

## Effect of Social Security on Fertility and Savings : An Overlapping Generations Model\*

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### ABSTRACT

This paper studies the general equilibrium effects of various social security programs on the rates of population growth and capital accumulation within an overlapping generations framework with endogenous fertility and savings. It also shows that if the rate of inter-generational transfers of income from old to young or child care cost is low, a competitive equilibrium follows a path of over-population and capital accumulation in a modified Pareto optimal sense; a social security program in such a case is Pareto improving. A fully funded system is not neutral if it is financed by child-taxes. It also shows that unlike in the case of exogenous fertility where competitive equilibrium attains steady-state only asymptotically, when fertility is endogenous it may attain a unique globally stable steady state in finite time.

*JEL Classification No* : E21, I38, J13.

### 1. INTRODUCTION

The institution of family internalizes many roles of incomplete or inefficient markets. In most developing countries, capital markets are imperfect, and social security, old-age pension schemes are non-existent. In these economies parents depend on their children for old-age support and they treat children as investment good; in such a set-up, the institution of family plays the inter-generational income transfers role of social security program. It has been argued that the economies with imperfect capital markets and no social security programs have inefficiently high rates of population growth (Neher [1971], Willis [1980], Raut [1985, 1990]). Thus introduction of a social security program will have impact on both savings and number of children and thus on the long-run growth rate of population and income.

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\*Conversations with T.N. Srinivasan and Ken Wolpin were very useful. I also benefited from the participants of seminars at the Indian Statistical Institute, and Calcutta University.

<sup>1</sup> For instance, husband and wife through marriage can internalize the absence of life insurance market (Kotlikoff and Spivak (1981)).

The studies on developed countries have focused mainly on the effects of social security on saving and labor supply taking fertility decisions as exogenous (Barro [1974, 1978], Feldstein [1974, 1978] and Kotlikoff [1979]) and found controversial effects. Being based on macro, partial equilibrium analysis, these studies (with the exception of Barro) have ignored the general equilibrium effects of social security on saving through its impact on wage rates and interest rates. The studies on developing countries have shown that social security reduces fertility by reducing the demand for children as means of old age supports (see, Swidler [1983], Hohm [1975], Entwisle and Winegarden [1984], and Gillaspay and Nugent [1983]). Due to unavailability of appropriate data, the studies on developing countries have not investigated the simultaneous effects of social security on fertility and savings, and thus on the rates of population growth and capital accumulation. The problem is particularly important for developing countries with rapidly growing population; our theoretical investigation will throw light on the issue of whether these countries could reduce their population growth rates and improve their capital labor ratios by means of introducing appropriate social security programs.

The starting point of endogenizing fertility and saving in the Samuelson [1958]-Diamond [1965] overlapping generations growth framework is the question as to what is the motive for having children and saving. Barro and Becker [1988] consider a growth model in which the motive for children and saving is that parents derive utility from children's welfare. The welfare of their children depends on the amount of bequest that the children receive from their parents, since the bequest from parents determine their earnings and the maximum welfare that they can attain with their earnings. Thus in Barro-Becker framework the motive for savings and children is altruism. Too few children or too little bequest will not be utility maximizing since by having some children and leaving some positive bequest the parents can increase their utility as a result of increase in the utility of their children. However, having too many children will reduce parent's life time consumption and hence will result in lower utility. The balance of all these opposing forces determine the number of children and the amount of savings for the purpose of bequest that parents would like to choose.

Raut [1985, 1990] considers an overlapping generations growth model in which demands for children, saving, and bequest in the human capital of children are motivated by the life-cycle permanent income hypothesis. In this framework, adults provide a fraction of their income to their old parents. This transfer is assumed to be determined by some social norms and I will talk about the mechanisms by which it is determined in a later section. Parents have three means of providing for old-age consumption: investing on skilled children, unskilled children and savings. All these become inputs to production in the next period, and they are all essential inputs in production. If parents decide not to have children and likes to save only on physical capital, then returns from children will exceed the returns from capital; in that case parents would like to have a few children. Similar is the case with investment in human capital of children. Using Cobb-Douglas utility functions, Raut studied the long-run and short-run effect of introducing a pay-as-you-go social security on growth rates of income, population and income distribution.

In the present paper we extend Raut's framework to study the general equilibrium effects of pay-roll tax financed and child-tax financed social security programs. The

main motivation for the latter kind of social security programs arises from the considerations that private and social costs of children could differ, especially in LDCs (see McNicoll [1985] and Raut [1985, 1990]). Later I consider the endogenous determination of the level of inter-generational income transfers.

In section 2, I set up the model and provide a proof for the existence and uniqueness of perfect foresight competitive equilibrium under quite general assumptions on production and utility functions. Section 3 highlights in the exogenous fertility case how the micro effects and the general equilibrium effects with production may differ. In sections 4 and 5, I study the short-run properties of the above social security programs. Section 6 studies the dynamic properties of the systems. In section 7, I derive the levels of child cost and social security income transfers that cause the decentralized economy to follow a Pareto inefficient path of over population and capital accumulation. In Section 8, I consider two approaches to endogenize the intergenerational income transfers in stationary environment. Finally section 9 concludes the paper.

## 2. THE BASIC FRAMEWORK

Consider an economy in which there are three goods in each period. One of them is producible and could be either consumed or invested for future production. The other two are two factors of production, namely, labor and capital.

I shall distinguish the consumer good, capital good, and the labor available at different dates as different commodities. Let  $K_t$ ,  $L_t$  denote the aggregate stock of capital, and labor available for production at time  $t$ . Let  $P = \{p_t = (p_t, q_t, w_t) \geq 0, t \geq 0\}$ , where  $p_t$ ,  $q_t$ ,  $w_t$  represent respectively the present value of a unit of consumer good, capital good, and labor available in period  $t$ ,  $t \geq 0$ . Let  $w_t = W_t/p_t$ , be the wage rate in terms of  $t$ -th period consumer good, i.e. the real wage rate; and let  $1 + r_t = q_t/p_t$  be the rate of return from capital in period  $t$  in terms of the consumer good of the same period.

### Firms

The production technology in the model is as follows. Capital lasts for one period and has zero scrap value. Once formed, capital cannot be consumed. In each period, the production process is represented by a production function that instantaneously transforms capital and labor into a flow of consumer goods. Let  $F(K, L)$  be the aggregate production function of the economy, where  $K$  is the stock of capital and  $L$  is the amount of labor. There is no technological change.

*Assumption A.1:*  $F$  exhibits constant return to scale and  $F(K, L) = 0$ , if  $K$  or  $L = 0$ , and  $f' > 0$  and  $f'' < 0$ , where  $f(k) = F(K, 1)$ .

The producer's problem at time  $t$  is to choose non-negative  $K_t$ ,  $L_t$  so as to maximize profit

$$p_t F(K_t, L_t) - q_t K_t - w_t L_t \quad (2.1)$$

The solution of this maximization problem yields demands for capital,  $K_t^d$ , labour,  $L_t^d$ , profit  $\Pi_t$ , and supply of total output  $Y_t = F(K_t^d, L_t^d)$ ,  $t \geq 0$ . Let us denote by

$$a^F(P) = \{(K_t^d, L_t^d) \mid (2.1) \text{ is maximized for all } t \geq 0\}$$

Note that  $a^F(P)$  is in general a correspondence. The following national income identity holds

$$p_t Y_t = q_t K_t^d + W_t L_t^d + \Pi_t \geq 0 \quad (2.2)$$

If the markets are perfectly competitive then under assumption A.1, we have

$$\Pi_t = 0, \text{ for all } t \geq 0. \quad (2.3)$$

### Households

In our economy households are of one type. Each person in this economy lives for three periods—young, adult, and old. I do not distinguish between sexes in this model. Imagine that people of different generations have an implicit joint family transfer arrangements among themselves of the following nature. When an individual is young, he is dependent on his parents and costs his parents  $\theta$  unit of consumption.<sup>2</sup> When he becomes an adult, he enjoys parenthood and participates in the labor market to support his family. He supplies one unit of labor in elastically. He gives a constant fraction of his income to his old parent who is too old to participate in the labor market and he also pays social security taxes. Out of the rest of his income, he makes a decision about how much to invest on physical capital and how many children to have in order to maximize intertemporal utility subject to budget constraints. I assume that the gestation period for capital is one generation and the firms are owned by the olds in each period. When he is old he receives the social security benefits, transfers (remittances) from his children, the returns from his capital, and profit from his firm.

Our model is an extension of the "life cycle hypothesis" for saving (Fisher [1930], Modigliani and Brumberg [1954]). According to this theory, each individual consumes his lifetime income. Over the individual's lifetime, his consumption is spread evenly, whereas his income is earned during pre-retirement years. He therefore saves and dissaves in such a way that his networth is never negative. I extend this life cycle hypothesis to include investment in number of children. This is tantamount to assuming that parent's utility depends only on their own lifetime consumption. Suppose a person is born in period  $t-1$ . We denote his utility function by  $U^*(C_t^1, C_{t+1}^2)$ , where  $C_t^1$  and  $C_{t+1}^2$  are respectively his consumption when he becomes adult in period  $t$  and old in period  $t+1$ . For most of our analysis, we will assume the utility function to be time separable as follows:

$$\text{Assumption A.2: } U^*(C_t^1, C_{t+1}^2) = U(C_t^1) + V(C_{t+1}^2), \quad U_1, V_1 > 0, \text{ and } U_{11}, V_{11} < 0 \quad (2.4)$$

<sup>2</sup> The child rearing cost in each period could be function of wage rates. I take it as constant to simplify the analysis.

where  $U_1, V_1$  are the first order and  $U_{11}, V_{11}$  are the second order derivatives of  $U$  and  $V$  respectively.

A more appropriate approach would be to postulate the utility function of the  $t$ -th generation adult to depend on his own consumption as well as the consumption of his parent who are still alive, i.e., as  $U^*(C_t^1, C_t^2, C_{t+1}^2)$ . I will consider it in section 8.

The budget constraints of a representative parent will depend upon the nature of social security taxations and benefits. We shall study two types of social security taxes, namely, payroll tax and child tax, and two types of benefits, namely, fully funded and pay-as-you-go. As our economy is closed, I will further assume that in the case of fully funded system the social security tax revenues are invested in the capital market. I consider pay-roll tax financed programs in this section. The child-tax financed programs are considered in a later section.

Assume that each person in our economy works when he is adult and retires when old. Each parent of generation  $t \geq 0$  is taxed a fraction  $\tau$  of his wage income  $W_t$  as social security taxes and when he is old, he gets social security benefits  $B_{t+1}$ . The budget constraints of a parent of generation  $t \geq 0$  are given by

$$p_t C_t^1 = (1 - a - \tau) W_t - p_t s_t - p_t \theta x_t \quad (2.5)$$

$$p_{t+1} C_{t+1}^2 = B_{t+1} + q_{t+1} s_t + a W_{t+1} x_t + \pi_{t+1} \quad (2.6)$$

where,  $\pi_{t+1} = \Pi_{t+1}/L_t$ ,  $s_t$  = savings and  $x_t$  = number of children, the actuarially fair benefits  $B_{t+1}$  are computed as follows:

$$B_{t+1} = \tau W_{t+1} x_t \quad : \text{for pay-as-you-go system} \quad (2.7)$$

$$B_{t+1} = q_{t+1} \tau W_t \quad : \text{for fully funded system} \quad (2.8)$$

In a closed economy, it is clear from (2.5), (2.6) and (2.8) that for a small  $\tau > 0$ , a fully funded system will have no effect on the national saving rate; the only effect it will have is that government saving will replace household saving, one-for-one. Therefore, we will consider only the pay-roll tax financed pay-as-you-go social security program.

A representative parent of generation  $t$  chooses non-negative  $s_t$ , and  $x_t$  to maximize his utility function (2.4) subject to the budget constraints (2.5)–(2.6). The solution of this problem constitutes the supplies of labor and capital and the demand for consumption in each period. Let us denote the solutions of this problem by

$$a^H(P) = \{(C_t^1, C_{t+1}^2, s_t, x_t) \mid \text{parents' problems are maximized for } t \geq 0\}$$

For given  $(C_t^1, C_{t+1}^2, s_t, x_t) \in a^H(P)$ , and for given initial conditions  $L_{-1}, s_{-1}, x_{-1}$ , define recursively the aggregate supplies of labor, and capital, and the aggregate demands and supplies of consumption goods in each period as follows :

$$\left\{ \begin{array}{l} L_{-1}^s \equiv L_{-1} \\ L_t^s = L_{t-1}^s x_{t-1} \\ K_t^s = L_{t-1}^s s_{t-1} \\ C_0^s \equiv B_0 + q_0 s_{-1} + aW_0 x_{-1} + \pi_0 \\ C_t^d = L_t^s C_t^1 + L_{t-1}^s C_t^2 \\ C_t^s = Y_t - L_t^s \cdot (s_t + \theta x_t) \\ \text{= total income-investment in period } t, t \geq 0, \end{array} \right. \quad (2.9)$$

where, superscripts  $s$  and  $d$  denote respectively supply and demand. We can now define the excess supplies of commodities in each period by,

$$\left\{ \begin{array}{l} \eta_t^K = K_t^s - K_t^d \\ \eta_t^L = L_t^s - L_t^d \\ \eta_t^C = C_t^s - C_t^d, t \geq 0. \end{array} \right. \quad (2.10)$$

Define the excess supply correspondence,  $\eta(P)$  by

$$\eta(P) = \{(\eta_t^K, \eta_t^L, \eta_t^C)_0^\infty \mid \eta_t^s \text{'s are defined as above for each element of } a^H(P) \text{ and } a^F(P)\}$$

**Definition 2.1:** A perfect foresight competitive equilibrium is a non-negative price vector  $P = (p_t, q_t, W_t)_0^\infty$  and  $(C_t^1, C_t^2, s_t, x_t)_0^\infty \in a^H(P)$ , and  $(K_t^d, L_t^d)_0^\infty \in a^F(P)$  such that  $\Pi_t(P) = 0$  and  $\eta_t^K = \eta_t^L = \eta_t^C = 0$ , for all  $t \geq 0$ ; and if there is an excess supply of any commodity in any period, then its price is zero.

Note that the existence of an equilibrium is equivalent to finding a non-negative  $P = (p_t, q_t, W_t)_0^\infty$  such that  $0 \in \eta(P)$ , and  $\Pi_t(P) = 0, t \geq 0$ .

**Proposition 2.2 :** (Walras Law) For  $\tau > 0$ , non-negative  $P = (p_t, q_t, w_t)_0^\infty$ , and

$$(\eta_t^K, \eta_t^L, \eta_t^C)_0^\infty \in \eta(P), \eta_t^K = \eta_t^L = 0, \Rightarrow \eta_t^C = 0 \text{ for all } t \geq 0.$$

*Proof :* Suppose  $\eta_t^K = \eta_t^L = 0$ , then  $L_t^s = L_t^d$  and  $K_t^s = K_t^d$ . Note that  $C_t^d = L_t^s C_t^1 +$

$$L_{t-1}^s C_t^2$$

$$\begin{aligned}
&= \frac{1}{p} \cdot \{L_t^s ((1-a-\tau)W_t - p_t(\theta x_t + x_t)) + L_{t-1}^s (aW_t x_{t-1} + q_t s_{t-1} + B_t + \pi_t)\} \\
&= \frac{1}{p} \cdot \{W_t L_t^s + q_t K_t^s + \Pi_t - p_t L_t^s (\theta x_t + s_t)\} \\
&= Y_t - L_t^s (\theta x_t + s_t) = C_t^s, \text{ for all } t \geq 0, \text{ (by (2.2) \& (2.9)).}
\end{aligned}$$

Q.E.D.

Define  $p_t = (p_t, q_t, w_t)$  then  $P = (p_t)_{t=0}^{\infty}$ . The following is an implication of the assumption that consumers are liquidity constrained.

**Proposition 2.3:** Let  $\{\lambda_1, \lambda_2, \dots\}$  be a sequence of positive numbers, and let  $P^* = (\lambda_t p_t)_{t=0}^{\infty}$ . Then  $x \in \eta(P)$  if and only if  $x \in \eta(P^*)$ .

*Proof;* Obvious from (2.1) and (2.5)–(2.8).

Proposition 2.3 implies that we can normalize  $p_t$  in each time period  $t \geq 0$ . From proposition 2.2 and 2.3 it follows that for all  $t \geq 0$ , if equilibrium  $L_t, p_t, q_t$ , and  $W_t$  are given then we can compute equilibrium quantities  $C_t^1, C_{t+1}^2, s_t, x_t$ , and thus equilibrium  $L_{t+1}, K_{t+1}$ , and prices  $p_{t+1}, q_{t+1}$ , and  $W_{t+1}$  from a one-period Arrow-Debreu model. This provides a basis for finding conditions and a proof for the existence of a competitive equilibrium for our overlapping generations model as follows.

**Assumption A.3:** For all  $x, y$ ,  $\lim_{z \rightarrow 0} U_1^*(z, y) = \infty$ , and  $\lim_{z \rightarrow 0} U_2^*(x, z) = \infty$ .

**Assumption A.4:**  $U^*(\cdot)$  is strictly quasi-concave, and monotonic.

**Theorem 2.4:** Under assumptions A.1, A.3 and A.4, there exists a unique perfect foresight competitive equilibrium. Or in other words, there exists a unique sequence of equilibrium prices,  $P = (p_t, q_t, W_t)_{t=0}^{\infty} \Rightarrow 0$ , and two unique sequences of equilibrium quantities  $(C_t^1, C_t^2, s_t, x_t)_{t=0}^{\infty} \in a^H(P)$ , and  $(K_t^d, L_t^d)_{t=0}^{\infty} \in a^F(P)$  such that  $(\cdot) \in \eta(P)$ , and  $\Pi_t = 0$ , for all  $t \geq 0$ .

*Proof;* The existence part follows from Raut [1985].

I prove now the uniqueness part. From the first order necessary conditions of profit maximization it follows that

$$q_t(\tau) = p_t f'(k_t(\tau)) \quad (2.11)$$

$$W_t(\tau) = p_t (f(k_t(\tau)) - k_t(\tau) f'(k_t(\tau))) \quad (2.12)$$

Note that since there is no uncertainty in the model, in each period the equilibrium rates of returns from both the assets should equalize, i.e.,

$$q_t(\tau) = \frac{a}{\theta} W_t(\tau), \text{ for all } \tau, \text{ and } t \geq 0. \quad (2.13)$$

This implies

$$f'(k_t(\tau)) = \frac{a}{\theta} (f(k_t(\tau)) - k_t(\tau)f'(k_t(\tau))) \quad (2.14)$$

*Lemma 2.5:* The equation (2.14) has a unique solution,  $k^*$  for all  $\tau, t \geq 0$ .

*Proof:* In (2.14) suppressing  $\tau$  and the subscript  $t$ , we get

$$0 = f(k) - (\theta/a + k)f'(k)$$

Denote the right hand side of the above as  $g(k)$ . Note that

$$\lim_{k \rightarrow 0} g(k) = -\infty, \lim_{k \rightarrow \infty} g(k) = \infty, \text{ and } g'(k) = -(\theta/a + k)f''(k) > 0$$

That is,  $g(k)$  is an increasing function of  $k$  that changes sign within its domain. Therefore it has a unique solution.

Q.E.D.

*Proof of theorem 2.4:* Using  $s_t/x_t = k^*$  in (2.5)-(2.6) we can eliminate  $s_t$ . Strict quasi-concavity of  $U^*(\cdot)$  guarantees the uniqueness of  $x_t$ ,  $C_t^1$ , and  $C_{t+1}^2$ . That prices are unique follows from (2.11) and (2.12).

Q.E.D.

*Definition 2.6:* A dynamic system  $x_t \in \mathbf{R}^2, t \geq 0$  is in steady state from period  $t_0$  if  $X_{t+1} = X_t$ , for all  $t \geq 0$ .

*Theorem 2.7:* For any  $\tau \geq 0$  the perfect foresight competitive equilibrium prices and quantities are in steady state from  $t \geq 1$ . Moreover, steady state prices and quantities are unique and independent of  $\tau$  and initial conditions, and hence the steady state is globally stable.

*Proof:* Follows from lemma 2.5.

We want to study the properties of the equilibrium quantities,  $(x_t, s_t)$  and the prices  $(p_t, q_t, W_t)$  as we vary  $\tau$ . By proposition 2.3, I can take the equilibrium  $p_t = 1$ , for  $t \geq 0$ . Since all the prices could be computed from (2.11) and (2.12) once  $k_t$  is known, we will study only the properties of  $k_t, s_t$ , and  $x_t, t \geq 0$ . We shall distinguish between short-run effects, i.e. the effects in the period when the program is introduced, and the long-run effects, i.e., the steady state effects. The magnitudes of the short-run and the long-run effects differ mainly due to the fact that while in the short-run the wage rate of a decision maker is not affected, in the long-run, more precisely a generation later, the wage rate will be affected by a social security program.

### 3. EXOGENOUS FERTILITY AND THE EFFECTS OF SOCIAL SECURITY

The effect of social security on savings has been studied mainly within the micro partial equilibrium framework (see, Barro [1978] for an exception). Within a general equilibrium model with production, a social security program that affects saving will also effect the general equilibrium interest and wage rates in opposite directions. An increase



in interest rate will have income and substitution effects on savings in the opposite directions. The net effect will in general be ambiguous. In this section I compare the direction and magnitude of partial equilibrium effect with general equilibrium effect of a social security program on saving under the assumption that fertility is exogenously given.

Suppose a pay-as-you-go social security system financed by pay-roll taxes is introduced in period  $t \geq 0$ . I shall now assume that parents' utility functions are time separable as in (2.4). The first order necessary condition for an interior solution to the parent's problem is

$$-U_1 + (1 + r_{t+1}) V_1 = 0$$

Totally differentiating (3.1) and (2.7) with respect to  $s_t$ ,  $\tau$ ,  $B_{t+1}$ , and eliminating  $\partial B_{t+1}$  we have

$$\frac{\partial s_t(\tau)}{\partial \tau} \Big|_{\tau=0} = - \frac{U_{11} W_t + (1 + r_{t+1}) W_{t+1} x_t V_{11}}{U_{11} + (1 + r_{t+1}) V_{11} (1 + r_{t+1})} < 0, t \geq 0. \quad (3.2)$$

(3.2) implies that the micro partial equilibrium effect of a social security program on saving is unambiguously negative,

We now consider the general equilibrium effect. Totally differentiating equations, (2.7), (2.11), (2.12) and (3.1) with respect to  $B_{t+1}$ ,  $s_t$ ,  $1 + r_{t+1}$ ,  $W_{t+1}$ , and  $\tau$  at  $\tau = 0$  and eliminating  $\partial B_{t+1}$ ,  $\partial(1 + r_{t+1})$ , and  $\partial W_{t+1}$ , one gets

$$\frac{\partial s_t(\tau)}{\partial \tau} \Big|_{\tau=0} = - \frac{U_{11} W_t + (1 + r_{t+1}) W_{t+1} x_t V_{11}}{U_{11} + (1 + r_{t+1}) V_{11} (K_{t+1} f'' + f') + V_1 f''/x_t}, t = 0 \quad (3.3)$$

and

$$\frac{\partial s_t(\tau)}{\partial \tau} \Big|_{\tau=0} = - \frac{U_{11} (W_t + (1-a) k_t k'_t f'(k_t)) + (1 + r_{t+1}) W_{t+1} x_t V_{11}}{U_{11} + (1 + r_{t+1}) V_{11} (k_{t+1} f'' + f') + V_1 f''/x_t}, t \geq 1$$

where,  $k_{t+1} = s_t/x_t$ ,  $k'_t = \partial k_t(\tau)/\partial \tau$ , and the derivatives  $f'$ ,  $f''$  are evaluated at  $k_{t+1}$  unless mentioned otherwise. (3.3)' is derived by eliminating  $\partial W_t$  also. Note that (3.3) is greater than (3.3)'.

The sign of (3.3) is indeterminate. Two sufficient conditions for (3.3) to be negative are

$$f'(k_t) + k_t f''(k_t) \geq 0 \quad (3.4)$$

and

$$f'(k_t) s_t V_{11} + V_1 \geq 0. \quad (3.5)$$

The following proposition which follows from (3.2)–(3.5) compares the general equilibrium effect with the partial equilibrium effect of the social security program on saving.

**Remark 3.1:** The condition (3.4) is related to the curvature of the production function, and is equivalent to the assumption that the elasticity of interest rate with respect to capital-labor ratio, i.e., is  $-\frac{d \log f'(k)}{d \log k}$  is less than one. It could be shown that (3.4) is satisfied by Cobb-Douglas production function,  $f(k) = k^\sigma$ ,  $0 < \sigma < 1$ .

*Proposition 3.2:*

- (a) If (3.5) holds with equality then the general equilibrium and partial equilibrium effects are the same.
- (b) If (3.4) or (3.5) holds then the direction of general equilibrium effect on saving is the same as the partial equilibrium effect, *i.e.* negative.
- (c) If (3.5) is true then the magnitude of the general equilibrium effect is smaller than the partial equilibrium effect.

It is easy to see that the above social security system is equivalent to the joint family transfer scheme with  $a$  replaced by  $a + \tau$ . If we further assume that with the introduction of the program children reduce their support to their old parents exactly by the amount provided by the public program,<sup>3</sup> the social security program will have no effect. To the extent the public transfers offset the voluntary transfers within a household, the effect of social security on saving will be undermined. The issue needs to be resolved empirically.

#### 4. EFFECTS OF SOCIAL SECURITY ON FERTILITY AND SAVINGS

I assume now that fertility is endogenous, and study the impact of a social security program on fertility and saving.

As the lemma 2.5 holds for all  $t \geq 0$  it follows that the equilibrium wage and interest rates are unchanged and that both equilibrium fertility and saving rates will either increase, decrease or remain unchanged when a pay-as-you-go social security program is introduced. Denote the solution of (2.14) by  $k^*$ . Note that

$$k_{t+1}(\tau) \equiv \frac{s_t(\tau)}{x_t(\tau)} = k^* \text{ for all } t, \tau \geq 0 \quad (4.1)$$

Using this I eliminate  $s_t$  in (2.5) and (2.6). The first order condition for an interior maximum becomes

$$-(k^* + \theta) U_1 + (1 + r_{t+1})(k^* + \theta) V_1 = 0 \quad (4.2)$$

Totally differentiating (4.1), (4.2), and (2.7) with respect to  $x_t$ ,  $s_t$ ,  $B_{t+1}$ , and  $\tau$ , and eliminating  $\partial B_{t+1}$ , we have

$$\frac{\partial x_t(\tau)}{\partial \tau} \Big|_{\tau=0} = - \frac{W_t U_{11} + (1 + r_{t+1}) W_{t+1} x_t V_{11}}{(k^* + \theta)(U_{11} + (1 + r_{t+1})^2 V_{11})} \quad (4.3)$$

and

$$\frac{\partial s_t(\tau)}{\partial \tau} \Big|_{\tau=0} = - \frac{k^*}{k^* + \theta} \cdot \frac{W_t U_{11} + (1 + r_{t+1}) W_{t+1} x_t V_{11}}{U_{11} + (1 + r_{t+1})^2 V_{11}} \quad (4.4)$$

<sup>3</sup> Empirical evidence on crowding-out issue is none too less controversial. For instance, Munnel (1974) in her econometric analysis found a strong crowding out effect. Summerizing many empirical evidence. Northworth and Motley (1974) concluded that there is no significant reduction in the private transfer as a result of publicly provided social security program.

Therefore from (4.3) and (4.4) it follows that the general equilibrium effects of the program will be negative on both saving and fertility rates. Moreover, (4.4) and (3.2) show that the magnitude of the general equilibrium effect is smaller than the partial equilibrium effect with exogenous fertility.

## 5. CHILD-TAX FINANCED SOCIAL SECURITY PROGRAMS

In LDCs, where the cost of raising children is very low and the markets for old age pension are absent, it has been argued that the parents tend to have large family (Willis [1980], Nehar [1971], larger than Pareto optimal size (see Raut [1985], Nerlove, Razin and Sadka [1987]). The divergence between social and private costs of children arising mainly from market failures motivates one to consider child tax financed social security programs.

In this scheme, parents of generation  $t \geq 0$  are taxed at the rate of  $\tau$  per child in their working years and given a benefit,  $B_{t+1}$ , when they are old. The budget constraints of a parent of generation  $t$  in this case are given by

$$p_t C_t^1 = (1 - a) W_t - p_t s_t - p_t (\theta + \tau) x_t \quad (5.1)$$

$$p_{t+1} C_{t+1}^2 = B_{t+1} + q_{t+1} s_t + a W_{t+1} x_t + \pi_{t+1} \quad (5.2)$$

where the actuarially fair benefits  $B_{t+1}$  are given by

$$B_{t+1} = p_{t+1} \tau x_{t+1} : \text{for pay-as-you-go-system} \quad (5.3)$$

$$B_{t+1} = q_{t+1} \tau W_t : \text{for fully funded system} \quad (5.4)$$

Note that due to no uncertainty in our model, the after tax equilibrium rates of returns from children and capital should equalize, i.e.,

$$(1 + r_{t+1}(\tau)) = \frac{a}{\theta + \tau} W_{t+1}(\tau) \text{ for all } t \geq 0 \quad (5.5)$$

From lemma 2.5, it follows that the after tax equilibrium capital labor ratio  $k(\tau)$  will be the same in all periods. Applying implicit function theorem to (2.14) it follows that

$$\frac{\partial k(\tau)}{\partial \tau} \Big|_{\tau=0} = - \frac{f'(k^*)}{(\theta + a k^*) f''(k^*)} > 0, \quad (5.6)$$

Thus  $k(\tau) > k^*$ , for all  $\tau > 0$ , where  $k^*$  is the equilibrium capital labor ratio before the social security program was introduced. Note that

$$k_{t+1}(\tau) \equiv \frac{s_t(\tau)}{x_t(\tau)} = k(\tau), \text{ for all } t \geq 0 \quad (5.7)$$

Eliminating  $s_t$  in (5.1) and (5.2) using (5.7), the first order condition for an interior maximum becomes

$$-U_1 + f'(k(\tau)) V_1 = 0 \quad (5.8)$$

*Fully funded system*

Totally differentiating (5.4), (5.7), and (5.8) with respect to  $x_t$ ,  $s_t$ ,  $\tau$ ,  $B_{t+1}$ , and eliminating  $\partial B_{t+1}$ , we have for a fully funded program

$$\frac{\partial x_t(\tau)}{\partial \tau} \Big|_{\tau=0} = - \frac{x_t(k' + 1) U_{11} + f'' k' V_1 + f' x_t V_{11} \Psi}{(k^* + \theta) (U_{11} + (1 + r_{t+1})^2 V_{11})}, t = 0 \quad (5.9)$$

$$= - \frac{\{x_t(k' + 1) + (1-a) k^* k' f''\} U_{11} + f'' k' V_1 + f' x_t V_{11} \Psi}{(k^* + \theta) (U_{11} + (1 + r_{t+1})^2 V_{11})}, t \geq 1 \quad (5.9)'$$

and

$$\frac{\partial s_t(\tau)}{\partial \tau} = \frac{\partial k(\tau)}{\partial \tau} x_t(\tau) + \frac{\partial x_t(\tau)}{\partial \tau} k(\tau), t \geq 0 \quad (5.10)$$

where

$$\Psi = 1 + \frac{(a-1) k^* f'' - f'}{(\theta + a k^*) f''}, \text{ and } k' = dk(\tau)/d\tau$$

(5.9)' is derived eliminating  $\partial W_t$  also. Note that the sign of (5.9) will in general be indeterminate. A sufficient condition for it to be negative is that  $\Psi > 0$ , which after simplification becomes

$$a > \frac{1}{2} \left\{ \left( 1 - \frac{\theta}{k^* f''} \right) \right\} \quad (5.11)$$

Whether or not this is satisfied will depend on the curvature of the production function. For example, Cobb-Douglas function satisfies this. Note that if the second term in (5.10) is positive then undoubtedly the saving rate will be increased as a result of introducing the social security program. If, however, the second term is negative, then the effect on saving will depend on the relative strengths of the two terms, and the net effect is in general indeterminate. Interestingly, unlike the fully funded program of the previous section, the present program will affect parents' decisions regarding both savings and number of children.

*Pay-as-you-go system*

Following exactly the same procedure for the pay-as-you-go system, it can be shown that

$$\frac{\partial x_t(\tau)}{\partial \tau} \Big|_{\tau=0} = - \frac{x_t(k' + 1) U_{11} + f'' k' V_1 + f' x_t V_{11} \Phi}{(k^* + \theta) (U_{11} + (1 + r_{t+1})^2 V_{11})}, t = 0 \quad (5.12)$$

$$= - \frac{\{x_t(k' + 1) + (1-a) k^* k' f''\} U_{11} + f'' k' V_1 + f' x_t V_{11} \Phi}{(k^* + \theta) (U_{11} + (1 + r_{t+1})^2 V_{11})}, t \geq 1 \quad (5.12)$$

where

$$\Phi = x_{t+1} + \frac{(a-1) k^* f'' - f'}{(\theta + a k^*) f''}$$

The sign of the above derivative is also indeterminate. A sufficient condition for (5.12) to be negative is that the second term of  $\Phi > 0$ , or,

$$a \geq 1 + \frac{f'(k^*)}{k^* f''(k^*)} \quad (5.13)$$

Note that while Cobb-Douglas function  $f(k) = k^\sigma$ ,  $0 < \sigma < 1$  satisfies (5.13), the CES production function  $f(k) = (k^\rho + 1)^{1/\rho}$  does not for  $0 < \rho \leq 1$ .

The effect of the program on saving is given by the expression (5.10), and it is indeterminate in this case also. On remark that warrants at this point is that unlike in the case of payroll tax financed program, a child tax financed fully funded program is not neutral.

*Example 5.1: (Cobb-Douglas Economy)*

Suppose the utility function is given by

$$U^*(C_t^1, C_{t+1}^2) = \log C_t^1 + \log C_{t+1}^2 \quad (5.14)$$

and the production function is given by

$$F(K, L) = K^\sigma L^{1-\sigma}, \quad 0 < \sigma < 1 \quad (5.15)$$

Note that for a fully funded system (5.5) yields

$$F_1 = \frac{a}{\theta + \tau} F_2, \text{ implies, } \frac{s_t + \tau x_t}{x_t} = \frac{\sigma(\theta + \tau)}{(1 - \sigma)a} \equiv (k(\tau)) \quad (5.16)$$

Using this to eliminate  $s_t$  from (5.1) and (5.2), and maximizing (5.14) with respect to  $x_t$ , subject to these new budget constraints yields

$$x_t(\tau) = \frac{(1 - a) W_t}{2(k(\tau) + \theta)} - \frac{B_{t+1}}{2f'(k(\tau) + \theta)}$$

Substituting  $1 + r_{t+1} = f'(k(\tau))$  in (5.4) and the above expression, one gets

$$x_t(\tau) = \frac{(1 - \sigma)a(1 - a)W_t}{2\theta(\sigma + a(1 - \sigma)) + \tau(2\sigma + a(1 - \sigma))} \quad (5.17)$$

From this one derives that

$$\frac{\partial x_t(\tau)}{\partial \tau} \Big|_{\tau=0} = - \frac{(1 - \sigma)a(1 - a)(2\sigma + a(1 - \sigma))}{4\theta^2(\sigma + a(1 - \sigma))^2} W_t < 0 \quad (5.18)$$

using this (5.16) one gets

$$\frac{\partial s_t(\tau)}{\partial \tau} = - \frac{(1 - \sigma)a(1 - a)}{2\theta(2\sigma + a(1 - \sigma))} W_t < 0 \quad (5.19)$$

From (5.16) it follows that the after tax capital labor ratio will increase by  $\frac{\partial k(\tau)}{\partial \tau} \Big|_{\tau=0} = \frac{\sigma}{(1 - \sigma)a}$ . Therefore, the wage rate will rise. If we define the total savings as  $S_t(\tau) = s_t(\tau) + \tau x_t(\tau)$  then we have

$$\frac{\partial S_i(\tau)}{\partial \tau} \Big|_{\tau=0} > 0$$

What we have proved is the following proposition.

**Proposition 5.2:** In a Cobb-Douglas economy, a child tax financed fully funded social security system reduces household fertility and savings. However, it increases the aggregate (i.e., government+private) saving rate and the wage rate.

Now for a pay-as-you-go system, note that (5.5) implies

$$\frac{s_t}{x_t} = \frac{\sigma(\theta + \tau)}{(1-\sigma)a} \equiv ((k\tau)) \quad (5.20)$$

Proceeding exactly the same way as above one derives that

$$x_t(\tau) = \frac{(1-a)}{2(k(\tau) + \theta + \tau)} W_t - \frac{\tau x_t(\tau) x_{t+1}(\tau)}{2f'(k(\tau) + \theta + \tau)} \quad (5.21)$$

From which we derive

$$\frac{\partial x_t(\tau)}{\partial \tau} \Big|_{\tau=0} = - \frac{1}{2\theta(\sigma + (1-\sigma)a)} \cdot \{a(1-a)(1-\sigma)/\theta + x_t x_{t+1}/f'\} < 0 \quad (5.22)$$

and

$$\begin{aligned} \frac{\partial s_t(\tau)}{\partial \tau} &= k'(\tau) x_t(\tau) + k^* \frac{\partial x_t(\tau)}{\partial \tau} \\ \frac{\partial s_t(\tau)}{\partial \tau} \Big|_{\tau=0} &= - \frac{\sigma x_t x_{t+1}}{2(1-\sigma)a[\sigma + (1-\sigma)a]f'} < 0. \end{aligned}$$

Therefore, in this case both fertility and saving rates decline. However, the saving rate declines less than the fertility rate since by (5.6) the capital-labor ratio is larger after the introduction of the program.

## 6. LONG RUN EFFECTS OF SOCIAL SECURITY PROGRAMS

In the case of exogenous fertility, it is clear from (3.3) and (3.3)' that the effect of a pay-roll tax financed social security system on saving and hence on capital labor ratio will diminish over time. Assuming that the economy is converging to a steady state, and using the steady-state budget constraints in the derivation of (3.3)', I can derive the long-run effect as

$$\frac{\partial s(\tau)}{\partial \tau} \Big|_{\tau=0} = - \frac{(U_{11} + xf' V_{11})W}{U_{11}((1-a)f'' \frac{k}{x} - 1) + f'' V_1/x + f' V_{11}(f' + (1-a)kf'')} \quad (6.1)$$

where,  $W$ ,  $s$ ,  $k$  are steady-state wages, savings, and capital-labor ratio.

When both saving and fertility are endogenous, however, (4.1) and (4.2) imply that the steady-state is attained at  $t \geq 1$ . While in the case of a pay-roll tax financed program,

the capital-labor ratio, and hence wage rate are unaffected, in the case of a child-tax financed social security program they increase more in the short-run and less in the long-run. Moreover, the effect of the former program will be to decrease both fertility and saving by the same proportion, and the effects of the latter would be to reduce fertility proportionately more than saving.

## 7. OPTIMAL POPULATION GROWTH AND SOCIAL SECURITY

In this section we ask whether a decentralized economy results in an over population and if so, would a social security program improve the situation? It is apparent that an answer to these questions will depend upon the welfare criterion.

The issue of optimal population growth has been addressed in the literature using utilitarian, *i.e.*, social welfare approach. The Benthamite criterion is based on total utility, and Millian criterion is based on average utility. Dasgupta [1969] was the first to point out that optimality of population could not be separated from that of savings. While he derived the joint optimality for the rate of population growth and capital accumulation, his analysis was based on an aggregate model with no mechanism for individual fertility and savings decisions. More precisely, he assumed that population could be instantaneously controlled by government to any desired level (Lane [1975] refined this analysis). In a model with endogenous fertility and savings decisions, Nerlove, Razin, and Sadka [1987] showed that while Benthamite criterion leads to a higher rate of population growth than the Millian criterion, a laissez-faire equilibrium need not lead to a smaller population growth than the Benthamite criterion, or to a larger population than the Millian criterion. I modify the Pareto optimality criterion below and show that if the exogenously determined amount of inter-generational income transfers are not large enough, a laissez-faire equilibrium leads to over-population and capital accumulation.

**Definition 7.1:** A feasible program is a sequence  $\{(C_t^1, C_t^2, s_t, x_t) \geq 0\}_0^\infty$  that satisfies

$$C_t^1 + \frac{C_t^2}{x_{t-1}} + k_{t+1} x_t + h_t \leq f(k_t)$$

where,

$$k_t = \frac{L_{t-1} s_{t-1}}{L_t} = \frac{s_{t-1}}{x_{t-1}} : \text{capital-labor ratio}$$

$$h_t = \theta x_t : \text{investment in children}$$

$$L_t = L_{t-1} x_{t-1}$$

and  $L_{-1}$ ,  $s_{-1}$ , and  $x_{-1}$  are given.

**Definition 7.2:** A feasible allocation  $\{(C_t^1, C_t^2, s_t, x_t)\}_0^\infty$  is Pareto optimal if there does not exist another feasible allocation  $\{(\bar{C}_t^1, \bar{C}_t^2, \bar{s}_t, \bar{x}_t)\}_0^\infty$  such that

$$U(\bar{C}_t^1, \bar{C}_{t+1}^2) \geq U(C_t^1, C_{t+1}^2) \text{ for all } t \geq 0$$

with strict inequality for some  $t$ .

*Remark 7.3:* This definition of optimality is limited in that it compares the welfare of living members of a cohort as criterion for comparing consumption streams. The population size does not matter in this comparison. More specifically, if two allocations give exactly the same consumption to everybody, they are equivalent by this optimality criterion regardless of their relative population sizes. This criterion has been also used by Nerlove *et. al.* [1987].

*Proposition 7.4:* If the voluntary joint family transfer scheme satisfies

$$a < \frac{\theta}{2\theta + k^* f'(k^*)} \quad (7.1)$$

and the utility function satisfies A.3 then a pay-roll tax financed pay-as-you-go social security program introduced in period  $t_0 \geq 1$  will improve the welfare of every living beings of generations  $t \geq t_0 - 1$ .

*Proof:* Suppose the social security program is introduced in period  $t_0 > 0$ . Obviously the welfare of the  $t_0 - 1$ —th generation will be higher as the social security program provide them with higher consumption at their old age. For  $t \geq t_0$ , define the indirect utility function of the  $t$ -th generation by

$$\Delta(\tau) = \text{Max } U(C_t^1, C_{t+1}^2) \text{ subject to (2.5)-(2.7)} \quad (7.2)$$

By (4.1),  $k_t(\tau) = k^*$ , for  $t \geq 1$ . Therefore, the wage rate  $W_t(\tau) = f(k^*) - k^* f'(k^*) = \text{constant}$ , for all  $t \geq 1$ ,  $\tau \geq 0$ . Hence,  $\Delta_t(\tau)$  is the same for all generations  $t \geq t_0$ . By assumption A.3, (7.2) has an interior solution which allows us to combine all three budget constraints into one, and to express the Lagrangian of (7.2) as

$$L(C_t^1, C_{t+1}^2, \lambda, \tau) = U(C_t^1, C_{t+1}^2) - \lambda (C_t^1 + \frac{C_{t+1}^2}{1 + \rho_{t+1}(\tau)} - (1 - a - \tau) W_t)$$

where,  $\rho_{t+1}(\tau) = \frac{k^* r_{t+1} + \theta \tau / a}{k^* + \theta}$ . By the envelope theorem, we have

$$\frac{d\Delta(\tau)}{d\tau} = \frac{\partial L}{\partial \tau} = - \left\{ - \frac{C_{t+1}^2}{[1 + \rho_{t+1}(\tau)]^2} \cdot \frac{\theta}{a(k + \theta)} + W_t \right\}$$

which simplifies to

$$\frac{d\Delta(\tau)}{d\tau} \Big|_{\tau=0} = \lambda \left\{ \left[ \frac{(1-a)\theta}{a[\theta + k^* f'(k^*)]} - 1 \right] W_t + \frac{\theta C_t^1}{a[\theta + k^* f'(k^*)]} \right\}$$

and it is positive by (7.1). Thus this social security program will increase the welfare of all generation  $t \geq t_0 - 1$  without reducing the welfare of the previous generations.

Q.E.D.

*Corollary 7.4.1:* If (7.1) and A.3 are satisfied then the decentralized economy results



in a higher rate of population growth and capital accumulation than a Pareto optimal rate.

*Proof;* Follows from (4.3), (4.4) and proposition 7.4.

## 8. DETERMINATION OF INTER-GENERATIONAL TRANSFERS OF INCOME

I will consider two alternative approaches to endogenize  $a$  and then show that both lead to identical problem in a stationary environment. One approach is in terms of social norms. Individuals form societies and devise social norms in order to perform certain economic activities which are otherwise not credible. Each individual contributes to the formation of the social norms in the society he lives in, but individually he is powerless to change any norm. The threats to violation of a social rule are mutual sanction, and outcaste from the society.<sup>4</sup> I continue to assume that parents care only about their own consumption. A set of social rules will be evolved such that it will sustain a rate of inter-generational transfers  $a$  in the following problem:

$$\text{Max}_{a_t} \{ \text{Max } U(C_t^1, C_{t+1}^2) \} \quad (8.1)$$

subject to

$$C_t^1 = (1 - a_t) W_t - s_t - \theta x_t$$

$$C_{t+1}^2 = (1 + r_{t+1}) s_t + a_{t+1} W_{t+1} x_t$$

Another way to look at this problem is that adults will provide  $a_t W_t$  to his parents so that he could set a good example to his children as well as to other neighbours. In economic terms, the above amounts to assuming that the adults in time period  $t$  will have adaptive expectations about  $a_{t+1}$  as a function of his own transfer to his parents,  $a_t$ , namely  $a_{t+1} = \pi(a_t)$ , and we assume that  $\pi$  is an identity function. The problem here is a forward induction on  $a_t$ ,  $x_t$  and  $k_t$ . Notice that in a stationary state for all  $t \geq 0$  we have  $x_t = x_{t+1} \equiv x$ ,  $k_t = k_{t+1} \equiv k$ ,  $a_t = a_{t+1} \equiv a$  and  $C_t^2 = C_{t+1}^2 \equiv C^2$  and  $C_t^1 \equiv C_{t+1}^1 = C^1$  the problem reduces to

$$\text{Max}_a \{ \text{Max } U(C^1, C^2) \}$$

subject to

$$C^1 = (1 - a) W - s - \theta x$$

$$C^2 = (1 + r) s + a W x$$

where  $1 + r = f'(k)$

$$W = f(k) - k f'(k).$$

<sup>4</sup> An excellent description of this phenomenon could be found in Sarat Chatterjee's Bengali novel "Palli Samaj" (meaning Village Society).

The second approach to make  $a$  endogenous is to postulate a utility function of the form  $U^*(C_t^1, C_t^2, C_{t+1}^2)$  in problem (8.1). Note that solution here will involve backward induction on  $a_t$  and forward induction on  $x_t$  and  $k_t$ . This is basically a rational expectations approach. Note that in stationary state for all  $t \geq 0$ , we have  $x_t = x_{t+1} \equiv x$ ,  $k_t = k_{t+1} \equiv k$ ,  $a_t = a_{t+1} \equiv a$  and  $C_t^2 = C_{t+1}^2 \equiv C^2$  and  $C_t^1 = C_{t+1}^1 \equiv C^1$ . Then the utility function reduces to  $U(C^1, C^2)$  and the budget constraints are the same as in problem (8.2). Thus these two problems are indistinguishable from each other.

In both cases it is apparent that a social security program will be neutral since a voluntary transfers will adjust in such a way that combined transfers will be the same as the transfers without a social security program. It is also apparent that decentralized economy will lead to Pareto optimal allocation. Note that for these results we have not assumed interdependent utility functions as in Barro [1974].

## 9. CONCLUSION

The paper has studied the properties of various social security programs in an overlapping generation general equilibrium model. It has provided a simpler proof for the existence and uniqueness of perfect foresight competitive equilibrium using recursively the equilibria of a sequence of one period Arrow-Debreu economies. The following results have been shown.

In the case of exogenous fertility, while micro, partial equilibrium effect of a social security program is to reduce savings, general equilibrium effect is indeterminate. Discrepancy between the two effects arise because general equilibrium analysis takes into account the effect of a program on future capital-labor ratios, and hence the interest rates and wages; partial equilibrium analysis ignores these effects. In the case of fertility and savings both endogenous, the direction of general equilibrium and partial equilibrium effects is to reduce population growth and capital accumulation.

The paper also introduced child-tax financed pay-as-you-go and fully funded social security programs. It has shown that while both programs increase the future capital-labor ratios, the effects on fertility and saving rates are dubious and depend upon the choice of utility and production functions. For instance, when both utility and production functions are Cobb-Douglas, a fully funded system reduces both fertility and saving rates (fertility rate being reduced more than saving rate), and a pay-as-you-go system reduces fertility rate but the saving rate may increase or decrease depending upon the parameters of the functions.

When fertility is exogenous, a steady state is attained only asymptotically, but when fertility is endogenous the steady state is attained in finite time and hence steady-state in this case is unique.

If the rate of inter-generational transfers of income and costs of raising children are low, a laissez-faire equilibrium leads to over-population and capital accumulation in a modified Pareto optimal sense. A social security program is Pareto improving in such a case.

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