

## Chapter 7

### Costs

#### What to read?

- Introduction
- Section 7.1 (economic cost, business cost, opportunity cost, sunk cost)
- Section 7.2 (all)
- Section 7.3 (p.198-204, p. 206 factor price changes, figure 7.6, The shape of the long-run cost curves on p.207-212)
- Section 7.4(p.214-215, figure 7.9)

## Two reasons to study costs

1. understanding relationship between costs of inputs and production helps us determine least costly way to produce
2. relationship between output and costs determines nature of an industry
  - how many firms are in the industry
  - how high price is relative to cost

## Business vs. economic costs

- *business costs*: only explicit costs (out of pocket)
- *economic costs*: explicit cost + implicit cost = *opportunity cost*
- *opportunity cost*
  - value of best alternative use of the resource
  - classic example: "There's no such thing as a free lunch"
  - "What have you given up to study opportunity costs"

## Cost of running your own firm

- *explicit cost*: \$40,000 per year (rent, materials, wage payments)
- instead of paying yourself a salary, you keep any profit at year's end
- your labor *opportunity cost* = \$25,000/year you could have earned working for another firm
- business cost = \$40,000
- economic cost = \$65,000 = \$40,000 + \$25,000

## Capital costs

- capital is a *durable* good: a product that is usable for years
- capital may be rented or purchased

## Short-run cost measures (section 7.2)

- fixed cost ( $F$ ): production expense that does not vary with output
- variable cost ( $VC$ ): production expense that changes with quantity of output produced
- Total cost ( $C$ ):

$$C = VC + F$$

## Sunk fixed cost

- usually assume fixed cost is *sunk*: expenditure that you cannot be recovered
  - opportunity cost of capital is zero
  - because you can't get this expenditure back no matter what you do, so ignore it when making decisions
- example: walk out of a bad movie early, regardless of what you paid to attend
- otherwise, fixed cost is called *avoidable*

## Numerical Example

- Next table shows costs involved to produce various levels of output in the short-run.
- Short-run means capital is fixed and that involves fixed cost of \$48.
- How do you find fixed cost? It is the cost you incur if your output level is zero.
- In the numerical example, try to understand what happens to average fixed cost, average variable cost, average cost and marginal cost as the output level increases.

**Table 7.1 Variation of Short-Run Cost with Output**

Output, $q$	Fixed Cost, $F$	Variable Cost, $VC$	Total Cost, $C$	Marginal Cost, $MC$	Average Fixed Cost, $AFC = F/q$	Average Variable Cost, $AVC = VC/q$	Average Cost, $AC = C/q$
0	48	0	48				
1	48	25	73	25	48	25	73
2	48	46	94	21	24	23	47
3	48	66	114	20	16	22	38
4	48	82	130	16	12	20.5	32.5
5	48	100	148	18	9.6	20	29.6
6	48	120	168	20	8	20	28
7	48	141	189	21	6.9	20.1	27
8	48	168	216	27	6	21	27
9	48	198	246	30	5.3	22	27.3
10	48	230	278	32	4.8	23	27.8
11	48	272	320	42	4.4	24.7	29.1
12	48	321	369	49	4.0	26.8	30.8

## Average cost concepts

- average fixed cost:

$$AFC = F / q$$

- average variable cost:

$$AVC = VC / q$$

- average (total) cost:

$$AC = C/q = AFC + AVC$$

## Total Cost (numerically from the table, analytical example)

- Total cost  $C = \text{Fixed cost (F)} + \text{variable cost (VC)}$
- In the table, fixed cost  $F=48$ , and variable cost varies with output level.
- Total cost for  $q = 0$  is?
- Total cost for  $q = 1$  is ?
- Since total cost  $C$  to produce output level  $q$  varies with the output level,  $C$  is a function of  $q$ , and we will sometimes explicitly write it as  $C(q)$ .
- Example:  $C(q) = 125 + 2q$
- Example:  $C(q) = 125 - 2q + 4q^2$

## Marginal cost (MC)

- Change in total cost,  $\Delta C$ , when output changes by  $\Delta q$
- $MC = \Delta C / \Delta q$  (or  $dC/dq$ )
- Numerically from the table, MC
- Note that MC varies with the output level  $q$ , i.e., MC is a function of  $q$ , and we may sometimes use the notation  $MC(q)$  when we do things analytically.

## A simpler solved problem

- If short-run cost function is

$$C(q) = 125 + 2q$$

- what are the:
  - fixed cost
  - variable cost
  - average cost
  - average fixed cost
  - average variable cost?
  - Marginal cost?

## Answer

- if  $C(q) = 125 + 2q$
- fixed cost =  $F = 125$ , which you obtain by calculating  $C(0)$ .
- variable cost =  $VC(q) = 2q$
- average cost =  $AC(q) = C(q)/q = 125/q + 2$
- average fixed cost =  $AFC(q) = 125/q$
- average variable cost =  $AVC(q) = 2$
- note: marginal cost =  $MC(q) = 2$

## Numerically for the same example

- if  $C(q) = 125 + 2q$ , what is the average cost of producing 5<sup>th</sup> output
- $AC(5) = C(5)/5 = 135/5 = 27$
- variable cost when producing 5 units =  $VC(q) = 2q$
- average fixed cost at producing 5 units =  $AFC(q) = 125/q$
- average variable cost =  $AVC(q) = 2$
- note: marginal cost =  $MC(q) = 2$



## Solved problem

- if short-run cost function is

$$C(q) = 125 - 2q + q^2$$

- what are the:
  - fixed cost
  - variable cost
  - average cost
  - average fixed cost
  - average variable cost?

## Answer

if  $C(q) = 125 - 2q + q^2$

fixed cost =  $F = 125$

variable cost =  $VC(q) = -2q + q^2$

average cost =  $AC(q) = C(q)/q = 125/q - 2 + q$

average fixed cost =  $AFC(q) = 125/q$

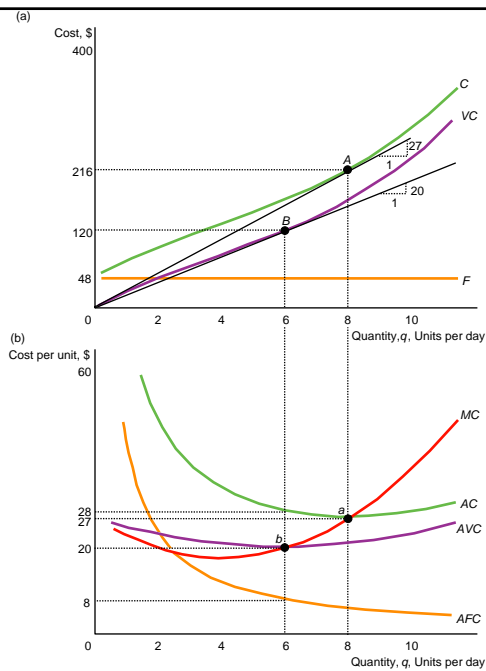
average variable cost =  $AVC(q) = -2 + q$

note: marginal cost =  $MC(q) = -2 + 2q$

## Graphically:

- Use the data in table 7.1 to plot various cost curves and study the relationship between them.
- Here is how: [animated graph](#)

**Figure 7.1**  
Short-Run Cost Curves



## *MC* curve cuts *AC* and *AVC* at their minimum points

- Note that *MC* cuts *AC* and *AVC* curves at their minimum points. This is a general property.

## Relationship between short-run marginal cost and marginal product of labor (p.193)

- How does the cost function relate to the production function? Or in other words, if the technology is expressed in terms of production function, how do we derive the cost function? We do this here for the case of short run, later in this chapter do it for long-run. In the short-run capital is fixed.
- *MC* is the change in variable cost as output increase by one unit, i.e.,

$$MC = \frac{\Delta VC}{\Delta q} = \frac{w\Delta L}{\Delta q} = \frac{w}{MP_L}$$

$$MC = \frac{w}{MP_L}$$

### Example:

- Suppose wage rate is \$5 per hour. Suppose it is producing 5 units of output. To produce the 6<sup>th</sup> unit of output it requires 4 hours of labor. What is the marginal cost at the output level 5?
- The marginal product of an hour of labor is  $\frac{1}{4}$  unit units of output.
- $MC = 5 \text{ divided by } \frac{1}{4} = \$20$
- Suppose short-run production function is  $q = 2L^{0.5}$
- Suppose the wage rate is \$5 per unit of labor. What is the marginal cost at the 5<sup>th</sup> unit of output?

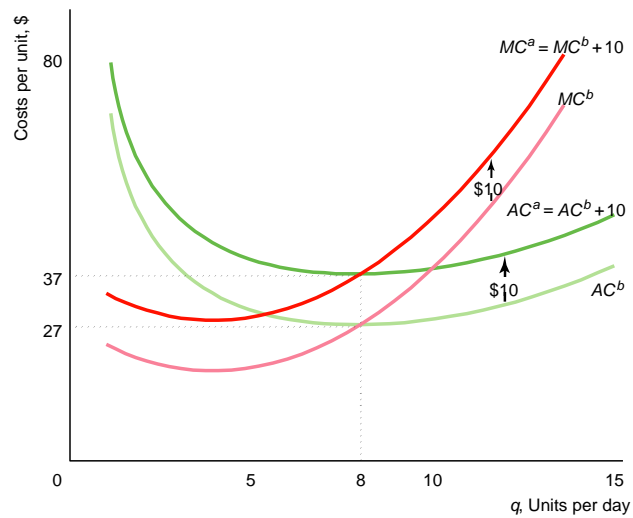
### Effects of specific tax on cost curves

- Suppose there is a specific tax of \$10 charged per unit of output. How does it affect fixed cost, average variable cost and marginal cost? How does it shift the curves?

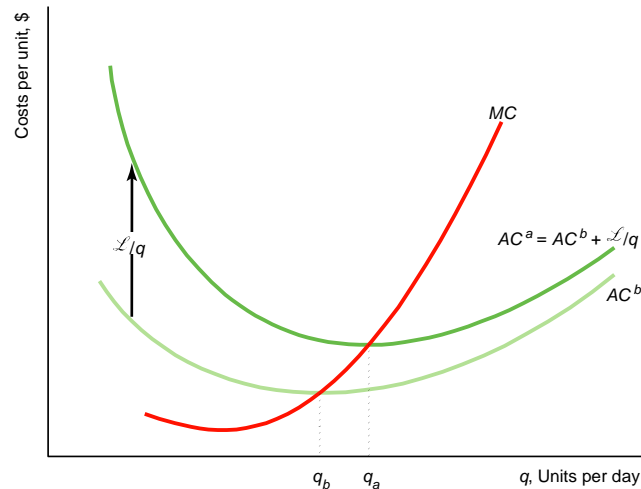
**Table 7.2** Effect of a Specific Tax of \$10 per Unit on Short-Run Costs

$Q$	$AVC^b$	$AVC^a = AVC^b + \$10$	$AC^b = C/q$	$AC^a = C/q + \$10$	$MC^b$	$MC^a = MC^b + \$10$
1	25	35	73	83	25	35
2	23	33	47	57	21	31
3	22	32	38	48	20	30
4	20.5	30.5	32.5	42.5	16	26
5	20	30	29.6	39.6	18	28
6	20	30	28	38	20	30
7	20.1	30.1	27	37	21	31
8	21	31	27	37	27	37
9	22	32	27.3	37.3	30	40
10	23	33	27.8	37.8	32	42
11	24.7	34.7	29.1	39.1	42	52
12	26.8	36.8	30.8	40.8	49	59

**Figure 7.3** Effect of a Specific Tax on Cost Curves



## Page 197 Solved Problem 7.1



## Long-run costs

- All factors are variables, there are no fixed factors of production.
- The main problem is to determine the minimum cost of producing a certain level of output. The manager has to choose the levels of capital and labor that can produce a given level of output. There are many combinations of  $K$  and  $L$  that can produce a given level of output, i.e., all the combinations on an iso-quant that we studied earlier. The manager should choose that combination which minimizes the cost. Denote that cost by  $C$ . As you vary the output level, the minimum cost  $C$  will also vary. This dependence could be represented by  $C(q)$ .
- Apart from output level, the cost of production also depends on the input prices. If the price of capital or labor changes, how does it affect the cost and the choice of the cost minimizing levels of capital and labor that can produce a given level of output?
- To study this, we need to understand first the iso-cost line (iso means same)
- $C = wL + rK$
- We also need to know what is the slope of this iso cost line
- We will also need the concept of MRTS for iso-quant that we did in the previous chapter. And understand the condition  $MRTS = -w/r$  (for cost minimization)

## Manager's problem

- Draw an isoquant for  $q = 100$ , show various input combinations that can produce this output level, for instance, point  $x = (50, 100)$ ,  $y = (24, 303)$  point  $B = (116, 28)$ . Compute the costs for A and for B. Pose the problem:  $x$ ,  $y$  and  $z$ , all can produce  $q = 100$ , which of these the manager should choose? Obviously the one with lower cost.
- $x$ ,  $y$ , and  $z$  are only two choices on the isoquant. But there are infinite number of points on the isoquant, each such combination will produce the output level  $q = 100$ , how do choose one that cost the least?
- Iso-cost line makes this choice easier.
- Iso-quant next few slides)

## Table 7.3 Bundles of Labor and Capital that Cost the Firm \$100

Suppose the wage rate  $w = \$5$  and rental rate  $r = \$10$

**Table 7.3 Bundles of Labor and Capital That Cost the Firm \$100**

Bundle	Labor, $L$	Capital, $K$	Labor Cost, $wL = \$5L$	Capital Cost, $rK = \$10K$	Total Cost, $wL + rK$
<i>a</i>	20	0	\$100	\$0	\$100
<i>b</i>	14	3	\$70	\$30	\$100
<i>c</i>	10	5	\$50	\$50	\$100
<i>d</i>	6	7	\$30	\$70	\$100
<i>e</i>	0	10	\$0	\$100	\$100

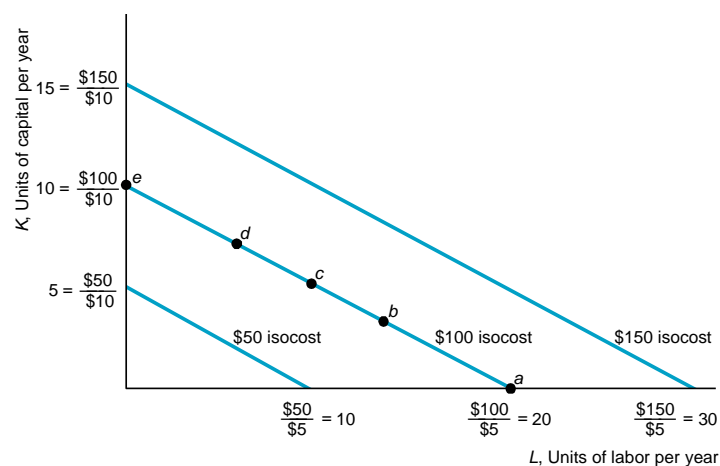
Equation: If we use  $K$  units of capital and  $L$  units of labor then the total cost is  $C = wL + rK$ . Suppose we plot all those combinations of  $L$  and  $K$  that cost the same \$100, we have one iso cost corresponding the cost level  $C=100$ . We get other iso-cost lines for other cost levels. Remember, the slope of iso-cost lines depend on the ratio of the  $w/r$ .

## Steps:

- First draw an isocost line corresponding to  $C = 100$ , and fixed wage rate  $w = \$5$  and rental rate  $r = \$10$ . The equation of this iso-cost line is
- $100 = 10K + 5L$
- It is an equation of a line. To draw this line and study its properties, express it in the familiar form ( $y = mx + b$ ), i.e.,
- $K = (-5/10)L + 100/10$
- For the general case  $C = rK + wL$ , this becomes
- $K = (-w/r)L + C/r$
- What is the slope? Intercept etc. How does the iso-cost line change as you change  $w$  keeping everything else fixed?

## Figure 7.4:A Family of Isoquant Lines

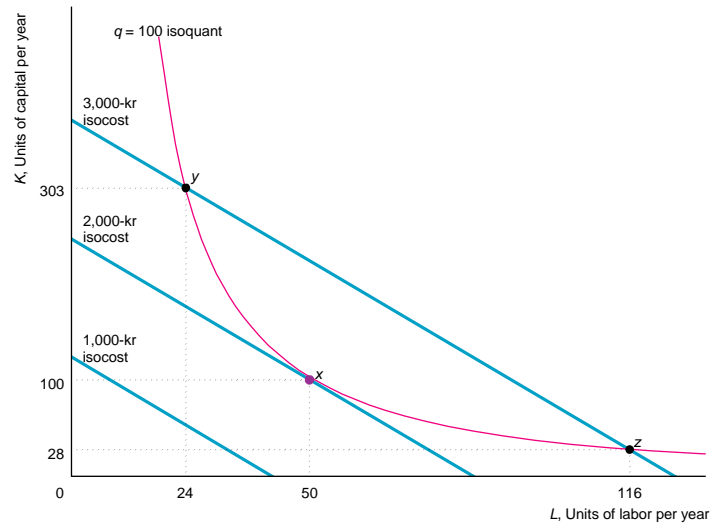
Suppose the wage rate  $w = \$5$  and rental rate  $r = \$10$





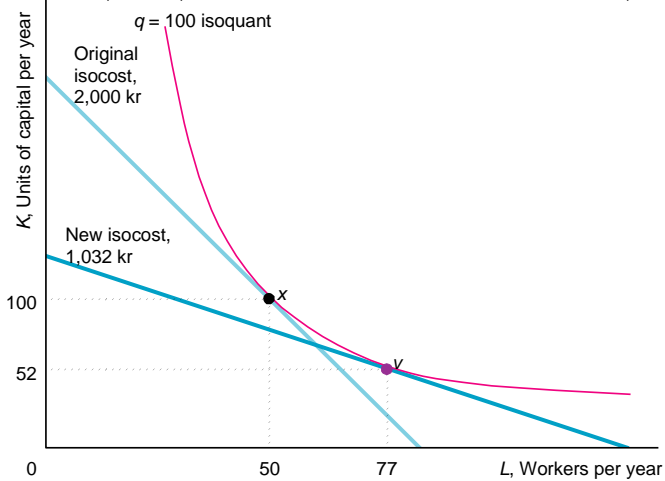
## Figure 7.5 Cost Minimization

Now I am going to show you how to choose the cost minimizing Output level  $q = 100$ , using the isoquant for  $q = 100$  and iso-cost lines in the following diagram.

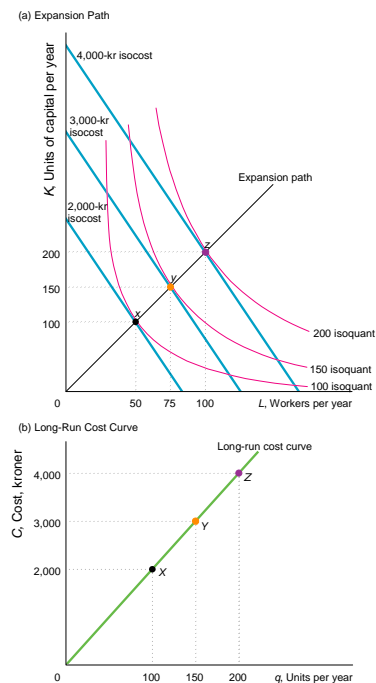


## Figure 7.6 Change in Factor Price

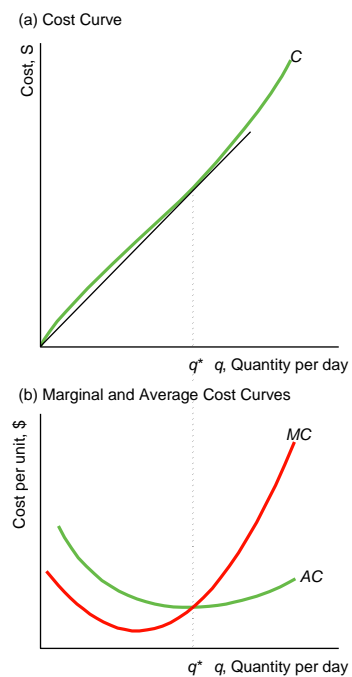
Suppose we still want to produce the output level  $q = 100$ . Now I am going to show you how to your cost minimizing input choices and also the total cost change when wage rate  $w$  falls from \$24 to \$8 and rental rate remains constant at  $r = \$8$ .



**Figure 7.7**  
Expansion Path  
and Long-Run  
Cost Curve



**Figure 7.8**  
Long-Run  
Cost Curves



## Table 7.4 Returns to Scale and Long-Run Costs

Output, $Q$	Labor, $L$	Capital, $K$	Cost, $C = wL + rK$	Average Cost, $AC = C/q$	Returns to Scale
1	1	1	12	12	
3	2	2	24	8	Increasing
6	4	4	48	8	Constant
8	8	8	96	12	Decreasing

$w = r = \$6$  per unit.

## Figure 7.9 Long-Run Average Cost as the Envelope of Short-Run Average Cost Curves

