MATHEMATICS

January 4, 2024

1. The HCF of two numbers a and b is 5 and their LCM is 200. Find the product ab.
2. Find the value of k for which $x = 2$ is a solution of the equation $kx^2 + 2x - 3 = 0$.
3. Find the value/s of k for which the quadratic equation $3x^2 + kx + 3 = 0$ has real and equal roots.
4. If in an A.P ., $a = 15$,$d = -3$ and $a_n = 0$, then find the value of n.
5. If $\sin x + \cos y = 1$; $x = 30°$ and y is an acute angle, find the value of y .
6. Find the value of $\cos 48° - \sin 42°$.
7. The area of two similar triangles are $25 sq.cm$ and $121 sq.cm$. Find the ratio of their corresponding sides.
8. Find the value of 'a' so that the point $(3, a)$ lies on the line represented by $2x - 3y = 5$.
9. If S_n, the sum of the first n terms of an A.P. is given by $S_n = 2n^2 + n$,then find its n^{th} term.
10. If the 17^{th} term of an A.P. exceeds its 10^{th} term by 7, find the common difference.
11. The mid-point of the line segment joining $A(2a, 4)$ and $B(-2, 3b)$ is $(1, 2a + 1)$. Find the values of a and b.
12. A child has a die whose 6 faces show the letters given below :

    $\boxed{\text{A}}$ $\boxed{\text{B}}$ $\boxed{\text{C}}$ $\boxed{\text{A}}$ $\boxed{\text{A}}$ $\boxed{\text{B}}$

13. Find the HCF of 612 and 1314 using prime factorisation.
14. Show that any positive odd integer is of the form $6m + 1$ or $6m + 3$ or $6m + 5$, where m is some integer.

15. Cards marked with numbers 5 to 50 (one number on one card) are placed in a box and mixed thoroughly. One card is drawn at random from the box. Find the probability that the number on the card taken out is

    (a) a prime number less than 10,

    (b) a number which is a perfect square.

16. For what value of k, does the system of linear equations

$$\begin{aligned} 2x + 3y &= 7 \\ (k-1)\,x + (k+2)\,y &= 3k \end{aligned}$$

have an infinite number of solutions ?

17. Prove that $\sqrt{5}$ is an irrational number.

18. Find all the zeroes of the polynomial $x^4 + x^3 - 14x^2 - 2x + 24$, if two of its zeroes are $\sqrt{2}$ and $-\sqrt{2}$.

19. Point P divides the line segment joining the points $A\,(2,1)$ and $B\,(5,-8)$ such that $\frac{AP}{AB} = \frac{1}{3}$.If P lies on the line $2x - y + k = 0$, find the value of k.

20. For what value of p, are the points $(2,1)$, $(p,-1)$ and $(-1,3)$ collinear ?

21. Prove that :

$$\frac{\tan\theta}{1-\tan\theta} - \frac{\cot\theta}{1-\cot\theta} = \frac{\cos\theta + \sin\theta}{\cos\theta - \sin\theta}$$

22. If $\cos\theta + \sin\theta = \sqrt{2}\cos\theta$, show that $\cos\theta - \sin\theta = \sqrt{2}\sin\theta$.

23. A part of monthly hostel charges in a college hostel are fixed and the remaining depends on the number of days one has taken food in the mess. When a student A takes food for 25 days, he has to pay ₹4, 500, whereas a student B who takes food for 30 days, has to pay ₹5, 200. Find the fixed charges per month and the cost of food per day.

24. In $\triangle ABC$, $\angle B = 90^\circ$ and D is the mid-point of BC. Prove that $AC^2 = AD^2 + 3CD^2$.

25. In Figure **??**, E is a point on CB produced of an isosceles $\triangle ABC$, with side $AB = AC$. If $AD \perp BC$ and $EF \perp AC$, prove that $\triangle ABD \sim \triangle ECF$.

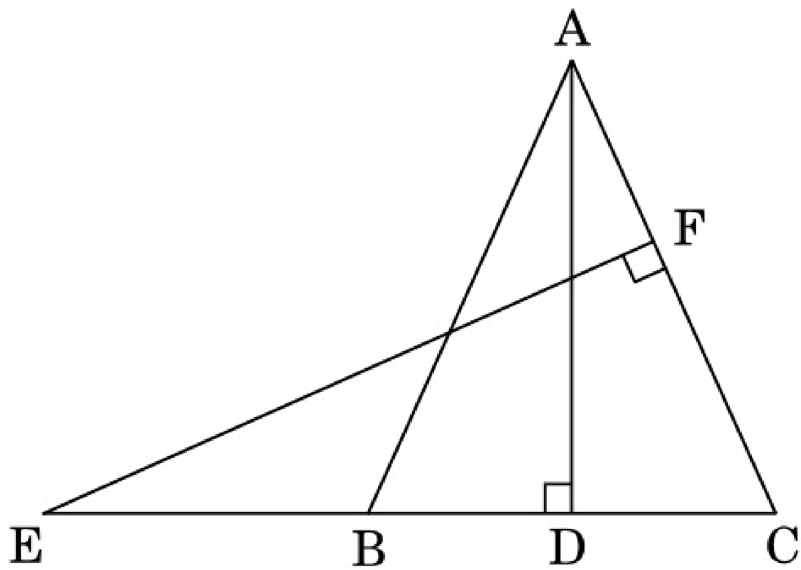

Figure 1: Triangle

26. Prove that the parallelogram circumscribing a circle is a rhombus.
27. In Figure **??**, three sectors of a circle of radius $7cm$, making angles of $60°$, $80°$ and $40°$ at the centre are shaded. Find the the area of the shaded region.

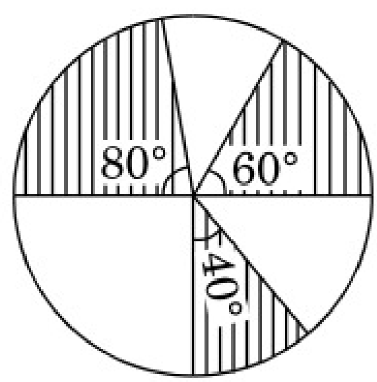

Figure 2: Circle

28. A juice seller was serving his customers using glasses as shown in Figure **??** . The inner diameter of the cylindrical glass was $5cm$ but bottom of the glass had

a hemispherical raised portion which reduced the capacity of the glass. If the height of a glass was $10cm$, find the apparent and actual capacity of the glass. (Use $\pi = 3.14$)

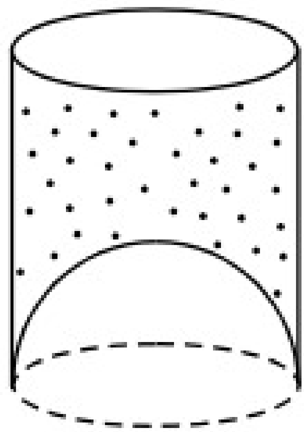

Figure 3: Hemisphere

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29. A girl empties a cylindrical bucket full of sand, of base radius $18cm$ and height $32cm$ on the floor to form a conical heap of sand. If the height of this conical heap is $24cm$, then find its slant height correct to one place of decimal.

30. A train travels $360km$ at a uniform speed. If the speed had been $5km/hr$ more, it would have taken $1hr$ less for the same journey. Find the speed of the train.

31. Solve for x :

$$\frac{1}{a+b+x} = \frac{1}{a} + \frac{1}{b} + \frac{1}{x}; a \neq b \neq 0, x \neq 0, x \neq -(a+b)$$

32. If the sum of the first p terms of an A.P. is q and the sum of the first q terms is p; then show that the sum of the first $(p+q)$ terms is $\left\{-(p+q)\right\}$.

33. In a triangle, if the square of one side is equal to the sum of the squares of the other two sides, then prove that the angle opposite to the first side is a right angle.

34. Construct an isosceles triangle whose base is $8cm$ and altitude $4cm$ and then another triangle whose sides are $\frac{3}{4}$ times the corresponding sides of the isosceles triangle.

35. A boy standing on a horizontal plane finds a bird flying at a distance of $100m$ from him at an elevation of $30°$. A girl standing on the roof of a $20m$ high building, finds the elevation of the same bird to be $45°$. The boy and the girl

are on the opposite sides of the bird. Find the distance of the bird from the girl. (*Given* $\sqrt{2} = 1.414$)

36. The angle of elevation of an aeroplane from a point A on the ground is $60°$. After a flight of $30 seconds$, the angle of elevation changes to $30°$. If the plane is flying at a constant height of $3600\sqrt{3} metres$, find the speed of the aeroplane.

37. Prove that :

$$\frac{(1 + \cot\theta + \tan\theta)(\sin\theta - \cos\theta)}{(\sec^3\theta - \csc^3\theta)} = \sin^2\theta\cos^2\theta$$

38. An open metallic bucket is in the shape of a frustum of a cone. If the diameters of the two circular ends of the bucket are $45cm$ and $25cm$ and the vertical height of the bucket is $24cm$, find the area of the metallic sheet used to make the bucket. Also find the volume of the water it can hold. ($Use \pi = \frac{22}{7}$)

39. Write the common difference of the A.P. $\sqrt{3}$, $\sqrt{12}$, $\sqrt{27}$, $\sqrt{48}$, ...

40. Find the coordinates of a point A, where AB is a diameter of the circle with centre $(3, -1)$ and the point B is $(2, 6)$.

41. Prove that tangents drawn at the ends of a diameter of a circle are parallel.

42. Apply division algorithm to check if $g(x) = x^2 - 3x + 2$ is a factor of the polynomial $f(x) = x^4 - 2x^3 - x + 2$.

43. Evaluate :

$$\frac{\csc^2(90° - \theta) - \tan^2\theta)}{2(\cos^2 37° + \cos^2 53°)} - \frac{2\tan^2 30° \sec^2 37° \cdot \sin^2 53°}{\csc^2 63° - \tan^2 27°}$$

44. Construct a right triangle in which sides (other than the hypotenuse) are $8cm$ and $6cm$. Then construct another triangle whose sides are $\frac{5}{3}$ times the corresponding sides of the right triangle.

45. Find the sum of all the two digit numbers which leave the remainder 2 when divided by 5.

46. Find the values of x for which the distance between the points $A(x, 2)$ and $B(9, 8)$ is 10 units.

47. The mid-point of the line segment joining $A(2a, 4)$ and $B(-2, 3b)$ is $(1, 2a + 1)$. Find the values of a and b.

48. Find the probability that a number selected at random from the numbers $3, 4, 4, 4, 5, 5, 6, 6, 6, 7$ will be their mean.

49. In two concentric circles, prove that all chords of the outer circle which touch the inner circle, are of equal length.

50. Prove that $\left(5 - 3\sqrt{2}\right)$ is an irrational number, given that $\sqrt{2}$ is irrational number.

51. In an A.P., the n^{th} term is $\frac{1}{m}$ and the m^{th} term is $\frac{1}{n}$. Find

    (a) $(mn)^{th}$term ,

    (b) sum of first (mn) terms.

52. Prove that in a right triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.

53. Construct a pair of tangents to a circle of radius $4cm$ which are inclined to each other at an angle of $60°$.