

MATHEMATICS

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1 Discrete

1. The HCF of two numbers a and b is 5 and their LCM is 200. Find the product ab .
2. Find the HCF of 612 and 1314 using prime factorisation.
3. Show that any positive odd integer is of the form $6m + 1$ or $6m + 3$ or $6m + 5$, where m is some integer.
4. Prove that $\sqrt{5}$ is an irrational number.
5. Prove that $(5 - 3\sqrt{2})$ is an irrational number, given that $\sqrt{2}$ is irrational number.
6. Find the sum of all the two digit numbers which leave the remainder 2 when divided by 5.
7. If in an A.P., $a = 15, d = -3$ and $a_n = 0$, then find the value of n .
8. If S_n , the sum of the first n terms of an A.P. is given by $S_n = 2n^2 + n$, then find its n^{th} term.
9. If the 17^{th} term of an A.P. exceeds its 10^{th} term by 7, find the common difference.
10. If the sum of the first p terms of an A.P. is q and the sum of the first q terms is p ; then show that the sum of the first $(p + q)$ terms is $\{-(p + q)\}$.
11. Write the common difference of the A.P. $\sqrt{3}, \sqrt{12}, \sqrt{27}, \sqrt{48}, \dots$
12. In an A.P., the n^{th} term is $\frac{1}{m}$ and the m^{th} term is $\frac{1}{n}$. Find
 - (a) $(mn)^{\text{th}}$ term,
 - (b) sum of first (mn) terms.

2 Algebra

13. Find all the zeroes of the polynomial $x^4 + x^3 - 14x^2 - 2x + 24$, if two of its zeroes are $\sqrt{2}$ and $-\sqrt{2}$.
14. Apply division algorithm to check if $g(x) = x^2 - 3x + 2$ is a factor of the polynomial $f(x) = x^4 - 2x^3 - x + 2$.
15. A train travels 360km at a uniform speed. If the speed had been 5km/hr more, it would have taken 1hr less for the same journey. Find the speed of the train.
16. Find the value of k for which $x = 2$ is a solution of the equation $kx^2 + 2x - 3 = 0$.
17. Find the value/s of k for which the quadratic equation $3x^2 + kx + 3 = 0$ has real and equal roots.
18. Solve for x :

$$\frac{1}{a+b+x} = \frac{1}{a} + \frac{1}{b} + \frac{1}{x}; a \neq b \neq 0, x \neq 0, x \neq -(a+b)$$

19. If $\sin x + \cos y = 1$; $x = 30^\circ$ and y is an acute angle, find the value of y .
20. Find the value of $\cos 48^\circ - \sin 42^\circ$.
21. Prove that :

$$\frac{\tan \theta}{1 - \tan \theta} - \frac{\cot \theta}{1 - \cot \theta} = \frac{\cos \theta + \sin \theta}{\cos \theta - \sin \theta}$$

22. If $\cos \theta + \sin \theta = \sqrt{2}\cos \theta$, show that $\cos \theta - \sin \theta = \sqrt{2}\sin \theta$.
23. Prove that :

$$\frac{(1 + \cot \theta + \tan \theta)(\sin \theta - \cos \theta)}{(\sec^3 \theta - \csc^3 \theta)} = \sin^2 \theta \cos^2 \theta$$

24. Evaluate :

$$\frac{\csc^2(90^\circ - \theta) - \tan^2 \theta}{2(\cos^2 37^\circ + \cos^2 53^\circ)} - \frac{2 \tan^2 30^\circ \sec^2 37^\circ \cdot \sin^2 53^\circ}{\csc^2 63^\circ - \tan^2 27^\circ}$$

3 Matrices

25. For what value of k , does the system of linear equations

$$\begin{aligned} 2x + 3y &= 7 \\ (k-1)x + (k+2)y &= 3k \end{aligned}$$

have an infinite number of solutions ?

26. A part of monthly hostel charges in a college hostel are fixed and the remaining depends on the number of days one has taken food in the mess. When a student A takes food for 25 days, he has to pay ₹4,500, whereas a student B who takes food for 30 days, has to pay ₹5,200. Find the fixed charges per month and the cost of food per day.

4 Vectors

27. Find the value of ' a ' so that the point $(3, a)$ lies on the line represented by $2x - 3y = 5$.
28. The mid-point of the line segment joining $A(2a, 4)$ and $B(-2, 3b)$ is $(1, 2a + 1)$. Find the values of a and b .
29. Point P divides the line segment joining the points $A(2, 1)$ and $B(5, -8)$ such that $\frac{AP}{AB} = \frac{1}{3}$. If P lies on the line $2x - y + k = 0$, find the value of k .
30. For what value of p , are the points $(2, 1)$, $(p, -1)$ and $(-1, 3)$ collinear ?
31. Find the coordinates of a point A , where AB is a diameter of the circle with centre $(3, -1)$ and the point B is $(2, 6)$.
32. Find the values of x for which the distance between the points $A(x, 2)$ and $B(9, 8)$ is 10 units.
33. The mid-point of the line segment joining $A(2a, 4)$ and $B(-2, 3b)$ is $(1, 2a + 1)$. Find the values of a and b .

5 Construction

34. Prove that tangents drawn at the ends of a diameter of a circle are parallel.
35. The area of two similar triangles are 25sq.cm and 121sq.cm . Find the ratio of their corresponding sides.
36. In Figure 1, E is a point on CB produced of an isosceles $\triangle ABC$, with side $AB = AC$. If $AD \perp BC$ and $EF \perp AC$, prove that $\triangle ABD \sim \triangle ECF$.

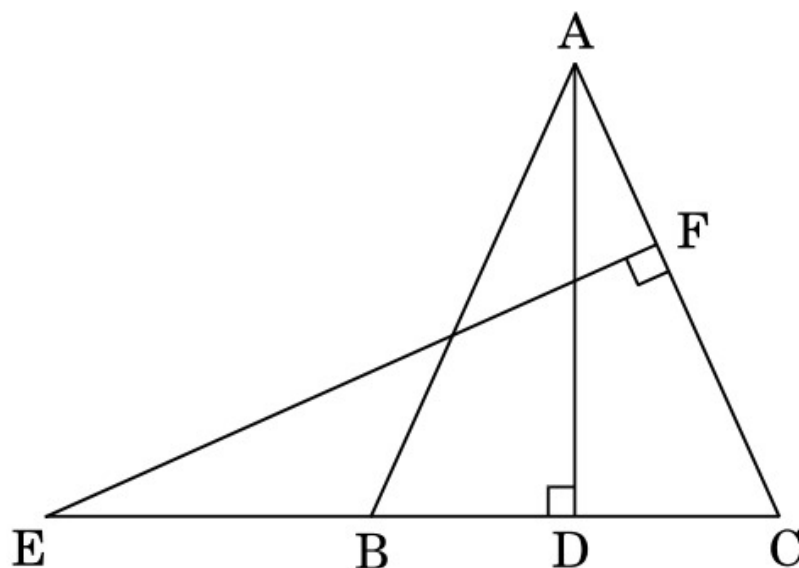


Figure 1: Triangle

37. Prove that the parallelogram circumscribing a circle is a rhombus.
38. In a triangle, if the square of one side is equal to the sum of the squares of the other two sides, then prove that the angle opposite to the first side is a right angle.
39. In two concentric circles, prove that all chords of the outer circle which touch the inner circle, are of equal length.
40. Construct an isosceles triangle whose base is 8cm and altitude 4cm and then another triangle whose sides are $\frac{3}{4}$ times the corresponding sides of the isosceles triangle.
41. Construct a right triangle in which sides (other than the hypotenuse) are 8cm and 6cm . Then construct another triangle whose sides are $\frac{5}{3}$ times the corresponding sides of the right triangle.
42. Construct a pair of tangents to a circle of radius 4cm which are inclined to each other at an angle of 60° .
43. In $\triangle ABC$, $\angle B = 90^\circ$ and D is the mid-point of BC . Prove that $AC^2 = AD^2 + 3CD^2$.
44. Prove that in a right triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.

6 Trigonometry

45. A boy standing on a horizontal plane finds a bird flying at a distance of $100m$ from him at an elevation of 30° . A girl standing on the roof of a $20m$ high building, finds the elevation of the same bird to be 45° . The boy and the girl are on the opposite sides of the bird. Find the distance of the bird from the girl. (Given $\sqrt{2} = 1.414$)
46. The angle of elevation of an aeroplane from a point A on the ground is 60° . After a flight of 30seconds, the angle of elevation changes to 30° . If the plane is flying at a constant height of $3600\sqrt{3}$ metres, find the speed of the aeroplane.

7 Geometry

47. In Figure 2, three sectors of a circle of radius $7cm$, making angles of 60° , 80° and 40° at the centre are shaded. Find the the area of the shaded region.

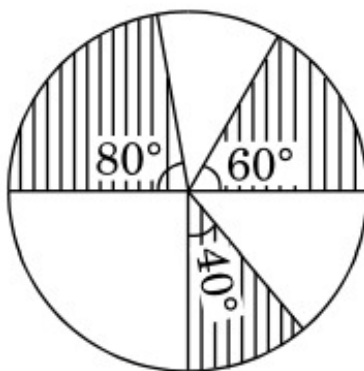


Figure 2: Circle

48. A juice seller was serving his customers using glasses as shown in Figure 3. The inner diameter of the cylindrical glass was $5cm$ but bottom of the glass had a hemispherical raised portion which reduced the capacity of the glass. If the height of a glass was $10cm$, find the apparent and actual capacity of the glass. (Use $\pi = 3.14$)

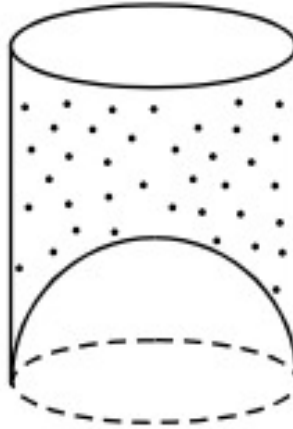


Figure 3: Hemisphere

49. A girl empties a cylindrical bucket full of sand, of base radius 18cm and height 32cm on the floor to form a conical heap of sand. If the height of this conical heap is 24cm , then find its slant height correct to one place of decimal.
50. An open metallic bucket is in the shape of a frustum of a cone. If the diameters of the two circular ends of the bucket are 45cm and 25cm and the vertical height of the bucket is 24cm , find the area of the metallic sheet used to make the bucket. Also find the volume of the water it can hold. (Use $\pi = \frac{22}{7}$)

8 Probability

51. A child has a die whose 6 faces show the letters given below :

A

B

C

A

A

B

52. Cards marked with numbers 5 to 50 (one number on one card) are placed in a box and mixed thoroughly. One card is drawn at random from the box. Find the probability that the number on the card taken out is

- (a) a prime number less than 10,
- (b) a number which is a perfect square.

53. Find the probability that a number selected at random from the numbers 3, 4, 4, 4, 5, 5, 6, 6, 6, 7 will be their mean.