R&D Investment in Evolving Market Conditions

by
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Abstract:

This paper formulates a dynamic model of R&D investment of private firms operating in evolving market conditions. Using the dynamic programming approach, the paper derives the closed form optimal R&D investment rule in the presence of exogenous variables that Granger cause the marketing environment. Conditions for identification and econometric techniques for estimation of the structural parameters are given. The paper provides a tractable approach to evaluation of policies regarding R&D subsidies, firm size, market concentration that is free from the Lucas critique on econometric policy evaluation.

Keywords: *Research and development, Rational Expectations, Granger Causality.*

JEL Classification Numbers: D21, L1

1.   Introduction

 The in-house R&D investment of a private firm produces a process or a product innovation. The timing of an innovation from a given R&D investment is uncertain and it depends on the R&D capability of the firm and the scientific complexity of the innovation that the firm pursues. The profitability of a firm's new innovation depends on market conditions such as how competitive the firm's industry is, the size of the firm, the strength of patent protection and the ease of imitation of the innovation by other rival firms. Government policies also affect these market conditions. These market conditions evolve over time and influence the profitability of a firm's R&D investment. Thus the Lucas critique [1976] on policy evaluation applies to R&D investment. The main point of the critique in the present context is that if a firm's R&D investment decision under uncertainty depends upon its expectations about the future market conditions and policy changes, then, instead of estimating a R&D decision rule by throwing in arbitrarily some policy variables as regressors, one should model and estimate the parameters of the firm's objective function and the stochastic processes that govern the future environments in which the firm operates.

Hansen and Sargent [1981] pioneered such a line of research by modeling the US aggregate labor supply decisions over time for a representative agent. They gave up the dynamic programming approach for solving their cost-of-adjustment model of labor supply decisions, and followed the variational approach that uses Euler equation, Transversality condition and the Wiener-Kolmogorov prediction formula to compute a close form optimal decision. I have shown in Raut [2004] that it is possible to derive a close form solution following the dynamic programming approach and using only the matrix Riccati equation. The Wiener- Kolmogorov prediction formula is not required for this purpose. I have also shown that in the linear quadratic problems, the

hypothesis of rational expectations impose cross equations parameter restrictions which are useful to identify some of the structural parameters but not all. To identify all structural parameters, further structure is needed. More specifically, I have shown that when the firm's environment variables follow first order autoregressive processes, the cross equations restrictions imposed by the rational expectation hypothesis help to identify only a subset of the structural parameters. When some of the environmental variables are assumed to follow autoregressive processes of orders higher than two, not only that all the structural parameters are identified from the cross equation restrictions, there are also over-identifying restrictions that could be used for more efficient estimation of the parameters and for testing of the model. I showed these in Raut [2004] under the assumption that technological knowledge is observable. In this paper, I extend the analysis to the case when technological knowledge is imperfectly observed and when there are other exogenous variables that Granger cause the firm's environment variables.

There are many hypotheses about factors that affect the accumulation process of technological knowledge. Some of these factors are as follows:

(1) Firm size, intensity of rivalry or competition (Schumpeter [1934, 1950]).
(2) Complete uncertainty about the profitability of a new product, if it is a product innovation, and partial uncertainty about the shifts in demand for the product if it is a process innovation (Schmookler [1966]).
(3) The R&D capabilities or the "strength of knowledge" for efficient R&D search for firms having different "science bases" (Rosenberg [1976], Nelson [1982], Nelson and Winter [1977,1978], Evenson and Kislev [1976]).
(4) Government policies such as R&D tax credits, anti-trust policies to restrict monopoly power of a firm, licensing schemes, patent laws, and investments in basic research affecting (1)-(3) above.

Theoretical models of R&D have considered most of the above factors and studied the effect of government policies regarding R&D subsidies, market concentration, and firm size (Dasgupta and Stiglitz [1980a&b] among others, see Kamien and Schwartz [1981] for a survey of these papers). The empirical research, on the other hand, has been carried out mainly in two lines, ignoring many of the above aspects of R&D. One set of studies is concerned with testing the Schumpeterian hypothesis regarding the effects of firm size and intensity of rivalry on the pace of R&D investments within a static framework (see Levin and Reiss [1984], Kamien and Schwartz [1981] for an account of these studies). The other set of studies is concerned with the effect of R&D investment on productivity growth (Griliches [1984], Mohnen [1992], Mairesse and Sassenou [1991], for accounts of studies on developed countries, and Raut [1995] for an account of studies on developing countries). Although many studies are directed toward policy analysis, these studies do not formulate R&D investment decisions using a dynamic economic model and then estimate the model parameters.[1]

The rest of the paper is organized as follows: In section 2, I formulate the R&D investment decision problem of a private firm as a dynamic programming problem. In section 3, I derive explicitly the optimal R&D decision rule with cross equation restrictions. In section 4, I use these restrictions and the optimal decision rule to address the identification issues. In section 5, I

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[1]Pakes [1984], however, goes a step closer in this direction; instead of deriving the reduced form solution with cross equations restrictions imposed by rational expectations, he, however, parameterizes the reduced form solution for estimation.

introduce exogenous variables that Granger cause the market conditions of the firm. In section 6, I deal with issues related to unobserved technological knowledge. In section 7, I discuss the econometric estimation and testing issues.

2.   The Basic Model

Technological knowledge has been conceptualized in the literature in many ways. For instance, Arrow[1962] treats technological knowledge as information about the states of nature and the role of R & D is to acquire more knowledge about the states of nature to improve one's subjective beliefs about the possibility of reaping an innovation. Nelson [1982] views technological knowledge as "capability for efficient search" and R&D helps to search for a given target, say for instance, a product innovation or a process innovation. The main point of this notion of technological knowledge is that it is the strength of knowledge that determines how much R&D efforts are expected to be successful as opposed to Schmookler's viewpoint [1966] that the pay-off determines R&D investments. Griliches [1979,1984] treated the stock of technological knowledge as one of the factors of production, analogous to stock of physical capital. Like capital stock, it depreciates and becomes obsolete over time, but can replenish over time with R&D investments.

My definition of technological knowledge draws from all three notions. I view accumulation of technological knowledge as acquisition of more information about the states of nature related to product improvements or process improvements. I also view it as a deliberate economic activity similar to investment in physical capital. R&D input adds to the stock of knowledge which might be immediately used or might be useful for further information production. However, unlike in the case of investment in physical capital, I assume here that the marginal rate, b, at which a unit of R&D adds to the stock of knowledge varies from industry to industry depending on the R&D capability or strength of knowledge or the science base of that industry. There are various sources for spillover effects, e.g., government's investment in basic research, technological knowledge of other domestic or foreign firms, the strength of which depends on the patent law. I assume that all these constitute a constant amount of spillover knowledge in each period. More formally,

(1) $z_{1t+1} = a_1 z_{1t} + bR_t + c_1 + w_{1t}, \ t \geq 0$

where

z_{1t} = our firm's stock of knowledge at the beginning of period t

R_t = R & D investment of the firm in period t

$1 - a_1$ = depreciation rate for knowledge

b = technological capability or a measure of strength of knowledge

c_1 = a constant measuring the spill-over effect

w_{1t} = a random shock in period t.

The specification (1) of the technology knowledge production is general enough to encompass various empirical findings on differential lagged effects of R&D investments on production of knowledge.

Technological knowledge is an intangible, indivisible, inappropriable, i.e., difficult to

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institute a property right on, and involves externalities in production and its use. Following an analogous strategy in statistical decision theory, I impute an indirect private value to a stock of technological knowledge in the following way:

The timing of innovation is uncertain, but the likelihood of its taking place in any period is higher, the greater is the stock of accumulated knowledge in the beginning of that period. Let $P(z_{1t})$ be the probability that the firm will reap the innovation in period t if its stock of knowledge is z_{1t}, given that it has not achieved it yet. I assume that $P(z_{1t})$ takes the following form:

(2) $$P(z) = \frac{1}{\mu - \gamma} \cdot (\mu z - \gamma z^2), \ \ 0 < z \le 1, \ \ \mu > \gamma > 0.$$

The value of an innovation at time *t* depends on firm size z_{2t}, intensity of rivalry z_{3t}, and the market condition, or the profitability from the current line of research ξ_t. Market concentration or intensity of rivalry is an industry level attribute. More rivals in an industry lead to a higher chance for imitation of an innovated product or process, and also to a higher chance for another firm's innovation to arrive before the current innovation has reaped its maximum monopoly rent. Furthermore, more rivals in an industry may reduce the market share of a firm. All these factors lead to a lower value for an innovation, the higher is the intensity of rivalry.[2]

The effect of firm size on the value of technological knowledge may come through different channels. Following Nelson's [1959] interpretation, I argue that the larger firms having already established name and reputation in the market can appropriate higher benefits from an innovation. Therefore, a larger firm envisages a bigger return from a given stock of technological knowledge than a smaller firm.[3]

Another important factor in the determination of value of technological knowledge is the completely unknown demand for new products in the case of product innovation and the shift in the demand in the case of process innovation. Higher are these uncertainties, the lower will be the value of technological knowledge. This is sometimes referred to as Schmookler's hypothesis or demand pull or market opportunity hypothesis. I denote the value of an innovation as a function of the environment variables at time *t* as follows:

(3) $\eta(z_{2t}, z_{3t}, \xi_t)$

For simplicity, I am assuming that (3) gives the present value at time t of the stream of cash-flows that the innovation will bring, and it depends only on the market condition prevailing at that time, but not on the future market conditions. For instance, this will be the case if the innovation is patented and sold to another firm for an amount of royalty payments, whose value is

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[2] Also greater monopoly power reduces the incentive for innovation as the firm with monopoly power can continue to earn the monopoly rent without venturing into a new technological innovation. It is generally argued that an intermediate level of market concentration is most conducive to rapid technological innovation.

[3] It should, however, be noted that the smaller firms are not necessarily restricted to use their knowledge only in their own production units as they can always sell it to another firm with licensing arrangements.

determined by the market conditions prevailing then.[4]

Let R_t be the R&D input used in period t. Assume that the cost of R&D is quadratic in input use. One period expected reward from a stock of knowledge z_{1t} in period t is then given by

(4) $\quad v(z_{1t}) = \eta(z_{2t}, z_{3t}, \xi_t)P(z_{1t}) - 0.[1 - P(z_{1t})] - \theta R_t^2$

plus a stock of technological knowledge, z_{1t+1} as given by (1).

Assume that after reaping the targeted innovation, the firm will venture into another innovation that will use the knowledge of the previous pursuit. The firm then faces an infinite horizon for its R&D investment decisions. Given the sequences, $\{z_{2t}\}$, $\{z_{3t}\}$, and $\{\xi_t\}$, that characterize the environment facing the firm, the expected value of a sequence of technological knowledge $\{z_{1t}\}$ obtained by using the sequence of R&D investments $\{R_t\}$ is given by

(5) $\quad V_0 = E_0 \sum_{t=0}^{\infty} \beta^t \left(\eta(z_{2t}, z_{3t}, \xi_t)P(z_{1t}) - \theta R_t^2 \right)$

where $E_t(x)$ denotes the conditional expectation of x given information set Ω_t. I assume the following linear specification for the reward function.

(6) $\quad \eta(z_{2t}, z_{3t}, \xi_t) = r_0 + r_1\xi_t + r_2 z_{2t} + r_3 z_{3t}$

where $r_0, r_1, r_2 > 0$ and $r_3 < 0$. Substituting (2) in (6) and disregarding the terms with powers greater than two, one gets

$$\eta(z_{2t}, z_{3t}, \xi_t) \cdot P(z_{1t}) = (r_0 + r_1\xi_t + r_2 z_{2t} + r_3 z_{3t}) \cdot \frac{\mu}{\mu - \gamma} \cdot z_{1t} - \frac{r_0\gamma}{\mu - \gamma} \cdot z_{1t}^2$$

Substituting the above in equation (4) and regarding $r_0 + r_1\xi = \zeta$, equation (4) can be rewritten as,

(7) $\quad v(z_{1t}) = Z_t^{'} Q Z_t + H R_t^2$

where, $Q = (q_{ij})_{i,j=1,...4}$, $\quad q_{11} = -r_0\gamma/(\mu - \gamma), q_{12} = r_2\mu(\mu - \gamma), q_{13} = r_3\mu/(\mu - \gamma)$
$q_{14} = \mu/(\mu - \gamma)$ other q_{ij}'s are zero, $H = -\theta$ and $Z_t = (z_{1t}, z_{2t}, z_{3t}, \zeta_t)'$. I further assume that z_{2t}, z_{3t}, and ζ_t follow a first[5] order auto-regressive process given by

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[4] Kamien and Schwartz [1981], explicitly modeled rivalry using a subjective hazard function, and then derived a functional relationship between rivalry and the present value of an innovation.

[5]In a later section I consider higher order auto-regressive processes.

$$z_{2t+1} = a_2 z_{2t} + c_2 + w_{2t}$$

(8) $\quad z_{3t+1} = a_3 z_{3t} + c_3 + w_{3t}$

$$\zeta_{t+1} = a_4 \zeta_{4t} + c_4 + w_{4t}$$

where (w_{2t}, w_{3t}, w_{4t}) is a three dimensional vector of white noise processes. Writing (1) and (8) together, one has,

(9) $\quad Z_{t+1} = A Z_t + B R_t + c + w_t$

where

$$A = \begin{pmatrix} a_1 & 0 & 0 & 0 \\ 0 & a_2 & 0 & 0 \\ 0 & 0 & a_3 & 0 \\ 0 & 0 & 0 & a_4 \end{pmatrix}, \quad B = \begin{pmatrix} b \\ 0 \\ 0 \\ 0 \end{pmatrix}, \quad c = \begin{pmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \end{pmatrix}, \quad w_t = \begin{pmatrix} w_{1t} \\ w_{2t} \\ w_{3t} \\ w_{4t} \end{pmatrix}$$

I assume w_t to be a 4-dimensional Gaussian process with mean 0 and variance-covariance matrix Σ. Notice that I am assuming here for simplicity that R&D activities affect neither the firm size nor the intensity of rivalry. While this assumption is innocuous in the short-run,[6] for a medium to long-run analysis this may not be the case, see Landes [1969] for historical evidence.

Assume that the manager of the firm knows the parameters of his objective function in equation (5) and the parameters of the stochastic processes in equation (9). At the beginning of each period t, he observes the realization of the variables in his information set, Ω_t. The variables in his information set include any stochastic process that Granger causes either z_{1t}, z_{2t}, z_{3t}, or ζ_t. I assume for now that the manager of the firm can observe z_{1t} and the information set Ω_t does not contain any variables. In a later section, I deal with the case of unobserved technological knowledge.

The R&D investment choice problem of the manager can be treated precisely as follows: Given Ω_t in period t, he chooses a R&D investment R_t so as to maximize (5) subject to (9). To solve this problem using dynamic programming approach, the corresponding Bellman's equation for a slightly general case is given by,

$$(10)\, V_t(Z_t) = \max_{R_t} \left(Z_t^{'} Q Z_t + q' Z_t + R_t^{'} H R_t + h' R_t + \beta E_t V_{t+1}(A Z_t + B R_t + c + w_t) \right)$$

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[6]However, see Levin [1981], and Levin and Reiss [1984] for studies of the simultaneity of R&D expenditures and market concentration in a static framework.

In the present case, q and h are zero vectors. Solution of (10) gives the optimal R_t as a function of Ω_t. The next two sections closely follow Raut [2004]. I derive the close form solution to the above problem in the next section.

3.   Optimal R&D Investment Decision Rule

Following Bertsekas [1976], and Chow [1975,1981], it is easy to show that under certain conditions[7] on A, B, c, Q, and H, there exists an optimal stationary solution to (10) which is given by

(11) $R_t = -(GZ_t + g)$

where

(12) $G = \beta(H + \beta B'KB)^{-1}B'KA$

(13) $g = (H + \beta B'KB)^{-1}\left(\beta B'Kc + \dfrac{\beta}{2}B'k + \dfrac{h}{2}\right)$

K is a positive definite solution of the following matrix Riccati equation:

(14) $K = Q + \beta A'[K - \beta KB(H + \beta B'KB)^{-1}B'K]A$

and k is the solution to the following vector Riccati equation:

(15) $k'(I - \beta(A - BG)) = q' + 2g'(H + \beta B'KB)G + 2\beta c'K(A - BG) - h'G - 2\beta g'B'KA$

Note that the vector Riccati equation (15) involves K, G and g, whereas the matrix Riccati equation K does not involve k and g. To find the explicit optimal decision rule from equation (11), note that

$$\beta B'KB + H = b^2\beta k_{11} + \theta$$

and

$$B'KA = (bk_{11}a_1,...,bk_{14}a_4)$$

Substituting these in equation (11), one gets

(16) $R_t = -\dfrac{b\beta}{b^2\beta k_{11} + \theta}(k_{11}a_1z_{1t} + k_{12}a_2z_{2t} + ... + k_{14}a_4z_{4t}) - g$

Compute k_{11}, $k_{12},....k_{14}$ from the Riccati equation (14) as follows:

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[7]These conditions are controllability and observability as stated in Bertesekas [1976].

$$K \quad = \quad A^{'} \left[\beta K - \frac{\beta^2 KBB'K}{\beta b^2 k_{11} + \theta} \right] A + Q$$

$$= \quad A'DA + Q$$

(17)

$$= \quad \begin{pmatrix} a_1^2 d_{11} & a_1 d_{12} a_2 & \ldots & a_1 d_{14} a_4 \\ & & & \\ \ldots & \ldots & \ldots & \ldots \\ & & & \\ a_4 d_{42} a_1 & a_4 d_{42} a_2 & \ldots & a_4^2 d_4 \end{pmatrix} + Q$$

where

$$D \quad = \quad \left(d_{ij} \right)_{i,j=1,2,3,4}$$

$$= \quad \beta K - \frac{\beta^2 KBB'K}{\beta b^2 k_{11} + \theta}$$

(18)

$$= \quad \beta K - K \begin{pmatrix} m & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} K$$

where $m = \frac{\beta^2 b^2}{\beta b^2 k_{11} + \theta}$. It is now easy to compute d_{ij} 's as follows:

$$d_{11} = \beta k_{11} - m k_{11}^2 = \frac{\theta \beta k_{11}}{b^2 \beta k_{11} + \theta}, \text{ after substituting the value of } \boldsymbol{m}$$

$$d_{12} = \beta k_{12} - m k_{11} k_{12} = \frac{\theta \beta k_{12}}{b^2 \beta k_{11} + \theta}, \text{ after substituting the value of } \boldsymbol{m}$$

....

Substituting these in the right hand side of the last equality in the expressions (17) and then equating the matrix elements of both sides, one gets the following:

(19) $\quad k_{11} \left(1 - \frac{a_1^2 \theta \beta}{b^2 \beta k_{11} + \theta} \right) = q_{11}, i.e., k_{11}(1 - a_1 \lambda) = q_{11}$

(20) $\quad k_{12} \left(1 - \frac{a_1 a_2 \theta \beta}{b^2 \beta k_{11} + \theta} \right) = q_{12}, i.e., k_{12}(1 - a_2 \lambda) = q_{12}$

where, $\lambda = \frac{a_1\theta\beta}{b^2\beta k_{11}+\theta}$. Substituting these in equation (16) and simplifying the expression (13) in a similar fashion, the close form decision rule (21) in the following proposition obtains.

Proposition 1: *A closed form solution to the firm's problem is given by*

(21) $\quad R_t = -g + \alpha_1 Z_{1t} + \alpha_2 Z_{2t} + \alpha_3 Z_{3t} + \alpha_4 \zeta_t$

where,

$$g = \frac{b\lambda}{a_1\theta(1-\lambda)} \sum_{j=1}^{4} c_j (1 - a_j\lambda)^{-1} q_{1j}$$

$$\alpha_i = -\frac{b\lambda}{a_1\theta} a_i (1 - a_i\lambda)^{-1} q_{1i}, \quad i = 1,...4$$

$$\lambda = \frac{a_1\theta\beta}{b^2\beta k_{11} + \theta}$$

and k_{11} is a positive solution of the quadratic equation:

$$k_{11}\left(1 - \frac{\theta\beta a_1^2}{b^2\beta k_{11} + \theta}\right) = q_{11}$$

Equation (21) and the system of equations (9) for the motion of the environment constitute the firm's decision rule. The assumption of rational expectations and a particular specification of the stochastic processes in equation (9) have generated cross equations parameter restrictions in the decision function of the firm. These restrictions are generally used for identification of the structural parameters and also for testing the rational expectations hypothesis assuming the model (9) is correct or for testing the specification of the model (9) assuming the rational expectations hypothesis is correct. I shall take up the identification issues in the next section and the estimation and testing issues in a later section.

For identification of structural parameters, the environment variables must follow higher order autoregressive processes than what I have assumed. I now extend the above analysis by assuming a third order autoregressive process for the environment variables and then discuss how the structural parameters can be identified for this model. Assume that

$$
\left.
\begin{aligned}
Z_{1t+1} &= a_1 Z_{1t} + bR_t + c_1 + w_{1t} \\[2ex]
Z_{2t+1} &= \delta_2(L)Z_{2t} + c_2 + w_{2t} \\[2ex]
Z_{3t+1} &= \delta_3(L)Z_{3t} + c_3 + w_{3t} \\[2ex]
\zeta_{t+1} &= \delta_4(L)\zeta_t + c_4 + w_{4t}
\end{aligned}
\right\}
$$

(22)

where
$$
\delta_i(L) = a_{i1} + a_{i2}L + a_{i3}L^2, \, i = 2,3,4
$$

and L is the lag operator, i.e., $LX_t = X_{t-1}$. Note that Z_{1t} is assumed to have the same form as before. For the above system, one can derive the optimal solution by expanding the state space Z_t to contain all the lag values of Z_t's and extending the definition of A, Q, B and c appropriately, see Raut [2004] for details. One obtains the following proposition:

Proposition 2: *A closed form solution of the optimization problem (10) subject to the law of motion of the environment (22) is given by*

(23)
$$
R_t = -g + \alpha_1 Z_{1t} + \alpha_2(L)Z_{2t} + \alpha_3(L)Z_{3t} + \alpha_4(L)\zeta_t
$$
where

(24)
$$
g = \frac{b\lambda}{a_1\theta(1-\lambda)}\left[\frac{c_1 q_{11}}{1 - \lambda a_1} + \sum_{j=2}^{4}\frac{c_j q_{1j}}{1 - \lambda\delta_j(\lambda)}\right]
$$

(25)
$$
\alpha_1 = -\frac{b\lambda}{a_1\theta}a_1(1 - a_1\lambda)^{-1}q_{11}
$$

(26)
$$
\lambda = \frac{a_1\theta\beta}{b^2\beta k_{11} + \theta}
$$

and k_{11} is a positive solution of the quadratic equation:

(27)
$$
k_{11}\left(1 - \frac{a_1^2\theta\beta}{b^2\beta k_{11} + \theta}\right) = q_{11}
$$

(28)
$$
\begin{aligned}
\alpha_j(L) &= -\frac{b\lambda}{a_1\theta} \cdot \frac{a_{j1} + a_{j2}(\lambda + L) + a_{j3}(\lambda^2 + \lambda L + L^2)}{1 - a_{j1}\lambda - a_{j2}\lambda^2 - a_{j3}\lambda^3} \cdot q_{1j} \\[2ex]
&= -\frac{b\lambda}{a_1\theta}q_{1j}\sum_{i=0}^{2}\frac{\left(\delta_j(\lambda)/\lambda^i\right)_+}{1 - \lambda\delta_j(\lambda)}L^i, \quad j = 2,3,4
\end{aligned}
$$

$()_+$ *denotes the **annihilation operator** that tells us to ignore negative powers of L.*

Hansen and Sargent [1981] gave a similar close form solution using Wiener - Kolmogorov prediction formula. Following the dynamic programming approach, however, I have derived equations (23) and (24) directly from the matrix Riccati equation of the problem.

4.   Identification of Parameters: Need for More Lags

The structural parameters are $\theta, \beta, q_{1j}, \ j = 1, 2, ...4)$ from the objective function, $\sum, b, a_1, a_{ij}, c_i, i = 2...4, j = 1..3$ from the stochastic processes (22). The second set of coefficients could be estimated from the system of equations (22). The estimation of the decision rule (22) will give the estimates for the reduced form parameters g, and α_1, and $\alpha_{ij}, i = 2...4, j = 1..3$. Rewriting the expressions for α_{21}, α_{22} and α_{23} from equation (28), we have

(29) $\alpha_{21} = -\dfrac{b\lambda q_{12}}{a_1\theta} \cdot \dfrac{\delta_2(\lambda)}{1 - \lambda\delta_2(\lambda)}$

(30) $\alpha_{22} = -\dfrac{b\lambda q_{12}}{a_1\theta} \cdot \dfrac{a_{22} + \lambda a_{23}}{1 - \lambda\delta_2(\lambda)}$

(31) $\alpha_{23} = -\dfrac{b\lambda q_{12}}{a_1\theta} \cdot \dfrac{a_{23}}{1 - \lambda\delta_2(\lambda)}$

There are similar expressions for α_{ij}, $i = 3, 4$ and $j = 1..3$. Substituting the values of α_{23} from equation (31) and analogous values of α_{33}, and α_{43} in equation (24) we have the following:

(32) $c_1\alpha_1 / a_1 + \displaystyle\sum_{j=2}^{4} c_j\alpha_{j3} / a_{j3} = (1 - \lambda)g$

Note that from equation (32), one can get an estimate of λ. Substituting the value of q_{11} from equation (25) in equation (27) and then substituting the value of k_{11} in equation (26) one gets

(33) $\lambda = \dfrac{a_1\beta}{1 + b^2\beta\alpha_1 / \lambda}$

From equation (33) one can estimate β..

Also note that substituting in equation (24) the values of q_{11} from equation (25), q_{12} from equation (30) and q_{13} and q_{14} from the equations that parallel equation (30), one can get an estimate of θ. Now from equation (30), one gets q_{12}, and from equations parallel to equation (30) for Z_{3t} and ζ_t, one can estimate q_{13} and q_{14}. Finally, from equation (25) one can estimate q_{11}. So, all the structural parameters could be recovered in this case. Note that I have never used the equations that are parallel of equations (29) and (30) corresponding to the other two

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variables, Z_{3t}, ζ_t in this identification strategy. These are over-identifying restrictions across equations which could be used to test the validity of the model or to design more efficient GMM estimators.

5. Granger Causality and Choice of Exognous Variables in Ω_t

So far implicitly I have assumed that Ω_t contains only Z_{1t}, Z_{2t}, Z_{3t}, ζ_t and their lag values. In fact, Ω_t should include all observable variables that Granger cause either Z_{1t}, Z_{2t}, Z_{3t}, or ζ_t. In this section, I consider the nature of the close form solution and the cross equations restrictions for this case. For expositional ease, I continue to assume third order auto-regressive processes for Z_{2t}, Z_{3t} and ζ_t. Assume that there is an extra stochastic process X_{2t} which Granger causes Z_{2t} and which is related to Z_{2t} process as follows

(34)
$$
\left.
\begin{array}{rcl}
Z_{2t+1} & = & a_2(L)Z_{2t} + \mu_2(L)X_{2t} \\[2em]
X_{2t+1} & = & a_2'(L)Z_{2t} + \mu_2'(L)X_{2t}\zeta_{t+1}
\end{array}
\right\}
$$

where
$$a_2(L) = a_{21} + a_{22}L + a_{23}L^2$$
and
$$\mu_2(L) = m_{21} + m_{22}L + m_{23}L^2.$$
Similarly $a_2'(L)$ and $\mu_2'(L)$ are defined.

Assuming the same first order autoregressive processes for Z_{1t}, Z_{3t}, ζ_t as before, and expanding the state space variable Z_t to accommodate X_t and its lags and appropriately modifying the matrices, A, B, C and using the matrix Riccati equation, the close form solution can be shown to be as stated in the following proposition:

Proposition 3: *A close form solution to the firm's problem is given by*

(35) $\quad R_t = -g + \alpha_1 Z_1 t + \alpha_2(L)\begin{pmatrix} Z_{2t} \\ X_{2t} \end{pmatrix} + \alpha_3(L)Z_{3t} + \alpha_4(L)\zeta_t$

where g, α_1, $\alpha_3(L)$ and $\alpha_4(L)$ are as in (23), and $\alpha_2(L)$ is given by

$$\alpha_2(L) = \left(\alpha_{21} + \alpha_{22}L + \alpha_{23}L^2 \quad \alpha_{21}' + \alpha_{22}'L + \alpha_{23}'L^2 \right)$$

$$\alpha_{21} = -\frac{\lambda b}{a_1 \theta} \cdot \frac{a_2(\lambda) + a_2'(\lambda)\rho(\lambda)}{\psi(\lambda)} q_{12}$$

$$\alpha_{22} = -\frac{\lambda b}{a_1 \theta} \cdot \frac{a_{22} + a_{22}'\rho(\lambda)}{\psi(\lambda)} q_{12}$$

$$\alpha_{23} = -\frac{\lambda b}{a_1 \theta} \cdot \frac{a_{23} + a_{23}' \rho(\lambda)}{\psi(\lambda)} q_{12}$$

$$\rho(\lambda) = \frac{\lambda \mu(\lambda)}{1 - \lambda \mu'(\lambda)}$$

$$\psi(\lambda) = 1 - a(\lambda) - a'(\lambda)\rho(\lambda)$$

α_{2j}' *'s have the similar expression as* α_{2j} *'s after we replace* a_{2j} *'s and* a_{2j}' *'s respectively by* m_{2j} *'s and* m_{2j}' *'s.*

The identification of the structural parameters in this case follows the same steps as in the previous model. Notice that the above could be generalized for higher order auto-regressive processes and for other Z-variables easily.

6.   Unobserved Technological Knowledge

I now extend the analysis to the case of unobserved Z_{1t}. I assume that there is a set of "noisy measurements", X_t 's for Z_{1t}. Denote by $\hat{Z}_{1t} = E(Z_{1t} \mid X_t)$. It is well known in the control theory literature that the same closed form solutions hold if we replace Z_{1t} by $\hat{Z}_{1t}$ in any of the equations (21), (23) or (35). The problem still remains, however, how to evaluate $E(Z_{1t} \mid X_t)$.

Two approaches could be followed to estimate $E(Z_{1t} \mid X_t)$. One approach, popular in the optimal control literature, is based on the Kalman-filtering formula (see Chow [1981] for an exposition of Kalman filtering). While this is an appropriate approach, it assumes that initial stock of knowledge is known, which is rather a strong assumption if one would like to use panel data. Even when one could obtain some noisy estimates of the initial stock of technological knowledge, Kalman filtering algorithm when combined with an algorithm of maximum likelihood estimation of the structural parameters become highly non-linear and may not converge.

An alternative approach followed by approach followed by Griliches [1979] and Pakes and Griliches [1984] is to take changes in the stock of knowledge at time *t* as the weighted sum of past five years' R & D investments and then to relate it to the number of patents applied for by the firm in any period. Although, their purpose was not the estimation of stock of knowledge, their method could be adopted to generate an empirical measure of knowledge up to a scale factor. From their productivity analysis, one could get a direct estimate of the weights for different lags of R&D expenditures and hence a measure of knowledge with measurement errors (see Griliches [1979]). Following this line of research, I postulate that

(36) $Z_{1t} = \beta' X_t + u_t$

where β is a vector of regression coefficients and X_t includes past R & D expenditures, and other technological variables such as royalty and technical fee payments domestically and abroad, the number of scientific and engineering personnel, etc., and $u_t \sim iid(0, \sigma^2)$. The stock of knowledge variable Z_{1t} is not observed, what one observes is P_t the total number of patents the

firm has applied for up to period t. Assume that

(37) $P_t = k$ if and only if $\alpha_k < Z_{1t} < \alpha_{k+1},$

 where $k = 0, 1, 2, ...m$ (a large positive number), $\alpha_0 = 0,\quad \alpha_j \le \alpha_{j+1},\ j = 1, 2,...m$, and $\alpha_{m+1} = \infty$.

        Let F be the distribution function of u. From equations (36) and (37), it follows that

(38) $\text{Prob}\{\ P_t = m\ \} = F(\alpha_{m-1} - \beta^{'} X_t) - F(\alpha_m - \beta^{'} X_t)$

 These models are known as ordered qualitative response models (see Amemiya [1985] and Maddala [1983] for more about these models).

        One can use Logit or Probit specification to estimate equation (38). The parameter β in equation (38) could, however, be estimated only up to a scale factor, i.e., one can estimate only β / σ .

7.   Estimation and testing of the model

I discuss the estimation and testing issues for the model (22) and (23). Equation (23) is not a regression equation since it does not have an error term. An error term naturally arises, however, as follows: View ζ_t as a random process which is observed by the manager of the firm but not by the econometrician. Assume that $c_4 = 0$ and that ζ_t follows a first order auto-regressive process, i.e., $\delta_4(L) = a_4$. So, the disturbance term in equation (23) is

$$e_t = -\frac{b\lambda}{a_1 \theta}(1 - \lambda a_4)^{-1} a_{14}\zeta_t$$

$$= -\frac{b\lambda}{a_1 \theta}.q_{14}(1 - \lambda a_4)^{-1}(1 - a_4 L)^{-1} w_{4t}$$

 i.e.,

(39) $(1 - a_4 L)e_t = -\dfrac{b\lambda}{a_1 \theta}.q_{14}(1 - \lambda a_4)w_{4t}$

        It is clear that the error term in equation (23) follows a first order auto-regressive process. For higher order auto-regressive processes, $\delta_4(L)$, it is straightforward to derive expressions similar to equation (39). Treating estimated Z_{1t} as observed technological knowledge, one can now use the method of maximum likelihood to estimate all the parameters.

        Assuming that the model (22) is true for the Z_t - processes, the cross equation parameter restrictions could be used to test the hypothesis of rational expectations. Let L_1 be the likelihood of the sample of observations on R_t 's when α 's and g are unrestricted in equation (23). This involves estimating 9 parameters. Let L_2 be the likelihood of the sample on R_t 's when α 's and g are estimated as function of the structural parameters in equation (23). There are now 6

parameters to be estimated. Note that the Neyman - Pearson's likelihood ratio test criterion

$$-2(logL_2 - logL_1) \sim \chi^2_{9-6}$$

$9 - 6$ = number of restrictions under the hypothesis that the cross equation parameter restrictions are true. In fact, the same test could also be used for testing the specifications of the model (22), under the assumption that the rational expectations hypothesis is true.

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