ESSAY I

MYOPIC TOPOLOGIES ON GENERAL COMMODITY SPACES

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1. INTRODUCTION

The problem of development planning is how to allocate resources among different activities over time and under uncertainty so as to maximize social welfare. In the economic literature, this is studied under the heading of optimal economic growth. The availability of natural and human resources at different points of time, together with the exogeneously given technology determines the set of feasible programs. A social ordering--either partial or complete--is defined over the feasible set. Then the problem of economic growth reduces to finding a program that is maximal with respect to this social preference ordering. Whether such a preference ordering should exhibit impatience or myopia is a long standing controversy in the planning literature.

Ramsey[1928] in his seminal attempt to give a rigorous treatment of the optimal economic growth problem took the undiscounted sum of utility streams for the entire future to represent the social preference ordering. Pigou in his "Economics of Welfare" had advocated the same long before Ramsey, but Ramsey was the first to apply it formally to the optimal growth problem. Taking the undiscounted sum of utilities to represent the social preference ordering, however, runs into several conceptual as well as mathematical problems. First, if the original intention of having a zero discount rate lay in the ethics of treating all generations equally, then, as shown by Koopmans[1965] and Chakravarty[1969], this might not be the case in general. In fact, in the presence of population growth and technological change, an equal treatment of all generations would rather require a non-zero discount rate; moreover this discount rate depends on the rate of population growth and technological change. But then, the choice of a welfare function could not be separated from the technological considerations in that case. Second, the undiscounted total utility criterion does not define a functional over the entire feasible program space; it becomes unbounded in most parts of the feasible program space. This could not, however, be an argument for rejecting this viewpoint as there are several modifications suggested in the literature to solve this technical problem. For instance, the finite bliss assumption of Ramsey himself, and the overtaking criteria of Von Weizsacker, Gale, and McFadden, could tackle this problem to a great extent. Both these modifications, however, define only partial orderings over the entire space of programs. (See Chakravarty[1962] for an account of other modifications suggested in the literature, and for a critical evaluation of them).

Araujo[1985] as well as the present essay show that the existence of competitive equilibrium in infinite horizon economies in general requires that individual preferences be myopic or impatient. So the question arises: Should a welfare function exhibit impatience so that it is consistent with individual behaviors? How do we choose a social welfare function for practical purposes?

Marglin[1963] argues that consumption externalities and schizophrenic individual behaviors are some of the reasons why a social discount rate should be lower than the individual discount rates. The issue of how to arrive at a social welfare function for practical purposes is rather a political matter. Sen[1961] effectively argues that 'a popular ratification of a utility function seems to be out of the question'. This remains true even if the interests of the unborn children are taken care of by the self-concern of the educated parents. Indeed the reason lies in Arrow's impossibility theorem which rules out democratic aggregation of the individual preferences to arrive at a social preference ordering. Marglin[1963, p.110] argues that 'definition of entire preference map is needlessly ambitious; all we really want to find out is the marginal time preference in the neighbourhood of the optimal rate of investment". Even here, Arrow's impossibility theorem applies; a value judgement is, in any case, necessary. But then, should the value judgement be in the form of taking weighted average of individual discount rates, or imputing it from the policy maker's preferences in an iterative way as suggested by Marglin, or should it be based on postulating a set of axioms on behavior of a rational government which may logically lead to a positive discount rate?

The last viewpoint was followed by Koopmans[1960], Diamond, Koopmans, and Willamson[1964], and Diamond[1965]. Precisely, they presume that choice among programs by the policy makers could be represented by a complete preference ordering. They laid down a set of axioms on the preference ordering and then showed that if a rational behavior means satisfying these axioms, then a social welfare function representing the preference ordering is bound to exhibit impatience or time perspective.

The standard sufficient condition for the existence of an optimal program is compactness of the feasible set and continuity of the preference ordering. Of course, both compactness and continuity are predicated on some notion of topology on the program space. Having chosen a topology, prices are then defined as continuous linear functionals. These prices, as in the finite dimensional case, are used to decentralize the optimal allocation and price out efficient allocation.

Koopmans[1960] was the first to show that the topology on the sequence space, ℓ_∞, imposes behavioral restrictions on continuous preferences. These behavioral restrictions he referred to as myopia or impatience. Diamond[1965] introduced a notion of myopia, calling it eventual impatience, and proved that the product topology on ℓ_∞ imposes eventual impatience on continuous monotonic preferences. Bewley[1972] attributed to Hildenbrand for the notion of asymptotic impatience on ℓ_∞ and for the observation that all Mackey continuous preferences over ℓ_∞ are asymptotically impatient. Brown and Lewis[1981] introduced the concepts of strong and weak myopic preferences and the strong (resp. weak) myopic topologies on ℓ_∞. The strong (resp. weak) myopic topologies are such that all continuous preferences are strongly (resp. weakly) myopic. They showed that Hildenbrand's observation about the Mackey topology on ℓ_∞ characterizes it in the following sense: The Mackey topology on ℓ_∞ with respect to the pairing $<\ell_\infty, \ell_1>$ is the finest strongly myopic locally convex Hausdorff topology on $<\ell_\infty, \ell_1>$ (Brown and Lewis[1981, theorem 4a]).

Using a characterization of myopic topologies on ℓ_∞, due to Brown and Lewis, Araujo[1985] proved that the Mackey continuity of preferences is a necessary condition for the existence of a competitive equilibrium in ℓ_∞, i.e., for any topology on ℓ_∞ finer than the Mackey topology and coarser than the sup norm topology, there exists a pure exchange economy with two agents where the core is empty, and hence no competitive equilibrium.

All these results are proved in ℓ_∞. A unified treatment of time and uncertainty, however, calls for a state space richer than the integers. This is the case even when the time is discrete, and there are only two states of nature at each point of time. So the need for a generalization of the above results to L_∞ is apparent.

In this paper I extend the notions of strong and weak myopia due to Brown and Lewis to L_∞. The characterization of strong myopic topology on L_∞ is used to extend Araujo's necessity theorem to L_∞. This extension of Araujo's theorem together with Bewley's existence theorem supports Bewley's intuition that the Mackey topology is the appropriate topology for infinite dimensional commodity spaces.

In section 2, all the concepts and notation are defined. Section 3 summarizes the main results of the essay. Other important observations are included as remarks in section 4. Section 5 puts all the proofs together.

2. CONCEPTS AND TERMINOLOGIES

Let $(W,\mathfrak{B},\mu)$ be a σ-finite measure space. The set W could be viewed here as the set of states of nature or the set of time points or both. An event is a subset of W. The set of all possible events are assumed to form a σ-algebra, $\mathfrak{B}$. Let μ be a positive σ-finite measure on $(W,\mathfrak{B})$.

Let L_∞ be the space of all μ-essentially bounded real-valued measurable functions on $(W,\mathfrak{B},\mu)$. L_∞ is viewed here as the space of state- and time- contingent commodity bundles. Let L_1 be the space of all integrable functions.

For x and y in L_1 or L_∞, define $\geq$ on L_1 or L_∞ by $x \geq y$ if $x(w) \geq y(w)$ a.e.

Definition 2.1: A **preference ordering** is a transitive binary relation on L_∞. A preference ordering $\succsim$ is **complete** if x, y in L_∞ implies either $x \succsim y$ or $y \succsim x$, and $\succsim$ is **monotonic** if $x \geq y$ a.e. implies $x \succsim y$.

Definition 2.2: A real linear vector space L is called an **ordered vector space with an order $\leq$ if L is** partially ordered by $\leq$ in such a way that the partial ordering $\leq$ is compatible with the algebraic structure of L, i.e., for all x, y, and z in L, $x \leq y$ implies $x + z \leq y + z$, and $x \geq 0$ implies $ax \geq 0$ for every real number $a \geq 0$. Let L be an ordered vector space with $\leq$ its order. A **seminorm** on L is a function p: L --> R such that $p(x + y) \leq p(x) + p(y)$, $p(tx) = |t| p(x)$, for all x, y in L and all t in R. **A locally convex topology** is a topology generated by a family of seminorms P. A seminorm p is **monotonic** if $x \geq y \geq 0$ implies $p(x) \geq p(y)$. A seminorm p **dominates** a seminorm q if there exists a $c > 0$ such that $q(x) < c\, p(x)$ for all x in L. Let Q be a family of monotonic seminorms. Q is said to be a **base** for a family of seminorms P if every p in P is dominated by a seminorm from Q, and Q is a subset of P. A topology is said to be a **locally convex topology with a monotone base** if its associated family of seminorms has a base of monotonic seminorms. By L^+, we shall denote all x in L such that $x \geq 0$.

Definition 2.3: Let L be a topological vector space with τ its topology. A preference ordering, $\succsim$, is **τ-continuous** if for all x in L, both {y in L : $y \succsim x$} and {z in L : $x \succsim z$} are τ-closed.

I now extend the notions of strong and weak myopia from ℓ_∞ to L_∞.

Let $\P = \{e = \{E_n\} : \{E_n\} \subset \mathfrak{B},\ E_n \downarrow \phi\}$

For x in L_∞, e in $\P$, and w in W, define

$$x_n^e(w) = (1_{E_n} x)(w), \text{ and}$$

$$\check{x}_n^e(w) = x(w) - x_n^e(w),$$

where 1_E denotes the indicator function of E.

Definition 2.4: A preference ordering $\succsim$ on L_∞ is **strongly myopic** if for all x, y, z in L_∞, $x \succ y$ implies for all e in ¶, and for all sufficiently large n, $x \succ y + z_n^e$; and it is called **weakly myopic** if for all x, y, c in L_∞, where c is a constant vector, $x \succ y$ implies for all e in ¶, and for all sufficiently large n, $x \succ y + c_n^e$.

Note that when W is countable then ¶ is a singleton set, and these concepts are the same as that in Brown and Lewis.

Definition 2.5 A topology τ on L_∞ will be called **strongly myopic [resp. weakly myopic]** if all τ-continuous complete preference orderings on L_∞ are strongly myopic [resp. weakly myopic].

It is easy to note that the strong and weak myopia agree for monotone preferences.

Let τ_1 and τ_2 be two topologies on L_∞. The topology τ_2 is called **finer than** τ_1 if $\tau_1 \subset \tau_2$. We shall use the notation $(L,\tau)^*$ to denote the topological dual of L under the topology τ.

We study two topologies on L_∞, namely τ_{SM}^M,[1] the finest strongly myopic locally convex Hausdorff topology with a monotone base, and

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[1] Note that this is not what is studied in Brown-Lewis; in fact, they study the finest strongly myopic locally convex Hausdorff topology.

τ_{WM}, the finest weakly myopic locally convex Hausdorff topology. The questions are: Do they exist? If so, what are their basic properties?

Definition 2.6: Let E and F be two vector spaces over R. A **pairing** is an ordered pair $\langle\langle E,F\rangle\rangle$ together with a bilinear functional $\langle\ ,\ \rangle$ defined on E X F. A $\langle\langle E,F\rangle\rangle$ **dual topology on E** is a topology such that F is the topological dual of E. Let F be a subspace of linear functionals on E. Let $\sigma(E,F)$ denote the weakest topology on E such that F is its topological dual. And also let

$$\int f\, d\mu \text{ denote } \int_W f\, d\mu$$

Let us have the pairing $\langle\langle L_\infty, L_1\rangle\rangle$ with the bilinear functional defined for all f in L_∞, and g in L_1 by,

$$\langle f,g\rangle = \int fg\, d\mu = T_g(f) \text{ say,}$$

It is well known that $\sigma(L_\infty,L_1)$ is generated by the family of seminorms $\{|T_g(f)| : g \text{ in } L_1\}$, and is a Hausdorff locally convex topology with a monotone base. Let τ_m be the Mackey topology on L_∞ when paired with L_1, i.e. the topology of uniform convergence on $\sigma(L_1,L_\infty)$-compact, convex sets of L_1. Since, $\sigma(L_\infty,L_1) \subset \tau_m$, we note that τ_m is Hausdorff locally convex. In the proof of lemma 5.3 we shall show that, in fact, it has a monotone base.

3. STATEMENT OF THEOREMS

I assume the measure space $(W, \mathfrak{b}, \mu)$ to be σ-finite.

THEOREM 3.1: $\tau^{M}_{SM} = \tau_m$

COROLLARY 3.2: Let τ be a locally convex Hausdorff topology on L_∞ and let $\tau \subset \tau_m$, then τ is strongly myopic.

Let u denote the unit vector of L_∞, that is $u(w) = 1$ a.e.

THEOREM 3.3: τ_{WM} exists on L_∞. J is in $(L_\infty, \tau_{WM})^*$ **if and only if** for all e in ¶, $J(u^e_n)$ --> 0 as n --> ∞. **Moreover,** $(L_\infty, \tau_{WM})^{*^+} = L_1{}^+$.

Denote the $\|.\|_i$-topology on L_i by τ_i, for $i = 1$, and ∞.

Definition 3.4 A pure exchange economy on (L_∞, τ) is one which satisfies the following:

(a) the preferences of the agents are τ-continuous,

(b) the initial endowment of each agent is in L_∞,

(c) the consumption set of each agent is a subset of L_∞.

Now we have the following extension of Araujo's theorem.

THEOREM 3.5: Let $\sigma(L_\infty, L_1) \subset \tau \subset \tau_\infty$. Given any τ finer than τ^{M}_{SM}, there exists a pure exchange economy on (L_∞, τ) with two agents, for which the core is empty, hence no competitive equilibrium.

4. SOME USEFUL REMARKS

Remark 4.1: Let τ_{SM} be the finest strongly myopic locally convex Hausdorff topology on L_∞. $\tau_{SM} = \tau_m$ on ℓ_∞.

Remark 4.2 : Applying the last part of theorem 3.3 and the fact that every continuous linear functional in a locally convex Hausdorff topological vector space with a monotone base is the difference of two positive continuous linear functionals (Kelly et. al.[1963, theorem 23.6, p.228]), it can be shown easily that if a topology τ has a monotone base then τ is weakly myopic if and only if it is strongly myopic.

Remark 4.3: D.J.Brown pointed out that the Mackey topology on L_∞ is the finest strongly myopic locally convex Hausdorff topology in the family of topologies that are coarser than the sup norm topology, τ_∞. This follows easily from the proof of theorem 3.1.

Remark 4.4: From lemma 5.1 we know that $\tau_{SM}^M \subset \tau_{SM}$, but we still do not know whether or not $\tau_{SM} \subset \tau_{SM}^M$.

5. PROOFS

I now assume that the following lemmas are true and prove theorem 3.1. The lemmas will be proved later.

LEMMA 5.1: Let τ be a locally convex Hausdorff topology on L_∞. Then, τ is strongly myopic if and only if for all x in L_∞, and e in ¶, $x_n^e \dashrightarrow 0$ as $n \dashrightarrow \infty$.

LEMMA 5.2: τ_{SM}^{M} exists on L_∞.

LEMMA 5.3: $\tau_m \subset \tau_{SM}^{M}$.

LEMMA 5.4: $\tau_{SM}^{M} \subset \tau_\infty$.

LEMMA 5.5: $(L_\infty, \tau_{SM}^{M})^* = L_1$.

PROOF OF THEOREM 3.1: Lemma 5.2 asserts that τ_{SM}^{M} exists. By lemma 5.5 we have, $(L_\infty, \tau_{SM}^{M})^* = L_1$. But τ_m is the finest locally convex Hausdorff topology with a monotone base on L_∞ such that L_1 is its topological dual. Hence $\tau_{SM}^{M} \subset \tau_m$. But by lemma 5.3, $\tau_m \subset \tau_{SM}^{M}$. Thus $\tau_{SM}^{M} = \tau_m$.

Q.E.D.

Now I prove the lemmas. The lemmas 5.1 and 5.4 are needed to prove lemmas 5.2 and 5.5 respectively.

PROOF OF LEMMA 5.1: The same argument as in lemma 1b in Brown and Lewis holds.

PROOF OF LEMMA 5.2: Let Q be the family of seminorms on L_∞ such that q is in Q if and only if q is monotonic and for all e in ¶ and x in L_∞, $q(x_n^e) \dashrightarrow 0$ as $n \dashrightarrow \infty$. Let P be the set of all seminorms on L_∞ each of which is dominated by a member of Q. Note that P contains the family of seminorms of point wise convergence on L_∞, which separates points of L_∞. Hence P generates a Hausdorff locally convex topology on L_∞. By lemma 5.1, it is strongly myopic. That it is the finest follows from the definition of P.

Q.E.D.

PROOF OF LEMMA 5.3: Note that a typical seminorm of τ_m is given by

$$p_C(x) = \sup \{ \mid \int xy \, d\mu \mid : y \text{ in } C\}, \ x \text{ in } L_\infty,$$

where C is a $\sigma(L_1,L_\infty)$-compact, convex subset of L_1. We want to show that p_C is a seminorm of τ^M_{SM}. Fix x in L_∞, and e in ¶ arbitrarily. Note that,

$$p_C(x^e_n) = \sup \{ \mid \int 1_{E_n} xy \, d\mu \mid : y \text{ in } C\}$$

$$= \sup \{ \mid \int 1_{E_n} g \, d\mu \mid : g \text{ in } C^*\}$$

where $C^* = \{xy : y \text{ in } C\}$. Now note that the linear operator, $T : L_1 \dashrightarrow L_1$ defined by, $Ty = xy$, is $\sigma(L_1,L_\infty)$-continuous, for let p' be a seminorm of $\sigma(L_1,L_\infty)$. Then p' is given by,

$$p'(y) = \mid \int yz \, d\mu \mid, \text{ for some } z \text{ in } L_\infty.$$

$$= p'_z(y) \text{ say.}$$

Now,

$$p'_z(Ty) = \mid \int Tyz \, d\mu \mid$$

$$= \mid \int xyz \, d\mu \mid$$

$$= \mid \int y(xz) \, d\mu \mid$$

$= p'_{xz}(y)$, since xz is in L_∞.

Hence T is $\sigma(L_1, L_\infty)$-continuous (see Reed and Simon[1980, theorem V.2, pp.129]). Thus $C^* = T[C]$, the image of C under T, is $\sigma(L_1, L_\infty)$-compact. Hence by Dunford and Schwartz[1958, theorem 1, pp.430], C^* is weakly sequentially compact. Again by Dunford and Schwartz[1958, theorem 9, pp.292],

$$p_C(x_n^e) = \sup \{ \mid \int 1_{E_n} g \, d\mu \mid : g \text{ in } C^*\} \to 0, \text{ as } n \to \infty,$$

for all x in L_∞, e in ¶, and for all $\sigma(L_1, L_\infty)$-compact, convex subset C of L_1. Also note that p_C is a monotonic seminorm. Hence p_C is a seminorm defining τ_{SM}^M.

Q.E.D.

PROOF OF LEMMA 5.4:[2] Let p be a seminorm of τ_{SM}^M. I first assume that p is monotonic, and prove that there exists a $c > 0$ such that $p(x) \le c$ for all x in L_∞ with $\|x\|_\infty = 1$. If possible, suppose $p(x) > c$ for all $c > 0$. Then for all $m > 0$, there exists x^m in L_∞, $\|x^m\|_\infty = 1$ such that $p(x^m) > m$. Now by definition of p, for each e in ¶, $p(\check{x}_n^{me}) \to p(x^m) > m$. Hence there exists a $k(m,e) > 0$ such that $p(\check{x}^m_{k(m,e)}) > m$. Now let u be the unit vector of L_∞, that is $u(w) = 1$ for all w in W. Note that for all $m > 0$, $\|x^m\|_\infty = 1$ implies that $|x^m| \le 1$ a.e. Which implies that for all $m > 0$, $x^m \le u$ a.e. Now note that $\check{u}^e_{k(m,e)} \le u$ implies that $\check{x}^{me}_{k(m,e)} \le \check{u}^e_{k(m,e)} \le u$. This in turn implies that $m < p(\check{x}^{me}_{k(m,e)}) \le p(\check{u}^e_{k(m,e)}) \le p(u)$,

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[2] A discussion with Norman Wildberger was useful in proving this.

since p is monotonic. This implies, $p(u) > m$ for all $m > 0$. This is a contradiction to the fact that p is real valued. As all other seminorms of τ_{SM}^{M} are dominated by monotonic seminorms of τ_{SM}^{M}, the above fact is true for all seminorms of τ_{SM}^{M}. Thus all τ_{SM}^{M}-continuous seminorms are τ_{∞}-continuous.

Q.E.D.

PROOF OF LEMMA 5.5: I first prove that $(L_{\infty},\tau_{SM}^{M})^* \subset L_1$. It is well known that $(L_{\infty},\tau_{\infty})^* = ba(W,\mathfrak{B},\mu)$, the set of all bounded finitely additive set functions on $(W,\mathfrak{B})$, which are absolutely continuous with respect to μ. Let J be in $(L_{\infty},\tau_{SM}^{M})^*$. Then by lemma 5.4 above J is in $(L_{\infty},\tau_{\infty})^*$. So, there exists an η in $ba(W,\mathfrak{B},\mu)$ such that $J(x) = \int x\, d\eta$. Now we prove that η is countably additive. For, let $\{A_n\} \subset \mathfrak{B}$, and $\{A_n\}$ decreases to empty set. We have to show that $\eta(A_n) \to 0$ as $n \to \infty$. In fact, by τ_{SM}^{M}-continuity of J, we have $J(1_{A_n}) \to 0$ as $n \to \infty$. Hence $\eta(A_n) = J(A_n) \to 0$ as $n \to \infty$. So η is countably additive. Thus by Radon-Nikodym theorem, there exists a y in L_1 such that $J(x) = \int xy\, d\mu$. Hence J is in L_1.

I now prove that $L_1 \subset (L_{\infty},\tau_{SM}^{M})^*$. Let f be in L_1. Denote the corresponding induced linear functional on L_{∞} as $T_f(x) = \int xf\, d\mu$. We want to show that T_f is τ_{SM}^{M}-continuous, which is equivalent to showing that $p(x) = |T_f(x)|$ is a seminorm of τ_{SM}^{M}. This is true indeed, for note that $|x_n^e.f| \le |xf|$ for all n and e, $|xf|$ is in L_1, and $|x_n^e.f| \to 0$ a.e., as $n \to \infty$. Hence by Lebesgue's dominated convergence theorem,

$$\lim_{n \to \infty} p(x_n^e) \le \lim_{n \to \infty} \int |x_n^e . f| \, d\mu = 0, \text{ for all } e \text{ in } \P.$$

Q.E.D.

PROOF OF THEOREM 3.3: The proof of the first two parts of the theorem is exactly the same as in Brown and Lewis. I shall prove here that $(L_\infty, \tau_{WM})^{*+} = L_1^+$.

Let f be in L_1^+. Then note that the corresponding induced linear functional $T_f(x) = \int xf \, d\mu$ is positive, and applying Lebesgue's dominated convergence theorem, it is easy to show that, for all e in $\P$, $T_f(u_n^e) \to 0$ as $n \to \infty$. Hence by the second part of this same theorem T_f is τ_{WM}-continuous. Now let J be in $(L_\infty, \tau_{WM})^{*+}$. Then, following the proof of theorem 2a in Brown and Lewis, it can easily be shown that J is $\|.\|_\infty$-continuous. Now following the same argument as in the proof of lemma 5.5 above we establish that there exists a f in L_1 such that $J(x) = \int xf \, d\mu$. To show that $f \ge 0$, we note that J is positive implies that $J(x) \ge 0$ for all $x \ge 0$. Taking $x = 1_A$, A in $\mathfrak{B}$, we note that $\int 1_A f \, d\mu \ge 0$ for all A in $\mathfrak{B}$. Hence $f \ge 0$ a.e. Thus f is in L_1^+.

Q.E.D.

PROOF OF THEOREM 3.5

I follow Araujo's argument to prove the theorem. Suppose τ is finer than τ_{SM}^M. Then there exists a purely finitely additive measure $\lambda > 0$ on W such that λ is bounded and absolutely continuous with respect to μ. Now I construct a pure exchange economy with two agents for which the core is empty.

First note that there exists w in L_∞ such that $\int w \, d\lambda > 0$. Let the initial endowments of the two agents be $w_1 = w_2 = w$, and their consumption sets be L_∞^+. Let the preferences of these consumers be represented by the following utility functions:

$$u_1(x) = \int x \, d\lambda, \text{ for } x \text{ in } L_\infty,$$

$$u_2(x) = \int xy \, d\mu, \text{ for some } y \text{ in } L_1^+, \text{ for all } x \text{ in } L_\infty.$$

It is easy to check that the above is a pure exchange economy. If possible, let us assume that this economy has non-empty core. Let (x'_1, x'_2) be in the core, where x'_1 and x'_2 are in L_∞. Now I show that $u_1(x'_1) = 0$; but by assumption, $u_1(w_1) > 0$; this leads to violation of individual rationality property of a core allocation, and thus to a contradiction.

In order to prove that $u_1(x'_1) = 0$, appealing to the Yosida-Hewitt theorem (see the mathematical appendix of Bewley[1972]), and to the fact that λ is absolutely continuous with respect to μ, we note that, for all $n > 0$, there exists E_n such that $\mu(E_n) < 1/n$, $\lambda(E_n^c) = 0$, $E_n \downarrow \phi$. It is now trivial to note that for all $n > 0$, $x'_1 \cdot 1_{E_n^c} = 0$, as $\lambda(E_n^c) = 0$, and this is so because the consumption of commodities in E_n^c does not contribute to the utility of the first consumer, whereas, it contributes to the second consumer's utility. Now we note that for any $r > 0$,

$$\begin{aligned}\{w: |x'_1| > r\} &\subset \{w: |x'_1 - x'_1 \cdot 1_{E_n^c}| > r/2\} \cup \{w: |x'_1 \cdot 1_{E_n^c}| > r/2\} \\ &= \{w: |x'_1 - x'_1 \cdot 1_{E_n^c}| > r/2\} \\ &\subset E_n\end{aligned}$$

So,

$$\mu\{w: |x'_1| > r\} \leq \limsup \mu(E_n) \text{ as } n \to \infty,$$
$$= 0.$$

Thus $x'_1 = 0$ a.e.(μ). But λ is absolutely continuous with respect to μ. So $x'_1 = 0$ a.e.(λ). Thus, by theorem 20.d in Dunford and Schwartz[1958], $u_1(x'_1) = 0$.

Q.E.D.