Mortality and Disability Risk Sharing under an OASDI Program in a Stochastic Overlapping Generations Framework[*]

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Abstract

Individuals develop adverse health conditions as they get older, which limit their earnings, resources, and well-being. Workers also face uncertainty about their life spans, which affect their savings and consumption decisions. Many well-known economic characteristics of these risks prevent private markets from pooling these risks adequately. Pay-as-you-go social insurance programs – by levying payroll taxes on workers and paying benefits to disabled workers, retirees, and survivors – can substitute for these missing markets. In addition to sharing these two types of risks, such programs will affect the aggregate capital stock, consumption, and the elderly poverty rate. This paper models the risk sharing and the aggregate effects using a calibrated stochastic overlapping generations general equilibrium model. The paper uses as a benchmark the stationary equilibrium of an economy in which individuals self-insure themselves against these risks by adjusting their own saving, and compares this benchmark to the stationary equilibriums of economies with simplified old-age insurance (OASI) and disability insurance (DI) programs, alone and in combination (OASDI). The representative worker's lifetime welfare is computed in the benchmark and the

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alternatives. Optimal replacement rates for the model's OASI and DI programs are also calculated.

The main findings of the paper are that the OASI and DI programs in the model economy together improve the lifetime welfare of workers, reduce the variation of consumption and asset holdings over the life cycle, and virtually eliminate elderly poverty that may arise from these two sources of income shocks. The welfare-equivalent increase in consumption at all ages in the benchmark would be about 14 percent for the DI program alone or 11 percent for the combined programs. The OASI program in the model, however, does not improve individual well-being. This might be due to several factors. First, similar to other models in the literature, this model assumes that the accidental bequest of an individual is redistributed equally among living individuals, which effectively mimics an actuarially fair annuity market with some progressive redistribution. This undermines the annuity role of the OASI program. Second, the model disregards the survivors' benefits components of the OASI program, which would increase the welfare of an individual from an increase in welfare of his survivors. The paper finds that the optimal replacement rate to be zero percent for the OASI program without the survivors' benefit component and with equal sharing of unintended bequest, and the optimal replacement rate for the DI program to be 90 percent.

1 Introduction

In this paper, I use a calibrated stochastic overlapping generations model of the US economy and study the nature of mortality and disability risk-sharing provided by an OASDI program to workers in the long-run stationary equilibrium. I also study how the program improves elderly poverty prevalence and examine the effect of the program on aggregate capital accumulation.

Disability and mortality risks are two significant risks that affect an individual worker's labor supply, savings and well-being. Over time, an individual worker may develop work-limiting disabilities with a small but positive probability, reducing earnings to zero and leading, unless there is DI insurance or sufficient savings, to very low levels of consumption. A disability insurance program that charges a reasonable premium to an able worker and pays out a certain fraction of his productive year's earnings when he becomes disabled can smooth his consumption and improve his well-being. The DI component of the OASDI program provides such insurance. Without the OASDI program or disability insurance markets, an individual will self-insure against such risks by saving more and working longer hours when able so that he can spend resources from his savings to endure the period of disability.

In the case of mortality risk, if an individual knew exactly how long he was going to live, he would plan his asset holdings and labor supply over time in such a way that he would enjoy a smooth stream of consumption over his lifespan. However, when the timing of death is uncertain, on the one hand, he would like to save more while young as a precaution against the event that he lives longer than he expected. On the other hand, since he is more likely to survive when young and there is only a small chance that he will survive to old age, he would like to consume more out of his asset holdings while young and save very little for his old age. The relative magnitudes of these two opposite effects would determine asset holdings and consumption patterns over his life cycle. If the second effect is strong, however, he will end up with few resources for his old age in the self-insured economy. This outcome is not because individuals are irrational and myopic so they do not save enough for their old age, as argued by some to justify the existence of an OASDI program. Rather, it is the optimal decision under rational expectations. An annuity scheme that allows an individual to save while young and get a stream of fixed income during his retirement years until he dies, can pool mortality risks across the population. Thus it can assure a higher level of consumption during older ages. The OASI part of the OASDI program provides such annuities to workers. However, the fixed income that the OASI program pays to a retired individual is computed on an actuarially fair pay-as-you-go basis, which will be lower than a private annuity financed from the same taxes. This is especially likely when the interest rate is higher than the growth rate of the economy.

Many well-known economic problems[1] associated with these two types of risks prevent well-functioning private disability and annuity markets from emerging. The OASDI program plays the role of these missing markets. Workers, who earn insured status by working and contributing to the OASDI Trust Fund a certain minimum amount of time, are eligible to get annuities and survivors benefits from the OASI program and disability benefits from the DI program. At this stage of the analysis, following the computational simplifications adopted in other research in the literature, I focus only on the annuity component of the OASI program. I plan to include survivors benefits in my future work.

Compared to an economy that does not have an OASDI program or private markets for sharing disability and mortality risks, how does the availability of an OASDI program affect the means and variances of consumption, asset holdings and earnings over the life cycle of a representative individual? How does it affect elderly poverty? In addition, how does it affect aggregate capital accumulation? What is the value of the OASDI program to individuals? What are the optimal tax and benefit rates for the OASI and DI programs that maximize a representative consumer's average lifetime welfare?

Almost all previous studies of the US economy that use an overlapping generations, general equilibrium framework find negative effects of the OASI program on individual life time welfare and long-term capital accumulation. For instance, Feldstein, 1974 finds that unless a portion of the population is completely myopic and thus lacks foresight to provide for their own old age, an OASI program does not improve welfare. Hubbard and Judd, 1987, using an overlapping generations model with mortality risks and liquidity constraints that prevent individuals from borrowing against future labor income (the same constraints apply in the present paper), find that an OASI system reduces welfare of a representative individual in the long run. Auerbach and Kotlikoff, 1987, using a non-stochastic overlapping generations model but relaxing the liquidity constraint assumption, find that the OASI program significantly reduces long-run individual welfare and the aggregate capital stock. On the other hand, İmrohoroglu et al., 1995, using an overlapping generation model with mortality risks but with an assumption that individuals value utility from future consumption more than utility from current consumption, find that the OASI program improves the

[1] The problems are moral hazard and adverse selection.

welfare of a representative individual.

Mortality risk leads to difficult choices in modeling of unintended bequests in computable overlapping generations models, and may significantly influence the policy conclusions. At the end of a period if an individual dies, his savings become an unintended bequest. Who gets this unintended bequest? It generally goes as bequest to survivors of the individual. Incorporating such bequests in a computable dynamic general equilibrium model poses serious computational problems. To simplify computational burden, most models in the literature assume that unintended bequests are distributed uniformly across all living individuals in a period. This, together with private savings, however, can be thought of as annualized annuity and thus undermines the annuity role of the OASI program. For instance, consider an annualized annuity in which an individual puts his savings during a period when he is alive and he gets an income at the beginning of the next period if he survives and gets nothing if he dies. Individuals will demand for such annuities, and competitive markets for such annuities will emerge and will efficiently allocate the mortality risks of individuals. The income an individual gets from this annuity is higher than what he could get by self-insuring himself investing in physical capital, i.e., with his savings. If unintended bequests were allocated to the surviving individuals proportionate to their savings, we would have reproduced exactly the above annualized annuity market. The assumption of equal distribution can be viewed as the above annualized annuity with a progressive redistribution to those who save lower amounts, maybe because they are poorer, from those who save higher amounts. The OASI program also intends to perform the same annuity role with a progressive redistribution. Thus, the assumption that is commonly made in the literature undermines the role of the OASI program. This draft also makes the equal distribution assumption and comes to the same conclusion. However, preliminary results show that other allocation rules may change the policy conclusion. I will examine this in a revised draft.

No previous studies have examined the risk-sharing properties of the DI program within an overlapping generations, general equilibrium model. Chandra and Samwick, 2009, however, consider a partial equilibrium framework without taking into account the feedback effects of the DI program on the macro economy. They exclude mortality risks and the OASI component of the OASDI program. They find that the DI program improves the welfare of a representative worker; furthermore, a representative worker is “willing to pay about 5 percent of expected consumption to eliminate the disability risk” that he may face; and “no more than 20 percent of the mean assets accumulated before voluntary retirements are attributable to disability risks."

In this study, I incorporate both disability and mortality risks, and consider both the OASI and the DI components of the OASDI program. I examine the long-run equilibrium and take as the benchmark an economy in which individuals self-insure themselves against these risks by adjusting their own savings and labor supply without access to an OASDI program or private markets for sharing these risks. The main findings of the paper are that the OASI and DI programs together improve lifetime welfare of workers, reduce variations of consumption and asset holdings over their life cycles, and virtually eliminate elderly poverty that arise from these two sources of risks to income. The value of the OASDI program to a representative worker is equivalent to having 10.56 percent higher consumption at all ages in the benchmark equilibrium. The value is even higher for the DI program. For the OASI program, however, disregarding the survivors' benefit component of the program, and assuming equal sharing of unintended bequests, the actuarially fair pay-as-you-go (PAYGO) annuity component of the program does not improve individual lifetime well-being. Since the OASDI program mitigates motives for savings, the macroeconomic costs of the program are that it reduces the aggregate capital stock and per capita consumption and income.

For this paper, I define the replacement rate as the percentage of the working population's average yearly wage that the program will pay out to an individual in case the risky event covered by the program occurs. The replacement rate that maximizes the lifetime well-being of an individual turns out to be zero percent for the OASI program without the survivors' benefit component, and 90 percent for the DI program. Under a less plausible assumption adopted in a few studies, that individuals value the utility of future consumption more than the utility of present consumption, the optimal OASI replacement rate becomes a positive number (around 55 percent for the chosen parameter values).

The rest of the paper is organized as follows. Section 2 briefly describes the overlapping generations framework and the values of the parameters used to calibrate the model. Section 3 reports the findings of the paper. Section 4 concludes the paper and outlines the extensions of this basic framework for future work.

2 The Basic Overlapping Generations Framework

In this overlapping generations model, each individual lives a finite number of periods. Each period can be viewed as a year. I assume that starting from age 21 individuals supply a fixed number of hours of labor each period and decide only how to divide current income between current consumption and saving. An individual pays OASDI taxes and anticipates receiving DI benefits if he becomes disabled during any period before his mandatory retirement at age 65. He anticipates receiving OASI benefits starting at the mandatory retirement age of 65 until he dies. The benefits that he receives from each program are constant fractions of the average earnings of the working population during the period. I take these rates as policy parameters, calculated from benefits data in the 2006 Trustees Report. However, the OASI and DI tax rates are calculated on an actuarially fair, pay-as-you-go basis. As mentioned in the introduction, I assume that unintended bequests are allocated to all living individuals in equal share. Sensitivity analysis with respect to alternative rules for distributing the unintended bequests will be carried out in the revised draft.

Mortality assumptions in the current version follow those of İmrohoroglu et al., 1995. Individuals are assumed to live to at most age 85. Survival probabilities are taken from Faber, 1982. The unconditional probability of dying at various ages, derived from these survival probabilities, are plotted in Figure 1. The assumption that all workers die at 85, although it reduces computational complexity and allows comparison with İmrohoroglu et al., 1995, is quite restrictive for the analysis of old-age insurance. In a later version the maximum age will be extended to 110 and the more recent mortality data from Bell and Miller, 2005 will be used.

The disability incidence rate is assumed to be 0.0058, drawn from table V.C5 in the 2006 Trustees Report (equal to the ultimate intermediate incidence rate, 7 percent higher than the base period rate of 5.4 per thousand). The probability of recovery from disability is assumed to be 0.017, averaging the age-specific recovery rates of males and females in table 18 of Zayatz, 2005. In the current model the incidence and recovery rates are assumed to be independent of age. In a revised draft, I will allow these probabilities to be age dependent. To avoid computational complexity, mortality probabilities before retirement are assumed to be independent of disability status.

As is standard, I assume that the gross domestic product Y is produced with an aggregate capital stock K and aggregate labor hours measured in "efficiency units" (explained below) L by using the following Cobb-Douglas production function:

$$Y = F(K, BL) = K^{\sigma}(BL)^{1-\sigma}, 0 < \sigma < 1.$$

The parameter B denotes the productivity level of a reference worker in a given year. The real value of output that a worker unit can produce is his productivity level. The productivity level of a worker in a given year will depend on his level of experience. Therefore, a unit

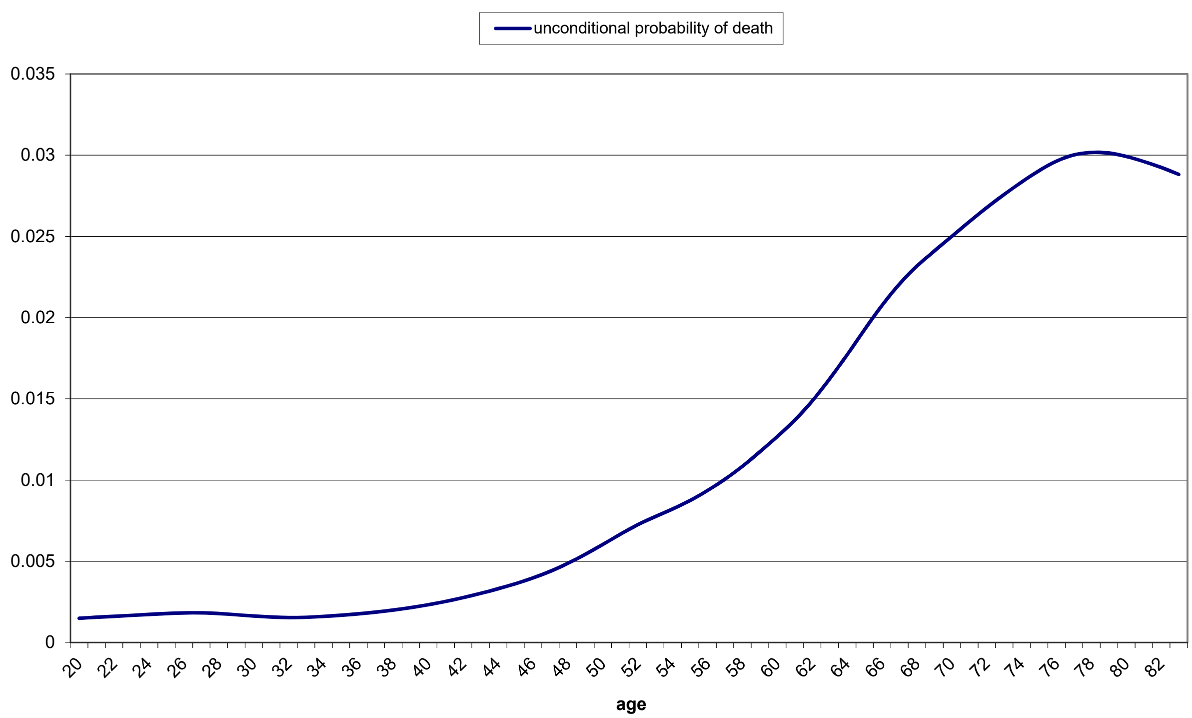

Figure 1: Probability of dying at the end of an age.

of more experienced labor can produce more than a unit of less experienced labor. We assume that the age of a worker is a good surrogate for his experience, and we take the least experienced worker as the reference worker. The productivity level of the reference worker also increases over time. The unit of reference labor hours in a base period is known as the *labor in efficiency units*. In the above production function, in aggregating labor hours of workers from different age groups, I have weighted their raw labor hours by their age-specific productivity levels. I tried several estimates of the age-specific labor productivity parameters. I report results only for the estimates of age-specific labor productivity levels from Hanse, 1993. The parameter σ in the above production function measures the output elasticity of capital; i.e., the percentage increase in output from a one percent increase in capital. In competitive markets, σ also measures the share of rental income in national income. Following much of the academic research, I take $\sigma = 0.35$.

Individuals derive utility from consuming goods and leisure time, which determines their savings and labor supply behavior. I assume that if an individual in any period consumes c units of consumer goods and the portion ℓ of a period as leisure, his well-being in that period is measured by $u\left(c, \ell\right) = \frac{\left[c^{\gamma}\ell^{1-\gamma}\right]^{1-\eta}}{1-\eta} 0 < \gamma \leq 1, \eta \neq 1$. Throughout his life, an individual will experience various random health and/or mortality shocks. Individuals do not know in advance which shocks they will experience, but they do understand the distribution or relative likelihood of future events. The measure of lifetime well-being is the expected value of the sum of well-being in each period t, discounted by the factor β . That measure of lifetime well-being is given by

$$U = E \sum_{t=0}^{T} \beta^t \psi_t u\left(c_t, \ell_t\right)$$

where T is the maximum age that an individual can live, ψ_t is the probability that he is alive during age t, and the expectation is taken over all possible sequences of disability shocks an individual may encounter over the working years.

In the above specification, the parameter γ measures how an individual values consumption of material goods relative to leisure - the higher the value of γ , the more the individual prefers consumption over leisure. Following İmrohoroglu et al., 1995, I assume labor supply is exogenously fixed at $1 - \ell_t = 0.45$ in each period during working age and $1 - \ell_t = 0$ during retirement. This average labor hours of 0.45 “assumes that individuals devote 45 hours a week (out of possible 98 hours) to work.” Without loss of generality, I assume that $\gamma = 1$. The parameter η measures an individual's preference for substituting consumption between two periods - the higher the value of η , the lower is the individual's preference to substitute consumption of one period over another period. In the present context with risks, η also measures an individual's attitude towards risk. When $\eta = 0$, the individual is risk neutral. A higher value of η means he is more risk averse. This parameter affects savings.

The estimates of η for US workers vary from study to study. For this analysis, following İmrohoroglu et al., 1995, I assume for the benchmark analysis, $\eta = 2.0$, the depreciation rate for capital = 0.045, and the discount factor for aggregating inter-temporal utility β = 0.967. (A second, lower, discount rate used by İmrohoroglu et al., β = 1.011, will also be tested.) Furthermore, I assume that the population growth rate is 1.2 percent and the productivity growth rate is 1.65 percent. In a revised draft I will carry out sensitivity analysis with respect to critical parameters such as β and η .

From the 2006 Trustees Report, I find the average wage index (AWI) for 2005 to be \$36,953[2] and average OASI benefit per retired worker to be \$10,852, which produces a replacement rate of 29 percent and for DI the average benefit per recipient is \$10,271, which produces a replacement rate of 28. (The OASI and DI benefits include benefits paid to family members.) In this paper, I take the replacement rate for both programs to be 29 percent. The parameter B in the production function is a scale factor, calibrated so that the long-run average yearly earnings from the model match the 2005 AWI in the Trustees Report.

At the beginning of a period, an individual of age a has assets of amount k_a , which are carried from the previous period. He is given an equal share of aggregate unintended bequest, q. He gets income of $r(k_a + q)$ from his assets and inheritance. This income combined with his assets and inheritance $k_0 + q$ is one component of his budget for this period. If he is not disabled during the period, he works $1 - \ell$ hours. Given the wage rate and his productivity level, he earns wages w_a . He pays OASI and DI taxes out of his earnings. If he is disabled, he gets disability benefits D_a , which equals the DI replacement rate times AWI. During his retirement years, he does not work and gets only OASI benefits equal to the OASI replacement rate times AWI. From his total income, he decides to the amount to consume and the amount of assets to hold for the next period to maximize his expected utility.

I have used the dynamic programming approach to solving the individual's choice problem. I use a grid search method to find optimal asset holdings by restricting the search to

[2] Reported in the link http://mwww.ba.ssa.gov/OACT/COLA/AWI.html

4097 grid points. I implemented the numerical computations of this model using Sun's public domain, object-oriented Java programming language.

3 Findings

3.1 The OASDI Program and the life-cycle Pattern of Consumption and Asset Holdings

The source of variations in consumption, asset holdings and earnings in this model is differential disability shocks. Other shocks that influence earnings and hence consumption and asset holdings are ignored to simplify the computational complexity. In this economy, all individuals begin their lives from the same initial position of zero asset holdings and good health status. As time progresses, individuals vary in their experiences of disability shocks over the life cycle. Those disability shocks affect their earnings and hence consumption and asset holdings. In the real world, many other types of shocks also affect earnings. In this paper, I consider only disability health shocks that prevent one from engaging in substantial gainful activity in the labor market. While individuals all start equal, they drift away from each other over time in terms of their asset holdings and consumption levels. In the absence of the OASDI program, and in the absence of disability insurance and annuity markets, risk-averse individuals will self-insure themselves to cope with mortality and disability risks. Figures 2 and 4 show the average levels of their consumption and asset holdings. Figures 3 and 5 show variances of consumption and asset holdings, which measure how far they drift apart from each other as they get older, as mentioned above. For comparison, I have plotted these variables for two scenarios: one with the current OASDI program, and the other without any OASDI program or markets for sharing these risks; i.e., the benchmark scenario. For further comparison, I have also reported in figures 2 and 4 the level of consumption and asset holdings for the same economy (i.e., with same parameter values) when there are no mortality and disability risks. This I refer to as no risks scenario in the figures and in Table 1.

As illustrated by the figures, levels of consumption and asset holdings are higher in the self-insured case than in the economy with an OASDI program. Moreover, an OASDI program reduces variations in both asset holdings and consumption across individuals of all age groups. The variances and averages of both variables peak at the retirement age, with a notable difference that under the self-insurance economy, consumption peaks at an earlier age and at a higher value. Thus under self-insurance, the precautionary motive for savings dominates and individuals save higher amounts and consume at a higher rate when they are young, as discussed in the introduction.

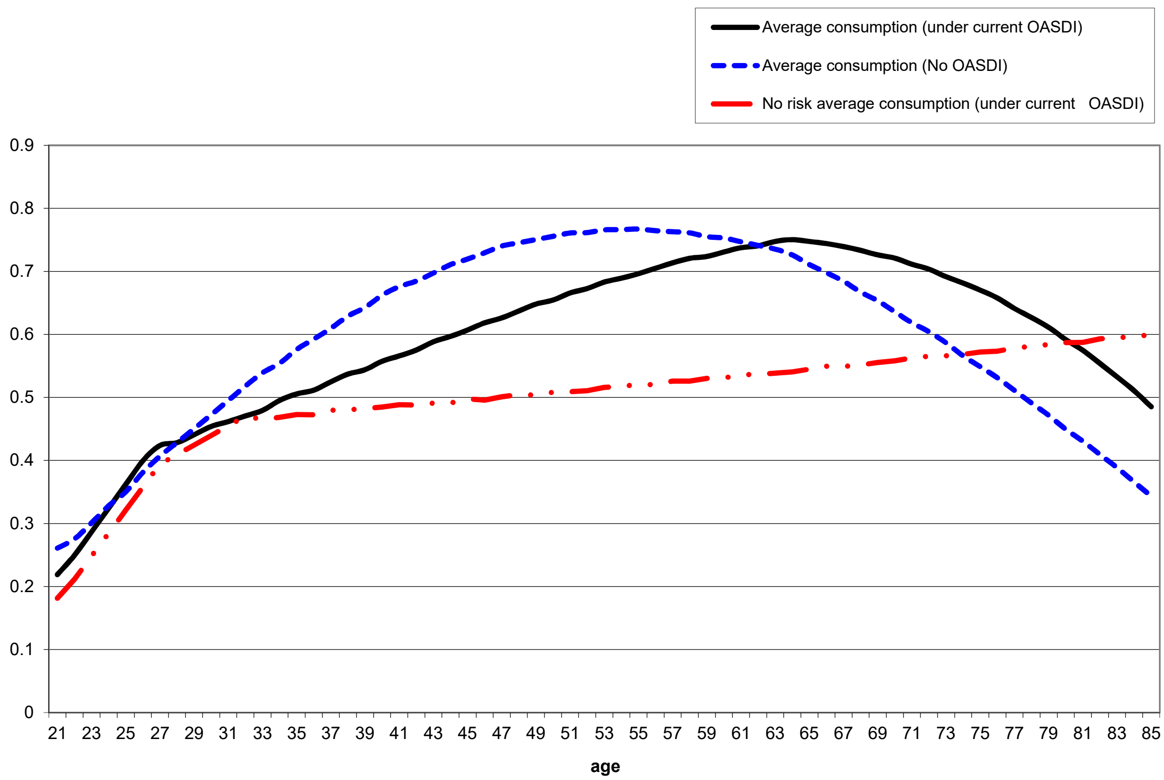

Figure 2: Average consumption at each age.

3.2 Effect of the OASDI Program on Individual Lifetime Welfare, the Elderly Poverty Rate and Aggregate Capital

How does an OASDI program affect an individual's lifetime welfare? What is the value of an OASDI program to individuals from the point of view of risk sharing? How does the program affect elderly well-being and poverty rates? Finally, how does the program affect the long-run capital stock, per capita income, and consumption? I compare results with the benchmark scenario in which individuals self-insure themselves against mortality and disability risks without having access to any social insurance or private insurance to share the mortality and disability risks.

I first compare equilibrium aggregate effects of the OASI and DI programs with the benchmark case (Table 1). I compute the average lifetime welfare of an individual for two values of the time preference parameter, $\beta = 0.967$ and $\beta = 1.011$. İmrohoroglu et

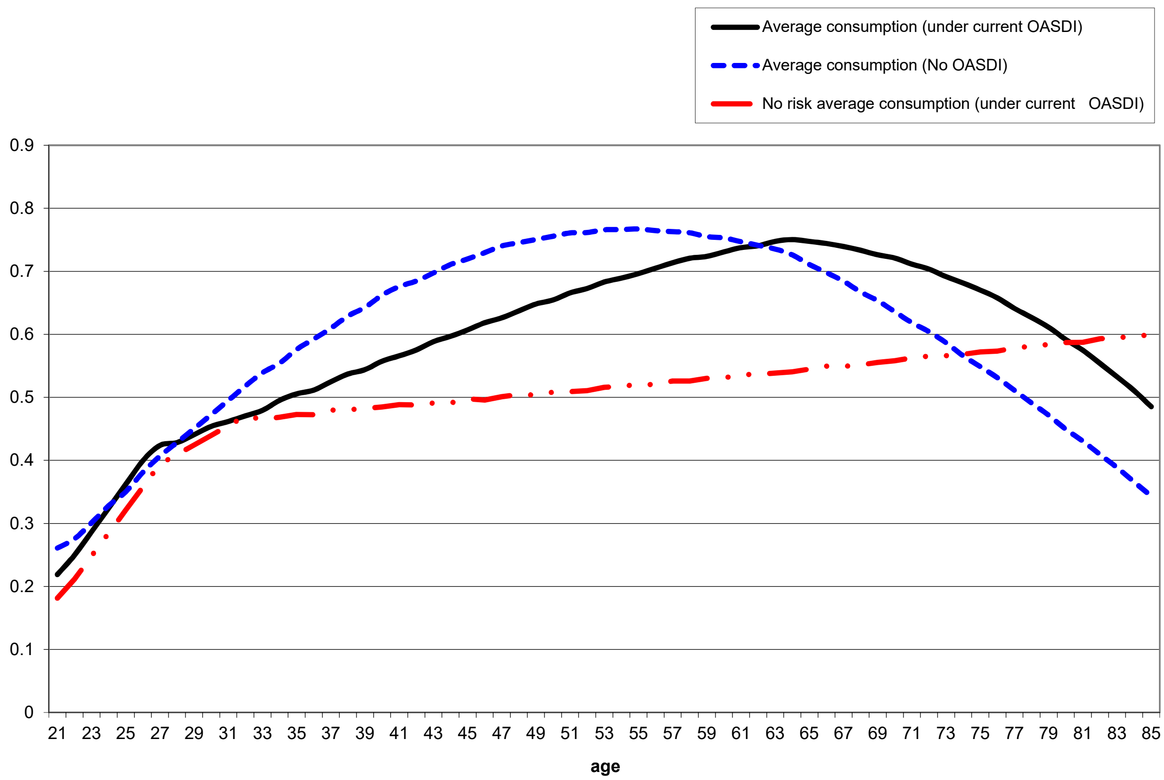

Figure 3: Average consumption at each age.

al., 1995 refer to empirical studies that found the value of β to be around 1.011. For both parameter values, individual welfare is higher in the case of the OASDI program than in the self-insurance case. Welfare units are abstract, and it is hard to interpret the significance of those welfare gains. Therefore, to get a better perspective, I compute the equivalent variation for each welfare gain or loss. The equivalent variation measures welfare gain as equivalent to a percentage increase in consumption at every age in the benchmark scenario. The equivalent variation for the gain in welfare from the OASDI program of the model is 10.56 percent. This means that the OASDI program makes an individual feel as well off as in the self-insurance situation with 10.56 percent higher consumption at each age. The qualitative nature of the welfare gains is similar for $\beta = 1.011$.

To examine the contributions of each component program to the welfare gain, I examine each program separately. I report results for the OASI program in column 3 and the DI program in column 4. The DI program improves welfare for both values of β. The PAYGO, actuarially fair annuity component of the OASI program does not improve welfare when $\beta = 0.967$. It does improve welfare, however, when $\beta = 1.011$.

So what positive effect does the OASI program have on the welfare of workers? To address this, I report in Table 2 the percentage of the population of different ages in retirement that will fall below a model-defined poverty threshold in the presence of the OASI program and in the absence of the OASI program. The official poverty threshold for an individual age 65 or over was \$9,367 in 2005, about 30 percent of per-capita income in 2005.[3]

Accordingly, I define a poverty threshold in long-run equilibrium under each policy regimes to be 30 percent of the equilibrium per capita income in that regime. It is clear from Table 2 in the benchmark simulation (with no OASDI program) a substantial portion of the population falls under this threshold at older ages but that with an OASDI program almost no one in the model population falls below the threshold. The actual poverty rate in the U.S. population is considerably higher than in the model. See Table 8.2 in Social Security Administration, 2004, in which the poverty rate is about 10 percent for the oldest beneficiaries or 30 percent for the oldest non beneficiaries. The model may underestimate the actual poverty rate for two reasons: First, the assumption of receiving an equal share of the unintended bequest means each individual is ensured a basic minimum level of income

[3] For official threshold levels, see the US Census Bureau link http://www.census.gov/hhes/www/poverty/threshld.html. I have calculated current per capita income for the year 2005 dollars as per capita income in 2000 dollars from the White House link http://www.whitehouse.gov/fsbr/income.html, multiplied by the GDP inflater, i.e., the ratio of current GDP in 2005 over current GDP in 2000 from BEA link http://www.bea.gov/national/xls/gdplev.xls. This gives 2005 per capita income to be \$31,160.

in all policy regimes. Second, there are many other sources of income fluctuations that can throw an elderly into poverty and the model does not incorporate those sources to avoid computational complexity.

It is important to note that this finding is different from the traditional arguments for the OASI program. According to those arguments, individuals are myopic or irrational and thus do not save enough for their retirement, and the OASI program performs a paternalistic role of saving for the retirement of the myopic non-optimizing people. In this model, individuals have rational expectations, meaning that individuals know that with some small but positive probability they will survive to old age. However, because the present is more certain than the future, their optimal behavior is to consume most of their savings when they are young and more certain to live, and leave very little for older ages, when the likelihood of surviving is very low.

Another important role played by the OASI program is that it provides survivors' benefits to dependents of a deceased worker. When I include this in future work, the model may show welfare gains from the OASI program for the parameter value $\beta = 0.967$. This positive role will be in addition to virtually eliminating elderly poverty that I mentioned earlier. The computation of the equilibrium becomes more complex in this case, and that is why very little has been investigated in the literature. I will incorporate this in a follow-up study.

The OASDI program of the model reduces the variance of asset holdings and consumption levels and reduces elderly poverty, but it also reduces the aggregate capital stock, consumption levels and per capita income in the long run as compared to the self-insured benchmark case.

The results of the model show the macroeconomic and individual welfare costs of providing income guarantees through OASI to the elderly population, without which a large percentage of the elderly population would fall into poverty even when they have rational expectations and make perfectly rational asset-holding decisions.

3.3 Optimal OASDI Replacement Rates

In the previous subsection we saw that given the current replacement rate of 0.29, the model's OASI program reduces lifetime welfare when $\beta = 0.967$ and improves lifetime welfare when$\beta = 1.011$. What would be the OASI replacement rate for each β that would maximize a representative individual's lifetime welfare? I find that for $\beta = 0.967$, the optimal OASI replacement rate is 0, and for $\beta = 1.011$, the optimal replacement rate is 0.55,

Table 1: Evaluation of Insurances Provided by the OASI and DI Programs

	In the Presence of Mortality and Disability Risks				No Risks	
	Without publicly provided OASDI, i.e., replacement rates for OASI = 0, and DI = 0	Publicly provided OASDI with replacement rates for OASI = 0.29 and DI = 0.29	Publicly provided OASDI with replacement rates for OASI = 0 and DI = 0.29	Publicly provided OASDI with replacement rates for OASI = 0.29 and DI = 0	Without publicly provided OASDI with replacement rates for OASI = 0 and DI = 0	Publicly provided OASDI with replacement rates for OASI = 0.29 and DI = 0.29
Welfare						
b =0.967	66.27	74.10	77.35	61.74	78.46	73.73
b =1.011	49.49	80.75	78.81	56.60	82.53	83.27
Equivalent Variation						
b =0.967		10.56	14.33	-7.34	15.54	10.12
b =1.011		38.71	37.21	12.57	40.04	40.57
Per capita income						
In 2005 dollars	68,084.04	57,705.57	60,561.81	64,172.08	56,210.00	53,135.64
Interest Rate	5.03	8.29	7.24	6.09	6.71	7.89
Asset holdings						
Mean (2005 dollars)	264,566.69	167,084.55	191,113.20	224,463.96	185,687.04	158,629.30
Variance (*E-6)	6,585.34	2,869.45	3,050.67	5,003.71		
Consumption						
Mean (2005 dollars)	51,923.40	49,176.49	50,248.09	50,929.21	43,499.50	42,307.20
Variance(*E-6)	155.70	69.65	75.38	143.41		
Earnings						
Mean (2005 dollars)	43,599.07	36,953.00	38,782.05	41,093.97	35,974.40	34,006.81

Notes: 1) The equivalent variation is the percentage increase in bench mark consumption stream that makes him as well off at the benchmark situation as he is in the present situation.

instead of the currently assumed rate of 0.29 (see Table 3 for details).

I also computed the optimal replacement rates for OASI and DI jointly that would maximize a representative individual's lifetime welfare. When $\beta = 0.967$, the replacement rates come out to be 0 for the OASI program and 0.9 for the DI program. When $\beta = 1.011$, the replacement rates come out to be 0.55 for the OASI program and 0.9 for the DI program.

4 Conclusions

This paper uses a stochastic overlapping generations model and calibrates it to the US economy to examine the nature of mortality and disability risk-sharing protections that are provided by an OASDI program to individual workers. The model incorporates both disability and mortality risks. The paper assumes that unintended bequests are redistributed equally among the living individuals, as assumed by most previous studies. The paper argues that this assumption in effect provides annuity type protection against mortality risks and also redistributes income from the low earners to high earners, both of which are purposes of OASI programs. The paper examines properties of long-run equilibrium, taking as the

Table 2: Elderly Poverty with and without the OASDI Program.

Age Group	**Percentage of population in poverty**	
	Under the current OASDI Program	**Without an OASDI Program**
65	0.00	0.20
66	0.00	0.22
67	0.00	0.24
68	0.00	0.26
69	0.00	0.29
70	0.00	0.33
71	0.00	0.37
72	0.00	0.42
73	0.00	0.48
74	0.00	0.56
75	0.00	0.65
76	0.00	0.78
77	0.00	0.93
78	0.00	1.14
79	0.00	1.41
80	0.01	1.80
81	0.01	2.27
82	0.01	2.97
83	0.01	3.94
84	0.02	5.43
85	0.03	7.77

Table 3: Equilibrium results for various OASI replacement rates given a fixed DI replacement Rate 0.29.

OASI Replacement Rate	Wage rate	Interest rate	Capital-Labor Ratio	Capital stock in efficiency unit	Per capita income	Average yearly Earnings (in 2005 dollars)	Average consumption (in 2005 dollars)	Welfare $\beta = .967$	Welfare $\beta = 1.011$
0.00	1.20	0.07	5.76	2.11	69,540.49	44,531.74	57,697.69	78.065	77.939
0.05	1.19	0.07	5.62	2.06	68,919.09	44,133.82	57,473.53	77.532	78.466
0.10	1.18	0.08	5.50	2.01	68,384.64	43,791.57	57,284.16	77.030	78.911
0.15	1.17	0.08	5.36	1.96	67,771.69	43,399.05	57,049.99	76.467	79.265
0.20	1.16	0.08	5.25	1.92	67,250.76	43,065.47	56,846.73	75.934	79.555
0.25	1.15	0.08	5.13	1.88	66,706.30	42,716.81	56,644.78	75.381	79.764
0.30	1.14	0.08	5.03	1.84	66,213.07	42,400.96	56,449.51	74.838	79.931
0.35	1.14	0.08	4.93	1.80	65,746.91	42,102.44	56,275.03	74.292	80.047
0.40	1.13	0.09	4.84	1.77	65,289.23	41,809.35	56,095.61	73.741	80.052
0.45	1.12	0.09	4.74	1.73	64,823.90	41,511.37	55,908.64	73.176	79.657
0.50	1.11	0.09	4.66	1.70	64,388.74	41,232.71	55,749.23	72.617	79.273
0.55	1.11	0.09	4.57	1.67	63,980.56	40,971.32	55,569.19	72.068	78.812
0.60	1.10	0.09	4.50	1.65	63,611.65	40,735.08	55,423.09	71.511	78.349
0.65	1.09	0.09	4.43	1.62	63,274.79	40,519.37	55,293.50	70.972	77.845
0.70	1.09	0.10	4.36	1.60	62,917.36	40,290.48	55,147.37	70.401	77.343
0.75	1.08	0.10	4.29	1.57	62,556.18	40,059.19	54,998.57	69.827	76.807
0.80	1.08	0.10	4.23	1.55	62,216.53	39,841.69	54,877.94	69.250	76.221
0.85	1.07	0.10	4.16	1.52	61,858.21	39,612.23	54,709.78	68.664	75.646
0.90	1.06	0.10	4.11	1.50	61,558.17	39,420.09	54,589.78	68.085	75.062
0.95	1.06	0.10	4.05	1.48	61,270.34	39,235.78	54,473.02	67.520	74.444
1.00	1.05	0.10	4.00	1.46	60,966.10	39,040.95	54,369.71	66.925	73.786

benchmark an economy in which individuals self-insure themselves against these risks by adjusting their own savings and labor supply without access to an OASDI program or private markets for sharing these risks.

The main findings of the paper are that the OASI and DI programs of the model together improve the lifetime welfare of workers, reduce the variation of consumption and asset holdings over the life cycle, and virtually eliminate elderly poverty that arise from the income fluctuations due to mortality and disability risks. The value of the model's OASDI program to a representative worker is as high as if consumption were 10.56 percent higher at all ages in the benchmark equilibrium. The value is even higher for the model's DI program. For the OASI program, however, assuming equal sharing of unintended bequests, and disregarding the survivors' benefit component of the program, the actuarially fair pay-as-you-go annuity component of the program does not improve individual lifetime well-being. Because the OASDI program mitigates the precautionary motive for savings, the macroeconomic costs of the program are that it reduces the aggregate capital stock and per capita consumption and income.

The replacement rate is defined here as the percentage of the working population's average yearly wage that the program will pay an individual in case the risky event covered by the program occurs. The replacement rate that maximizes the lifetime well-being of an individual turns out to be zero percent for the OASI program without the survivors' benefit component, and 90 percent for the DI program. Under a less plausible assumption adopted in a few studies that individuals value future utility of consumption more than the present utility of consumption, the optimal OASI replacement rate becomes a positive number (around 55 percent for the chosen parameter values).

It is possible that when the survivors' benefit component of the OASI program is incorporated and the assumption about the allocation of unintended bequests is modeled more realistically, the model's OASI program will produce welfare gains. It is also important to remove the assumption of age invariant probability of disability and recovery from disability, which may also change the results. Another important issue that I am examining in this framework is which economic group - differentiated by gender, race and education - benefits by how much from these programs.

5 Appendix

I consider an overlapping generations model with uncertain life time and disability risk. Each person's active life begins at age 21, his first period of life, and he lives a maximum of 65 periods, i.e., up to age 85. I assume that at age 65, i.e. in period 45, he retires.

Let $\mathcal{S} = \{s_d, s_m, s_a\}$, where s_d = disabled and eligible for disability benefits, s_m = somewhat disabled which lowers productivity but not qualified for disability benefits, and s_a = able or normal health. Asset levels are discrete values from the set $A = \{a_0 = 0, \ldots . a_{n_a}\}$, where n_a is the number of grid points in the asset dimension. The labor supply choices are discrete fractions of maximum possible working hours from the set $\Lambda = \{\ell_0 = 0, \ldots \ell_{n_l}\}$, where n_ℓ is the number of grid points in the labor supply dimension. The age-specific disability indexed producitivity measure $e(j, s)$, $j = 1 \ldots J$, $s \in \mathcal{S}$ and the age-specific Markov transition probability matrices $\Gamma_j(s'|s)$, $s \in \mathcal{S}$, $j = 2, \ldots, J$ are computed form the SIPP dataset.

Individual's choice problem:

$$E_1 \sum_{j=1}^{J} \beta^{j-1} \left(\prod_{k=1}^{j} \psi_k \right) u\left(c_j, 1 - \ell_j\right) \tag{1}$$

where E_1 denotes the Expectation of a random variable in period 1 given all the information available prior to that.

Notations:

a	=	asset at the beginning of a period
s	=	health status during a period
a'	=	asset at the end of a period
c	=	consumption during a period
$w(j, s)$	=	earnings function of a worker of age j and health status s
B_j	=	Social Security benefits received at age j
$D_j(s)$	=	Disability benefits received at age j, given health status s
b	=	accidental inheritance
τ_o	=	OASI tax rate
τ_d	=	DI tax rate

The Bellman equation of the choice problem in Eq. (1) is given by

$$V_j(a, s) = \max_{c_j, \ell_j} u(c, 1 - \ell) + \beta \psi_{j+1} \sum_{s'} V_{j+1}\left(a', s'\right) \Gamma_{j+1}\left(s'|s\right) \tag{2}$$

$$
\begin{aligned}
c + a' &= (1+r)a + (1-\tau_o-\tau_d)w\cdot e(j,s)\cdot \ell + B_j + D_j(s) + b \\
&\quad \text{where} \\
B_j &= \begin{cases} 0 & \text{if } j < J_R \\ B & \text{if } j \geq J_R \end{cases} \\
D_j(s) &= \begin{cases} D & \text{if } j < J_R, s = s_d \\ 0 & \text{otherwise} \end{cases}
\end{aligned}
$$

Denote the optimal policy function of the Bellman equation in Eq. (2) as $a' = \alpha_j(a,s)$, $\ell = \lambda_j(a,s)$. Let $\pi_j(a,s)$ be the share of time invariant population of age j holding asset a and facing health shock s.

Definition 1. A *stationary recursive equilibrium* for a given set of policy parameters (τ_o, τ_d) is a collection of value functions $V_j(a,s)$, individual policy functions for optimal asset holding $a' = \alpha_j(a,s)$, and for labor supply $\ell = \lambda_j(a,s)$, invariant distribution $\pi_j(a,s)$, $j = 1,..J$ of the agent types, the capital labor ratio k, interest rate r, and wage-rate per efficiency unit labor w, OASI benefit rate B, and DI benefit rate D, and per capita accidental bequest b such that

1. $r = f'(k)$, $w = f(k) - kf'(k)$
2. Given r, w, tax rates τ_o, τ_d and transfers b, B, D, the policy rules $a' = \alpha_j(a,s)$, $\ell = \lambda_j(a,s)$ and value functions $V_j(a,s)$ solve the Bellman equation, Eq. (2), corresponding to individual's choice problem.
3. Invariant distributions satisfy:
$$\pi_{j+1}(a',s') = \frac{\psi_{j+1}}{1+n}\cdot \sum_{s\in\mathcal{S}} \Gamma_{j+1}(s'|s) \sum_{a:\alpha_j(a,s)=a'} \pi_j(a,s), j = 1,...J-1, \tag{3}$$
and
$$\pi_1(a,s) = \begin{cases} 1 & \text{if } a = 0, s = s_n \\ 0 & \text{otherwise} \end{cases} \tag{4}$$
4. Aggregate capital
$$K = \sum_{j=1}^{J} \sum_{a\in\mathcal{A}} \sum_{s\in\mathcal{S}} a\pi_j(a,s) \tag{5}$$
and aggregate labor
$$L = \sum_{j=1}^{J} \sum_{a\in\mathcal{A}} \sum_{s\in\mathcal{S}} e(j,s)\lambda_j(a,s)\pi_j(a,s), \tag{6}$$

and the capital-labor ratio $k = K/L$.

5. Accidental bequest allocation rule

$$b = \sum_{j=1}^{J} \sum_{a \in \mathcal{A}} \sum_{s \in \mathcal{S}} \alpha_j (a, s)\, \pi_j (a, s) \left(1 - \psi_{j+1}\right) \tag{7}$$

6. Actuarially fair defined contribution OASI benefit rate or the annuity:

$$B \equiv \frac{\tau_o \sum_{j=1}^{J_R - 1} \sum_{a \in \mathcal{A}} \sum_{s \in \mathcal{S}} w(j, s)\, \lambda(a, s)\, \pi_j (a, s)}{\sum_{j=J_R}^{J} \sum_{a \in \mathcal{A}} \sum_{s \in \mathcal{S}} \pi_j (a, s)} \tag{8}$$

7. Actually fair defined contribution DI benefit rate:

$$D \equiv \frac{\tau_d \sum_{j=1}^{J_R - 1} \sum_{a \in \mathcal{A}} \sum_{s \in \mathcal{S} \setminus \{s_d\}} w(j, s)\, \lambda(a, s)\, \pi_j (a, s)}{\sum_{j=J_R}^{J} \sum_{a \in \mathcal{A}} \pi_j (a, s_d)} \tag{9}$$

8. (product market clears by Walras law, so no need to write it down.)

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