Development Economics [1]

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[1]To all my students at the University of California- San Diego, University of Hawaii-Manoa, Yale University, and Cal State Fullerton, interaction with whom through teaching formed this book. Your comments are most welcome.

Contents

Chapter 3

Basic Models of Growth and Development

Apart from the stylized facts and development indicators that we subscribed during our tour around the globe, especially from the less developed countries, let us briefly describe the six stylized facts that Kaldor found in 1948. Kaldor's stylized facts led to systematic development of grwoth models.

3.1 Kaldor's six stylized empirical facts

Kaldor in 1948 found six stylized facts about a few macro variables for developed countries. To describe them, let us have the following notations:
$Y(t)$ = total output or GDP at time t
$L(t)$ = labor force which is same as population at time t
$K(t)$ = stock of capital at t
$r(t)$ = rate of profit = total profit at $t/K(t)$
$v(t)$ = $K(t)/Y(t)$, capital output ratio
$S(t)$ = saving rate $= [Y(t) - C(t)]/Y(t)$, where $C(t)$ total consumption at t
$y(t)$ = per capita income $= Y(t)/L(t)$
$k(t)$ = capital- labor ratio $= K(t)/L(t)$

The six stylized facts that Kaldor found are as follows:

I. $y(t)$ grows exponentially more or less at a constant rate over a long period of time possibly with short-run fluctuations, known as business fluctuations.

II. Both $K(t)$ and $L(t)$ are growing exponentially over a long period of time, and the growth rate of $K(t)$ exceeds that of $L(t)$, and hence $k(t)$ is also growing over a long period of time.

III. $K(t)$ and $Y(t)$ are both growing over the long-period of time at the same rate so that $v(t)$ is more or less constant over time.

IV. $r(t)$ is more or less constant over time.

V. Different countries have different growth rates for $y(t)$.

VI. Economies with higher share of profit to income tend to have higher saving rate.

3.1.1 Need for an economic growth model

We have seen that based on the historical data, the growth accounting that we studied in the previous chapter can tell us the sources of variations in the observed growth rates. What is missing in that framework in so far as we are concerned with the growth and development process of less developed countries and with policies that can speed up the development process? The main deficiency is that it cannot tell us how those historical data were generated and cannot tell us where the economy is heading to. It cannot tell us how various economic factors such as labor, capital and total income interact with each other to generate the observed series of K_t, L_t, and Y_t that are used in the grwoth accounting formula. For instance, if for some reason the labor supply chnages in a particular time, say for instance due to some disease epidemics, or due to ban on child labor; it will affect wage rate, and wages or total income, and as a result it will affect the total savings and hence capital stock, which will affect future income and so on. The grwoth accounting formula does not tell us such interaction of various economic forces that generate the observed data on K_t, L_t, and Y_t. We need economic modeling of grwoth process to take these interactions. Various models incorporate specific features to explain certain phenomena. An important criterion for a theoretical model to be useful is that these models should be able to predict some of the empirical stylized facts listed above. There are several models available in the literature. Probably the first serious attempt at modeling economic growth process was by Harrod and Domar who used Leotieff production function. As you know Leontieff production function does not allow for factor substitutions in production, which is an unrealistic feature of Harrod-Domar model. We will first develop a simplified version of the Harrod-Domar growth model. We will examine the kind of predicitons this model produce and use cross country data to see if the predicitons are supported by observed expereinces of the countries. We will then develop Lewis model of grwoth and development and then finally the Solow grwoth model.

3.2 Harrod-Domar model

Their main assumptions are as follows:

A.1 The production function is Leontieff: $Y_t = \min\{vK_t, bL_t\}$

To simplify our exposition we make a stronger assumption that labor is abundant so that labor is not binding in production. Then the assumption A.1 implies that $Y_t = vK_t$, where v is the output-capital ratio, generally found to be of the magnitude $1/3$.

A.2 Let us denote by S_t the total savings in peirod t.They assumed a Keynesian savings function. $S_t = sY_t$, where $0 < s < 1$.

A.3 Assume closed economy, so that $I_t = S_t$, where $I_t = \Delta K_t = K_{t+1} - K_t$.

Notice that from assumption A.1, $Y_t = vK_t$ implies $\Delta Y_t = v\Delta K_t$. From assumption A.3 we have $I_t = S_t$, i.e., $\Delta K_t = S_t$.

$$\begin{array}{ccccl}
I_t & = & S_t & & \text{by assumptionA.3} \\
|| & & || & \longleftarrow & \text{by assumption A.2} \\
\Delta K_t & = & sY_t & & \\
|| & & || & & \\
\Delta Y_t/v & = & sY_t & & \quad \text{.... (*)}
\end{array}$$

from (*) we have $\frac{\Delta Y_t}{Y_t} = v.s$. So long as s and v is constant over time, we have constant grwoth rate. The only way a country can have higher grwoth rate in income is by having higher savings rate, or by making The growth rate in output is increasing in savings rate s and output capital ratio v. In this model the variations in the growth rate of total income among countries is due variations in their savings rate. For instance, suppose $v = 1/3$, then if a less developed country has savings rate 6%, then its growth rate is $g_Y = \frac{6\%}{3} = 2\%$. If the country wants to increase its grwoth rate to 9%, it should increase its savings rate to 27%. According to this model, the development problem boils down to the question of how an underdeveloped economy with low savings rate of 6% can increase it to say 27%. It might seems a easy thing to achieve, but in a poorer country with most people living in subsitence it is impossible to introduce any incentive scheme to increase the economy's savings rate. The policy makers in the fifties and sixties recommended that the developing countries should be given foreign aids which increase their investment rate. But it has not worked to achieve this goal. Furthermore, the current savings rates of many developing countries are as high as 27% or even more but their grwoth rates of total income is much lower than what is predicted by the Harrod-Domar model.

We can even put the model to empirical testing directly. We regress the growth rate of output

$$g_Y = \alpha + \beta.s + \epsilon$$

we look at the significance and magnitude of the estimate for β and R^2. of the above regression model. R^2 measures the goodness of fit, and it lies between 0 and 1. If the Harrod-Domar model is a good model of grwoth and development, then the R^2 should be high, and the estimate of β will be significant (i.e., t-statistics will be 1.96 or higher say to be significant at 5%). What do you find?

3.3 Lewis dual economy growth model

We follow closely Todaro's exposition of the Lewis model.

"One of the best-known early theoretical models of development that focused on the structural transformation of a primarily subsistence economy was that formulated by Nobel laureate W. Arthur Lewis in the mid-1950s and later modi-fied, formalized, and extended by John Fei and Gustav Ranis. The Lewis two-sector model became the general theory of the development process in surplus-labor Third World nations during most of the 1960s and early 1970s. It still has many adherents today, especially among American development economists. In the Lewis model, the underdeveloped economy consists of two sectors: (1) a traditional, overpopulated rural subsistence sector characterized by zero marginal labor productivity-a situation that permits Lewis to classify this as surplus labor in the sense that it can be withdrawn from the agricultural sector without any loss of output- and (2) a high-productivity modem urban industrial sector into which labor from the subsistence sector is gradually transferred. The primary focus of the model is on both the process of labor transfer and the growth of output and employment in the modem sector. Both labor transfer and modern-sector employment growth are brought about by output expansion in that sector. The speed with which this expansion occurs is determined by the rate of industrial investment and capital accumulation in the modern sector. Such investment is made possible by the excess of modem-sector profits over wages on the assumption that capitalists reinvest all their profits. Finally, the

p1

p2

p3

p4

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p6

terms and relative to average rural incomes, even in the presence of rising levels of open modern-sector unemployment and low or zero marginal productivity in agriculture. Institutional factors such as union bargaining power, civil service wage scales, and multinational corporations hiring practices tend to negate whatever competitive forces might exist in Third World modern-sector labor markets. We conclude, therefore, that when one takes into account the laborsaving bias of most modem technological transfer, the existence of substantial capital flight, the widespread nonexistence of rural surplus labor, the growing preva-lence of urban surplus labor, and the tendency for modern-sector wages to rise rapidly even where substantial open unemployment exists, the Lewis two-sector model-though extremely valuable as an early conceptual portrayal of the devel-opment process of sectoral interaction and structural change-requires consider-able modification in assumptions and analysis to fit the reality of contemporary Third World nations." (Todaro [1995])

3.4 Solow Growth Model

Assume that population is growing exogenously as: $L_t = L_0(1+n)^t, t = 0, 1, \ldots$. Assume that economy exhibits an exogenously given constant rate of Harrod neutral technological change in the aggregate production process over time. That is assume that L_t workers are equivalent to $\tilde{L}_t \equiv (1+b)^t L_t$ number of base period workers and there is no improvement in the efficiency of capital. Then as we have seen before, in the period tK_t and L_t amount of capital and labor is going to produce $Y_t = F(K_t, \tilde{L}_t)$ total output, where F is a base period production function and is assumed to be constant returns to scale. Solow further assumes Keynesian savings function: $S_t = sY_t, 0 < s < 1$, where S_t is the aggregate savings. If we further assume that our economy is closed, then in equilibrium $S_t = I_t$, where I_t is aggregate investment. He further assumes that investment takes one period to gestate, and δ is the rate of depreciation per period, then we have

$$
\begin{aligned}
K_{t+1} &= (1-\delta)K_t + I_t \\
&= (1-\delta)K_t + S_t, \text{(since } I_t = S_t \text{ in a closed economy)} \\
&= (1-\delta)K_t + sY_t, \text{ (because savings function is Keynesian)} \\
&= (1-\delta)K_t + sF(K_t, \tilde{L}_t) \\
&= (1-\delta)K_t + \tilde{L}_t sF\left(\frac{K_t}{\tilde{L}_t}, 1\right), \text{ (since F(,.,) is CRS)}
\end{aligned}
$$

Now dividing both sides by $\tilde{L}_t$, and denoting by $\tilde{k}_t = K_t/\tilde{L}_t$, i.e. the capital-labor ratio in efficiency unit, we have the following

$$\frac{K_{t+1}}{\tilde{L}_{t+1}}.\frac{\tilde{L}_{t+1}}{\tilde{L}_t} = (1-\delta)\frac{K_t}{\tilde{L}_t} + sF\left(\frac{K_t}{\tilde{L}_t}, 1\right)$$

which implies

$$\tilde{k}_{t+1}.(1+b)(1+n) = (1-\delta)\tilde{k}_t + sf(\tilde{k}_t)$$

which implies,

$$\tilde{k}_{t+1} = \tfrac{(1-\delta)\tilde{k}_t + sf(\tilde{k}_t)}{(1+b)(1+n)} \equiv g(\tilde{k}_t) \text{ say}$$

$$t = 0, 1, 2, ... \tag{3.1}$$

with L_0, K_0 and hence $\tilde{k}_0 = \frac{K_0}{L_0}$ given

Equation (3.1) is known as the **fundamental difference equation of Solow growth model.** The above difference equation contains all the information about the Solow economy. More specifically notice that once you know $\tilde{k}_0$, then using this in the above equation with $t = 0$ we can get $\tilde{k}_1$. Once we know $\tilde{k}_1$, again using this in the above equation with $t = 1$, we get $\tilde{k}_2$. In fact if we know $\tilde{k}_0$, then we know all future values of $\tilde{k}_t$. But once we know all $\tilde{k}_t$ values as the following formula shows, we know wage rate, interest rate, per capita income total income capital stock and labor in all periods:

$$r_t = f'(\tilde{k}_t) \tag{3.2}$$

$$w_t = (1+b)^t \left[f(\tilde{k}_t) - \tilde{k}_t f'(\tilde{k}_t)\right] \tag{3.3}$$

Since $L_t = (1+n)^t L_0$ $K_t = \tilde{k}_t.\tilde{L}_t = \tilde{k}_t.(1+b)^t L_t$, we can calculate per capita income in each period by

$$y_t = Y_t/L_t = F(K_t, \tilde{L}_t)/L_t = \left(\tilde{L}_t/L_t\right) f(\tilde{k}_t) = (1+b)^t f(\tilde{k}_t) \tag{3.4}$$

Question: Where does this economy lead to over a long period time? What are the long-run effects of an increase in savings rate (s), population growth rate (n), and the rate of technological change (b)? What are their effects in the short-run? The following phase diagram tells the story:

Long-run Implications:

In the long-run the $\tilde{k}_t$ reaches the steady-state level $\tilde{k}^*$, and the steady-state is globally stable. Thus looking back in the formula in the box, we see that in the long-run per capita income and wage rate is growing at the same rate as the rate of technological change, and interest rate is constant. Thus if there is no technological change, i.e., $b = 0$, there is no long-run growth in the per capita income and this counter to one of the six stylized facts listed above.

Now if n is decreased or s is increased, the g-curve in the phase diagram is going to shift upward, this will increase the value of $\tilde{k}^*$. Now looking at the formulas in the box again we can see that such policies have no effect on the long-run growth rate which depends only on the rate of technological change. However, as is obvious, if b increases, i.e., if there is higher rate of Harrod neutral technological change then higher is the rate of growth of per capita income, and wage rate.

Notice that if $b = 0$, then the long-run effect of a decrease in the population growth rate or an increase in saving rate will be to increase the level of capital labor ratio and hence level of per capita income; however they will not affect the long-run growth rates.

3.4.1 Transition dynamics or out of steady-state dynamics

Let us consider the above Solow model without technological change, In this case, $b = 0$, and $\tilde{k}_t$ is same as k_t. In the phase diagram (see figure 1, above) we see that starting from any initial capital-labor ratio k_0, the economy converges to the steady-state capital labor ratio k^* in the long-run. The steady-state is globally stable.

Notice that for $k_0 < k^*$, if the economy starts at k_0, over time the economy's capital labor ratio increases, and hence the per capita income, $y_t = f(k_t)$ also increases (see figure 3.1), but marginal product of capital, $r_t = f'(k_t)$, decreases over time, the growth rate of per capita income g_t between period t-1 and t decreases over time, see the numerical example below for further illustration of these points.

To study the effect of policy parameters such as savings rate, s, and population growth rate, n, on the long-run and short-run growth rates, notice that these variables do not affect long-run growth rates, but they affect the long-run level of capital-labor ratio k^* and the short-run growth rate. To illustrate the the effect on short-run growth rate, suppose countries A and B both start at the same k_0, but suppose country A has a higher savings rate, then its $g(k_t)$ curve will shift upward, and we can see now that if we take average annual growth for 5 years or 10 years, country A with a higher savings rate will have a higher average growth rate of in per capita income over a period of time. Similarly, if country A has a lower population growth, then its $g(k_t)$ curve will shift upward, and over a period of time the country A will have higher average growth rate than country B. Let us illustrate this further with a numerical example.

Example 3 *Suppose $n = .02$, $b = 0$, $\delta = .2$, the production function is Cobb-Douglas type, i.e., $F(K, L) = 6K^\sigma L^{1-\sigma}$, $0 < \sigma < 1$. For this production function we have $f(k) = 6k^\sigma$. Let us take $\sigma = .4$. Suppose both countries begin with $k_0 = 1.5$, and suppose country A has savings rate, $s = .20$ and for country B, savings rate $s = .10$, compute the per capita income of these countries for next 5 years and compute the average annual discrete growth rate as the average of year to year discrete growth rates. Which country has a higher average growth rate of per capita income? What are the long-run growth rates of these countries? How are the long-run growth rates of these countries affected by population growth rate and savings rate?*

To answer these questions, recall, the dynamical system or the difference equation associated with Solow model without technological change is given by

$$k_{t+1} = \frac{sf(k_t) + (1 - \delta)k_t}{(1 + n)} = g(k_t)$$

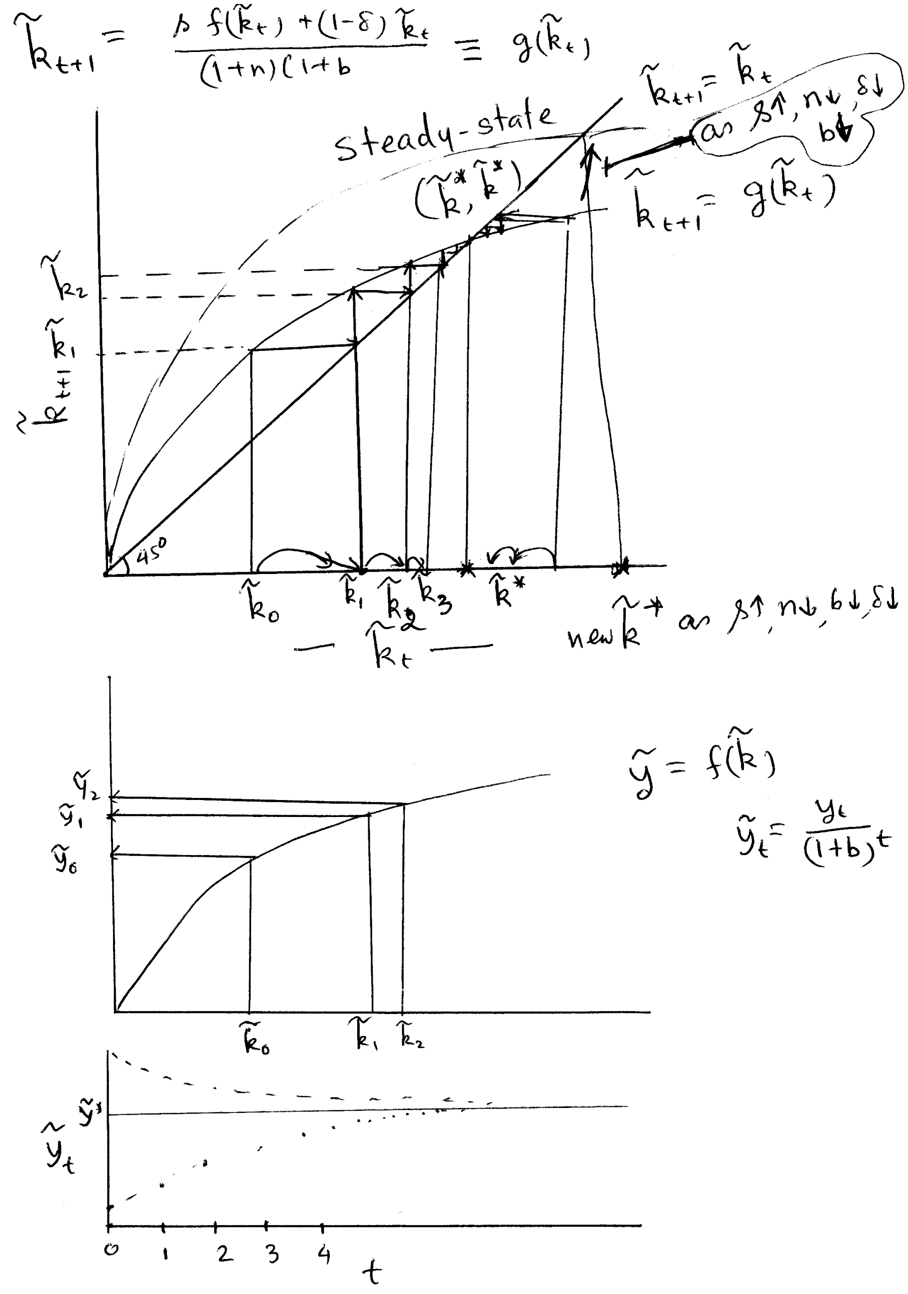

Figure 3.1: Phase diagram and the dynamics of per capita income in Solow growth model

the above for country A becomes:

$$k_{t+1} = \frac{1.2k_t^{.4} + .8k_t}{1.02}, k_0 = 1.5$$

the above for country B becomes:

$$k_{t+1} = \frac{.6k_t^{.4} + .8k_t}{1.02}, k_0 = 1.5$$

Table 3.1 gives for each country the next 5 year's capital labor ratio k_t, and per capita income y_t.

Period	$s = .2$:Country A			$s = .1$:Country B		
	k_t	y_t	g_t	k_t	y_t	g_t
0	1.50	07.05	–	1.50	7.05	–
1	2.56	08.74	0.24	1.87	7.70	0.09
2	3.72	10.15	0.16	2.22	8.26	0.07
3	4.91	11.34	0.12	2.55	8.73	0.06
4	6.07	12.34	0.09	2.85	9.13	0.05
5	7.18	13.20	0.07	3.13	9.48	0.04

Average growth rate of per capita income is
for country A $= 0.136$ and
for country B $= 0.06$.

Table 3.1: Simulated values of the Solow growth model

You can similarly do the exercise when both countries have common savings rate, say $s = .20$, but one country has higher population growth rate than the other. Furthermore, you can do the same analysis when there is Harrod neutral technological change, i.e., $b > 0$. The steps for that will be you begin with $\tilde{k}_0$, and use the dynamical system $\tilde{k}_{t+1} = g(\tilde{k}_t)$, to generate the future values of $\tilde{k}_t$; then use the formula $k_t = (1+b)^t \tilde{k}_t$, and $y_t = (1+b)^t f(\tilde{k}_t)$ and $r_t = f'(\tilde{k}_t)$ to generate the future values of of capital-labor ratio, k_t, per capita income, y_t, and rental rate r_t.

Convergence hypothesis: Notice that solow model predicts a kind of convergence of growth rates of all the countries. In fact, we find that while the countries with higher savings rate or lower population growth rate might have higher growth rates in the short-run, in the long-run the growth rate is the same as the rate of technological change, b. If all countries share the same technology, then in the long-run all countries will converge to the same growth rate, namely to the growth rate of Harrod neutral technological change.

Poverty Trap: In the above standard Solow model we have assumed a concave production function. However, suppose the production function is not concave, and it rather shows first increasing and then decreasing returns to scale. This will lead to the following phase diagram. It explains the poverty trap phenomenon.

Cycles and Chaotic dynamics of per capita income:

If the graph of the g-function looks as in figure 3.2, then as we can see that the capital-labor ratio and hence the per capita income will cycle around the steady state value. For cycles and chaos, the g-function should satisfy some conditions, which is beyond the scope of this course.

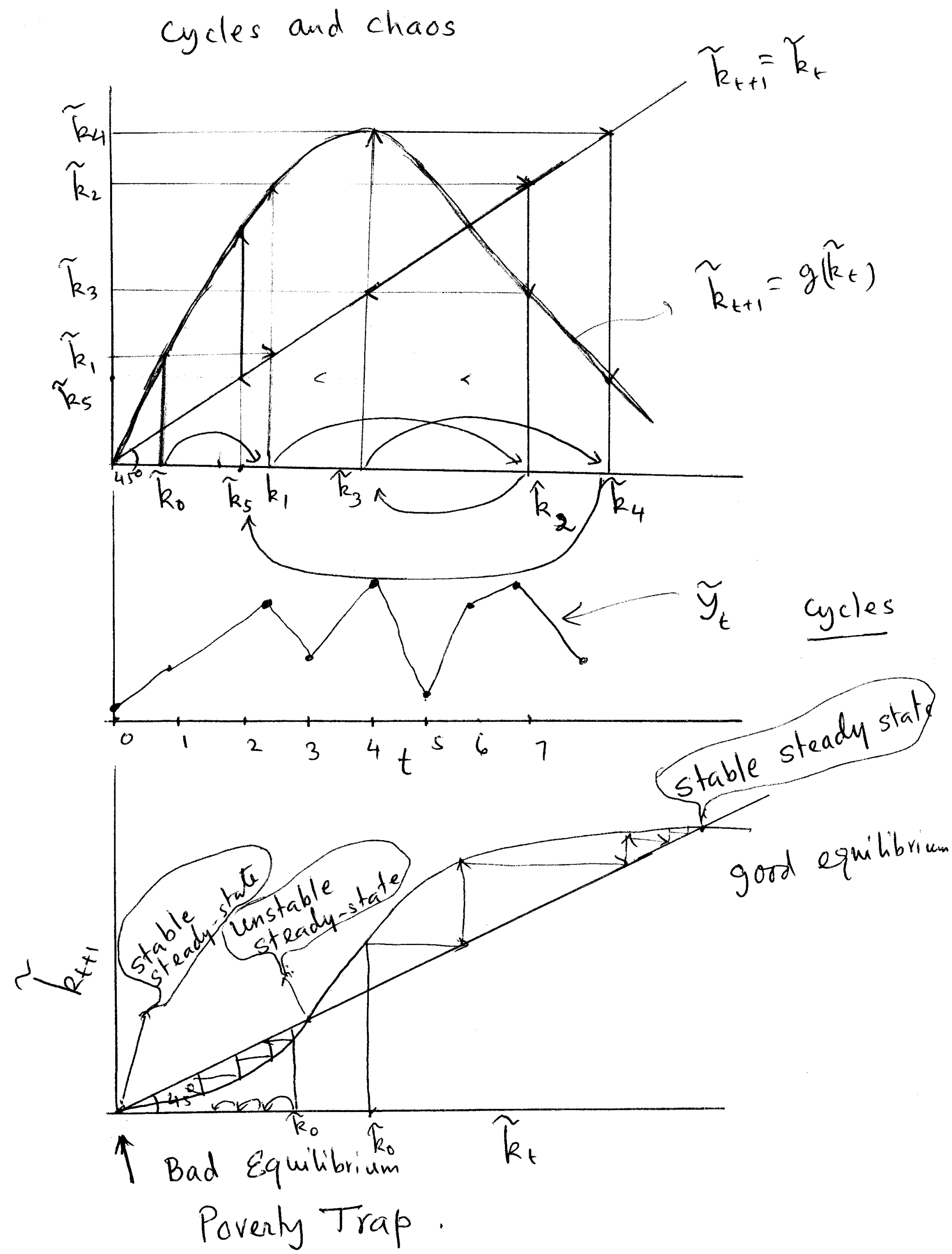

Figure 3.2: Phase diagram and the dynamics of per capita income in Solow growth model