conducted over the span of 1,500 simulated seconds (or approximately 500 real seconds). Cars were allowed to move at up to 0.2 times the length of the vector between its current and target location. This was a reasonable choice to make in the context of the simulation as this allowed for variance in step sizes, which is reasonable since vehicles do not typically move at uniform rates (think freeway versus urban street). Finally, the fare structure is as follows:

- A flat fare of $2 is collected by the driver who initiates the pickup at the rider's start location.
- A $0.05 per unit distance fee is also charged to the rider for every unit distance on the DIRECT path from the current to the end location within a zone.
- Fees are collected by each driver who transports a rider through a zone after the rider is dropped off at the zone destination (which could be a transfer point).

A preliminary experiment was first conducted, however, to determine a reasonable number of cars to allocate to each zone. A uniform number of cars were allocated to each zone since the rider pick up and drop off location were drawn from a uniform distribution, and thus it was equally likely, at least in the uniform cases, that a rider could spawn in or desire to go to a location in any of the sixteen zones. The number of cars per zone were swept from 1 to 6, and therefore anywhere from 16 to 96 cars could have been in service at any time. A plot of the three metrics of interest - average driver earnings, standard deviation of driver earnings, and average rider wait times - is displayed in Figure 1. The experiments were repeated for the remaining permutations, though their plots have been excluded even from the Appendix in order to conserve pages.

Using this plot, the optimal number of cars to allocate per zone was determined as the number at which the average rider wait time experienced diminishing returns (lower is better) and at which the earnings per driver began to drop needlessly as well. This value turned out to be 4 cpz (cars per zone) for high spawn rates, 3 cpz for medium spawn rates, 2 cpz for low spawn rates. Furthermore the decline in standard deviation indicated a more even distribution of earnings, which was desired as well for the sake of fairness. For concentrated spawn simulations, a single extra car was added to every zone which had a concentrated spawn location probability.

Figures 2 and 3 display the earnings distribution, average wait time, completed trips plots as well as additional statistics which resulted from a typical run of a simulation for high rider spawn rate and

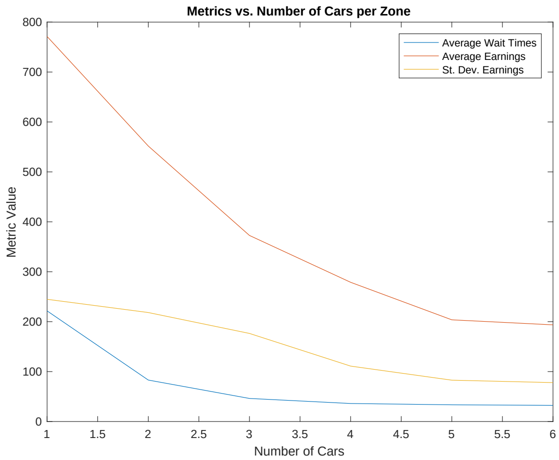

Fig. 1. Plots of average rider wait time, average driver earnings, and standard deviation of rider earnings as a function of the number of cars per zone using a high rider spawn rate and uniform rider location distribution

uniform rider spawn location. We note here the asymptotic behavior of the rider wait time and the relatively even earnings distribution (albeit with some spikes) among 64 independently operating cars. Note that even though very little complexity was involved in determining the zone routing and that grouping was done somewhat indiscriminately, we still end up seeing global behavior that is beneficial to both riders and drivers, including over \$300 in average earnings for drivers and an asymptotic wait time of only about 38 simulation seconds over the span of 641 completed trips. The results of the remaining runs, which can all be observed to exhibit similar behavior, can be found in Figures 4 - 13 in the Appendix. The combined results of these findings demonstrate that our model indeed proved to be reasonably successful in performing well overall.

V. CONCLUSION

We have presented a novel model of ride sharing that simultaneously tackles numerous problems presented in the carpool and ride sharing literature through simply breaking down the larger problem into smaller parts. As a result, our model significantly reduces the need to perform expensive optimization computations offline and is therefore fairly scalable even in large real time systems. Furthermore, our model has serious promise with regards to fairness towards riders and drivers by both actively mitigating the risk of a driver receiving no business and simultaneously influencing the average rider wait time to exhibit asymptotic behavior. Our results remain fairly consistent experiment after experiment in the multi-agent simulation, and therefore further highlight the robustness of the findings.

We have also discovered through experimentation that there exists a natural tradeoff between having too many and too few cars on the road, with the former being more ideal for riders while the latter being more ideal for drivers. That being said, there typically will exist some critical point at which having more cars does not substantially improve rider wait times but does reduce profits.

Finally, as a consequence of the need to simulate our experiment, we also have constructed a generalizable framework that can extend to more sophisticated control schemes should the need arise. Our framework essentially can be considered a tool on which future researchers can build and test their own hypotheses on, should sufficient interest manifest along this line of work.

VI. FUTURE WORK

Due to the limited time constraints of the quarter, there remains a considerable amount of further work to be conducted in order for the findings to become publishable. The largest of these would be to apply our model framework on a real dataset, such as the NYC taxi dataset cited throughout the literature. This would require a fair amount of reformatting of our current framework in order to achieve reasonably accurate results. For example, in order to get more accurate time estimates between real locations, we would need to make use of a mapping service such as Google Maps or an equivalent.

Additionally, in this generic framework we assume a general 4x4 grid structure; however, this may not be the best partition of a real city such as New York City. This same logic applies to the location of transfer points between zones. In this regard, the zone partitions and transfer point locations would need to be tailored to the physical layout of the city itself and the approximate population density. Arriving at optimal locations for these points would probably require a data-driven machine learning approach, and thus in

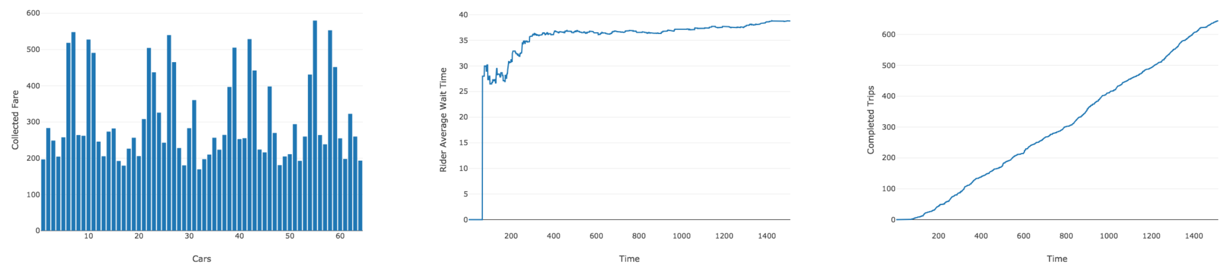

Fig. 2. Plots depicting (from left to right): Distribution of Earnings from all Cars, Average Wait Time of Riders, and number of completed trips over 1500 simulation time steps. Spawn Location Probabilities: Uniform, Spawn Frequency: High

Metric	Value
Min. Earnings	169.34
Max Earnings	579.85
Avg. Earnings	306.10
St. Dev. of Earnings	119.19
Asymptotic Rider Wait Time	38.79
Completed Trips	641

Fig. 3. Additional Statistics - Spawn Location Probabilities: Uniform, Spawn Frequency: High

itself would take about another quarter's worth of work.

Finally, there remains the task of determining whether the model will be able to operate robustly throughout the span of an entire day or more. This includes operating during peak and off-peak hours, which may introduce some additional complications not necessarily present in an idealized model. These and the other tasks aforementioned must be implemented in order to perform a more direct comparison to the proposals in the literature and, subsequently, gauge the commercial viability of this model. We therefore view the further work as having paramount importance and being the crucial next steps of this work.

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APPENDIX

The remaining pages contain figures and statistics from the remaining 5 simulations mentioned in Section IV.

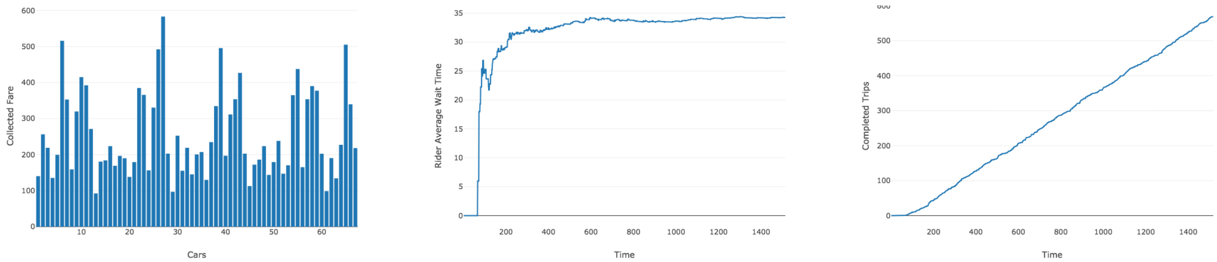

Fig. 4. Plots depicting (from left to right): Distribution of Earnings from all Cars, Average Wait Time of Riders, and number of completed trips over 1500 simulation time steps. Spawn Location Probabilities: Concentred, Spawn Frequency: High

Metric	Value
Min. Earnings	92.35
Max Earnings	583.27
Avg. Earnings	252.47
St. Dev. of Earnings	117.10
Asymptotic Rider Wait Time	34.02
Completed Trips	564

Fig. 5. Additional Statistics - Spawn Location Probabilities: Concentred, Spawn Frequency: High

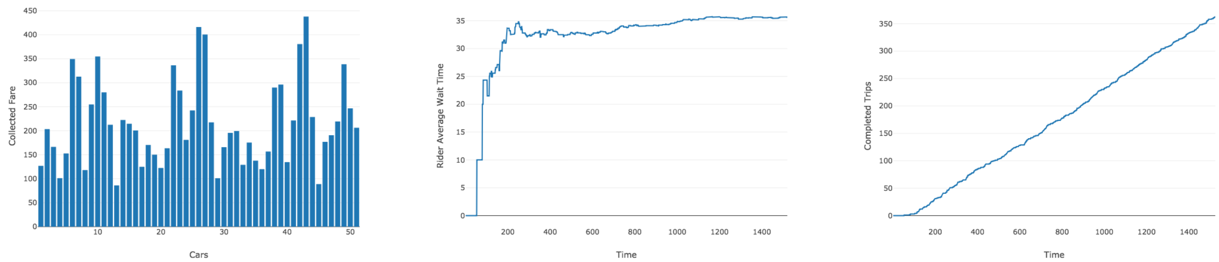

Fig. 6. Plots depicting (from left to right): Distribution of Earnings from all Cars, Average Wait Time of Riders, and number of completed trips over 1500 simulation time steps. Spawn Location Probabilities: Concentred, Spawn Frequency: Medium

Metric	Value
Min. Earnings	84.09
Max Earnings	437.94
Avg. Earnings	213.85
St. Dev. of Earnings	89.29
Asymptotic Rider Wait Time	35.65
Completed Trips	358

Fig. 7. Additional Statistics - Spawn Location Probabilities: Concentred, Spawn Frequency: Medium

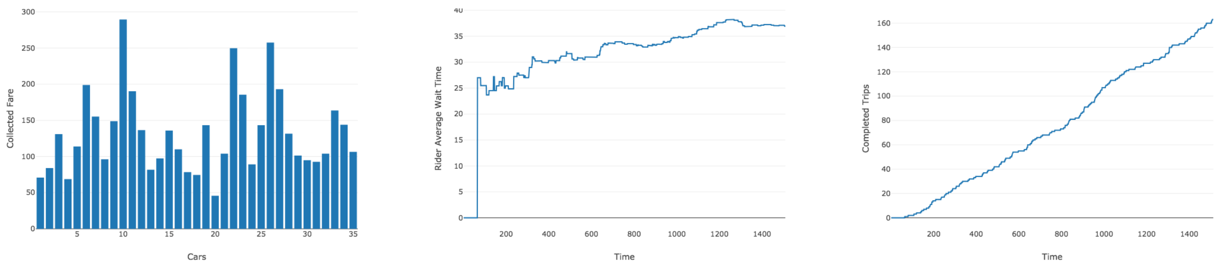

Fig. 8. Plots depicting (from left to right): Distribution of Earnings from all Cars, Average Wait Time of Riders, and number of completed trips over 1500 simulation time steps. Spawn Location Probabilities: Concentred, Spawn Frequency: Low

Metric	Value
Min. Earnings	45.72
Max Earnings	289.45
Avg. Earnings	131.31
St. Dev. of Earnings	55.86
Asymptotic Rider Wait Time	37.11
Completed Trips	160

Fig. 9. Additional Statistics - Spawn Location Probabilities: Concentred, Spawn Frequency: Low

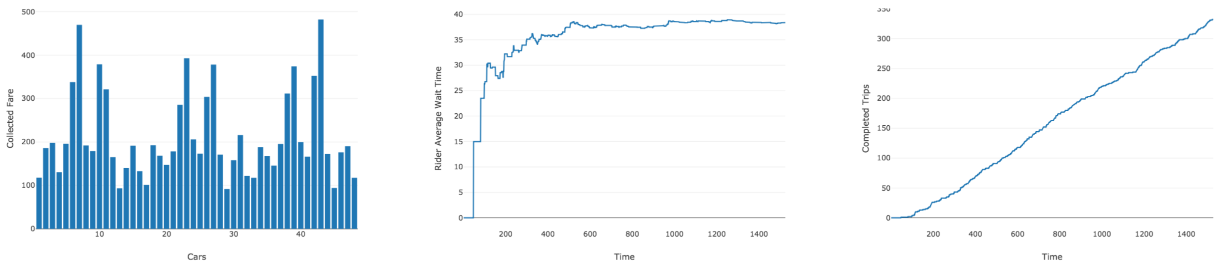

Fig. 10. Plots depicting (from left to right): Distribution of Earnings from all Cars, Average Wait Time of Riders, and number of completed trips over 1500 simulation time steps. Spawn Location Probabilities: Uniform, Spawn Frequency: Medium

Metric	Value
Min. Earnings	91.68
Max Earnings	482.06
Avg. Earnings	208.37
St. Dev. of Earnings	98.10
Asymptotic Rider Wait Time	38.28
Completed Trips	327

Fig. 11. Additional Statistics - Spawn Location Probabilities: Uniform, Spawn Frequency: Medium

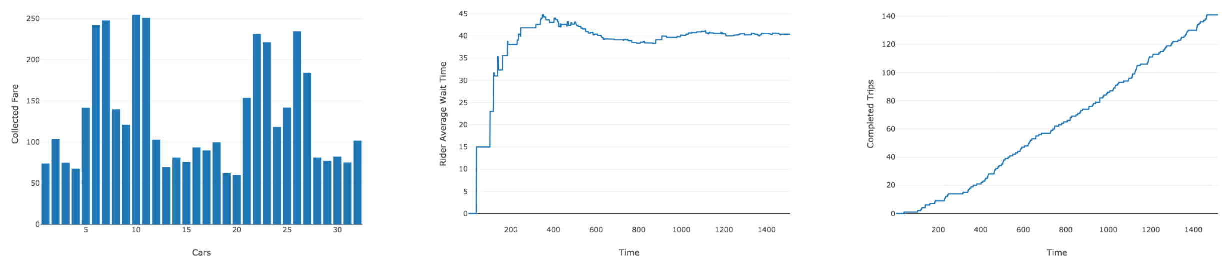

Fig. 12. Plots depicting (from left to right): Distribution of Earnings from all Cars, Average Wait Time of Riders, and number of completed trips over 1500 simulation time steps. Spawn Location Probabilities: Uniform, Spawn Frequency: Low

Metric	Value
Min. Earnings	50.79
Max Earnings	254.77
Avg. Earnings	129.37
St. Dev. of Earnings	65.05
Asymptotic Rider Wait Time	40.41
Completed Trips	141

Fig. 13. Additional Statistics - Spawn Location Probabilities: Uniform, Spawn Frequency: Low