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School Choice and the Intergenerational Poverty Trap

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Abstract

This paper formulates a dynamic altruistic model of parental choice of school quality and intergenerational social mobility. It shows that when there are many school qualities, the earnings of children as a function of parental schooling investment is a non-concave function, which leads to multiple steady-state equilibria. The paper studies the intergenerational dynamics of parental schooling investment and gives conditions on the rate of return from parental schooling investment under which some families are stuck in an intergenerational poverty trap. The policy implications are also discussed.

1. Introduction

In this paper we formulate a model of parental school choice and provide an alternative explanation for the existence of the poverty trap.

Poverty is prevalent in all parts of the world. Even in developed countries, where the incidence of absolute poverty is less prevalent, the incidence of relative poverty is widespread. For instance, the World Bank (2004, table 2.7) reports that the relative poverty rate measured as a percentage of total income earned by the poorest 20 percent of the population is 10.6 percent in Japan, 9.6 percent in Norway, and is as bad as 6.1 percent in the UK and 5.4 percent in the US. The prevalence of relative poverty is much worse in some of the developing countries. For instance, it is as low as 1.1 percent in Sierra Leone, 2.0 percent in South Africa, and 2.2 percent in Brazil. Two types of factors contribute to poverty. Market factors such as bad luck in the labor market or labor market discrimination against race or gender can cause people to be in poverty in spite of their possession of marketable skills and innate ability. The second types of factors are familial and intergenerational, which are transmitted from parents to children. For instance, individuals from poor family background may fall easily into poverty because they do not acquire sufficient marketable skills as their parents invest inadequately in such skills. This type of intergenerational link can lead to an intergenerational poverty cycle and low intergenerational social mobility.

Long ago Malthus (1798) formulated an aggregate growth model producing multiple steady-state equilibria. He used these multiple steady-state equilibria phenomena to argue that the poorer countries with per capita income below a threshold level would be caught in a poverty trap and would remain poor for ever; and the richer

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countries with per capita income above the threshold level would enjoy a high level of per capita income. His focus was to find a causal link between the high fertility rate and the high incidence of poverty that prevailed during his time. There was no technological progress in his model, nor any productivity growth from human capital accumulation. His model predicted that the only way for the poorer countries to come out of the poverty trap was to reduce population growth by controlling fertility rate. He formulated his model and the relationship between income and population growth at the aggregate level, and human capital consisting of health and education did not play any role in his model. While the idea of multiple steady-state equilibria leading to poverty cycle has been around for a long time, especially in the development economics literature, the focus of the recent research has been to explore mechanisms that lead to such multiple equilibria. We provide a new mechanism in this paper based on parental choice of schools.

There are many mechanisms that can lead to intergenerational poverty cycles. Sociologists emphasize how poor family background and family connection can hinder intergenerational social mobility. For instance, teenage pregnancy, drug addiction, and crimes that are generally prevalent among the socially disadvantaged parents lead their children to acquire similar social attitudes and hence to the perpetuation of the disadvantaged status across generations. Economists emphasize mechanisms based on parental investment in the human capital of children,[1] which is also our focus.

Since the seminal work of T. W. Schultz on the role of human capital on earnings, economists emphasize human capital as the main determinant of individual earnings and the public provision of education to all as a socially viable means of reducing income differences across social groups. Becker and Tomes (1979) formulated a microeconomic model of intergenerational transmission of income inequality, incorporating parental investment in children's education, and in a later model (Becker and Tomes, 1986) they incorporated interaction between fertility and parental human capital investment. They showed that as long as the transmission of family endowments including genetic, financial, and human capital is not too large, families will rise and fall, and will regress towards the mean within a few generations. The degree of social mobility and the time it takes a family to converge to the mean depend on the degree of heritability of these endowments and on market luck: the higher is the degree of heritability, the lower is the degree of social mobility. They found empirical evidence for regression towards the mean within three generations for industrialized countries. These models cannot explain the intergenerational poverty cycle since equilibrium leads to convergence to the mean. It should be noted that a statistically significant estimate of convergence to the mean in three generations does not preclude the possibility that a fraction of the population is stuck in an intergenerational poverty cycle.

Becker et al. (1990) extended the Becker and Tomes framework to an aggregate growth model that generated multiple steady-state equilibria, similar to Malthus' multiple steady-state equilibria, with the property that the less developed economies would be stuck in a low level equilibrium. In their model, a representative agent of a less developed economy invests very little in human capital of children and chooses a high fertility level. This leads their children to have low per capita income. A representative parent in the richer countries, on the other hand, will do the opposite and thus will be around a more prosperous equilibrium. The main mechanism in their model that leads to this result is that the rate of returns from human capital is low when the stock of human capital is low, and it increases for higher stocks of human capital and eventually the rate of return starts falling. It was technically difficult in their setup to study the dynamics of the optimal solution.[2]

Raut (1990) provided another mechanism for intergenerational transmission of economic inequality taking into account interaction between fertility and human capital investment. He showed that, even in a unique equilibrium (as opposed to the multiple steady-state equilibria in other studies), there would be an intergenerational poverty cycle. More specifically, he showed that in the unique equilibrium, the poor parents would choose a high fertility rate and low investment in physical capital and in their children's human capital, and the rich parents would do the opposite. Thus in the same equilibrium, there is a fraction of the population that is stuck in a low level equilibrium. He also studied the effect of these choices on the aggregate growth path of the economy. An important assumption that generated this result is that the better quality schools are located in richer neighborhoods.

In this paper we provide a model of intergenerational transmission of economic inequality which extends both strands of the above models. We provide a model of school choice which leads to a non-concave optimization problem and provides an alternative explanation for the existence of the intergenerational poverty trap. We do not provide any new analytical results in economic theory; we do, however, use the known results to characterize the dynamics of optimal paths of our model. The main feature of our model is that an altruistic parent taking exogenously given fertility choices decides how much to invest in their children's human capital. In our model, parental choice is on school quality. The schools are of different qualities, and a higher quality school costs more. A given year of schooling from a better quality school yields higher earnings. Parents have to pay a fixed cost, for instance, for renting or buying a house in the richer neighborhood to be able to send their children to a better quality school. Thus, the better is the school quality, the higher is the cost to attend it. This creates a non-concavity in the relationship between investment in schooling and earnings, of the type that at a very low level of investment the quality of the school is very low and the rate of return from an extra dollar at a lower quality school is lower than the rate of return from a better quality school. We also assume that there is a best quality school involving highest fixed cost to enter. While parents can invest extra variable amounts on school activities which increase the human capital and hence earnings abilities of their children, the marginal increase in earnings from such investments is assumed to be decreasing.

2. The Basic Framework

School Quality and Earnings

Lifetime earnings or permanent income of an individual depend on innate ability, schooling level, school quality, and family background. The effect of family background may be direct when family connection is important for labor market success. The family background also indirectly influences lifetime earnings because it determines the parental schooling investment. In this paper our focus is on the parental choice of school quality for their children and how it influences intergenerational transmission of poverty.

We view that schools are of many qualities. Schools are either private or public in our setup. A better quality private school, which is assumed to be accessible to everyone irrespective of the residency status, costs more for admission as compared to a lower quality school. In a public school system, we assume that a better quality school is located in a richer neighborhood and to access it one must buy or rent a house in that neighborhood as in the US. For our main analysis, we need no distinction between the private and public school systems and simply assume that a better quality school costs more.

We assume that earnings depend on the school quality and the amount spent on school activities such as tutoring and buying other learning materials. Assume that there are $n+1$ types of schools. Each type of school requires a fixed cost c_i for admission, and a variable cost h on school activities. A higher quality school has a higher fixed cost to enter. We index schools by their fixed cost c. Let $0 = c_0 < c_1 < \ldots < c_n = h_1$ be the fixed costs of these schools, where h_1 is the fixed cost of the highest quality school. Let $w_i(h)$ be the earnings of an individual who attended a school of quality c_i and had schooling investment h. If the total schooling investment on an individual is $h < c_i$, the school of quality c_i is not accessible to him. We incorporate this information by assuming that $w_i(h)$ is zero when $h < c_i$. Also assume that when the total schooling investment is the fixed cost amount c_i, i.e. there is no investment in the school activities, the earnings from this schooling investment coincides with the earnings from investment of the same amount on a school of one grade lower, c_{i-1}. For all higher investment $h > c_i$, we assume that earnings from a higher quality school is higher than a lower quality school. These assumptions are represented more precisely as follows: For $i = 1, 2, \ldots, n$,

$$w_i(h)\begin{cases} = 0 & \text{if } h < c_i \\ = w_{i-1}(h) & \text{if } h = c_i\,. \\ > w_{i-1}(h) & \text{if } h > c_i \end{cases} \tag{1}$$

Taking into account variations in school quality, from the above we derive the following relationship between an individual's schooling investment h and earnings,

$$w(h) = \max\{w_i(h),\ i = 0, 1, \ldots, n\}. \tag{2}$$

In Figure 1, we have described the above for $n = 2$. The earnings function $w(h)$ is the envelop of the component curves, shown in the figure as the thick curve.

For analytical tractability, we further assume that there is a continuum of distinct school qualities, and indexed by the closed interval $[0, h_l]$, where h_l is the best available quality of school at any time. We assume that school quality does not change over time. Starting at the lowest quality school, if a parent likes to invest a higher amount on his child's education, he can choose a better quality school in the school quality ladder. The child enjoys a higher marginal increase in earnings, the higher is the quality of the school that he attends, until he reaches the school quality h_l. That means, the earnings function exhibits increasing returns or non-concavity up to this point. After reaching the best quality school h_l, investment higher than h_l goes towards the child's private tuition or other school activities, and the marginal increase in the child's earnings is a decreasing function of the investment amount. That means, the earnings function is concave for all investment higher than h_l.

More formally, let h be the parental investment in a child's human capital and let $w(h)$ be the corresponding lifetime permanent income of the child. We assume that $w(h)$ is an increasing function of h, and it exhibits non-concavity of the form that it increases at an increasing rate up to a threshold level h_l, and then it increases at a decreasing rate. That is:

Assumption 1. $w'(h) > 0$ *for* $h \geq 0$ *and* $w(0) = 0$.

Assumption 2. *There exists* $h_l > 0$ *such that* $w''(h) > 0$ *for* $0 < h < h_l$, $w''(h) < 0$ *for* $h > h_l$ *and* $w''(h_l) = 0$, *i.e.* h_l *is an inflection point.*

Assumption 3. *There exists* $\bar{h} > 0$ *such that* $w(\bar{h}) = \bar{h}$ *and* $w(h) < h$ *for* $h > \bar{h}$.

Figure 2 gives a graphical representation of these assumptions. The earnings function $w(h)$ in Figure 2 can be derived from equation (2) as the limiting case when the number of school qualities tends to infinity, and $c_i - c_{i-1} \to 0$ for all $i = 1, \ldots, n$ as $n \to \infty$. In general, the limiting function will not be differentiable at every point. But we assume it to hold approximately.

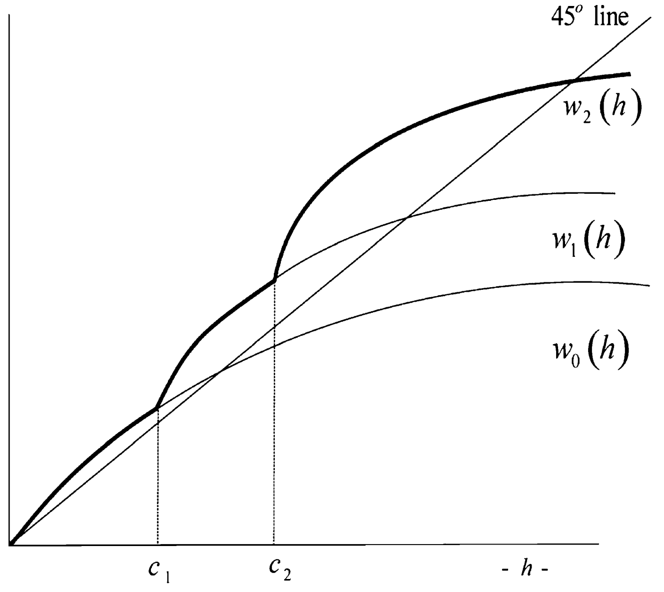

Figure 1. Finite Number of School Qualities and the Earnings Function $w(h)$

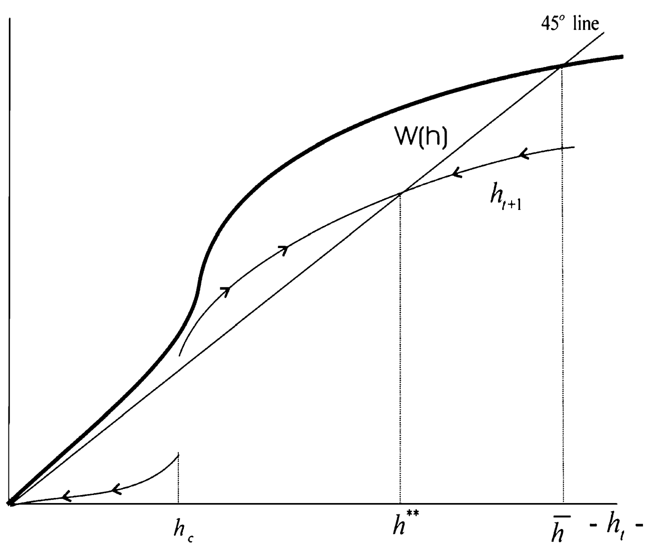

Figure 2. The Phase Diagram of Optimal Schooling Investment $h_{t+1} = \psi(h_t)$

School Choice

We assume that each parent has one child. A parent of generation t has an altruistic utility function of the form

$$U_t = u(c_t) + \rho U_{t+1}, \tag{3}$$

where c_t is his lifetime consumption, ρ is the degree of parental altruism and $u(c_t)$ is the lifetime felicity index. We assume that the felicity index function is a strictly concave function, i.e. $u' > 0$ and $u''(c) < 0$ for $c > 0$.

The capital market is intergenerationally imperfect in the sense that parents are limited to invest in their children by their own lifetime permanent income. The human capital investment problem of a representative parent of generation t with the level of human capital investment h_{t-1} and hence a permanent income level $w(h_{t-1})$ can be formulated as the following recursive dynamic programming problem:

$$V(h_{t-1}) = \max_{0 \le h_t \le w(h_{t-1})} \left[u(w(h_{t-1}) - h_t) + \rho V(h_t)\right] \tag{4}$$

where V is the value function. The state variable of the dynamic programming problem is the level of human capital investment h. The consumption of a representative parent of generation t is given by $c_{t-1} = w(h_{t-1}) - h_t$. Since $\rho < 1$, under Assumptions 1–3, there exists an optimal solution, and it can be seen easily that the value function is an increasing function of the level of human capital investment. We examine the dynamic properties of an optimal solution.

The Euler equation of the above problem is given by

$$u'(c_{t-1}) = \rho w'(h_t) u'(c_t). \tag{5}$$

A stationary state $h > 0$ satisfies

$$u'(w(h) - h) = \rho w'(h) u'(w(h) - h), \tag{6}$$

or equivalently

$$w'(h) = \rho^{-1}. \tag{7}$$

By Assumptions 1 and 2, $w'(h)$ is always positive and it first increases and then decreases, i.e. it has an inverted-U shape. Thus it follows from equation (7) that there may exist two positive stationary states. We denote them by h^* and h^{**}, and call them non-trivial steady states. Notice also that at $h = 0$, we have $c = 0$, and $u'(0) = \infty$. Hence equation (5) is also satisfied for $h = 0$. We call the steady state $h = 0$ as the trivial steady state. The trivial steady state $h = 0$ is a low level equilibrium and h^{**} is a prosperous equilibrium.

3. Dynamic Properties of the Optimal Solution

Along the optimal path a family may converge to one of the steady states or it may cycle around a steady state or may exhibit chaotic behavior. Applying the results of Dechert and Nishimura (1983) we characterize optimal paths.[3] We first note in the following theorem that all optimal solutions converge to a steady state, including the trivial steady state. Thus we do not have chaotic or cycling behavior for the optimal solution.

THEOREM 1. *The optimal path always converges to a steady state. The dynamics of the optimal path crucially depend on the shape of the earnings function $w(h)$ at $h = 0$.*

In Theorem 2 we show that when $w'(0) > 1/\rho$ holds, we have the standard neoclassical result that there is only one non-trivial steady state and all optimal solutions converge to this prosperous steady state.

THEOREM 2. *Suppose $w'(0) > \rho^{-1}$. Then all optimal paths from $h_0 > 0$ converge to h^{**}.*

Theorem 3 below characterizes the properties of the optimal solution for the case $w'(0) < 1/\rho$. It shows that when ρ is sufficiently high, we have multiple steady-state solutions. For families with school quality below a threshold level for some generation will eventually converge to the low level equilibrium and the families with schooling level above the threshold level will converge to the prosperous steady state h^{**}. This is the situation when some families are stuck in a low level poverty cycle. For a more precise statement of our result in the next theorem, we introduce a notation $\tilde{h} = \arg\max \dfrac{w(h)}{h}$.

THEOREM 3. *Suppose $w'(0) < \rho^{-1} < \max\left[\dfrac{w(h)}{h}\right]$. There is a level of human capital $0 < h_c < \tilde{h}$ so that every optimal solution from $h_0 < h_c$ converges to the origin and every optimal solution from $h_0 > h_c$ converges to h^{**}.*

We discuss the policy implications of the above results in the next section. The proofs of these theorems are given in the Appendix.

4. Conclusion

We have formulated a dynamic altruistic model of parental school quality choice and its effect on intergenerational social mobility. We have shown that when there are many school qualities, the earnings of children as a function of parental schooling investment is a non-concave function, which leads to multiple steady-state equilibria. We have also studied intergenerational dynamics of parental schooling investment.

We have shown that even though the objective function of the associated dynamic programming problem is non-concave, the non-concavity of the earnings function generates a low level equilibrium and a prosperous equilibrium, both are locally stable when the rate of return is below a threshold level. One implication of the existence of locally stable multiple steady-state equilibria is that the poor parents will invest very little on their children's school quality and over time the family will converge to the low level equilibrium. That is, the poor will be stuck in an intergenerational poverty trap. It suggests that the way to break out of this intergenerational poverty trap is to improve the quality of the low-end schools and make these schools freely available to the children of poor family backgrounds.

One possible extension of our model would be to incorporate an additional physical capital input with variable returns to scale along the lines of Kemp et al. (1998), and find conditions for poverty trap in a two-sector growth model.

5. Appendix

We follow Dechert and Nishimura (1983) to sketch proofs of Theorems 1–3, after stating a few lemmas. Let $\{h_t\}$ and $\{h'_t\}$ be any two solutions of equation (4) starting respectively from h_0 and h'_0.

Lemma 1. *If* $h_0 >$ (resp. $=$ or $<$) h'_0, *then* $h_t >$ (resp. $=$ or $<$) h'_t, $t \geq 1$.

This implies that the optimal path of parental human capital investment level $\{h_t\}$ is a monotone increasing.

Lemma 2. *If* $h_0 >$ (resp. $=$ or $<$) h'_1, *then* $h_t >$ (resp. $=$ and $<$) h_{t+1}, *for all* $t \geq 1$.

Proof of Theorem 1

(i) Suppose $h_0 \geq \bar{h}$. If an optimal path $\{h_t\}$ starting from h_0 is non-decreasing, $w(h_{t-1}) - h_t \leq w(h_t) - h_t \leq 0$. This implies consumption is never positive. It is clearly a contradiction. Therefore h_t must be strictly decreasing. Then $\{h_t\}$ converges to 0 or some positive value $\hat{h} < \bar{h}$. $\hat{h}$ satisfies $u'(w(\hat{h}) - \hat{h} = \rho w'(\hat{h})u'(w(\hat{y}) - \hat{h})$. It must be one of non-trivial steady state.

(ii) Suppose $\bar{h} > h_0 \geq 0$. If an optimal path $\{h_t\}$ crosses $\bar{h}$ from below at $\tau \geq 1$, that is $h_\tau > \bar{h}$, then $\{h_t\}$ is never optimal from y_τ by the argument in (i). Therefore $\{h_t\}$ never crosses $\bar{h}$. Then $\{h_t\}$ converges to 0 or some positive value $\hat{h}$. The limit $\hat{h}$ that the sequence $\{h_t\}$ converges to must satisfy $u'(w(\hat{h}) - \hat{h}) = \rho w'(\hat{h})u'(w(\hat{h}) - \hat{h})$. It must be one of non-trivial steady state also. □

Proof of Theorem 2

Suppose that there exists an optimal path $\{h_t\}$ converging to 0. If $0 < h_t < h^{**}$, $w'(h_t) > \rho^{-1}$ and $u'(w(h_{t-1}) - h_t) = \rho w'(h_t)u'(w(h_t) - h_{t+1}) > u'(w(h_t) - h_{t+1})$.

Since $u''(c) < 0$, i.e. $u'(c)$ is a decreasing function, we have

$$0 < w(h_{t-1}) - h_t < w(h_t) - h_{t+1}. \tag{A1}$$

If $\{h_t\}$ from $0 < h_0 < h^{**}$ converges to 0, (A1) must hold along $\{h_t\}$, $t \geq 1$. This is not possible if $\{h_t\}$ converges to 0. Therefore, every optimal path from $0 < h_0 < h^{**}$ must converge to h^{**}. Also optimal paths from $h_0 > h^{**}$ converge to h^{**}. This completes the proof. □

Lemma 3. *All optimal paths from* $h_0 > \tilde{h}$ *converge to* h^{**}.

Lemma 4. *There is a level of* h_0, $0 < h_0 < \tilde{h}$ *such that all optimal paths from* h_0 *converge to the origin.*

Lemma 5. *If* $h_0 \neq h^*$, *an optimal path from* h_0 *never converges to* h^*.

Proof of Theorem 3

Let a be the supremum of the initial points from which all optimal paths converge to the origin and b be the infimum of the initial points from which all optimal paths converge to h^{**}. It is clear that $a \leq b$. If $a < b$, we choose h_0, h'_0 with $a < h_0 < h'_0 < b$. Each of h_0 and h'_0 has a solution that converges to the origin and one that converges to h^{**}. Let $\{h_t\}$ be an optimal solution from h_0 that converges to h^{**} and $\{h'_t\}$ be an optimal solution from h'_0 that converges to the origin. Then there is a sufficiently large T such that $h'_t < h_T$. This contradicts $h_0 < h'_0$ and the monotonicity. Therefore $a = b$ must follow. h_c is chosen to be the value $a = b$. □

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Notes

1. Raut and Heckman (2005), however, provide a mechanism based on parental preschool investment to produce motivational and social skills in children, affecting their school performance, earnings, and social mobility.
2. While Becker et al. (1990) incorporated endogenous fertility choices in their model, Kemp and Kondo (1986) formulated a model of endogenous fertility earlier and provided interesting results. See also Benhabib and Nishimura (1989).
3. Long et al. (1997) provide the characterization of solution paths of the non-concave optimal growth problem in a continuous time framework. We, however, follow the discrete time characterization of Dechert and Nishimura (1983) that suits better our analysis.