ESSAY II

CAPITAL ACCUMULATION, INCOME INEQUALITY, AND ENDOGENEOUS FERTILITY IN AN OVERLAPPING GENARATIONS GENERAL EQUILIBRIUM MODEL.

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1. INTRODUCTION

This essay examines the theoretical linkages among population growth, income inequality, quality composition of the labor force, and household income in an overlapping generations model aggregating household decisions about fertility and savings.

The post World War II LDCs have experienced an unprecedented and sharp fall in their mortality rates unaccompanied by any significant decline in fertility rates. This has led to exceptionally high rates of population growth -- growth rates which none of the present developed countries experienced during their demographic transitions. (See Kuznets[1979, 134-135], the World Development Report (from now on abbreviated as WDR) [1984, 56-73], and Coale[1983, 823-832] for brief expositions on the issue.) Development economists of the 1960's and 1970's clearly recognized the global impact of this high growth upon limited natural resources. Its negative impact on general economic development per se was pointed out by MacNamara[1973] and World Development Report[1980], among others. More recently, WDR[1984,73-105] discusses the issue taking into account the recent achievements of the LDCs in reducing general fertility levels. This 1960's resurrection of Malthusian concerns even led some economists to compare the pressures of rapid population growth with the threat of Nuclear War (MacNamara 1973,pp.31).

Although population growth (or fertility) interacts with the economic development of a society through various channels, the present analysis focuses upon only a few of the more important ones. First of all, faster population growth leads to a larger dependency ratio, i.e., each worker supports more children and old people. According to one theory, a higher dependency ratio increases the proportion of household income devoted to consumption, thereby reducing the proportion available for savings. Some empirical studies support this hypothesis. See for example, Coale and Hoover[1958], Mason and Suits[1981,255-272], Mueller[1976], Lewis[1983], for individual country experiences; Leff[1969], Gupta[1975], Mason[1981], Mason and Fry[1982], for international cross section studies; and Tobin[1967], Mueller[1976], for models based on simulation. Alternative views suggest that household savings may rise with the dependency ratio. Specifically, higher "economic" demand for more children may induce parents to work harder and save more. Furthermore, children's income may contribute to household savings (indeed, in most developing countries child labor participation is quite high). Higher population growth may also, by stimulating the demand side of the product market and possibly depressing the wage-rental ratio, generate a higher rate of profit to the capitalist class. Finally, given that the main sources of household monetary savings in the LDCs are provided by just a few wealthy families, a situation mainly due to poor development of banking and credit systems (cf. WDR[1984, p.82]), high population growth may not have any adverse effect on household savings. Many LDC studies support this view, such as Kelley[1976, 1980] & Simon[1975], which are based on household survey data of individual countries, and also Simon[1976], which is based on a simulation model. In short, plagued by lack of appropriate data and a unified theoretical basis, the empirical studies to date have failed to resolve the debate on the population-savings link. See also Mason[1984], and McNicoll[1984] for more extensive surveys on the issue.

Many recent empirical studies of both developed and developing countries strongly suggest a negative association between number of children and their "quality", where household expenditure on children or education level are typical measures of child quality. For some empirical evidence, see Becker[1981, chapter5] and the references cited there, WDR[1984, p.85], and Birdsall[1980]. Empirical evidence regarding the relationship between fertility and per capita income is less conclusive. Although most studies assert a negative relationship (see WDR[1984, 69-70] for cross country experiences, and Vaidyanathan[1974, especially tables 7&8],
for some individual country experiences), the results of other studies support a positive relationship (see for examples, McInnis[1977, Table5] for Canada in 1861, Bash[1955, table12] for the United States in 1865). On a macro level, this quality-quantity trade-off can be

explained as follows: Rapid population growth causes capital widening. For example, big families tend to absorb public goods like education and public health on a lower per capita basis than smaller families. Private consumption expenditure per capita is likewise lower for families with more children (see Vaidyanathan[1974]), which may mean that malnutrition goes hand in hand with bigger families. As a result of low per capita consumption and malnutrition, bigger families produce lower quality children, if quality is measured by productivity or learning capability. Similar reasoning applies to international cross sections. At the micro level, Beckerian household models explain the quantity-quality interaction and the negative income-quantity relationship by imposing some restrictive assumptions upon unobservable utility functions. Becker's interaction hypothesis has not been tested empirically. [1]

High population growth tends to reduce capital per worker and labor force quality. [2] This naturally increases the gap in incomes between profit earning households and wage earning households. As a result, high population growth may lead to great inequality in income. For empirical support of this hypothesis, see Adelman and Morris[1973], Ahluwalia[1976], and Chenery et al.[1974]. This same mechanism may also increase income inequality between large and small

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[1] However, see Rosenzweig and Wolpin[1980], where they use Indian Household Survey data to test the interaction hypothesis.
[2] For an argument against the capital shallowing effect of population growth, however, see Simon[1981].

households by reducing labor quality. Economic demographers, on the other hand, have tried to establish the causality of population-inequality link in the other direction, see Repetto[1979], and also see Boulier's[1982] criticism. Rodgers[1983], in an extensive study of cross sectional data, treats both population and inequality as endogenous variables and finds no significant effects of population growth on income inequality. (See also Winegarden[1982] in this connection).[3] Just as the empirical studies of the population-savings link are inconclusive, the LDC studies on the population-inequality link likewise suffer from insufficient data and theoretical inconsistencies. Moreover, conventional measures of income inequality are unable to distinguish between the "distributional effect" and "compositional effect". Any conclusion about the population-income inequality link based on these measures is therefore spurious, as pointed out by Lam[1984].

Clearly, empirical work has so far failed to establish conclusive causal links between rapid population growth and macro variables such as household savings, income inequality, quality composition of the labor force, and per capita income in an economy. A better understanding of the macroeconomic effects and causes of population growth is essential in addressing many policy questions such as the following. Can egalitarian income redistribution reduce population

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[3] For several other arguments regarding the population growth and income inequality link, see Lindert[1978, 6-12], McNicoll[1984, 226-229].

growth? Or does higher population growth lead to greater income inequality? Should one expect the quality subsidization policies of China, Taiwan, Korea and India to reduce or have already reduced fertility rates significantly? Might alternative investment opportunities and improvements in banking services in remote undeveloped areas reduce fertility by providing savings alternatives to investment in children? Improved theoretical models are clearly needed to address these issues. I now turn to theoretical models dealing with these issues to see how far they shed light on these causalities.

Apart from the early nineteenth century work of Malthus and Ricardo, most macro growth theory, spurred by Harrod[1939], Domar[1946], Solow[1956], Swan[1956], Robinson[1956], and Tobin[1955], has treated population growth as exogenous to the economic system. The explanation of population growth was left to demographers. The thread of Malthus and Ricardo was, however, picked up by Niehans[1963], Nelson[1956] and Liebenstein[1954,1957], who treated population as endogenous. These models postulated change in population level as a function of various macro variables, although explicit, individual fertility decision making mechanisms were not considered. As a result, the policy recommendations based on these models may not be compatible with individual incentives and hence, may be non-operational. For comparative evaluations of these models on other issues, see Pitchford[1974,1977].

In a planned economy, the concept of optimal savings is quite important. Dasgupta[1969] pointed out that the problem of determining optimal savings can not be divorced from that of optimal population growth. The bulk of the modern optimal growth literature, however, beginning with Ramsey's 1928 seminal paper, has treated population as exogenous. On the other hand, Meade[1966] considered the problem of determining optimum population growth, taking the savings rate as constant and the stock of capital at each moment as given. Dasgupta[1969] was first to break this trend. He considered the problem of optimality of both capital and population simultaneously, but imposed a very stringent controllability assumption about macro population levels. He assumes that at each instant, including the present, the population level of the economy could be adjusted to any desired level. Lane[1975] improves Dasgupta's analysis by assuming that today's population, which already exists, could not be changed, and the future growth rate, which is partly determined by endogenous factors, could not be completely controlled. He posits that the growth rate is a function of per capita income in the tradition of macro growth theorists. The critique applied to macro growth models in regard to effectiveness of the policies therefore applies to these models as well.

The new Home Economics, pioneered by the works of Becker[1960,1965,1981], Easterlin[1968,1969], Lancaster[1966] and Mincer[1958] treats individual fertility decisions in a consumer choice theoretic framework of maximizing utility subject to a budget constraint. For a discussion of conceptual problems associated with such an approach, see Willis[1974], Nerlove[1974], Ben-Porath[1977] and Arthur's[1982] review of Becker's 1981 book. Broadly, there are four major lines of analysis in this framework -- namely, (i) the allocation of investment between human capital and physical capital, (ii) the allocation of human time, (iii) the specification of the household production function, and (iv) the integration of consumer choices and household production decisions, including the bearing and rearing of children. For a brief account of these works, see T. W. Schultz[1974]. All these works are static partial equilibrium analyses in the sense that they do not integrate micro savings and fertility decisions in order to build macro economics of population growth and capital accumulation. The use of household utility functions which involve all the well known conceptual problems of aggregating preferences, constitutes a second limitation of these models.

In the context of the present analysis, the main contribution of the new Home Economics is the explanation of the observed quality-quantity trade-off and the negative quantity-income relationship. As pointed out earlier, these results, however, require special assumptions about unobservable utility functions. (See Rosenzweig and Wolpin[1980] and Arthur[1982].) Furthermore, the definition of child quality and the motive behind parents' interest in it are not quite clear in these models(also see Arthur[1982]).

Since decisions about fertility and savings are all individual decisions having influence on and being influenced by macro level variables, it is natural to build a micro based, macro theory of population growth, capital accumulation and income distribution. Such consideration is relevant to a highly controversial issue such as whether, and if so, why, parents, especially poor parents in developing countries, want to have too many children. Are there any economic motives behind this?

Different lines of thoughts appear in the demographic transition theory literature. (For a survey see Caldwell[1982, Chap.4].) I assume the motive for having children to be completely economic, namely the desire for security in old age. Ample empirical evidence supports this hypothesis. (WDR[1984, 51-52] and Caldwell[1982].) The apparent reason for skepticism among economists about this motive as a basis for economic analysis of population growth is that the rate of return from children generally turns out to be very low, often negative. This suggests that other motives must be operative. This paper will show, however, that when capital markets are imperfect, as in developing countries, a low or negative return is not inconsistent with an old age security motive. In this connection, also see Chandrasekhar[1967,55-56], Dov Chernichovsky[1982], Willis[1980], Nehar[1971].

To build a micro foundation for macro growth economics of population and capital accumulation, explicating the special role played by income distribution and market fragmentation, I combine two overlapping generations models, namely, that of Samuelson[1958], and of Diamond[1965]. I assume that the economy includes two income groups, poor and rich, and two qualities of labor, skilled and unskilled. The motive for having children reflects solely the desire for security in old age. Some models in the literature closely resemble mine in some important respects. (See Dov Chernichovsky[1982], Willis[1980], Nehar[1971], Nerlove, Razin and Sadka[1984].) Chernichovsky uses the Beckerian household production function framework under imperfect capital market considerations. The Willis[1980] model is closely related to mine in that he also studies the implications of old age security hypothesis on rate of population growth in an overlapping generations framework. However, his model is not amenable to studying some of the other issues with which this paper is concerned. The Nerlove, Razin and Sadka's[1984] models, also based on the overlapping generations framework, assume "Chicago" utility functions, in which the number as well as the welfare of the children are included as arguments. Their models apply mainly to two generation economies without production. These simple models are unable to study any interaction between income distribution and different family decisions. Recently, Eckstein and Wolpin[1982] proposed an overlapping generations model of capital accumulation and population growth. In their model, however, the motive for having children is not old age security, and no role is played by income distribution in determining the population of the economy.

The plan of the paper is as follows. Section 2 sets up the model, explains the terminology, and posits all the issues to be addressed in the paper. Section 3 states and proves the existence of the equilibrium as defined in section 2. In section 4, I present a simple economy to study the issues laid down at the end of section 2; in section 5, I deal with those issues in a more general economy. Section 6 deals with the problems associated with different optimality concepts in the literature. Section 7 points out the limitations and possible extensions of the model and sketches intuitive solutions to some of the excluded issues. Finally, section 8 concludes the paper with a summary of the results.

2. THE BASIC MODEL

I consider an economy in which there are two[4] classes of people -- rich and poor. Interchangeably at times, I will refer to rich as capitalist and to poor as proletariat. Each person, in this economy, lives for three periods -- young, adult, and old. I do not distinguish between sexes in this model. When an individual is young, he is dependent on his parent. When he becomes an adult, he enjoys parenthood and participates in the labor market to support his family. He gives a constant (over time and across classes)[5]

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[4] A more realistic model should have at least three classes, see section 7 on this issue.
[5] This assumption is made only for symmetry and simplicity; it is

fraction of his income to his old parent, who is too old to participate in the labor market. Out of the rest of his income, he makes a decision about how much to invest on physical capital and how many children to have, in order to maximize intertemporal utility subject to budget constraints. There are three goods in the economy. One of them is producible and could be either consumed or invested for future production. The other two are two types of productive labor -- namely, skilled labor and unskilled labor, each unit supplied inelastically by each member of the adults of that generation.

Formation of each unit of capital, skilled labor and unskilled labor costs a poor parent $1+c$, d_S+f, and d_U, and a rich parent 1, d_S, and d_U+g units of consumers good respectively; d_S, d_U, c, f, and g are all positive and constant over time, and $d_S > d_U$. I also rule out the possibility of borrowing and lending for both capital formation and skill formation.

Although several explanations for this assumption of differential human capital costs **are** plausible, I consider only two more compelling ones here. First, in most LDCs, the poor live in rural areas which are far away from the cities. In these countries schools and colleges, especially the good ones, are all located in the cities. For the sake of analysis, I assume that only low quality schools are located in the villages. I interpret $f > 0$ as the transport cost a poor parent

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nowhere essential in the present analysis. Also see remark 3.6 on this.

has to bear in order to send his child to a school in cities. The interpretation for $g > 0$ is similar. I visualize $c > 0$ as a broker's fee a poor parent has to pay in order to obtain and have processed technical information about the capital market. A second conceivable motivation for these differential costs concerns differences in attitudes towards risk bearing. The production processes of skilled labor and physical capital involve higher risk than that of unskilled labor. Poor parents are probably more risk averse than the rich. As I have not introduced uncertainty in the model explicitly, I assume that the poor would behave exactly like the rich in a certainty equivalent way, had they received subsidies c and f per unit of capital and skill they form. Furthermore, $g > 0$ could be interpreted as a psychological compensation needed by a rich parent who suffers status loss if he sends his children to a lower quality school. With this interpretation, I do not need to restrict the better quality schools solely to the cities.

The technology of the model is simple. Capital lasts for one period and has zero scrap value.[6] Once formed, capital cannot be consumed. In each period, the production process is represented by a production function that instantaneously transforms capital and the two types of labor into a flow of consumer goods. There is no technological change.

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[6] This assumption helps to evade the problems associated with bequest and irreversibility of capital.

I shall distinguish the consumer good, capital good, and the labor of each type available at different dates as different commodities. Let K_t, L_t^S, L_t^U denote the macro level stock of capital, skilled labor, and unskilled labor available for production at time t. Let $P = \{(p_t, q_t, W_t^S, W_t^U) \geq 0,\ t = 0,1,2,\ldots\}$, where p_t, q_t, W_t^S, W_t^U represent respectively the present value of a unit of consumer good, capital good, skilled labor, and unskilled labor available in period t, for $t = 0,1,2,\ldots$. Let $w_t^S = W_t^S/p_t$, and $w_t^U = W_t^U/p_t$, be the wage rates in terms of t-th period consumer good, i.e. the real wage rates; and also let $1 + r_t = q_t/p_t$ be the rate of return of capital in period t in terms of the consumer good of the same period. Let me denote by $P_t = (p_t, q_t, W_t^S, W_t^U, p_{t+1}, q_{t+1}, W_{t+1}^S, W_{t+1}^U)$ the vector of prices in period t and t + 1 associated with P. Let F(x,y,z) be the aggregate production function of the economy, where x is the stock of capital, y is the amount of skilled labor, and z is the amount of unskilled labor.

The **producer's problem** at time t, for given P_t, is to choose non-negative K_{t+1}, L_{t+1}^S, L_{t+1}^U so that

$$(2.1)\ \text{Max}\ p_{t+1}F(K_{t+1}, L_{t+1}^S, L_{t+1}^U) - q_{t+1}K_{t+1} - W_{t+1}^S L_{t+1}^S - W_{t+1}^U L_{t+1}^U$$

The solution of this maximization problem yields capital and labor demand. We denote them by $K_{t+1}^d(P_t)$, $L_{t+1}^{Sd}(P_t)$, and $L_{t+1}^{Ud}(P_t)$.

I assume that the individuals belonging to the same class are identical. A proletarian adult of the t-th generation starts his decision making with a disposable income, $(1-a)W_t^U$, after giving away aW_t^U of his earned wage income to his old parent. Similarly, a capitalist adult begins his decision making with $(1-a)W_t^S$ amount of disposable income.

A **proletarian decision maker's problem**, for given P_t, is to choose non-negative s_t^P, x_t^{PS}, x_t^{PU} to

(2.2) Max $U^P(C_1, C_2)$

Subject to

$$(1 + ex_t^{PS} + ex_t^{PU})p_t C_1 \le (1-a)W_t^U - p_t(1+c)s_t - p_t(d_S+f)x_t^{PS} - p_t d_U x_t^{PU}$$

$$p_{t+1}C_2 \le q_{t+1} \cdot s_t^P + a(W_{t+1}^S \cdot x_t^{PS} + W_{t+1}^U \cdot x_t^{PU})$$

where,

e = child to adult convertion factor with respect to consumption in a household,

s_t^P = investment in physical capital of a poor parent,

x_t^{PS} = number of skilled children of a poor parent,

x_t^{PU} = number of unskilled children of a poor parent,

C_1, C_2 are consumption of the decision maker when he is young and old respectively, and

$U^P(,)$ is a typical proletarian's intertemporal utility function.

A representative **capitalist's problem**, for given P_t, is to choose non-negative s_t^R, x_t^{RS}, x_t^{RU} to

(2.3) Max $U^R(C_1, C_2)$

Subject to

$$(1 + ex_t^{RS} + ex_t^{RU})p_t \cdot C_1 \leq (1-a)W_t^S - p_t \cdot s_t^R - p_t \cdot d_S \cdot x_t^{RS} - p_t(d_U+g)x_t^{RU}$$
$$p_{t+1} \cdot C_2 \leq q_{t+1} \cdot s_t^R + a(W_{t+1}^S \cdot x_t^{RS} + W_{t+1}^U \cdot x_t^{RU})$$

where,

e = child to adult convertion factor with respect to consumption in a household,

s_t^R = investment in physical capital of a rich parent,

x_t^{RS} = number of skilled children of a rich parent,

x_t^{RU} = number of unskilled children of a rich parent,

C_1, C_2 are consumption of the decision maker when he is young and old respectively, and

$U^R(\ ,\ )$ is a typical capitalist's intertemporal utility function.

Solutions of (2.2) and (2.3) constitute the supply side of the markets for the capital and the two types of labor.

Remark:2.1: Due to the assumption of imperfection in the capital market, there are two separate budget constraints for each decision maker.

Let me now define the aggregate supply funtions[7] for the t+1-th period capital and labor as follows,

$$K_{t+1}^s(P_t) = L_t^R \cdot s_t^R(P_t) + L_t^P \cdot s_t^P(P_t)$$
$$L_{t+1}^{Ss}(P_t) = L_t^R \cdot x_t^{RS}(P_t) + L_t^P \cdot x_t^{PS}(P_t)$$

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[7] This assumption is made here to simplify the exposition. In general, these are correspondences. All the concepts and the existence theorem, however, could be generalized in a natural way to such cases.

$$L^{Us}_{t+1}(P_t) = L^R_t . x^{RU}_t(P_t) + L^P_t . x^{PU}_t(P_t)$$

for all $t = 0,1,2,\ldots$, where L^R_0, L^P_0, K_0 are given, and K_{t+1}, L^R_{t+1}, and L^P_{t+1} are solution of the following market clearing conditions:

$$\begin{aligned} K^s_{t+1}(P_t) &= K^d_{t+1}(P_t) \ (= K_{t+1}) \\ (2.4)\quad L^{Ss}_{t+1}(P_t) &= L^{Sd}_{t+1}(P_t) \ (= L^R_{t+1}) \\ L^{Us}_{t+1}(P_t) &= L^{Ud}_{t+1}(P_t) \ (= L^U_{t+1}) \end{aligned}$$

In other words, in equilibrium, the aggregate supplies of labor and capital emerging from consumer decisions equal the aggregate demands emerging from the producer's decision problem.

Definition 2.2: A **myopic**[8] **rational expectations equilibrium** is a sequence of non-negative price vectors $P = \{(p_t, q_t, W^S_t, W^U_t), t = 0,1,2,\ldots\}$ and a sequence $\{X_t = (s^R_t, s^P_t, x^{RS}_t, x^{RU}_t, x^{PS}_t, x^{PU}_t), t = 0,1,2,\ldots\}$, such that given P_t -- associated with P -- X_t solves (2.1) - (2.4); and if there is an excess supply of any commodity in any period, then its price is zero.

Definition 2.3: **A steady state** population growth and capital accumulation path is a sequence $\{X_t: t = 0,1,2,..\}$ in which $X_{t+1} = X_t$, for all $t = 0,1,2,\ldots$

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[8] In general, the parents' utility functions may include the welfare of their children; but we assume our parents to be myopic in the sense that while maximizing their utilities, they do not take welfare of the future generations into account.

Definition 2.4: A sequence $\{X_t = ((s_t^R, s_t^P, x_t^{RS}, x_t^{RU}, x_t^{PS}, x_t^{PU}) | (C_t^{R1}, C_t^{R2}, C_t^{P1}, C_t^{P2}), t = 0,1,2,\ldots\}$ is said to be a **feasible allocation** if it satisfies the following:

$$C_t + K_{t+1} + I_{t+1} + h_{t+1} = F(K_t, L_t^R, L_t^P) \text{ for } t = 1,2,\ldots,$$

where K_0, L_0^R, and L_0^P are given, and

$$\begin{aligned} C_t = {} & L_t^R(1+e(x_t^{RS} + x_t^{RU})).C_t^{R1} \\ & + L_t^P(1+e(x_t^{PS} + x_t^{PU})).C_t^{P1} \\ & + L_{t-1}^R.C_{t-1}^{R2} + L_{t-1}^P.C_{t-1}^{P2} \end{aligned}$$

$$K_{t+1} = s_t^R.L_t^R + s_t^P.L_t^P; \; h_{t+1} = cs_t^P.L_t^P$$

$$I_{t+1} = d_S(L_t^R.x_t^{RS} + (1+f)L_t^P.x_t^{PS}) + d_U((1+g)L_t^R.x_t^{RU} + L_t^P.x_t^{PU})$$

$$L_t^R = L_{t-1}^R.x_{t-1}^{RS} + L_{t-1}^P.x_{t-1}^{PS}$$

$$L_t^P = L_{t-1}^R.x_{t-1}^{RU} + L_{t-1}^P.x_{t-1}^{PU}$$

C_t^{ij} = consumption of the t-th generation adult at his working age (j = 1) and at his old age (j = 2), for i = R and P.

C_t = aggregate consumption of the economy

I_{t+1} = total investment in human capital at time t

K_{t+1} = aggregate investment in physical capital made in time t.

Definition 2.5: A feasible allocation, $\{X_t = ((s_t^R, s_t^P, x_t^{RS}, x_t^{RU}, x_t^{PS}, x_t^{PU}) | (C_t^{R1}, C_t^{R2}, C_t^{P1}, C_t^{P2}), t = 0,1,2,\ldots\}$ is **cohort-wise Pareto optimal** if there does not exist another feasible allocation X_t^* such that

$$c_t^{ij*} \geq c_t^{ij}$$

for all $t = 1,2,\ldots,$ for all $i = R,P,$ and $j = 1,2,$ with at least one strict inequality.

Remark 2.6 : This definition of optimality is limited in that it uses consumption of a representative member of each cohort as the criterion for comparing different consumption streams. The number of persons in the economy does not matter in this comparison. To be more specific, if two allocations give exactly the same consumption streams to each member of cohorts of different generations, then they are equivalent by this optimality criterion, regardless of relative population sizes.

Remark 2.7 : The myopic rational expectations equilibrium or what is known as temporary general equilibrium when there is no uncertainty, is a natural framework to use if one is concerned about integrating individual fertility and savings decisions. The Arrow-Debreu complete market framework only makes sense if the population at each point in time is exogenously given. The framework of sequential games with Nash Equilibrium as its solution concept is another possibility. However, I do not pursue this line of research in this paper.

One would like to ask the following questions. [1] When does a myopic rational expectations equilibrium exist? [2] Is it cohort-wise Pareto optimal? [3] What happens to income inequality, rates of population growth, and capital accumulation on a path of myopic rational expectations equilibria? [4] Does a myopic rational expectations equilibria path X_t, $t \geq 0$, converge to a steady state? [5] What is the concept of optimal population growth in this context? Could it be attained by decentralized myopic rational expectations decisions under 'realistic' policy interventions? What instruments should be used for it? [6] What are the effects of different types of technological change on income inequality, population growth, skill composition, and on capital accumulation? [7] Is there any trade-off between intra-generational equity and inter-generational equity?

Question [1] is addressed in the next section, while question[5] is addressed in section 6.Question [6] is not explored in detail here, but see section 7 for a few remarks on this. All other issues are discussed in sections 4 and 5.

3. EXISTENCE THEOREM

The following assumptions are sufficient to prove the existence of a myopic rational expectations equilibrium.

Assumption A3.1: $F(x,y,z) = 0$ if $x.y.z = 0$.

Assumption A3.2: F exibits constant return to scale.

Assumption A3.3:

$$\lim U_i^G(x,y) \dashrightarrow \infty \text{ as } x \dashrightarrow 0, \text{ for all } y,$$

and,

$$\lim U_i^G(x,y) \dashrightarrow \infty \text{ as } y \dashrightarrow 0, \text{ for all } x,$$

where U_i^G is the i-th partial derivative of U^G, for G = P, and R, and i = 1, 2.

Assumption A3.4: Utility functions are such that the functions, $s_t^R(.)$, $s_t^P(.)$, $x_t^{RS}(.)$, $x_t^{RU}(.)$, $x_t^{PS}(.)$, and $x_t^{PU}(.)$ are all differentiable functions, for all t = 0,1,2,...; the production function F is twice continuously (Frechet) differentiable; moreover, each element of the following matrix is < 1 in absolute value:

$$\begin{bmatrix} F_{11} & F_{12} & F_{13} \\ F_{21} & F_{22} & F_{23} \\ F_{31} & F_{32} & F_{33} \end{bmatrix} \cdot \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} = D$$

where, $F_{ij} = \partial F_i/\partial x_j$, $a_{ij} = \partial Z_i/\partial v_j$, i, j = 1, 2, 3

$Z_1 = \lambda s_t^R + s_t^P$

$Z_2 = \lambda x_t^{RS} + x_t^{PS}$

$Z_3 = \lambda x_t^{RU} + x_t^{PU}$

$v_1 = 1 + r_{t+1}$

$v_2 = w_{t+1}^S$

$v_3 = w_{t+1}^U$, $\lambda = L_t^R/L_t^P$.

THEOREM 3.5: Under assumptions A3.1 -- A3.4, a myopic rational expectations equilibrium exists. Moreover, in such an equilibrium, the poor do not form skilled labor and physical capital, and the rich do not form unskilled labor.

PROOF: First note that between period t and t + 1, three commodities, skilled labor, unskilled labor, and capital, are produced. For each of these commodities, there are two activities available--one by the rich and the other by the poor. It is easily seen that the unskilled labor production activity of the poor dominates that of the rich, and the skilled labor and physical capital formation activities of the poor are dominated by that of the rich. (An activity **dominates** another if, using the same amount of input, the first activity produces more good than the second).

Note that our one generation economy is not different from an Arrow-Debreu economy. Hence, if a myopic rational expectations equilibrium exists then it is production efficient. Production efficiency implies that all dominated activities do not operate. Therefore, s_t^P, x_t^{PS}, and x_t^{RU} are all zero in equilibrium.

Assumptions A3.1 and A3.3 imply that if an equilibrium exists then for all $t \geq 0$, $\pi_t = (p_t, q_t, W_t^S, W_t^P) > 0$. Suppose that any of them could be zero. Then although supply of that commodity will be zero, its demand will be positive. Hence this could not be an equilibrium.

Now I construct a sequence of equilibrium price vectors.

Define $p_0 = 1$, $q_0 = F_1(K_0, L_0^R, L_0^P)$, $W_0^S = F_2(K_0, L_0^R, L_0^P)$, and $W_0^U = F_3(K_0, L_0^R, L_0^P)$, where, K_0, L_0^R, and L_0^P are, respectively, the initial stock of capital, skilled labor and unskilled labor. Now suppose that equilibrium prices, η_t, and quantities, K_t, L_t^R, and L_t^P are found for time periods $\leq t$. we want to produce an equilibrium η_{t+1}. Note that if $(\eta_t|\eta_{t+1})$ solves (2.1)-(2.4), so does $(\eta_t|c\eta_{t+1})$, for $c > 0$. Thus, our problem is equivalent to finding $v = (1,\ 1+r_{t+1},\ w_{t+1}^S,\ w_{t+1}^U)$ such that $(\eta_t|v)$ solves (2.1)-(2.4). To that end, let us consider the mapping g: $R_+^3 \dashrightarrow R_+^3$, defined by

$$1 + r_{t+1} = F_1(\lambda s_t^R(v) + s_t^P(v),\ \lambda x_t^{RS}(v) + x_t^{PS}(v),\ \lambda x_t^{RU}(v) + x_t^{PU}(v))$$

$$w_{t+1}^S = F_2(\lambda s_t^R(v) + s_t^P(v),\ \lambda x_t^{RS}(v) + x_t^{PS}(v),\ \lambda x_t^{RU}(v) + x_t^{PU}(v))$$

$$w_{t+1}^U = F_3(\lambda s_t^R(v) + s_t^P(v),\ \lambda x_t^{RS}(v) + x_t^{PS}(v),\ \lambda x_t^{RU}(v) + x_t^{PU}(v))$$

where, $\lambda = L_t^R/L_t^P$, and arguments corresponding to the t-th period of the functions, $s_t^R(.)$, $s_t^P(.)$, $x_t^{RS}(.)$, $x_t^{PS}(.)$, $x_t^{RU}(.)$, $x_t^{PU}(.)$ are suppressed.

Without loss of any generality, assume that $\lambda < 1$. Since F is homogeneous of degree 1, the F_i's are homogeneous of degree zero. Multiplying each argument of F_i's by L_t^P, it is easy to note that a fixed point of $g(\ ,\ ,\ )$ does indeed solve (2.1)-(2.4).

Now I want to show that g is a contraction mapping so that it has a unique fixed point. Let us consider the following norm on R^3. For $x = (x_1,\ x_2,\ x_3)$ in R^3, let

$$\|x\| = \max\ \{|x_i| : i = 1,\ 2, \text{ and } 3\}.$$

Note that

$$\|g(x) - g(y)\| = \max\ \{|g_i(x) - g_i(y)|: i = 1,\ 2,\ 3\}$$

$$< b\|x - y\|, \text{ for some } b < 1.$$

For, by First Mean Value theorem of calculus,[9] we have

$$|g_i(x) - g_i(y)| = D_i(h).(x - y)$$
$$\leq \|D_i(h)\|.\|x - y\|,$$

where, $h = \theta x + (1-\theta)y$ for some $0 \leq \theta \leq 1$, and D_i is the i-th row of D in A3.4, and $\|D_i\| < 1$, by assumption A3.4. Thus g is a contraction mapping.

Plugging in the fixed point in s_t's and x_t's one obtains the equilibrium quantities.

Q.E.D.

Remark 3.6: F_{ij}'s are related to demand elasticities, and a_{ij}'s are related to supply elasticities of the three goods produced by the households. The assumption A3.4 corresponds to a form of stability condition for competitive equilibrium.

Remark 3.7: The proof of the above theorem holds even if the fraction, a, of the income to be transferred from the adult to his old parent varies over generations and across classes, so long as it is fixed and known to the decision makers. An interesting question is whether there is a sequence of these fractions that maximizes a social welfare function. One might also explore what rule generates a sequence that we observe in reality. A bargaining game theoretic

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[9] See for example, Rudin, W.[1953],"Principles of Mathematical Analysis", McGraw Hill Book Co., Inc., New York.

set-up would be useful in this context. However, I do not pursue these lines of research here.

Since, in equilibrium, the poor always supply unskilled labor, and the rich only skilled labor and physical capital, I denote the unskilled children of the poor as P, and the skilled children of the rich as R, and supress the superscripts S and U in the next two sections.

4. A SIMPLE ECONOMY

Suppose the production function of the economy is $F(K_t, L_t^R, L_t^P) = A(K_t)^{\theta}(L_t^R)^{1-\theta} + L_t^P$. The interpretation of this production function is as follows. Imagine that land is abundant, and poor people are agricultural laborers. Suppose that L_t^P units of unskilled labor can produce L_t^P units of rice on land. Working in manufacturing, the skilled laborers' work combine K_t units of physical capital with L_t^R units of skilled labor to produce $A(K_t)^{\theta}(L_t^R)^{1-\theta}$ units of iron. Now suppose that there is an international market place where rice could be exchanged for iron unit for unit.

Let me assume, for simplicity, that $e = 0$.

Let me further assume that $U^P(C_1, C_2) = U^R(C_1, C_2) = C_1.C_2$.

Note that in equilibrium, $w_t^P = 1$ for all $t \geq 0$. This implies that, in equilibrium,

$$x_t^P = (1\text{-}a).w_t^P/(2d_P) = (1\text{-}a)/(2d_P)$$

The first order conditions of (2.3) together with the market clearing conditions, yield the equilibrium quantities,

$$x_t^R = (1\text{-}a)(1\text{-}\theta)w_t^R/(2d_R),$$
$$s_t = (1\text{-}a)\theta w_t^R/2.$$

Also note that in equilibrium,

$$w_{t+1}^R = A^*, \text{ and}$$
$$(1+r_{t+1}) = A^*/d_R, \text{ for all } t \geq 1,$$

where,

$$A^* = d_R^\theta.A.\theta^\theta(1-\theta)^{1-\theta}$$

So, if the initial conditions are chosen suitably such that the starting wage rate for the rich is A*, one gets a steady state from the very begining. Such conditions are always possible to choose, e.g. by taking $L_t^R = 1$, and $K_t = \theta d_R/(1-\theta)$.

Now consider whether, for any arbitrary initial condition, the myopic rational expectations equilibrium $(x_t^R,\ x_t^P,\ s_t)$ converges to a steady state. It is clear from the formula for w_{t+1}^R above that, after $t > 1$, one gets a steady state income for both rich and poor. Thus, the equilibrium tends to the same steady state as is obtained by the suitably chosen initial conditions.

Now I study the properties of myopic rational expectations equilibrium assuming, without loss of much generality, that it is in steady state from the very beginning. Writing all equilibrium quantities together, I have

$$w_t^R = A^*$$
$$w_t^P = 1$$
$$1 + r_{t+1} = A^*/d_R$$
$$p_t = 1$$
$$x_t^R = (1-a)(1-\theta)A^*/(2d_R)$$
$$x_t^P = (1-a)/(2d_P)$$
$$s_t = (1-a)\theta A^*/2, \text{ for all } t \geq 0.$$

Let

$$1 + r_{t+1}^P = aw_{t+1}^P \cdot p_{t+1}/(d_P \cdot p_t), \text{ and}$$

$$1 + r_{t+1}^R = aw_{t+1}^R \cdot p_{t+1}/(d_R \cdot p_t),$$

where, r_{t+1}^R and r_{t+1}^P are the equilibrium interest rates enjoyed by respectively the rich and the poor from investment in their own children. This implies,

$$(1 + r_{t+1}^R)/(1 + r_{t+1}^P) = A^* d_P/d_R, \text{ and}$$

$$x_t^R/x_t^P = (1-\theta).(1 + r_{t+1}^R)/(1 + r_{t+1}^P).$$

By choosing a big enough A, the expression $(1 + r^R_{t+1})/(1 + r^P_{t+1})$ becomes slightly bigger than one, and by choosing θ suitably x^R_t/x^P_t could be made less than one. Now, $(1 + r^R_{t+1})/(1 + r^P_{t+1}) > 1$ implies that the rate of return from an unskilled child is less than that from a skilled child. If government intervenes in this economy by taxing the poor parent a little bit, and invests that on capital and skilled labor prudently, then in the new equilibrium, the net population of the economy is not increased, but the total output is bigger than the earlier equilibrium. Hence, I conclude that this decentralized economy results in a cohort-wise Pareto inefficient distribution of resources, having a tendency to invest too heavily in unproductive unskilled labor. And $x^R_t/x^P_t < 1$ implies that the poor like to have bigger families than the rich. Also note that, by choosing a or d_P, and d_R suitably, r^P_{t+1} could be made even negative, which implies a negative rate of return from unskilled children. So a low or negative rate of return from investment in children is not inconsistent with an old age security motive.

5. COBB-DOUGLAS ECONOMY.

Consider now an economy in which production and utility are all Cobb-Douglas functions. Formally, let

$$F(K_t, L^R_t, L^P_t) = AK_t^{\theta_1}(L^R_t)^{\theta_2}(L^P_t)^{\theta_3}, \; \theta_1 + \theta_2 + \theta_3 = 1, \; \theta_1,$$

θ_2, and $\theta_3 > 0$,

$U^R(C_1, C_2) = U^P(C_1, C_2) = C_1.C_2.$

The first order condition for the proletarian's problem of generation t, (2.2), implies

(5.1) $e(x_t^P)^2 + 2x_t^P - (1-a)w_t^P/d_P = 0.$

The only positive solution of this equation is given by,

(5.2) $x_t^P = (-1 + \sqrt{(1-e(1-a)w_t^P/d_P})/e$, if e is non-zero, and

$=(1-a)w_t^P/(2d_P)$, if $e = 0$.

The first order condition of the capitalist's problem of generation t, (2.3), together with the market clearing conditions implies

(5.3) $s_t = c[(1-a)w_t^R - dx_t^R(P_t)]$,

where $x_t^R(P_t)$ is the solution of the following equation,

(5.4) $e(x_t^R)^2 + dx_t^R - (1-a)w_t^R/d_R = 0$,

and where

$c = \theta_1/(1+\theta_1)$

$d = (1+ec/(1-c) + (1-\theta_2)e(1-a)w_t^R/d_R - 1)/\theta_2$ (> 2).

Proposition 5.1

[a] For the same level of income, the equilibrium family size is larger for a poor parent than for a rich one.

[b] If $w_t^R/d_R < w_t^P/d_P$, then in equilibrium, $x_t^R < x_t^P$.

PROOF: Note that in a quadratic equation

$$rx^2 + mx + n = 0$$

the only positive solution is

$$x(m,n) = (- m + (m^2 - 4rn)^{1/2})/(2r)$$

Observe that x is an increasing function of n and decreasing function of m, since $\partial x/\partial m < 0$, and $\partial x/\partial n > 0$. Also note that d in (5.4) is bigger than 2. So, the proposition easily follows if one compares the coefficients of equations (5.1) with (5.4).

Q.E.D.

For algebra to be simple assume from now on that e = 0. In this case, the following are the equilibrium quantities at time period t:

$$x_t^P = (1-a)w_t^P/(2d_P)$$
$$x_t^R = \theta_2/(1+\theta_1+\theta_2).(1-a)w_t^R/d_R$$
$$s_t = \theta_1/(1+\theta_1+\theta_2).(1-a)w_t^R$$

Let $k = 2\theta_2 d_P/((1+\theta_1+\theta_2)d_R)$. Note that $k < 1$.

For the moment, I assume that $L_t^P \geq L_t^R$. I shall show later that under certain conditions on d_R, and d_P, depending on the intial population sizes of the two groups, this is true. Let w_t^R, and w_t^P be the respective incomes of the rich and the poor parents of generation t, having population L_t^R, and L_t^P, and stock of capital K_t. From the profit maximization conditions of the producer's problem, (2.1), it follows that

$$L_t^R . w_t^R = \theta_2 . F$$

(5.5)

$$L_t^P . w_t^P = \theta_3 . F$$

where,

$$F = A(K_t)^{\theta_1}(L_t^R)^{\theta_2}(L_t^P)^{\theta_3}$$

The following proposition states that there is a trade-off between intra-generational equity and inter-generational equity of income.

Proposition 5.2: A tax transfer from the rich to the poor at time t results in a net increase in population and a decrease in capital stock and output at time t+1.

PROOF: Note that $(\partial x_t^P/\partial w_t^P)/(\partial x_t^R/\partial w_t^R) = 1/k > 1$, and $\partial s_t/\partial w_t^R = (1-a)\theta_1/(1+\theta_1+\theta_2) > 0$.

Now suppose that each of L_t^R rich parents are taxed by a lump sum amount τ, and the tax receipts are distributed equally among the L_t^P poor parents. So, the after tax equilibrium quantities are given by,

$$x_t^P(\tau) = (1-a)(w_t^P + \tau r)$$
$$x_t^R(\tau) = \theta_2/(1+\theta_1+\theta_2).(1-a)(w_t^R - \tau)$$
$$s_t(\tau) = \theta_1/(1+\theta_1+\theta_2).(1-a)(w_t^R - \tau)$$

where, $r = L_t^R/L_t^P$.

Denote the after tax equilibrium output by $F(\tau)$. Then $F(\tau)$ is given by,

$$F(\tau) = A(L_t^R s_t(\tau))^{\theta_1} (L_t^R x_t^R(\tau))^{\theta_2} (L_t^P x_t^P(\tau))^{\theta_3}$$

Note that

$$\begin{aligned} d\log F(\tau)/d\tau \,|_{\tau=0} &= -(\theta_1+\theta_2)/w_t^R + \theta_3 . r/w_t^P \\ &= -(\theta_1+\theta_2)/w_t^R + \theta_3 . L_t^R . w_t^R/(w_t^R . L_t^P . w_t^P) \\ &= -(\theta_1+\theta_2)/w_t^R + \theta_3 . \theta_2/(w_t^R . \theta_3), \text{ from (5.5)}, \\ &= -\theta_1/w_t^R < 0. \end{aligned}$$

Hence the proposition follows.

Q.E.D.

Proposition 5.3: By taxing the parents of any generation for having unskilled children, one can increase total output and decrease total population in period t+1, without any decline in the t-th period consumption of any parents, provided $(1-a) \geq 2\theta_2/(\theta_1+\theta_2)$.

PROOF: Suppose a tax rate of τ per unskilled child is imposed on generation t having population L_t^R , L_t^P, and stock of capital K_t. Note that the after tax equilibrium quantity of unskilled children for each of the poor parents is given by

$$x_t^P(\tau) = (1-a)w_t^P/(2d_P+2\tau).$$

So, the total tax revenue equals $\tau L_t^P . x_t^P(\tau)$. To avoid complications in computations, I assume that this tax revenue is given equally to each of L_t^R parents in a lump-sum fashion. Then, it is easy to note that the after tax equilibrium quantity of skilled children and capital for each of the rich parents are given by,

$$x_t^R(\tau) = \theta_2/((1+\theta_1+\theta_2)d_R)(1-a)(w_t^R + \tau x_t^P(\tau).L_t^P/L_t^R)$$

$$s_t(\tau) = \theta_1/((1+\theta_1+\theta_2)d_R)(1-a)(w_t^R + \tau x_t^P(\tau).L_t^P/L_t^R)$$

Clearly this policy diminishes the first period consumption of neither the poor nor the rich parents.[10] Now I want to show that, in the after tax new equilibrium in period t+1, the total output is larger and total population is smaller than the pre-taxed equilibrium. To that end, note that the after tax t+1 th period equilibrium output is given by,

$$F(\tau) = A(L_t^R s_t(\tau))^{\theta_1} (L_t^R x_t^R(\tau))^{\theta_2} (L_t^P X_t^P(\tau))^{\theta_3}$$

Note that

$$\begin{aligned} d\log F(\tau)/d\tau \,|_{\tau=0} &= (\theta_1+\theta_2)(1-a)w_t^P L_t^P/(2d_P w_t^R.L_t^R) - \theta_3/d_P \\ &= (\theta_1+\theta_2)(1-a)\theta_3/(2d_P\theta_2) - \theta_3/d_P, \text{ from (5.5)} \\ &= [(\theta_1+\theta_2)(1-a)/2\theta_2 - 1].\theta_3/d_P, \\ &\geq 0 \end{aligned}$$

Also note that

$$dL_t^R x_t^R(\tau)/d\tau)/(dL_t^P x_t^P(\tau)/d\tau \,|_{\tau=0} = -(1-a)k/2$$

whose absolute value is certainly less than one.

Q.E.D.

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[10] This, however, owes much to the Cobb-Douglas utility functions.

Remark 5.4: The proposition shows that the short run effect of taxing parents of any generation for having unskilled children could lead to a higher output per capita in the period after it, if the tax revenue is utilized properly. To study what bearing a fully funded social security system for the poor parents will have on savings, population growth, total output, the gap in wages of the poor and rich, and the skill composition of the labor force, one has to look at the new equilibrium when all of the tax revenue is judiciously invested on physical capital and on subsidization of skill formation. But analytic computations turn out to be cumbersome.

I now illustrate the convergence of the sequence of myopic rational expectations equilibria to a steady state equilibrium and discuss its properties. For $t \geq 0$, let

$$F = A(L_t^R.s_t)^{\theta_1}\,(L_t^R.x_t^R)^{\theta_2}\,(L_t^P.x_t^P)^{\theta_3}$$

It follows that $w_{t+1}^P = \theta_3 F/(L_t^P.x_t^P)$, and $w_{t+1}^R = \theta_2 F/(L_t^R.x_t^R)$.

So,

$$\begin{aligned} w_{t+1}^P/w_{t+1}^R &= \theta_3 L_t^R.x_t^R/(\theta_2 L_t^P.x_t^P) \\ &= \theta_3\theta_2 2d_P L_t^R.w_t^R/(\theta_2(1+\theta_1+\theta_2)d_R L_t^P.w_t^P), \\ &\quad \text{substituting equilibrium values of } x_t^P \text{ and } x_t^R \\ &= \theta_2 2d_P/((1+\theta_1+\theta_2)d_R), \text{ from (5.5)} \\ &= k \end{aligned}$$

Now, note that if K_0 is the initial capital stock, and L_0^R, and L_0^P

are the initial population sizes of the two groups, then

$$x_0^P/x_0^R = (1+\theta_1+\theta_2)d_R w_0^P/2\theta_2 d_P w_0^R, \text{ from the equations of the equilibrium quantities}$$
$$= (1/k).(\theta_3/\theta_2).(L_0^R/L_0^P)$$

For, from (5.5) $L_0^P.w_0^P/(L_0^R.w_0^R) = \theta_3/\theta_2$ implies, $w_0^P/w_0^R = \theta_3 L_0^R/\theta_2 L_0^P$.

Note that in the short run, for given initial conditions, one can find a set of d_R and d_P such that the poor will have bigger families than the rich in period 1. Also note that, $w_1^P/w_1^R = k < 1$. That is, a wage gap always exists between the rich and the poor of the first generation, regardless of whether or not the economy started with a completely egalitarian wages, i.e., $w_0^P/w_0^R = 1$. But also note that,

$$x_1^P/x_1^R = ((1+\theta_1+\theta_2)d_R/2\theta_2 d_P)\ w_1^P/w_1^R = 1 .$$

In fact, $x_t^P/x_t^R = 1$ for all $t \geq 1$. Also note that

$$(1+r_{t+1}^P)/(1+r_{t+1}^R) = (aw_{t+1}^P/d_P)/(aw_{t+1}^R/d_R)$$
$$= 2\theta_2/((1+\theta_1+\theta_2)d_R)$$

I have proved the following proposition:

Proposition 5.3:

[1] $w_t^P/w_t^R \text{ ---> } k, \text{ as } t \text{ ---> } \infty.$

[2] $x_t^P/x_t^R \text{ ---> } 1 \text{ as } t \text{ ---> } \infty.$

[3] $(1+r_t^P)/(1+r_t^R) \dashrightarrow 2\theta_2/((1+\theta_1+\theta_2)d_R)$ (< 1), as $t \dashrightarrow \infty$.

So, the long run equilibrium will always be characterized by a gap in wages as well as in the rates of intersts enjoyed by the two groups, the extent of which depends on the parameters of the economy. In the steady state the family sizes are equal for both the poor and the rich households.

6. OPTIMALITY

Two lines of thought prevail in the growth literature concerning optimality when there is only one type of person in the society. One criterion involves the maximisation of this typical person's utility function in the steady state. The other one involves the maximization of the (discounted or undiscounted) infinite sum of welfare levels of different generations. In the latter approach, a long standing dispute exists over whether to take per capita welfare or the total welfare as the welfare measure of a generation. See Pitchford[1977, sec.5] on this dispute. This problem is more important when population is a choice variable.

The first approach, originally proposed by Samuelson[1975], has flaws, as pointed out by Deardorff[1976]. The modifications adopted by economists, still pursuing this criterion to define the optimal level of population, is to postulate that the utility function of a parent includes the number of children as an argument. Razin and Ben

Zion[1975] were the first to introduce this modification and to prove that the applicability of Samuelson's criterion is still possible. Recently, Eckstein and Wolpin[1982] adopted the same point of view. This modification assumes away any economic motive for having children.

The other approach for optimality dates back to Ramsey[1928]. The leading proponent of the per capita welfare criterion is Koopmans, and that of the total welfare criterion is Meade. (See Pitchford[1977, sec.5] for details). The Koopmans-Meade controversy is based on a framework with exogenously determined population. A new problem, the problem of non-convexity, appears once fertility is subject to choice. The total welfare criterion involves non-convexity of the objective function as well as the feasible set, whereas the per capita criterion involves non-convexity of only the feasible set. Though the existence of an optimal allocation is not in much trouble (Chichilnisky[1981]), the decentralization of the optimal population growth and capital accumulation path through prices might be in danger. However, the total welfare approach still yields positive results in the literature, see Dasgupta[1969], Lane[1975], Pitchford[1977]. The problem with these results is that they are based on an aggregate macro relationship rather than individual decisions. Consequently, the optimum prices may not be decentralizable in a myopic rational expectations equilibrium model.

In a situation with two types of people, a new dimension is added to the controversy. One has to resolve first what should be the intra-generational welfare function of the society. Pareto optimality, Rawl's maxi-min criterion, and any welfare function involving inter-personal comparison, are some possibilities to consider. Once the objective function is defined, a more meaningful approach will be to try for a second best solution by restricting the search to feasible policies. Child tax and subsidy, tax-transfer, and old age support conditional on the number of children are possible 'realistic' policies in the case of endogenous fertility. Another related optimality problem is to find a sequence of transfer fractions, the fractions of income to be transferred from adults to their parents, in order to maximize the social welfare function.

As a result of these difficulties, I do not define optimality by means of a social welfare function. Instead, I employ per capita consumption criteria[11] to study the implications of certain policies like child taxation and redistribution of income within a generation.

The above results have a number of important implications for public policies, especially for policies designed to reduce rapid population growth of the LDCs. If the poor really face higher costs of skill and capital formation, as there are ample and compelling reasons to believe, then the public policy of reducing income disparities between the rich and the poor in any generation by means

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[11] this too involves value judgements but in a different sense.

of lump sum tax transfer will be ineffective in reducing the rapid population growth, let alone its adverse effects on such other variables as skill composition and capital accumulation. A policy of child quality subsidization will be equally ineffective. If the differences in these costs arise because the poor, unlike the rich, live far from where schools are located and transport is costly, then an appropriate public policy would be to build good quality schools in the rural areas where most of the poor live. As our results suggest, these schools should indeed be as good as the best ones in the cities to have any really significant effect. If, on the other hand, cost differences arise bacause of differences in attitude towards risk, then an appropriate public policy would be to provide the poor with less risky assets, probably riskfree assets, yielding higher rates of return than those earned from investment in their own unskilled children. Improvement in banking services, especially in the rural areas, will certainly be synergistic to such a policy. In actuality, however, the differences in those costs probably arise from both transport cost and attitude towards risk. So both of these policies are needed synergistically in the rural areas.

7. SOME REMARKS

I have shown that the assumption of differences in costs, no matter whether perceived or actual, will lead the poor to supply only unskilled labor, and the rich to supply human capital and physical capital. I also showed that the poor tend to have bigger families than the rich in equilibrium, in the short run at least. These findings are consistent with the widely observed negative relationships between household income and family size, and between child quality and quantity at the national level. This model therefore provides an alternative mechanism to the Beckerian interaction models which also try to explain these relationships.

I have not touched the issue of technological change and its impact on income inequality, capital accumulation, and population growth. It is intuitively clear that with Hicks neutral technological change income inequality will be aggravated, the family size of the poor parent will be even greater relative to that of the rich parent in the short run, and there will be an increase in the overall population level in the short run as well as in the long run. It is important to study the effects of labor augmenting technological change on these variables.

In the previous section I suggested informally that introduction of some riskless assets may reduce population growth of the LDCs. A rigorous study of such a policy requires the model to explicitly include assets such as money.

Another important extension of the model is to incorporate decreasing returns to scale in technology. To do this, one has to introduce another class into the model. People in this class will hold the fixed factors of the economy, e.g. landlords in the case of the fixed factor land.

Yet another important extension of the model would be to incorporate information and uncertainty. For that one has to lay down rules regarding dissemination of information between classes over time. In my opinion, this extended framework would be very useful in a study of the present pattern of demographic transition in most of the LDCs. In particular, proper formulation of information sets and the rules for expectations based on these sets could possibly explain why the fall in infant mortality has preceded the fall in birth rates with such a long lag. This in turn will explain how the income gap and family size gap move over time. This extended framework would also be helpful in undertstanding an interest of the capitalist class in withholding the dissemination of information to the rest of the society in order to perpetuate exploitation, see Arrow[1974, 172-173]. The education system, in particular, would be contaminated by this self-interest of the capitalist class.

8. CONCLUSIONS

Since the 1960's, empirical research has devoted considerable effort to the study of the causal relationships among population growth, household savings, income inequality, quality composition of the labor force, and per capita income as reflected in present LDC experiences. While a consensus has been reached, more or less, regarding the quality-quantity and income-quantity relationships, the studies failed to confirm even the direction of causality regarding the population-saving link or the population-income inequality link. A better understanding of these relationships is of pressing concern as it has bearing on different population policies of the LDCs. Incorporating population growth, as an economic choice variable, into a unified theoretical framework constitutes a vital first step.

Since savings and fertility decisions are made on a household level, simultaneously influencing and being influenced by different macro level variables, the macroeconomic implications of my model are derived from micro level considerations. Moreover, since much empirical evidence supports the "old age security hypothesis", I build my model around this hypothesis. In reality, of course, the desire for old age security is not the only reason for having children. The present analysis simply purports to investigate how much of the above mentioned interactions could be attributable to such a motive.

In the last two decades, some progress has been made toward understanding the link between population growth and saving in a micro based macro framework. On the other hand, the quality-quantity trade-off and inverse income-quantity relationships have been addresssed purely in a micro framework. These investigations have considered possible effects of income distribution upon those linkages or how these are related to the pace of population growth.

This essay has studied an overlapping generations model of an economy in general equilibrium, featuring two economic classes, rich and poor, and two labor qualities, skilled and unskilled, in each generation. Parents' fertility and savings decisions, in the aggregate, determine the macro level stock of capital and supplies of the two types of labor. It is assumed that the capital market imperfectly transfers consumption from the present to the future; and that the poor perceive higher costs in producing both capital as well as skilled children. The rich perceive a higher cost of producing unskilled children than the poor. The following results were derived:

This model provides an alternative mechanism to the Beckerian interaction models for generating a child quality-quantity trade-off and an inverse household income-quantity relationship such as that observed in most of the LDCs.

I showed formally that under quite general assumptions, a sequence of myopic rational expectations equilibria (which is the same as temporary general equilibrium) exists. In such an equilibrium path, the poor will specialize in supplying unskilled labour and the rich in supplying skilled labour and physical capital in every generation.

Regarding the different linkages and some of their policy implications, the following results hold for Cobb-Douglas (and Cobb-Douglas-like) production and utility functions:

In the short run, poor parents tend to have bigger families than the rich. The resulting economy exhibits wage inequality, regardless of the initial wage distribution. A tax transfer from the rich to the poor has several effects. First, population increases and total output for the next generation falls. Second, the next generation contains a higher proportion of unskilled laborers, earning a lower wage rate than before. Taxing parents for having unskilled children and utilizing the tax receipts prudently can, however, reverse these effects.

Apart from these short run results, I showed that the sequence of short run equilibria always converges to a steady state, regardless of initial conditions. This long run equilibrium creates a gap in incomes as well as in the rates of interest faced by the two groups. However, in the Cobb-Douglas case, the family size of the two classes is the same in the long run. This last result is specific to the Cobb-Douglas production function. The steady state family sizes need not be equal for other constant return to scale production functions.

These findings have the following implications for public policies, especially for population policies of the LDCs. If the poor really face a higher cost of skill and capital formation than the rich, as there appear to be ample and compelling reasons to believe, neither a policy of reducing income inequality by lump sum transfer from the rich to the poor, nor a policy of subsidizing the cost of skill formation for the rich and poor uniformly, will be able to reduce rapid population growth, let alone their adverse effects on capital accumulation and skill composition of the economy. The relevant public policy in this situation would be to build good quality schools in rural areas; in fact, they should be as good as the best ones in the cities. Moreover, this should be complemented by providing high yielding risk free assets, and improved banking services to implement it, if the poor are really more risk averse than the rich and the processes of skill and capital formation involves risks. However, this last policy is a tentative suggestion as our present model does not formally incorporate uncertainty.

This paper provides another step towards understanding the theoretical relationships between economic and demographic variables. It provides a framework suitable for studying important questions such as how a social security scheme for the poor might affect national household savings, the pace of population growth, and the quality composition of the labor force. Properly extended, it might be able to give an alternative explanation for the present pattern of demographic transition in the LDCs.