Problem set #4: Econ325
Due Date: April 4, 2000
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1. Consider the utility maximizing choice of fertility model we considered in the class. Suppose the utility function $u(n, z)$ of a household over the aggregate marketed good z and the desired number of surviving children n is given by

$$u(n, z) = \sqrt{n \cdot z}$$

Suppose the price of the aggregate marketed good is p_z and the cost of a surviving child is p_n. Suppose the household has income I. Solve for the household's optimal n, the demand for surviving children. What constitutes the child cost? Explain how the demand for children will be affected in each of the following cases:

(a) When mother gets higher education and thus have better chance of working and earning higher wages.

(b) When government introduces mandatory schooling for all children.

(c) When government prohibits employment of child labor.

(d) Government spends more money on health so that infant mortality rate goes down. (Here distinguish between the actual number of children the parents will have to meet a desired number of children).

(e) Government introduces social security program for all.

2. Suppose an worker with innate ability lives for two periods: period 1 (young) and period 2 adult. He derives utility from consumption of goods in these periods. Let c_1 be the consumption in period 1 and c_2 be the consumption in period 2. Let the price of the good is 1. His wage rate in period 1 is w_1 dollars per period, and his wage rate in period 2 depends on the amount of schooling s that the worker acquires when he is young. Each year of schooling requires θ dollars to invest in the first period, and s years of schooling yields $w_2(s, \tau)$ dollars as wages in the second period of life. He has utility function: $u(c_1, c_2)$. Formulate his utility maximizing problem of choosing his schooling level.

   (a) Show that his optimal schooling choice reduces to the maximization of the present value of his educational investment:

$$R(s; \tau) = \frac{w_2(s, \tau)}{1 + r} - \theta s.$$

   (b) Suppose the Mincer earnings function is given by $w_2(s, \tau) = 5 \ln(s) + s \cdot \tau$, where s is the schooling level of the worker,τis his ability level and the earnings in the second period does not depend on experience. Find the optimal schooling level s^* that maximizes his present value of educational investment $R(s; \tau)$.

(c) Suppose $r = .05$, and $\theta = .476$, what will be this worker's optimal schooling level? If another worker's r is .06, which is higher than the previous worker's interest rate for borrowing. What will be this individual's schooling level? Suppose family background is represented by family wealth which determines the ease of borrowing money for educational investment. Poorer family background would mean then higher interest rate. How would the education level of the children of poorer family background compare with the children of richer family background?

(d) How would you justify why individuals differ in their choices of schooling level s and given schooling level s, why individuals earn different wages in the labor market in each of the following scenarios?

(i) Individuals have equal opportunities but differ in abilities.

(ii) All individuals have the same ability but they have unequal opportunities.

(iii) Individuals have both unequal ability and opportunities..

(e) Suppose the ability parameter τ is not the innate ability but abilities acquired from the family environment and the neighborhood in which the kid grows up, say, for instance, skills such as motivation, socialization, which affect positively the earnings in the labor market, i.e., $w_2(s, \tau)$. What will be the effect of better family background on schooling level? If further we assume that less motivated kids have harder time in passing grades and they take longer to complete a grade. This, in our set-up, can be simply incorporated by assuming that kids of poorer family background will have higher cost of schooling, i.e., $\theta(\tau)$ is higher lower is τ. What will be the effect of family ground on the level of schooling?

(f) Suppose these skills τ can be altered by parental investment in pre-school. What would be the determinants of the level of parental pre-school investment?