Signaling Equilibrium, Intergenerational Social Mobility and Long Run Growth *

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Abstract

This paper provides a model of intergenerational social mobility and economic growth, in which innate ability of workers, and the type of their education and jobs determine the rate of technological progress and social mobility. The innate ability and hence productivity level of an individual is private knowledge. Education not only increases productivity level, more so for the higher ability individuals, it also acts as a signaling device for one's innate productive ability for the purpose of job matching in the labor market. It is shown that in economies with one-time non-renegotiable wage contracts, there are generally multiple signaling equilibria, all being far away from generating the maximum attainable rate of social mobility and economic growth. There are no natural economic grounds that can guide to select a particular equilibrium. Various labor market practices such as quit, layoffs and promotions based on worker's or employer's subjective assessment of on-the-job realized productivity, or explicit wage contracts contingent on some publicly observed noisy measurement of realized productivity, can improve some of the inefficiencies, and hence increase the rate of economic growth and social mobility. The remaining inefficiencies, however, can only be removed by intervening in the education system. The paper analyzes briefly a few education systems, and within the dual private-public education system, the paper examines the role of school vouchers or subsidies to the children of poorer family backgrounds in improving the rate of economic growth and social mobility.

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> [I]t is not a story that concludes, *Genius will out*-though Ramanujan's in the main, did. Because so nearly did events turn out otherwise that we need no imagination to see how the least bit less persistence, or the least bit less luck, might have consigned him to obscurity. In a way, then, this is also a story about social and educational *systems*, and about how they matter, and how they sometimes nurture talent and sometimes crush it. How many Ramanujans, his life begs us to ask, dwell in India today, unknown and unrecognized? And how many in America and Britain, locked away in racial or economic ghettos, scarcely aware of worlds outside of their own?

Robert Kanigel, *The Man who knew Infinity*, pp.3-4.

1 Introduction

It is a well established fact that an important source of growth in per capita income of modern economies is the increasing stock of tested and useful technological knowledge. Recent literature on endogenous growth is concerned with the process of such knowledge creation. Among others, Lucas [1988] provides a model in which identical workers spend part of their time acquiring skills or vocational training which increases their productivity; the new technologically useful knowledge that the individual workers acquire from their vocational or on the job training adds to the pool of social knowledge. In his model, factors that increase average education level of the work force also create higher long-run growth. In Romer's [1986] model, individual firms spend resources on R&D to create new technological knowledge; the individual firm's knowledge spills over to the rest of the economy creating the pool of social knowledge from which other firms benefit; this process generates increasing returns in production of knowledge and thus leads to long-run growth. Since machines cannot create new knowledge, and only humans working with machines can create new knowledge, implicitly in the Romer model also, the human capital of the R&D personnel plays an important role.

There is some empirical evidence (Adams [1990]) that accumulated basic scientific and engineering knowledge stocks filters through inter-industry spillovers with respective lags of 30 years and 10 years. Due to data limitation, his study, however, measured basic and engineering knowledge by the count of published articles in the respective fields. Both in developed and developing economies, there are also strong empirical evidence for spillovers of research knowledge generated by own R&D efforts of private firms (see Jaffe [1986] for a study on the US and Raut [1995] for a study on India).

While in this endogenous growth literature, creation of knowledge plays a great role, workers in this literature are generally assumed to be identical in their innate ability to create knowledge and in their productivity to generate income. But they do differ. This heterogeneity in innate ability may make a big difference in the process of knowledge creation; while average level of education is important, the distribution of education among the workers and the distribution of jobs among workers become a more important determinant of the rate of knowledge creation. For instance, suppose workers with high innate ability get educated in technical areas and work in jobs that are suitable to generate basic scientific and technological knowledge, and workers of somewhat lesser talent may get educated in engineering schools and work in jobs that generate applied knowledge, suitable for adopting basic scientific knowledge to industrial production purposes, and so on for other talent levels. This kind of assignment of talents of worker to jobs can generate faster growth in wages and total factor productivity.

The organization of the educational system in a society determines how individuals with different innate ability and family background decide their human capital investment. The mechanics of the labor market, which places workers of different schooling and talent to jobs determine the incentives structure for schooling choice of workers.

The well functioning of the labor market and school system is a critical determinant of social mobility and hence to total factor productivity growth. To see this simply, assume that the innate talent of an individual is independent of his or her family background.[1] Suppose the family background also influences individual schooling choice: the children with poorer family background have lower level of schooling (which has been widely found in many studies). Then it is quite clear that if talented individuals from poorer background did not obtain higher schooling, they are not tapped into the growth process. In such economies, therefore, higher social mobility through higher education of talented individuals from all backgrounds, and their assignment to the appropriate jobs in the growth enhancing modern sectors will lead to higher mobility and faster growth.

What then stands in the way of social mobility and growth?

There are many barriers. We focus on four main ones – (i) asymmetric information regarding the innate ability or talent of an individual (i.e., innate ability is observed by the individual but not by anyone else); (ii) dependence of cost of education on one's family background and innate

[1] There is no well established empirical evidence that the innate talent of individuals is in most part genetically inherited. The general wisdom on this is that there is probably a small correlation between the intelligence of parents and children, but the environmental factors are so dominant that it is very difficult to isolate the genetic inheritance, see Levine and Suzuki [1993].

ability; (iii) multiple equilibria resulting from asymmetric information, some equilibria with lower mobility and slower growth than the others, and this can happen in perfectly competitive markets with unprejudiced employers who hold self-fulfilling subjective beliefs relating the productivity level and the education level of the job market candidates; (iv) labor market practices: imperfection in the labor market may create institutions and complex wage contracts to cope with asymmetry in information on talent; whether such market responses emerge in an economy depends on many other factors such as existence of labor union.

To analyze the effects of these factors on social mobility and growth, we extend a Spence [1974] type signaling model of job matching and human capital investment to an overlapping generations framework.[2] This model of job matching and schooling decisions differ from the existing models on several counts. First, the most theoretical models of job matching frames the decision making in a search theoretic framework, in which, infinitely lived workers search over time for the best job that can produce the highest life-time earnings (see Jovanovich[1979] who explicitly allows worker's productivity to vary with the matched job, and the references cited there for its predecessors). In these models, however, employers are passive, they do not design complex wage contracts nor use any signals from workers to get an idea about worker's ability and negotiate the wage contracts accordingly. Furthermore, the assumption of agents living infinitely, which is suitable for studying unemployment duration, or labor mobility across jobs (these were the focus of the matching literature), turns out to be technically not suitable for studying intergenerational social mobility.

Second, in modeling schooling decisions, we also use a Spence type model. To begin with, we assume the influence of family background on cost of schooling is exogenously fixed, and study the properties of signaling equilibrium over time and in the stationary state, and later in section 4 we extend the model and endogenize this influence by assuming that parent's choice of pre-school investment level determines the cost of schooling of their children, and parent's investment decisions are determined by maximization of an expected altruistic utility function. We examine how some of the properties of signaling equilibrium change as a result of this endogenous parental pre-school investment decisions. Our model differs from the basic human capital theory (Becker [1962], Ben-Porath [1967], Mincer [1958], Rosen [1977] and Willis [1986,section 4]) in one main respect that since in our model agents live effectively only for one period, we do not consider life cycle issues such as on-the-job-training, and more importantly, the borrowing constraints for educational investment.

The rest of the paper is organized as follows: Section 2 describes the basic model where we

[2] There are alternative models of intergenerational social mobility, see for instance, Becker and Tomes [1979, 1986], and Loury [1981]. These models do not feature asymmetric information regarding innate productive ability of workers.

assume the relationship between family background and cost of education is exogenously given. In this section we define the signaling equilibrium with one-time non-renegotiable wage contracts, and a few concepts involving the equilibrium which are used later. In section 3 we study the properties of signaling equilibrium, including dynamics. In section 4, we examine how labor market practices such as quits, lay-offs and promotions based on subjective assessment of productivity, and how explicit contingent wage contracts based on some publicly observed noisy measurement of productivity, may reduce inefficiency due to asymmetric information. Section 5 briefly discusses the role of education system and education policies that can improve social mobility and economic growth. Section 6 extends the model to endogenize parent's pre-school investment in their children, which we assume to influence the cost of education of the children, and studies whether earlier properties of signaling equilibrium change significantly. In conclusion, section 7 discusses policy implications of our model for intergenerational social mobility and economic growth.

2 The Basic Model

We consider an overlapping generations model in which in each period one person is born to each parent. The gender of an individual is not important for this model, and we assume for ease of presentation the male gender. We denote by τ an individual's innate ability which affect one's productivity in the workplace as well as learning in school. There is some dispute in the sociology, psychometry and economics literature as to whether τ should consist of one component, which is then generally referred to as level of intelligence, or IQ, or should it be a vector allowing individuals to vary in their ability for various skills, such as verbal skill, mathematical or logical skills.[3] To keep our analysis manageable, however, we assume that τ is one dimensional, and children are born with a talent type represented by a finite set of discrete ordinal numbers, $\mathcal{T} = \{1, 2, ..., \hat{\tau}\}$. In $\mathcal{T}$, a higher number denotes a greater talent. The probability that a parent has a child of talent $\tau \in \mathcal{T}$ is $g(\tau)$.[4]

Another controversy which drew a lot of public debate is whether children's innate ability is ge-

[3] The arguments in the first strand is based on statistical analyses which show that generally many test scores exhibit the presence of a common factor, known as g-loaded factor which has been found to be a good predictor of many socio-political-economic outcomes, especially in the labor market success and educational attainment. (See, for instance, Herrnstein and Murray [1994], Gottfredson [1997]). There are many economic studies which find the opposite. (See, for instance, Heckman's critique of the Herrnstein and Murray book for arguments and references, and see Roy's [1951] model of earnings function that uses a multi-dimensional vector for ability).

[4] There are other controversies regarding talent, ability and intelligence. Some believe that one is born with a fixed level of intelligence, and training and environment has no effect on intelligence. Others do not agree with it, and believe that ability, intelligence and talent could be improved to some extent with better environment and training. Some believe that intelligence or innate ability is fixed when one is born, and less intelligent people can learn and do complex things that we face in our everyday life, in school curricula, and in modern jobs, except that they might take longer, and thus less productive; this is the view we take in this paper.

netically inherited from parent's innate ability. The general consensus is that there is some positive correlation. Although much of what we demonstrate will be valid for this general case, for ease of exposition and calculation, we assume that the probability mass function $g(\tau)$ does not depend on parent's talent type.

The talent type of an individual is private information, observed only by him and by no one else. Individuals, however, can choose an education level and its quality to signal his talent type. We denote a signal by a one[5] dimensional variable s, which we can view as quality adjusted number of years of schooling, a higher number representing either a better quality or higher schooling level. We further assume that the set of education levels, $\mathcal{S}$ is discrete and finite and is given by the ordinal numbers $\mathcal{S} = \{1, 2, ..., \hat{s}\}$, with a higher number representing a higher education level.[6]

We can think of each signal in $\mathcal{S}$ as a social class, social status, social rank according to earnings[7] or we can think of it simply as an occupation, depending on how we define the signals in $\mathcal{S}$. Furthermore, we assume that the parents' socio-economic status $s \in \mathcal{S}$ summarizes their children's family background or "environment". In each period the active members of the society will belong to one of the groups in $\mathcal{S}$. We are interested in modeling the intergenerational mobility of families across the groups in $\mathcal{S}$, without referring it as social mobility, class mobility or earnings mobility since they all coincide in our model. More specifically, let the probability mass function of the population in period t over the set of signals $\mathcal{S}$ be denoted as $\pi_t = \left(\pi_t^1, ..., \pi_t^{\widehat{s}}\right)$, $t \geq 0$. The economy begins at time $t = 1$ with an adult population whose parents' socio-economic status is distributed as $\pi_0 = \left(\pi_0^1, ..., \pi_0^{\widehat{s}}\right)$. The mobility matrix P_t consisting of transition probabilities of an individual born in a family background s_{t-1} will move to the family background s_t, in period t, for all values of $s_{t-1}, s_t \in \mathcal{S}$. Thus given π_0 and P_1, the distribution of population in period $t = 1$ is determined. The same process rolls over in the next period, and repeats until the end of time.

In what follows we provide an economic model of how individuals decide their human capital investment, and how they get matched with jobs in the labor market. Given the initial income distribution π_0, these two factors determine the mobility matrix P_t and the dynamics of the income distribution π_t over time, the nature of the intergenerational mobility, and how they affect the rate of technological progress and the growth rate of earnings. Our emphasis is to examine the effect

[5] Realistically, s is a vector, $s = \left(s^1, s^2, s^3\right)$, where s^1 represents the number of years of schooling, s^2 represents major specialization, such as Engineering, Medical Science, Physics, Chemistry, Business Administration, and other general subjects, and s^3 may represent quality of education, which is generally associated with the quality of the school from which s^1, and s^2 are obtained, for instance private, public, the average of the top 25% SAT scores of the school, student faculty ratio, and a host of such variables of the school (see (Daniel, Black and Smith [1995], and Heckman, Layne-Farrar and Todd [1995] for some of these variables in their empirical studies on school quality using NLSY data).

[6] The general practice in the human capital literature is, however, to treat S as continuous variable, more realistically it is a discrete set.

[7] We will see later that earnings are functions of $s \in \mathcal{S}$.

of asymmetric information on these decisions and the general equilibrium effect of these decisions. We begin with modeling of the production sector.

2.1 Production sector

There are some controversy as to whether years of education is a significant determinant of earnings, or is it that ability is the main determinant of earnings, and education just picks up the effect of ability (see Griliches and Mason [1972] and for more recent references, see Willis [1986]. It has been found that the effect of education on earnings is somewhat lower after ability is controlled for, but it is not insignificant. Much of these empirical studies are carried out in the Mincer [1958] earnings function framework, where they estimate market wage rate $w = \phi(s, \tau)$, where s is the number of years of schooling, and τ is an ability measure.[8] It is important to distinguish between an earnings function as above and the "productivity function" in order to understand where signaling theory of educational investment differs from human capital investment theory. *Productivity function*, $e(s, \tau)$, is the number of efficiency unit of labor that a worker with schooling level s and innate ability τ is equivalent to. The *earnings function* $w = \phi(s, \tau)$ is what the labor with education level s and ability τ gets paid in the market.

In our set-up, each job is characterized by its task and the employer in an industrial sector. We denote a job by η. We assume that each education level or school level is suitable for a particular task. If a worker with schooling level suitable for a particular task works in other tasks, his productivity will be lower. An employer is assumed to assign workers to the tasks according to what workers' schooling dictate. Thus when we talk about a different job, we are basically referring to a different employer or a different sector. We assume that η takes finite ordinal numbers from the set $\mathcal{H} = \{1, 2, ..., \hat{\eta}\}$, a higher index represents a more technical and growth enhancing. We assume that productivity function $e(s, \eta, \tau, \epsilon)$ of a worker depends on the worker-employer match η and there are random shocks to such productivity. For instance, the research output of a R&D worker is uncertain. Let $\mathcal{E} = \{e(s, \tau, \eta, \epsilon) | s \in \mathcal{S}, \tau \in \mathcal{T}, \eta \in \mathcal{H}\}$ denote the set of productivity levels. Let L_t be the total labor in efficiency units used in the production process. To simplify matters and without loss of much generality, we assume that the aggregate production in period t is represented by the following linear function:

$$F_t(L_t) = A_t L_t \tag{1}$$

where A_t is the total factor productivity parameter in period t.

In our model A_t is endogenously determined as follows: we presume that talented workers

[8] The original Mincer earnings function also includes an experience variable, x, which is generally taken to be number of years of work experience. Since it is not relevant for our issues, we drop it.

with higher education and working in higher up jobs in $\mathcal{H}$ can create more basic knowledge or new ideas about how to produce and distribute old products or new products cheaply. This basic knowledge is assumed to benefit future generations. More specifically, let $a(s, \tau, \eta)$ be the amount of basic scientific and engineering spillover knowledge created by a worker with education level s and innate ability τ when matched with the employer η. Let R_t denote the aggregate flow of spillover knowledge in period t in the economy, aggregated over all s, τ and η in the population. A_t evolves over time according to:

$$A_{t+1} = A_t \left(1 + \gamma(R_t)\right) \tag{2}$$

where $\gamma(R_t)$ is the growth rate of productivity level, assumed to be a time invariant function. If $R_t = 0, \gamma(R_t) = 0$ and γ may be assumed to be an increasing function as for instance, $\gamma(R_t) = R_t^{\mu}$, $\mu > 0$.

2.2 Employers' Problem

In the economics of imperfect information there are mainly two ways in which the workers' and employers' problems are formulated: the first approach is the Spence model where the employer announces a wage schedule, then workers make their decisions on education levels; the other approach due to Rothchild-Stiglitz is more appropriate in the insurance context, see for other schemes, Kreps [1990]. Spence's approach is more appropriate in our context, and we will follow this approach.

We assume that the production sector is competitive; the producer is risk neutral and he treats A_t as an externality when making his decisions. In each period $t \geq 1$, a producer η_t's role is to announce a wage schedule $\omega_t(s_t; \eta_t)$ for hiring purposes. He observes the education level s_t of a worker but not his talent type τ_t. The employer η_t holds a subjective belief about the conditional probability distribution of the productivity level $e\left(s_t, \tau_t, \eta_t, \epsilon_t\right)$ of an worker given his observed education level. We denote it by

$$q_t\left(e|s_t;\eta_t\right) = \text{Probability}\{e|s_t;\eta_t\}, e \in \mathcal{E}, s_t \in \mathcal{S}, \eta_t \in \mathcal{H} \tag{3}$$

Let $\omega_t(s_t; \eta_t)$ be the wage profile that the producer η_t announces. Perfect competition, and expected profit maximization imply that

$$\begin{aligned} \omega_t(s_t;\eta_t) &= A_t \textstyle\sum_{e\in\mathcal{E}} e \cdot q_t(e|s_t;\eta_t) \\ &\equiv A_t w_t(s_t;\eta_t), \text{ say} \end{aligned} \tag{4}$$

where $w_t(s_t; \eta_t) \equiv \sum_{e\in\mathcal{E}} e \cdot q_t\left(e|s_t; \eta_t\right)$. Notice that $w(.)$ depends on the producer's subjective conditional probability distribution, $q_t(e|s_t;\eta_t)$, and dependence of $w_t(.)$ on t is through the dependence of q on t.

2.3 Human capital investment

We consider only human capital investment in education, other important forms of human capital investment such as health and nutrition are not considered here. The investment in the level of education of an individual is a complex decision making process. Generally, parents make the initial investments such as pre-school investments and investments up to college or so, until an individual reaches enough maturity to make his own schooling decision. Family background can have great influence on educational attainment in several other ways. For instance, suppose that the quality of pre-school investment of parents' time at home affect children's motivation and persistence to continue schooling. Then, of course, more highly educated parents can provide better learning environment for their children at home. Similarly, more highly educated parents with their better knowledge base of child care, or simply because of their higher incomes can provide better pre-natal care, and health care for proper cognitive development of their children. Furthermore, innate ability will also affect the time and efforts it takes to complete an academic curriculum, and hence are important determinants of cost of schooling.[9]

We represent these effects of family background in our model simply by assuming that the cost, $\theta_t(s_t,\tau_t,s_{t-1})$, of obtaining a certain level of education $s_t\in\mathcal{S}$ for an individual in period t depends on his talent type $\tau_t\in\mathcal{T}$, and his parent's education level $s_{t-1}\in\mathcal{S}$, for all $s_t,s_{t-1}\in\mathcal{S}$, and $\tau_t\in\mathcal{T}$. In section 6, we consider the endogenous determination of family background. In this section we will assume that $\theta_t(s_t,\tau_t,s_{t-1})$ is increasing in s_t, and decreasing in τ_t and s_{t-1}. The assumption that $\theta_t(s_t,\tau_t,s_{t-1})$ varies with τ_t is necessary for education to act as signal for talent, see Spence [1974], or Kreps [1990] for a justification. We further assume that

$$\theta_t(s_t,\tau_t,s_{t-1}) = A_t\theta(s_t,\tau,s_{t-1})$$

where A_t is the productivity shift parameter of the aggregate production function defined earlier, and $\theta(.)$ is a time invariant function.

We assume that all individuals have identical linear[10] utility function $u(c_t)=c_t$ where c_t is the consumption of an adult of period t. An adult of period t with talent type $\tau_t\in\mathcal{T}$ and from a parent of education level s_{t-1} takes the announced wage function $w_t(s_t;\eta_t)$ of period t as given

[9] There are other ways education of parents can influence the educational achievement of their children, for instance, by providing role models. Also see section 5, Spence [1974] and Raut [1991] more on this.

[10] Thus we abstract away from bearings on our results from risk sharing between employers and workers.

and decides his education level $s_t \in \mathcal{S}$ and the expected job to perform by solving the following problem:

$$\max_{s_t \in \mathcal{S}, \eta_t \in \mathcal{H}} \; [\omega_t(s_t;\eta_t) - \theta_t(s_t,\tau_t,s_{t-1})]$$

which is equivalent to solving the following problem

$$A_t \max_{s_t \in \mathcal{S}, \eta_t \in \mathcal{H}} \; [w_t(s_t;\eta_t) - \theta(s_t,\tau_t,s_{t-1})] \tag{5}$$

Except for degenerate cases, there is a unique optimal solution s_t for each τ_t and s_{t-1}, which is independent of A_t. Notice that in our framework, all individuals with talent τ_t and family background s_{t-1} behave identically. We will refer to such an **agent as** (τ_t, s_{t-1}) . We denote the optimal solution of equation (5) for agent (τ_t, s_{t-1}) by $\sigma_t(\tau_t, s_{t-1})$ and $\eta_t(\tau_t, s_{t-1})$. We define a binary function $\chi_{\eta_t}^{\sigma_t}(s_t, \tau_t, s_{t-1}, \eta_t)$ which fully represents the optimal solution of (5) by

$$\chi_{\eta_t}^{\sigma_t}(s_t,\tau_t,s_{t-1},\eta_t') = \begin{cases} 1 & \text{if optimal solution for agent } (\tau_t,s_{t-1}) \text{ is } s_t \text{ and } \eta_t' \\ 0 & \text{otherwise} \end{cases} \tag{6}$$

for all $s_t, s_{t-1} \in \mathcal{S}$ and $\tau_t \in \mathcal{T}$, and $\eta_t' \in \mathcal{H}$. We refer to this $\chi_{\eta_t}^{\sigma_t}()$ as *optimal schooling and job choices in binary form.*

2.4 Signaling equilibrium

We assume that η is perfectly observed by the workers. The imperfect knowledge of η leads to matching with two-sided asymmetric information, for which the theory is very complicated, and not quite developed yet, see for instance Benabou [1993]. The equilibrium is recursively defined over time. At the beginning of time period t, π_{t-1} and A_t are already known. For given anticipated conditional probabilities $q_t(e|s;\eta_t)$ as in (3), the producer η_t announces the wage profile $A_t w_t(s_t;\eta_t)$ as in (4). Given $A_t w_t(s_t;\eta_t)$, the worker (τ_t, s_{t-1}) decides on optimal education level and the job $\chi_{\eta_t}^{\sigma_t}(s_t,\tau_t,s_{t-1},\eta_t)$ as in (6). The optimal decisions together with the distribution of their family background π_{t-1} and talent $g(\tau_t)$ generate the observed average value of the marginal product $A_t\hat{w}_t(s;\eta_t)$ of workers with education level s working in the job η_t as follows:

$$\hat{w}_t(s_t;\eta_t) = \frac{\sum_{\tau_t,s_{t-1}} \int e\left(s_t,\eta_t,\tau_t,\epsilon\right) f(\epsilon) d\epsilon \; \chi_{\eta_t}^{\sigma_t}\left(s_t,\tau_t,s_{t-1},\eta_t\right) g(\tau_t)\pi_{t-1}^{s_{t-1}}}{\sum_{\tau_t,s_{t-1}} \chi_{\eta_t}^{\sigma_t}\left(s_t,\tau_t,s_{t-1},\eta_t\right) g(\tau_t)\pi_{t-1}^{s_{t-1}}} \equiv \Psi_t\left(w_t\left(.\right)\right) \tag{7}$$

where, $f(\epsilon)$ is the density function of ϵ. Notice that optimal schooling choices $\chi_{\eta_t}^{\sigma_t}(s_t,\tau_t,s_{t-1},\eta_t)$ determines the transition probability $p_t(i,j)$ of an individual born in the family background $s_{t-1} =$

i, moves to the family background $s_t = j$, for all $i, j \in \mathcal{S}$ as follows:

$$p_t(i,j) = \sum_{\eta_t,\tau_t} \chi_{\eta_t}^{\sigma_t}(j,\tau_t,i,\eta_t)\, g(\tau_t) \tag{8}$$

Let $P_t = [p_t(i,j)]_{i,j\in\mathcal{S}}$ be the transition matrix in period t. Given π_{t-1}, P_t determines π_t according to the following equation

$$\pi_t = \pi_{t-1} P_t \tag{9}$$

and π_{t-1} and $\chi_{\eta_t}^{\sigma_t}(s_t,\tau_t,s_{t-1},\eta_t)$ determine R_t by

$$R_t = \sum_{s_t,\tau_t,\eta_t,s_{t-1}} a(s_t,\tau_t,\eta_t)\, \chi_{\eta_t}^{\sigma_t}(s_t,\tau_t,s_{t-1},\eta_t)\, g(\tau_t)\, \pi_{t-1}^{s_{t-1}} \tag{10}$$

and thus the growth rate, $\gamma(R_t)$ or equivalently, A_{t+1} according to equation (2). The economy moves to the next period with known π_t and A_{t+1}, and the above process starts all over again.

Definition 1 *Given an initial distribution π^0 of social groups $\mathcal{S}$, a* ***signaling equilibrium*** *of labor market is a sequence of anticipated distributions $\{q_t(e|s_t;\eta_t)\}_1^\infty$ defined in (3) and an associated sequence of wage schedules $\{w_t(s_t;\eta_t)\}_1^\infty$ defined in (4), optimal schooling and job choices in binary form $\left\{\chi_{\eta_t}^{\sigma_t}(s_t,\tau,s_{t-1},\eta_t)\right\}_1^\infty$ defined in (6), and an associated sequence of transition probability matrices over $\mathcal{S}, \{P_t\}_1^\infty$ defined in (8), such that the induced sequence of observed wage schedules $\{\hat{w}_t(s_t;\eta_t)\}_1^\infty$ defined in equation (7) coincides with the sequence of anticipated wage schedules $\{w_t(s_t;\eta_t)\}_1^\infty$, i.e., $w_t(s_t;\eta_t) = \hat{w}_t(s_t;\eta_t)$ for all s_t, η_t for which there exists some τ_t and s_{t-1} with $\chi_{\eta_t}^{\sigma_t}(s_t,\tau_t,s_{t-1},\eta_t) = 1$.*

Proposition 1 *A fixed point $w_t^*(.)$ of the mapping Ψ_t in (7) for all $t \geq 1$ is a signaling equilibrium and vice versa.*

I do not pursue the existence issues in this paper. In signaling literature, however, the existence of signaling equilibrium is not a problem, the problem is that there exist many of them. How to find them, and select one of them! One criterion that the mapping Ψ_t in (7) suggests is to use it as a learning algorithm with the interpretation that the employers begin with an initial guess of the relationship between productivity and schooling level and announce the wage schedule $w_t(.)$, and then they observe the realized average product of every schooling level and also the productivity levels of in the individual workers, and revise the wage schedule to $w_t'()$. and the process continues until we get convergence of beliefs and the observed distributions. Although we do not know if this learning algorithm will converge or not, but in the numerical simulations that we have considered, the above learning algorithm always converged. An important point to note is that the equilibrium that a learning process converges to depends on the initial beliefs of the employers, and the resultant signaling equilibria can differ in their properties drastically. A better approach to equilibrium selection would be to use some refinement arguments such as the notion of sequential equilibrium. For the specific economies with multiple equilibria that we consider, sequential equilibrium refinement is not applicable.

We are more interested in issues such as what kind of intergenerational mobility, wage growth and total factor productivity growth the model generates during the transition to a stationary equilibrium, and in the stationary state? Whether there exists multiple equilibria; whether the economy is generating maximum potential growth? Answers to these questions are mostly contained in the transition matrix P_t.

2.5 A few concepts

Given an equilibrium path of mobility matrices $\{P_t\}$, the income distributions $\{\pi_t\}$, the equilibrium wage schedules $\{w_t(.)\}$, and the flow of spillover scientific knowledge $\{R_t\}$, we define the **wage growth rate due to social mobility** between period t and $t+1$ by $\gamma_w = \sum_{i,j\in S}[(w_{t+1}(j) - w_t(i))/w_t(i)]P_t(i,j)\pi_t^i$, and **wage growth rate due to total factor productivity growth** between period t and $t+1$ by $\gamma(R_t)$. Notice that the average growth rate of earnings between period t and $t+1$ is the sum of γ_w and $\gamma(R_t)$.

It is possible to have different types of signaling equilibria. A **pure pooling signaling equilibrium** is a signaling equilibrium in which all types of agents from all economic backgrounds use the same signal, i.e., $\sigma_t(\tau_t, s_{t-1})$ is independent of τ_t and s_{t-1} for all $t \geq 1$. A **strict separating equilibrium** is a signaling equilibrium in which the agents of different talent type and family background use distinct schooling levels, and jobs i.e., $\sigma_t(\tau_t, s_{t-1}) = \sigma_t(\tau_t', s_{t-1}')$ or $\eta_t^*(\tau_t, s_{t-1}) = \eta_t^*(\tau_t', s_{t-1}')$ if and only if $\tau_t = \tau_t'$ and $s_{t-1} = s_{t-1}'$ for all $t \geq 1$. These are the kinds of equilibria generally studied in the game theory literature.

We define other kinds of equilibria relevant to our context. An **equal opportunity signaling equilibrium** is one in which $\sigma_t(\tau_t, s_{t-1}) = \sigma_t(\tau_t, s_{t-1}') \equiv \overline{s}_t(\tau_t)$ say, or $\eta_t^*(\tau_t, s_{t-1}) = \eta_t^*(\tau_t, s_{t-1}') \equiv \overline{\eta}_t(\tau_t)$ say $\forall s_{t-1}, s_{t-1}' \in \mathcal{S}$, i.e., all workers of the same talent type get the same education level or get the same job, and hence get paid the same wages, no matter what their family backgrounds are. An **equal opportunity separating equilibrium** is an equal opportunity equilibrium such that $\overline{s}_t(\tau) \neq \overline{s}_t(\tau')$ or $\overline{\eta}_t(\tau) \neq \overline{\eta}_t(\tau')$ if $\tau \neq \tau'$.

In our framework, the following result is straightforward.

Proposition 2 *It is impossible to have a strict separating equilibrium.*

In the next section we will consider implicit contracts and show that it is possible to get strictly separating equilibrium.

It is often difficult to compute the equilibrium path of an economic system, and we often like to study the properties of stationary equilibrium, which we define as follows:

Definition 2 *A* ***stationary signaling equilibrium*** *is a signaling equilibrium in which in every period* t*, aggregate flow of spillover knowledge* R_t*, subjective beliefs of the employers* $q_t(.)$ *, and hence the wage schedule,* $w_t(.)$ *, the transition probability matrix of the social groups,* P_t*, and the distribution of the social groups* π_t *are all independent of* t*.*

We denote these stationary variables without a time subscript. Notice that in a stationary equilibrium, we have $\pi = \pi P$, i.e., π is an *invariant probability distribution* with respect to P, and the stationary distribution of population over the social groups are the normalized non-negative eigenvectors of the stationary transition matrix P'.

There are some controversy regarding what is the best measure of mobility corresponding to a mobility matrix, P, see Conlisk [1990]. We do not use any of those criteria, instead we propose a criterion suitable in our framework: A **growth enhancing mobility measure** $\mu(P) \equiv R/R_{\max}$, where R is the flow of spillover knowledge and $R_{\max}$ is the maximum flow of spillover knowledge attainable in the economy out of all possible assignments of jobs and education levels to workers. Thus, this measure is bounded between 0 and 1, higher number represents higher growth enhancing mobility.

In a stationary equilibrium with a mobility matrix P and an invariant probability distribution π over $\mathcal{S}$, the distribution of income is stationary over time, and hence there is no wage growth due to mobility. The positive wage growth that has been reported in many empirical studies on geographical mobility, (see Jovanovich and Moffit [1990] and Sicherman and Galor [1990], among others) is a phenomenon along the transition path.

Note that not all economies will have stationary transition probabilities, nor all equilibria will converge to a stationary equilibrium. We will also examine the nature of equilibrium dynamics for various economies. We will assume in the rest of the paper except in section 4.2 that $\mathcal{H}$ is a singleton set, and drop η from the arguments of all the entities.

3 Nature of equilibrium mobility and growth

3.1 The basic economy

To make our points clear with least technicality, we consider this simpler economy for much of our analysis. Let $\mathcal{T} = \{1,2\}$, $S = \{1,2\}$. We assume that $a(s,\tau) = 1$ if $s = 2$ and $\tau = 2$, and

$a\,(s,\tau) = 0$ otherwise.

Whether there exists any signaling equilibrium, and if there exists one, whether there exist many equilibria some of which are Pareto superior, some of which are equal opportunity separating, some of which are growth maximizing separating, depend on the technology $e(\tau, s)$ and the cost function, $\theta(s_t, \tau_t, s_{t-1})$. We will illustrate our issues by fixing the following specification of the technology and assuming different forms for the cost function.

$$e(s,\tau) = \begin{cases} e_1 & \text{if } s = 1, \forall \tau \in \mathcal{T} \\ e_2 & \text{if } s = 2, \tau = 1 \\ e_3 & \text{if } s = 2, \tau = 2 \end{cases} \tag{11}$$

An interpretation of the above is that the workers with education level 1 are unskilled workers and the talent of the unskilled workers do not affect their productivity; however, higher educated talented workers have higher productivity than higher educated not-so-talented workers.

Assume that the cost function $\theta(s_t, \tau_t.s_{t-1})$ satisfies the following:

$$\left.\begin{array}{c} \theta(1, \tau_t, s_{t-1}) = 0 \; \forall \tau_t, s_{t-1}, \text{ and} \\ \theta(2,2,2) < \theta(2,1,2) < (e_2 - e_1) + p\,(e_3 - e_2) < \theta(2,2,1) < \theta(2,1,1) \end{array}\right] \tag{12}$$

Signaling equilibrium 1: Suppose the employers in period t hold the following subjective probability distribution $q_t\,(e|s)$)of productivity level e given his schooling level s, which in matrix form is given by

$$[q_t\,(e|s)]_{\substack{e=e_1,e_2,e_3 \\ s=1,2}} = \begin{bmatrix} 1 & 0 \\ 0 & 1-p \\ 0 & p \end{bmatrix}$$

According to (4), given the above expectations, the employer announces the following wage schedule:

$$w_t(s_t) = \begin{cases} 1 & \text{if } s_t = 1 \\ e_2.(1-p) + e_3.p & \text{if } s_t = 2 \end{cases} \quad \text{for all } t \geq 0$$

Given the above wage schedule, one can easily verify that the equilibrium schooling decisions $\sigma_t(\tau_t, s_{t-1})$ of an agent of talent type τ_t from the family background s_{t-1} is as follows:

$$\sigma_t(\tau_t, s_{t-1}) = \begin{cases} 1 & \forall \tau_t \in \mathcal{T} \; if \; s_{t-1} = 1 \\ 2 & \forall \tau_t \in \mathcal{T} \; if \; s_{t-1} = 2 \end{cases} \quad \text{for all } t \geq 0$$

It can be easily checked that given the above optimum solution, the observed conditional probability distribution of e given s_t will coincide with the anticipated one. Note that the the transition matrix associated with $\sigma_t(.)$ is the following:

$$P_t = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \forall t \geq 0$$

Thus in this economy there is no intergenerational mobility. Furthermore, the economy is in steady-state from the beginning. Thus, $R_t = p \cdot \pi_0^2$, and hence the productivity growth rate is given by $\gamma(p\pi_0^2)$ which is strictly less than $\gamma(p)$, the maximum attainable productivity growth rate for the economy when all talented individuals from all socio-economic groups obtain higher education.

This equilibrium is not equal opportunity separating, nor growth enhancing separating type. In this equilibrium, all talent types of the children are pooled within each type of family background of their parents.

Could there be any other equilibrium for the above economy? For a certain subclass of the above economies, there is another equilibrium, which is growth enhancing separating and is Pareto superior to the above equilibrium. To see this, consider the following:

Signaling equilibrium 2: let $v_t \equiv \frac{p}{p\pi_{t-1}^1 + \pi_{t-1}^2}$. Note that $v_t > p\, \forall t \geq 1$. At $t = 1$, v_1 is known. Let us suppose that apart from the assumption (11), the cost function also satisfies the condition:

$$\theta(2, 2, 1) < (e_2 - e_1) + v_1 (e_3 - e_2) < \theta(2, 1, 1)$$

Suppose the employer holds the following subjective probability distribution for the productivity type E_t given S_t:

$$\overline{q}_t (e|s) = \begin{bmatrix} 1 & 0 \\ 0 & 1 - v_t \\ 0 & v_t \end{bmatrix} \quad \text{for all } t \geq 1 \tag{13}$$

According to (4), given above expectations, the employer announces the following wage schedule:

$$\overline{w}(s_t) = \begin{cases} 1 & \text{if } s_t = 1 \\ e_2.(1 - v_t) + e_3.v_t & \text{if } s_t = 2 \end{cases}$$

Given the above wage schedule, the original $\sigma_t(\tau_t, s_{t-1})$ will be optimal for all (τ_t, s_{t-1}) except for $\tau_t = 2$, $s_{t-1} = 1$, who will choose $s_t = 2$. It can be easily checked that for this optimal solution, the observed conditional probability distribution of e_t given s_t will coincide with the anticipated one in equation (13). Note that the transition matrix associated with this new optimal schooling decision $\overline{s}_t^*(.)$ is as follows:

$$\overline{P}_t = \begin{pmatrix} 1 - p & p \\ 0 & 1 \end{pmatrix}$$

Thus in this economy there is intergenerational mobility. The proportion of population with higher education will go on increasing and the proportion of the population with lower education will go on decreasing. This process, however, cannot go on for ever, since in that case $v_t \to p$, as $t \to \infty$, which will mean that there will be some finite $t_0 > 1$ such that $v_{t_0} > \theta(2, 2, 1)$ for the first time and then on the equilibrium will switch on to the previous one with no mobility. Note, however that the new steady-state equilibrium growth rate will be $\gamma \left(\pi_{t_0}^2 \cdot p\right)$ since $\pi_{t_0}^2 > \pi_0^2$. Furthermore, the short-run growth rate up to period t_0, is higher in the second equilibrium than in the first type; and the second equilibrium is Pareto superior to the first.

Furthermore, notice that there will be a positive wage growth during all periods $t \leq t_0$, and after t_0, the source of growth is only from factor productivity growth.

Thus, in this economy there may exist multiple equilibria; which one will materialize depends on the expectations of the employers. The question is then, how the employer's expectations are formed? We need a theory of expectations formation of the producers to select an equilibrium, and we do not pursue this theory here.

Also note that the economy signaling equilibrium 1, will be in stationary state from time $t = 1$; whereas in signaling equilibrium 2, there will be upward mobility from social class $s = 1$ to $s = 2$ up to time $t = t_0$ according to the transition matrix $\overline{P}_t$, and during this period, there will be a positive wage growth due to upward mobility; after t_0, however, the process will revert to the mobility pattern of case 1. Under these two scenarios, however, the economy will have two different long-run income distributions.

3.2 An economy with more signals

Suppose $\mathcal{T} = \{1, 2\}$ and $\mathcal{S} = \{1, 2, 3\}$, and $a(s, \tau) = 1$ if $s = 2$ and $\tau = 2$, and $a(s, \tau) = 0$ otherwise. Let the productivity function be given by

$$[\, e(s, \tau)]_{\substack{s=1,2,3 \\ \tau=1,2}} = \begin{bmatrix} 1.0 & 1.0 \\ 2.0 & 3.0 \\ 3.5 & 4.0 \end{bmatrix}$$

Let the cost of education be as follows:

$$\theta(s_t, \tau_t, s_{t-1}) = \begin{cases} \frac{\rho \cdot s_t - 1}{\tau_t s_{t-1}} - .76 & \text{if } s_t = 2, \tau_t = 2, s_{t-1} = 1 \\ \frac{\rho \cdot s_t - 1}{\tau_t s_{t-1}} - .634 & \text{if } s_t = 2, \tau_t = 1, s_{t-1} = 3 \\ \frac{\rho \cdot s_t - 1}{\tau_t s_{t-1}} - .08 & \text{if } s_t = 2, \tau_t = 2, s_{t-1} = 3 \\ \frac{\rho \cdot s_t - 1}{\tau_t s_{t-1}} + .20 & \text{if } s_t = 3, \tau_t = 1, s_{t-1} = 3 \\ \frac{\rho \cdot s_t - 1}{\tau_t s_{t-1}} & \text{otherwise} \end{cases}$$

where $\rho = 3.5$.

Suppose the employers anticipate the following probabilities for productivity type e_t given the education level s_t of a randomly selected worker:

$$q_t\left(e_t | s_t\right) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{(1-p)\pi_{t-1}^3}{p\pi_{t-1}^1 + (1-p)\pi_{t-1}^3} & 0 \\ 0 & \frac{p\pi_{t-1}^1}{p\pi_{t-1}^1 + (1-p)\pi_{t-1}^3} & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \text{ for all } t \geq 1$$

Given his anticipated conditional probability distribution, he announces the following wage schedule:

$$w_t(s_t) = \begin{cases} 1 & \text{when } s_t = 1 \\ 2\frac{p\pi_{t-1}^1}{p\pi_{t-1}^1+(1-p)\pi_{t-1}^3} + 3\frac{(1-p)\pi_{t-1}^3}{p\pi_{t-1}^1+(1-p)\pi_{t-1}^3} & \text{when } s_t = 2 \\ 4 & \text{when } s_t = 3 \end{cases}$$

It can be easily checked that the optimal schooling decisions $\sigma_t(\tau_t, s_{t-1})$ of an agent (τ_t, s_{t-1}) is the following:

τ	s_{t-1}	$s_t^*(\tau_t, s_{t-1})$
1	1	1
2	1	2
1	2	1
2	2	3
1	3	2
2	3	3

(14)

It can be easily checked that the transition matrix associated with the above optimal decision, $\sigma_t(\tau_t, s_{t-1})$ is the following:

$$P_t = \begin{pmatrix} 1-p & p & 0 \\ 1-p & 0 & p \\ 0 & 1-p & p \end{pmatrix}$$

Given a transition probability matrix over $\mathcal{S}$, a schooling level (more generally a state variable) $s \in \mathcal{S}$ is said to **communicate** with another schooling level $s' \in \mathcal{S}$, if with a positive probability the system can transit from schooling level s to s' in some finite period. Notice that for the above transition probability matrix, every schooling level communicates with every other schooling level in $\mathcal{S}$. Hence, by a well-known theorem of Markov chains, we know that there exists a unique invariant distribution π over $\mathcal{S}$, to which the system converges to, starting from any initial distribution. This invariant distribution is given by the normalized eigenvector corresponding to the eigen value 1, we computed it to be $\pi = \left(\frac{(1-p)^2}{1-p+p^2}, \frac{p(1-p)}{1-p+p^2}, \frac{p^2}{1-p+p^2}\right)$.

Value of our mobility measure $\mu(P) = \frac{p}{1-p+p^2}$, which is less than 1. Notice that while the productivity growth rate $\gamma(R_t)$ and the wage growth rate γ_w in period t depend on the initial distribution, π_0, however, the long-run growth rates are independent of initial condition, and are given by $\gamma\left(\frac{p^2}{1-p+p^2}\right) < \gamma(p)$, the maximum attainable long-run productivity growth rate and $\gamma_w = 0$.

We can modify the above economy to generate equilibria with higher mobility leading to higher growth. It is possible to have multiple long-run equilibria, and the convergence to a particular equilibrium will depend on the initial distribution π_0 as in the previous example.

Example 1 *Numerical example*

Let us illustrate some of the above facts more clearly with this numerical example. Let us take $p = .27$, the schooling cost function as in table 1 below, and the rest of the parameters are as in model in section 3.2.

Table 1: The schooling cost of students by social background and ability, $\Theta(s_t, \tau_t, s_{t-1})$

	(τ_t, s_{t-1})					
s_t	(1,1)	(2,1)	(1,2)	(2,2)	(1,3)	(2,3)
1	0	0	0	0	0	0
2	3.5	0.99	1.75	0.875	0.533	0.503
3	7.0	3.5	3.5	1.750	2.533	1.167

The stationary signaling equilibrium quantities are then as follows:

$$[q(e_t|s_t)] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0.27 & 0 \\ 0 & 0.73 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

The stationary transition probability matrix $P = [p(s_{t+1} = j|s_t = i]_{i,j\in\mathcal{S}}$ over the social groups and the long-run equilibrium wage vector $w = [w(s)]_{s=1,2,3}$ corresponding to invariant income distribution $\pi = [\pi(s)]_{s=1,2,3}$, the flow of spillover knowledge R, and the total factor productivity growth rate $\gamma(R) = R^{\mu}$, $\mu = 1.5$ are as follows:

$$P = \begin{bmatrix} 0.73 & 0.27 & 0 \\ 0.73 & 0 & 0.27 \\ 0 & 0.73 & 0.27 \end{bmatrix}, \quad w = \begin{bmatrix} 1 \\ 2.73 \\ 4 \end{bmatrix}, \pi = \begin{bmatrix} 0.6637 \\ 0.2455 \\ 0.0908 \end{bmatrix} \quad \begin{matrix} R = 0.0908 \\ \gamma(R) = 2.74\% \end{matrix}$$

Remark 1 *:* We can show that even when the cost of education does not vary with one's family background, if the producer conditions his subjective expectations about a worker's ability on his family background (such as using the last name to find out if one is coming from such and such families with such and such ethnic background), then there are equilibria in which the children of the poorer family background will not invest in higher education, and thus mobility will be reduced and so will be economic growth.

4 Labor Market Practices

We have seen in the previous section that in our model we will always have pooling equilibrium, which restricts mobility and wage growth, and hence reduces growth. The main reason for the impossibility of having a strictly separated equilibrium is that there are $\hat{s} \cdot \hat{\tau}$ types of agents but there are only $\hat{s}$ signal classes, and the fact that employers are allowed only to use one-time non-renegotiable wage contracts during the tenure of the employment. Relaxing this assumption, and allowing other kinds of implicit or explicit contracts in which workers may be laid-off or demoted if his productivity is seen to be low, or the worker may be promoted or else he quits if he is found to be more productive worker after working with the firm for a while.

Suppose we allow those labor market practices. Since workers anticipate those practices, some workers with low talents in our earlier set-up who found a higher education more lucrative because he was pooled with high talent workers during his whole employment duration may find it is not lucrative any more because of the on the job screening. He will self-select not to have higher education. Thus the possibility of on the screening may lead to further separation. In the next two subsections, we consider our basic economy of section 3.1 and allow implicit contracts involving wage contracts as gambles, and allow more than one type of employers, and we then examine to what extent these labor market features can lead to separability of signaling equilibrium.

4.1 Quits, layoffs, and promotion

We assume that the productivity function of the worker $e(.)$ does not contain the random shock term ϵ and further assume that the production technology of the economy is such that $e(s,\tau) \neq e(s,\tau')$ whenever $\tau \neq \tau'$ for all s, τ, τ'. In that case, the employer can fully predict the talent type of the worker by observing his output. The employer would like to lay-off the low productive worker or demote him, and encourage the more productive workers to stay with higher pays; both types of workers may quit, however. The problem is that often the employer cannot observe the productivity level of the worker. He might, however, observe another random variable $Y(s,\tau)$ which he uses to predict the productivity level of the worker with school level s and ability τ. We can do the analysis using $Y(s,\tau)$. But again to simplify our point, we assume that $Y(s,\tau) \equiv e(s,\tau)$, i.e., the employer can observe productivity.

To proceed more formally, we assume that time period t is divided into two sub periods t and $t.5$. and that total wages during a period will be paid in two installments. We consider the following kind of sorting wage contracts:

> If the worker's schooling level is s then he will be paid w_0 in the first subperiod and during the second subperiod will be paid a gamble, $w_{t.5}$, which will take value w_* if his realized productivity level is less than $c > 0$, a constant (i.e., he is not promoted) and w^* if his realized output is larger than c (i.e., he is promoted).

Thus under this contract, a worker receives $w(s) = w_0 + Ew_{t.5}$. Notice that these kind of contracts also include the simpler contracts of the previous section, where $w_0 = w(s)/2$ and $w_{t.5} = w(s)/2$ with probability 1. What kind of labor contracts can evolve in the competitive markets, and could it lead to ability separating and growth enhanced separating equilibrium? As the following proposition shows, it depends on the nature of the schooling cost function, and the nature of the contracts and the value of c.

Let us consider a specific sorting wage contract w^*as follows:

> If the worker's schooling level is $s = 1$, he will be paid $e_1/2$ in both subperiods; if his schooling level is $s = 2$, he will receive $e_2/2$ in the first subperiod and in the second subperiod $e_2/2$ if his realized productivity level is $\tau = 1$ (i.e., if he is not promoted) and he will receive $e_3/2$, which is his true productivity, and also a bonus $(e_3 - e_1)/2$, if his realized productivity level is $\tau = 2$ (i.e., if he is promoted).
>
> Let us consider the following class of economies:

$$\begin{aligned} &\theta(2,2,1) < (e_3 - e_1) < \theta(2,1,1) \ldots.(a) \\ &\theta(2,2,2) < (e_2 - e_1) < \theta(2,1,2) \ldots(b) \end{aligned} \tag{15}$$

Proposition 3 *For economies satisfying conditions (a) in (15), and (12), the sorting wage contract w^* above is a signaling equilibrium wage contract. If, furthermore, conditions (b) in (15) are satisfied then the resulting equilibrium is equal opportunity ability separating.*

There could be other type of contracts which can produce the same result as in the above proposition, they are basically equivalent to the above contract w^* in the sense that no other contract can pay higher amount while maintaining the zero profit condition on the employers. The situation gets more complicated and interesting when we allow randomness in the productivity function. This aspect will be incorporated in future work.

Please notice that the class of economies satisfying conditions in the above proposition is a subclass of economies in section 3.1. So we also have other two equilibria of section 3.1. But if employers are competitive and allowed to announce above type of contracts, then perfect competition will rule out the pooling equilibrium of section 3.1.

4.2 Labor Market Signaling and Job Matching

We now like to investigate if the presence of various jobs in which productivity level varies for given ability and education levels, can help to achieve further separation. Furthermore, we would like to investigate if market signaling can help in matching workers to jobs, an aspect of job matching that is mostly ignored in the job matching literature.

The basic structure of our model is very similar to that of Jovanovich [1979]; the main essential difference between his model and the present model is that we assume signaling based matching, whereas Jovanovich assumes a random matching in a search theoretic framework. To keep our analysis to simplest form, we consider an economy with two productive sectors: $\eta = 1$ and $\eta = 2$. Both sectors employ skilled labor and unskilled labor. Assume that the sector $\eta = 2$ is more research intensive and sector $\eta = 1$ is less research intensive. We assume the flow of spillover knowledge depends on job as follows: $a(s, \tau, \eta) = 1$ if $s = 2$, $\tau = 2$, and $\eta = 2$, and $a(s, \tau, \eta) = 0$ otherwise. The following table summarizes the match specific productivities to various levels of schooling s and talent τ combinations.

The productivity function $e(s, \tau, \eta)$				
	$s = 1$		$s = 2$	
	$\tau = 1$	$\tau = 2$	$\tau = 1$	$\tau = 2$
$\eta = 1$	e_1	e_1	e_2	e_3
$\eta = 2$	e_1	e_1	$e_2 - \epsilon$	$e_3 + \frac{1-p}{p} \cdot \epsilon$

Notice that two signaling equilibria of section 3.1 are still available, and in these two equilibria the market is not generating incentive structure for best matching of workers to jobs. However, if the employers use on-the-job screening device as in the previous subsection, then the economy generates a better matching of workers to job. In fact, one can attain a growth-enhancing ability separating pooling equilibrium. To see this, consider the following wage contracts:

> For each employer $\eta = 1$ and $\eta = 2$, the wage contract is of the same form as in the previous subsection, with the exception that in place of e_2 and e_3 of the previous contract, we use the corresponding productivity values from the above table for η.

Proposition 4 *For the class of economies satisfying conditions (12), (a) in (15), and* $e_3 - e_1 > \theta(2, 1, 2)$*, job matching is achieved in the market through education signaling and self-selection, and it is more separated than the equilibrium in the previous proposition. In equilibrium nobody gets fired nor anybody quits.*

Please notice that we are not able to obtain strict separation in the above proposition because we assumed for $s = 1$ that $e(s, \tau, \eta) = e_1$ for all τ and η. Suppose instead there is an employer for whom $e(s, \tau, \eta)$ varies with τ, then we can find conditions on cost of schooling which will lead to strict separability.

Notice, however, that strict separability, or even growth enhancing separability are attained only when schooling costs satisfy the conditions in the above propositions. But there are economies which cannot achieve strict separability or even growth maximizing separability or an equal opportunity separability when talented children from poorer family background have higher costs than what the above propositions imply. Notice also that it is not possible for private employers to subsidize such socially disadvantaged children and tie them to work for the company after completing school, unless the company is subsidized by the government (because the net profit from such wage contract is always negative in equilibrium).[11] This problem can be handled only in the school system, which we turn next.

5 Education Systems: purely public, or private-public and school voucher

What should be the right kind of education policy that can lead to higher mobility and economic growth depends on the school system. There are various types of education systems adopted by various countries: in some systems, schools are completely public, for instance, in Korea; most countries have dual system with both public and private schools, for instance, in India and the US. Generally the public schools are free but their qualities depend a great deal on the budget it receives from the government, which in turn depends, in some countries such as the US, on the pool of property taxes raised in the neighborhood in which the school is located. Since children living in a particular neighborhood can only attain its public school, we can see immediately that children from poorer family background face higher cost of schooling. There are many other reasons for higher cost, we do not discuss them here.

If one of the objectives of an education system is to attain higher social mobility and higher income growth, from our analysis so far, it is clear that the talented children must get higher education and get placed in the growth enhancing sectors. Many times competitive labor market may not be able to produce enough economic incentives for the children of the poorer family background to achieve higher education. While higher education for all might be an important social goal by itself, but given resource constraints, a society might focus on identifying the more talented individuals of poorer family background, and give them only the required subsidy so that they can attain higher education and work in the growth enhancing technical sectors. Poorer the family background, the higher is the subsidy needed. Where would the money come from for such subsidies? Recall that these talented individuals can create higher income for the future generation by creating a positive knowledge; moreover, assuming that productivity function $e(s,\tau,\eta)$ represents productivity net of

[11] These kind of tied transfers are generally observed in army recruiting and rarely in private sector if there are no tax advantages to such subsidies.

pay-roll taxes, with higher mobility, there is higher income growth during the transition to long-run equilibrium, and with progressive or even with proportionate taxes on earnings, the government should be able to raise the required subsidy money. But all children look alike, and they do not wear a tag indicating their talent level, how could the talented ones be identified?

I argue that this could be done with school voucher system. To see it how, assume that only the private schools provide higher quality education and the public schools provide lower quality education. Any one who can afford can attain a private school; however, whether one will pass the curriculum of a private school, depends on the innate ability and family background of the student. These effects are, however, already reflected in the cost differences. If the voucher for a child of particular family background is such that the voucher reduces the cost of the talented child just enough so that he can attain the higher quality education, then the students or their parents will self-select to receive voucher only for talented children and not so talented ones would rather self-select to go to public school system. Alternatively, government can use the subsidy money to improve the quality of the public schools attained by the children of poorer family backgrounds, so much so that the public schools are of the same quality as the private school. This can also lead to higher social mobility and growth. What kind is a better education policy is an area of constant academic research and focus of continuing political debates in the US congress and elsewhere.

To get a better idea, consider the economy of section 3.2. Suppose the government announces a voucher of $\$0.50$ to a child of family background 1, which must be used for the sole purpose of attaining school level 3. This policy will move the economy to another stationary equilibrium in which the equilibrium subjective probability matrix expressing employer's beliefs, $q\left(e|s\right)$, the mobility matrix of the three income groups P, the associated invariant distribution of income π, and the income levels $w(s)$, $s=1,2,3$ are as follows:

$$q\left(e|s\right)=\begin{bmatrix}1&0&0\\0&1&0\\0&0&0\\0&0&0\\0&0&1\end{bmatrix}\quad P=\begin{bmatrix}1-p&0&p\\1-p&0&p\\0&1-p&p\end{bmatrix}\quad \pi'=\begin{bmatrix}(1-p)^2\\p(1-p)\\p\end{bmatrix}\quad w=\begin{bmatrix}1\\2\\4\end{bmatrix}$$

It is also clear that in this case our measure of mobility $\mu\left(P\right)=1$, and the growth rate is the highest possible. To get a better insight, let us consider the numerical example. For this specific economy, a subsidy of 0.50 per individual will achieve the goal. Total cost to subsidize all the talented individuals of the old stationary equilibrium is $\$0.059$; but it will improve the total factor productivity growth rate from 2.74% to 14.02%. Thus it might be worthwhile. Also notice that income distribution will become more egalitarian in the long run, which is given by $\pi'=(.5329,.1971,.27)$ as compared to the earlier one $\pi=\left(0.6637,0.24551,0.0908\right)$.

It should be noted that a highly regulated public education system with competitive bidding for entry into a limited number of higher education slots, can also achieve the same objective of growth and social mobility. This system was introduced by Park regime in South Korea and is believed to have attained a high degree of economic growth and social mobility. There are many other dimensions to the political and academic debate on educational policies, see for instance, Stiglitz [1975] in an imperfect information context, and Becker [1962] in a perfect information context.

6 Parental altruism, investment in human capital and mobility

So far, we assumed that cost of schooling depends on one's family background which we assumed to represent children's learning environment; individuals had no control over it. This assumption that the "destiny" of children are fixed by birth (as in the "Indian caste system") might seem very restrictive. Parents care about their children's welfare. Thus, they may like to incur pre-school human capital investment so that their children have better opportunities for learning. Would this change the basic nature of the equilibrium we studied in section 3? We examine these issues here.

We assume that the cost of producing signal s_t for an individual of talent type τ_t in period t depends on the level of parental pre-school investment h_t which is now decided by parents. We further assume that a pre-school investment of h_t on an adult of period t costs his parents $A_t h_t$ amount of t-th period resources; we assume as in the previous section that the cost of obtaining signal s_t for an adult with talent level τ_t and pre-school investment h_t is proportional to the t-th period productivity level as follows:

$$\theta_t(s_t, \tau_t, h_t) = A_t.\theta(s_t, \tau_t, h_t) \tag{16}$$

In this section, we identify an adult agent in period t by his pre-school investment h_t and talent level τ_t and **denote the agent by** (τ_t, h_t)**.** As in the previous section, the employers in period t, $t \geq 1$, form their subjective beliefs regarding the relationship between schooling level and the productivity level of workers who are in the job market, and announce a competitive wage schedule, $A_t w_t(s_t)$. A worker (τ_t, h_t) in period t takes this wage schedule $w_t(s_t)$ and his schooling cost function (16) as given and decides his own schooling level s_t, and the pre-school investment h_{t+1}on his child. We assume that the parents do not observe their children's ability when making the pre-school investment decisions, and thus h_{t+1} is not a function of his child's ability. The budget constraint of the agent (τ_t, h_t) is given by:

$$c_t = A_t\left[w_t(s_t) - \theta(s_t, \tau_t, h_t)\right] - A_{t+1}h_{t+1} \geq 0 \tag{17}$$

We assume that agent (τ_t, h_t)'s decisions are guided by the following Von Neumann-Morgenstern expected altruistic utility function:

$$E_{\tau_{t+11},\tau_{t+2},...}U_t\left(c_t, c_{t+1}...\right) = E_{\tau_{t+11},\tau_{t+2},...}\sum_{\iota=0}^{\infty}\beta^i u\left(c_{t+i}\right) \tag{18}$$

To keep our exposition simple, we assume that there are finite number of human capital investments choices from the set $\Xi = \left\{h^1, h^2, ..., h^{\widehat{h}}\right\}$. Let us denote the optimal schooling decision and optimal pre-school investment decision functions of agent (τ_t, h_t) by $\sigma_t(\tau_t, h_t)$ and $\psi_t(\tau_t, h_t)$ respectively. The corresponding decisions in binary form $\chi^{\sigma_t}\left(s_t, \tau_t, h_t\right)$ and $\chi^{\psi_t}\left(s_t, \tau_t, h_t\right)$ are defined respectively by

$$\chi^{\sigma_t}\left(s_t, \tau_t, h_t\right) = \begin{cases} 1 & \text{if } s_t = \sigma_t\left(\tau_t, h_t\right) \\ 0 & \text{otherwise} \end{cases}$$

and

$$\chi^{\psi_t}\left(h_{t+1}, \tau_t, h_t\right) = \begin{cases} 1 & \text{if } h_{t+1} = \psi_t\left(\tau_t, h_t\right) \\ 0 & \text{otherwise} \end{cases}$$

Let us denote by ξ_t the probability distribution of the agents (τ_t, h_t) in period t, i.e., $\xi_t\left(\tau, h\right) \equiv P\left\{(\tau_t, h_t) = (\tau, h)\right\}$, is the proportion of the adult population that belong to the group $(\tau_t, h_t) = (\tau, h)$. We assume that the initial distribution ξ_0 is given. Given the distribution ξ_t of agents (τ_t, h_t) in period t, $t \geq 1$, and given above optimal schooling and pre-school investment decisions $\chi^{\sigma_t}\left(s_t, \tau_t, h_t\right)$ and $\chi^{\psi_t}\left(h_{t+1}, \tau_t, h_t\right)$ in binary form for all agents (τ_t, h_t), we get the flow of spillover research knowledge R_t and distribution of agents $\xi_{t+1}(\tau_{t+1}, h_{t+1})$ for the next period as follows:

$$R_t = \sum_{s_t, \tau_t, h_t} a\left(s_t, \tau_t\right)\chi^{\sigma_t}\left(s_t, \tau_t, h_t\right)\xi_t\left(\tau_t, h_t\right) \tag{19}$$

$$\xi_{t+1}\left(\tau_{t+1}, h_{t+1}\right) = g\left(\tau_{t+1}\right)\sum_{\tau_t, h_t}\chi^{\psi_t}\left(h_{t+1}, \tau_t, h_t\right)\xi_t\left(\tau_t, h_t\right) \tag{20}$$

The economy moves over time to produce **a feasible path** as follows: The initial distribution of agents, ξ_1 over $\mathcal{T} \times \Xi$ is given; the employers announce a wage schedule $w_1\left(s_1\right)$; each agent $(\tau_1, h_1) \in \mathcal{T} \times \Xi$ chooses an own schooling rule $s_1 = \sigma_1\left(\tau_1, h_1\right)$, and a pre-school investment on his children decision rule $h_2 = \psi_1\left(\tau_1, h_1\right)$, such that he has non-negative consumption given in equation (17). determines his consumption, and the binary variables $\chi^{\sigma_1}\left(s_1, \tau_1, h_1\right)$ and $\chi^{\psi_1}\left(h_2, \tau_1, h_1\right)$ that are associated respectively with the decisions $\sigma_1(.)$ and $\psi_1(.)$ are determined. The spillover knowledge R_1 in period $t = 1$ and the distributions of agents for the next period $\xi_2\left(\tau_2, h_2\right)$ are determined from equations (19) and (20); this process iterates over time to get all the future quantities.

The concept of signaling equilibrium in the previous section could be modified as follows:

Definition 3 *A **signaling equilibrium with endogenous pre-school investment** is a sequence of feasible wage schedule, schooling and pre-school investment decision functions* $\{w_t(.), \sigma_t(.,.), \psi_t(.,.)\}$ *and an initial distribution of population* $\xi_1(\tau_1, h_1)$ *such that for all* $t \geq 1$*,for all agents* (τ_t, h_t) $\in \mathcal{T} \times \Xi$*, and given the wage schedule* $w_t(s_t)$*,*

(a) $\sigma_t(\tau_t, h_t)$ *is his optimal schooling function,*

(b) $\psi_t(\tau_t, h_t)$ *is his optimal pre-school investment function, and*

(c) the wage schedule $w_t(s_t)$ *satisfies the following self-fulfilling expectations condition:*

$$w_t(s_t) = \frac{\sum_{\tau_t, h_t} e(s_t, \tau_t)\, \chi^{\sigma_t}(s_t, \tau_t, h_t)\, \xi_t(\tau_t, h_t)}{\sum_{\tau_t, h_t} \chi^{\sigma_t}(s_t, \tau_t, h_t)\, \xi_t(\tau_t, h_t)}$$

for all $s_t \in \mathcal{S}$ *which are chosen by some agent* (τ_t, h_t)*.*

Given the optimal schooling decision $\sigma_t(\tau_t, h_t)$ and the population distribution $\xi_t(\tau_t, h_t)$ we can derive the distribution of social status s_t over $\mathcal{S}$ in period t for all $t \geq 1$ from

$$\pi_t^{s_t} = \sum_{\tau_t, h_t} \chi^{\sigma_t}(s_t, \tau_t, h_t)\, \xi_t(\tau_t, h_t), \quad s_t \in \mathcal{S} \tag{21}$$

and the transition function $P_t = [p_t(s_t, s_{t+1})]_{s_t, s_{t+1} \in \mathcal{S}}$ over $\mathcal{S}$ from

$$p_t(s_t, s_{t+1}) = \frac{\sum_{\tau_{t+1}, \tau_t, h_t} g(\tau_{t+1}) \cdot \chi^{\sigma_{t+1}}(s_{t+1}, \tau_{t+1}, \psi_t(\tau_t, h_t)) \cdot \chi^{\sigma_t}(s_t, \tau_t, h_t)\, \xi(\tau_t, h_t)}{\sum_{\tau_t, h_t} \chi^{\sigma_t}(s_t, \tau_t, h_t)\, \xi(\tau_t, h_t)} \tag{22}$$

Notice that since $w_t(.)$ is a one-one function of s_t, the above two also define the earnings distribution for each generation, and the transition matrix of intergenerational earnings mobility.

The basic question is then: How do we compute a signaling equilibrium as defined in definition 3? Could we use the recursive structure or a Markovian structure of the dynamic programming? This framework may not, however, produce the recursive structure generally used in standard neoclassical stochastic growth models . To see this notice that since there are generally multiple signaling equilibrium wage schedules in every period that we have seen in the previous section, the equilibrium wage schedule $w_t()$ may differ in two periods for the economy with everything else same if the employers use different equilibrium conditional probabilities $q(.)$ in two periods. We rule it out by assuming that employers hold the same conditional expectations and thus announce the same wage schedule when the economy consist of the same distributions of workers and their pre-school investment. Let us denote the maximized value of the von Neumann Morgenstern expected altruistic function of agent (τ_t, h_t) by $V_t(\tau_t, h_t)$. Under above assumptions, we can solve agent (τ_t, h_t)'s optimal decision problem using the following functional equation of a dynamic programming problem:

$$\tilde{V}_t(\tau_t, h_t) = \max_{h_{t+1}} \left[u\left(A_t \widehat{w}_t(h_t, \tau_t) - A_{t+1} h_{t+1}\right) + \beta E_{\tau_{t+1}} \tilde{V}_{t+1}(\tau_{t+1}, h_{t+1}) \right] \tag{23}$$

where,

$$\widehat{w}_t(\tau_t, h_t) \equiv w_t(\sigma_t(\tau_t, h_t)) - \theta(s_t(\tau_t, h_t), \tau_t, h_t)$$

We further assume that the instantaneous utility function u satisfies the property that $u(x.y) = u(x).u(y)$. Let us use a transformation $V_t(h_t, \tau_t) = \tilde{V}_t(h_t, \tau_t) / u(A_t)$.Then the problem (23) becomes

$$\begin{aligned} V_t(h_t, \tau_t) &= \max_{h_{t+1} \in \Xi} u\left(\widehat{w}_t(h_t, \tau_t) - \frac{A_{t+1}}{A_t} h_{t+1}\right) + \beta E_{\tau_{t+1}} V_{t+1}(h_{t+1}, \tau_{t+1}) \frac{u(A_{t+1})}{u(A_t)} \\ &= \max_{h_{t+1} \in \Xi} u\left(\widehat{w}_t(h_t, \tau_t) - (1 + \gamma(R_t)) h_{t+1}\right) + \beta u(1 + \gamma(R_t)) E_{\tau_{t+1}} V_{t+1}(h_{t+1}, \tau_{t+1}) \end{aligned} \tag{24}$$

Definition 4 *A* ***Markov perfect stationary signaling equilibrium*** *is a wage schedule $w(s), s \in \mathcal{S}$, the optimal schooling decision function $\sigma(\tau, h)$, the optimal policy function $h_+ = \psi(\tau, h)$ acting as optimal pre-school investment function, a probability distribution ξ over $\mathcal{T} \times \Xi$, and the flow of spillover knowledge $R > 0$, satisfy the following system of equations, (25)-(30):*

$$w(s) = \sum_{e \in E} e \cdot q(e|s), \tag{25}$$

for some subjective beliefs $q(e|s)$ held by the employers,

$$\sigma(\tau, h) = \arg\max_{s \in S} \left[w(s) - \theta(s, \tau, h) \right], \tag{26}$$

$$R = \sum_{s, \tau h} a(s, \tau) \chi^{\sigma}(s, \tau, h) \xi(\tau, h) \tag{27}$$

the function $h_+ = \psi(\tau, h)$ is the optimal policy function associated with the following functional equation or the Bellman equation of the dynamic programming:

$$V(\tau, h) = \max_{h_+} \left[u\left(\widehat{w}(\tau, h) - (1 + \gamma(R)) h_+\right) + \beta u(1 + \gamma(R)) E_{\tau_+} V(\tau_+, h_+) \right] \tag{28}$$

where, $\widehat{w}_t(\tau, h) \equiv w(\sigma(\tau, h)) - \theta(\sigma(\tau, h), \tau, h)$,

$$w(s) = \frac{\sum_{\tau, h} e(s, \tau) \chi^{\sigma}(s, \tau, h) \xi(\tau, h)}{\sum_{\tau, h} \chi^{\sigma}(s, \tau, h) \xi(\tau, h)} \tag{29}$$

and $\xi(,.,)$ is the invariant distribution of the transition matrix over $\mathcal{T} \times \Xi$ as follows:

$$\xi(\tau_+, h_+) = g(\tau_+) \sum_{\tau \in \mathcal{T}, h \in \Xi} \chi^{\psi}(h_+, \tau, h)) \, \xi(\tau, h) \tag{30}$$

where $\chi^{h}(h_+, \tau, h)$ is a binary variable taking value 1 if $h_+ = \psi(\tau, h)$ and taking value 0 otherwise.

We can use techniques from Stokey and Lucas [1990] to analyze the above functional equation and hence the nature of the stationary equilibrium for continuous state space. The following general result can be proved:

Proposition 5 *The value function $V(\tau, h)$, the optimal schooling function $\sigma(\tau, h)$ and the optimal pre-school investment function $h_+ = \psi(\tau, h)$ of agent (τ, h) are all increasing functions of τ and h, for all τ, and h.*

It is not trivial even to compute a Markov perfect stationary equilibrium. We can, however, use linear programming techniques to verify if a given wage schedule $w(s)$, schooling and pre-school investment functions $\sigma(\tau, h)$ and $\psi(\tau, h)$ constitute a Markov perfect stationary signaling equilibrium. We illuminate the effect that endogeneity of pre-school investment has on the nature of growth and social mobility with the basic economy that we studied in section 3.1. We turn to it next.

6.1 The basic economy (continued)

We assume the cost function $\theta(s_t, \tau_t, h_t)$ to be the same as in equation (12), with the understanding that h_t here plays the same role as s_{t-1} there, and the same productivity function. It is obvious that the optimal schooling function will be then

$$[\sigma_t(\tau_t, h_t)]_{\substack{\tau_t = 1,2 \\ h_t = h_1, h_2}} = \begin{bmatrix} 1 & 2 \\ 1 & 2 \end{bmatrix} \tag{31}$$

In section 3.1 we saw that those economies did not exhibit social mobility in the stationary equilibrium. We want to examine if that is also the case in the present set-up with endogenous pre-school investment. To that end, we have the following:

Proposition 6 *Suppose the felicity index function is linear of the form $u(c) = c$, i.e., agents are risk neutral. Suppose the schooling cost function $\theta(s, \tau, h)$ of the economy satisfies condition (12). Then there exists a stationary Markov perfect signaling equilibrium* ***if and only*** *if the pre-school investment levels h_1, h_2 satisfies condition (35) below. Whenever there exists one, in fact, there* *exist precisely four stationary Markov perfect equilibria, with varying degree of social mobility and economic growth and they are pareto ranked.*

To see this, notice that given our assumption on schooling cost in equation (12), optimal schooling function must be of the form in equation (31). Utilizing the properties of the optimal pre-school investment function in proposition 5, it is easy to show that the there are four possible form of $\psi(\tau,h)$:

$$1)\ \psi(\tau,h) = \begin{cases} h_1 & \forall\tau \text{ and } h=h_1 \\ h_2 & \forall\tau \text{ and } h=h_2 \end{cases} \quad 2)\ \psi(\tau,h) = \begin{cases} h_1 & \forall h \text{ and } \tau=1 \\ h_2 & \forall h \text{ and } \tau=2 \end{cases}$$

$$3)\ \psi(\tau,h) = \begin{cases} h_1 & \text{if } \tau=1 \text{ and } h=h_1 \\ h_2 & \forall\tau \text{ otherwise} \end{cases} \quad 4)\ \psi(\tau,h) = \begin{cases} h_2 & \text{if } \tau=2 \text{ and } h=h_2 \\ h_1 & \text{otherwise} \end{cases}$$

Let us consider the case 1). Let us denote by $\hat{h}_i = (1+\gamma(R))\,h_i$, and $\hat{\beta} = (1+\gamma(R))\,\beta$. The above $\psi(\tau,h)$ to be the optimal policy function of the Bellman equation (28), the following must be satisfied for each agent (τ,h) :

$$\begin{aligned} &\text{For } (\tau,h)=(1,h_1): \\ V(1,h_1) &= w(1)-\hat{h}_1+\hat{\beta}\left[(1-p)V(1,h_1)+pV(2,h_1)\right] ...(S1) \\ &\geq w(1)-\hat{h}_2+\hat{\beta}\left[(1-p)V(1,h_2)+pV(2,h_2)\right] ...(S2) \end{aligned}$$

...

$$\begin{aligned} &\text{For } (\tau,h)=(2,h_2): \\ V(2,h_2) &\geq w(2)-\theta(2,2,h_2)-\hat{h}_1+\hat{\beta}\left[(1-p)V(1,h_1)+pV(2,h_1)\right] ...(S7) \\ &= w(2)-\theta(2,2,h_2)-\hat{h}_2+\hat{\beta}\left[(1-p)V(1,h_2)+pV(2,h_2)\right] ...(S8) \end{aligned}$$

We have above system of eight inequalities in four unknowns $V(\tau,h)$, $\tau=1,2$ and $h=h_1,h_2$. If there exists a feasible solution to the above system of inequalities, then we can confirm that the postulated optimal policies $\sigma(\tau,h)$ and $\psi(\tau,h)$ and the wage schedule $w(s)$ defined in section 3.1 indeed constitute a Markov perfect stationary equilibrium. Notice that the above procedure is valid for verifying any other postulated equilibrium for the above basic economy, or for more general economies in which τ and h take finite number of values. For our basic economy, however, we can solve the above analytically. When τ and h take many more values, we can use an artificial linear programming to find a feasible solution to the above.

Using the equality constraints, we find the solution analytically as follows:

$$V(1,h_1) = V(2,h_1) = \frac{w(1)-(1+\gamma(R))\,h_1}{1-(1+\gamma(R))\,\beta}, \tag{32}$$

$$V(1,h_2)=\frac{w(2)-\theta(2,1,h_2)-(1+\gamma(R))h_2+(1+\gamma(R))\beta p\Delta\theta}{1-(1+\gamma(R))\beta} \tag{33}$$

and

$$V(2,h_2)=\frac{w(2)-\theta(2,2,h_2)-(1+\gamma(R))h_2-(1+\gamma(R))\beta(1-p)\Delta\theta}{1-(1+\gamma(R))\beta} \tag{34}$$

where $\Delta\theta=\theta(2,1,h_2)-\theta(2,2,h_2)$. Notice, however, that the above solution should also satisfy the inequality constraints of the system $(S1)-(S8)$. The constraints $(S1), S(2), (S7)$and $(S8)$, and the above solutions (32)-(34) imply that

$$h_2-h_1=\beta\left[w(2)-w(1)-E_\tau\theta(2,\tau,h_2)\right] \tag{35}$$

For the postulated $\sigma(\tau,h)$ and $\psi(\tau,h)$ to be an equilibrium, the economy should satisfy the linear constraint for h_1 and h_2 given in equation (35).[12] For such economies, however, we have all inequalities in $(S1)-(S8)$ as equalities, i.e., each agent would be indifferent between the pre-school investment choices h_1 and h_2. If we started with any other forms of $\psi(\tau,h)$, we will end up exactly with the above equality constraints. We still need to compute R and the observed conditional distribution of e given s. To that end, we must compute the transition matrix $P_1=[p((\tau,h),(\tau_+,h_+))]$ $(\tau,h),(\tau_+,h_+)\in\mathcal{T}\times\Xi$ and the associated invariant probability distribution, $\xi(\tau,h)$ over $\mathcal{T}\times\Xi$. These are given as follows:

$$P_1=\begin{array}{cl}\left(\begin{array}{llll}1-p & p & 0 & 0\\ 1-p & p & 0 & 0\\ 0 & 0 & 1-p & p\\ 0 & 0 & 1-p & p\end{array}\right) & \begin{array}{l}:(1,h_1)\\ :(2,h_1)\\ :(1,h_2)\\ :(2,h_2)\end{array}\end{array}$$

The above has two ergodic sets: $\{(1,h_1),(2,h_1)\}$ and $\{(1,h_2),(2,h_2)\}$ Notice that The fist ergodic set corresponds to the signal class $s=1$ and the second ergodic set corresponds to the signal class $s=2$. Let us denote by $\xi^1=(1-p,p,0,0)$ and $\xi^2=(0,0,1-p,p)$ The above system has a whole range of invariant distributions, given by $\xi=\left((1-\lambda)\xi^1+\lambda\xi^2\right)$, $0\leq\lambda\leq1$. Which particular one the system converges to, depends on the initial distribution. As before suppose that the initial proposition of population in the signal class $s=2$ is π_0^2. Then we have $\lambda=\pi_0^2$. We compute the stationary state flow of spillover knowledge from equation (27) as $R=\pi_0^2\cdot p$. The associated mobility matrix over the set of signals $\mathcal{S}$ can be easily seen to be

$$\tilde{P}=[p(s,s_+)]_{s,s_+=1,2}=\left(\begin{array}{cc}1 & 0\\ 0 & 1\end{array}\right)$$

This is precisely the signaling equilibrium 1 of section 3.1.

[12] We must note that such economies are generically impossible to exist.

Following the above steps, we can easily compute the signaling equilibria for the other cases as given below:

$$2)\ P_2 = \begin{pmatrix} 1-p & p & 0 & 0 \\ 0 & 0 & 1-p & p \\ 1-p & p & 0 & 0 \\ 0 & 0 & 1-p & p \end{pmatrix} \xi = \begin{pmatrix} (1-p)^2 \\ (1-p)\,p \\ (1-p)\,p \\ p^2 \end{pmatrix}, \tilde{P} = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix}, \quad \begin{array}{l} R = p^2 \\ \mu\,(P) = p \end{array}$$

$$3)\ P_2 = \begin{pmatrix} 1-p & p & 0 & 0 \\ 0 & 0 & 1-p & p \\ 0 & 0 & 1-p & p \\ 0 & 0 & 1-p & p \end{pmatrix} \xi = \begin{pmatrix} 0 \\ 0 \\ (1-p) \\ p \end{pmatrix}, \tilde{P} = \begin{pmatrix} 1-p & p \\ 0 & 1 \end{pmatrix}, \quad \begin{array}{l} R = p \\ \mu\,(P) = 1 \end{array}$$

and

$$4)\ P_2 = \begin{pmatrix} 1-p & p & 0 & 0 \\ 1-p & p & 0 & 0 \\ 1-p & p & 0 & 0 \\ 0 & 0 & 1-p & p \end{pmatrix} \xi = \begin{pmatrix} 1-p \\ p \\ 0 \\ 0 \end{pmatrix}, \tilde{P} = \begin{pmatrix} 1 & 0 \\ 1-p & p \end{pmatrix}, \quad \begin{array}{l} R = 0 \\ \mu\,(P) = 0 \end{array}$$

It is also clear (and we have numerical example to assert) that when agents are not risk neutral, i.e.,the felicity index $u(c)$ is non-linear, the proposition 6 is no longer true. In fact, we have a whole non-generic subclass of economies of section 3.1 that had no social mobility, which still have no social mobility even after pre-school investment is endogenized by parental altruism.

Notice that equalities of $(S1)-(S8)$ imply that we can have any policy function $h_+ = \psi\,(\tau, h)$ as optimal policy function. This make us ponder if any of those other policy functions together with the optimal schooling function constitute another equilibrium.

7 Policies and conclusions

In this paper, we have considered a model of intergenerational social mobility and economic growth, in which innate ability of workers, and the type of their education and jobs determine the rate of technological progress and social mobility. More talented individuals with higher education and working in the growth enhancing sectors can lead to higher rate of technical progress and wage growth. Moreover, higher is the rate of mobilization of these talented individuals to higher education and growth sectors, the higher is the rate of social mobility. The incentive structure in the labor market that matches workers to jobs, together with the talent level and cost of education of individuals of various social backgrounds, determine the incentives for various types of education for individuals. Important features of our model are that the innate ability of an individual is a private knowledge, i.e., (possibly) known only to the individual and that education not only increases productivity of the individual, more so for an higher ability individual, it also acts, at least at the time of initial hiring, as a signaling for individual's innate productive ability for the purpose of job matching in the labor market.

In economies with one-time non-renegotiable wage contracts[13], we show that the asymmetric information regarding innate ability (hence productivity level) of workers leads to existence of multiple signaling equilibria arising from multiplicities of unprejudiced employers' self-fulfilling beliefs regarding the relationship between schooling and productivity. These equilibria vary in the degree of social mobility and economic growth, and all of them could be inefficient in the sense of being far from generating the maximum possible rate of social mobility and economic growth. Furthermore, there are no natural ”refinements” based on economic grounds that can lead to selection of an equilibrium.

We then considered various labor market practices or institutions[14] such as implicit contracts involving quits, layoffs, promotions and demotion of workers based on the employer's or worker's assessments of realized productivity on the job, and explicit wage contracts contingent upon the outcome of some publicly observed indicator of one's actual productivity. We show that these institutions can lead to removal of some of the above inefficiencies and thus to a higher rate of economic growth and social mobility. The remaining impediments to economic growth and social mobility can only be removed by intervening in the education system.

We considered briefly various school systems – purely public systems such as in South Korea, and dual systems with both private and public education such as in India and the US. We argued that, within the dual private-public system, subsidies or school vouchers for higher education to children from poorer family backgrounds can lead to self-selection of the more talented ones with the poorer backgrounds into higher education and growth enhancing jobs, and hence to higher social mobility and economic growth in a cost effective way. It also follows from our analysis that increasing average education level of the population may not be the most effective way of raising the growth rate.

In most countries, especially in less developed countries, higher education is highly subsidized by the government. If the source of lower social mobility is due to higher cost of education faced by children of poorer family background, subsidizing higher education uniformly for children of all family backgrounds is not necessarily going to be effective in inducing the talented children from poorer family background to opt for higher education.

[13] By this we mean an employment contract in which wage is a function of schooling level only, which is offered during hiring, and not renegotiated later during the tenure of the employment upon receiving further information about the worker's innate ability.)

[14] These institutions themselves might be the result of asymmetric information.

Our analysis has the following implication for the proposed educational policy allowing children to borrow for college education. This will be effective only to the students on the margin who have enough pre-school investment so that the rate of return from college is higher than the interest rate on the loan. For those with poorer family background, they would need loans at lower interest rates, or their parents should be given targeted subsidies for the purpose of pre-natal and preschool investment to reduce their children's cost of higher education.

The practical relevance of the above types of policies hinges on important empirical questions: How to estimate the schooling cost as a function of schooling level, schooling type, school quality, family background and innate ability? How much more scope remains in an economy to reduce inefficiency by developing appropriate labor market practices? Another important empirical issue in this connection is to examine if observed educational attainment and job assignments of individuals in a society are according to their innate ability, or according to their family backgrounds. The existence of multiple equilibria arising from unprejudiced employer's self-fulfilling expectations also raises important empirical questions: How to verify whether an economy is stuck with a low level equilibrium where growth rate, and social mobility are low, and how to design policies that will allow the economy to move from a low level equilibrium to an equilibrium with higher growth and social mobility?

The framework proposed here could be calibrated using real data and be used to carry out various policy analyses regarding the cost of schooling subsidies, or school vouchers, and benefits that will accrue to the society in terms higher mobility, faster economic growth, and more egalitarian income distribution. In our future work, we plan to pursue some of these issues using the NLSY (National Longitudinal Surveys of Youths) data for the US.

References

Adams, James D. [1990], ”Fundamental Stocks of Knowledge and Productivity Growth”, *J. Polit. Econ.*, 98(4)673-702.

Becker, G. [1962], ”Investment in human capital: a theoretical analysis”, *J. Polit. Econ.*,(supplement) 70:9-49.

Becker, G. and N. Tomes [1979], ”An equilibrium theory of the distribution of income and intergenerational mobility”, J. Polit. Econ.

Becker, G. and N. Tomes [1986], ”Human Capital and the rise and fall of families”, J. Polit. Econ.

Benabou, Ronald [1993], ”Search Market Equilibrium, Bilateral Heterogeneity, and Repeat Purchases”, *J. Econ. Theory*, vol. 60:140-158.

Ben-Porath, Yoram [1967], ”The production of human capital and the life cycle of earnings”, *J. Polit. Econ.*, 75:352-365.

Conlisk, J. [1990], ”Monotone Mobility Matrices,” *J. Math. Sociol.*

Daniel Kermit, Dan Black and Jeffrey Smith [1995], ”College Quality and the Wages of Young Men”, (mimeo) The Wharton School of the University of Pennsylvania, Philadelhia.

Gottfredson, Linda S. [1997], ”Why g-matters: The complexity of everyday life,” *Intelligence*, vol.24(1):79-132.

Griliches, Z. and W.M.M. Mason [1972], ”Education, Income, and Ability”, J. Polit. Econ. vol.80(3):75-103.

Heckman, James J., Anne Layne-Farrar and Petra Todd [1995], ”Does Measured School Quality Really Matter? An Examination of the Earnings-Quality Relationship”, in Gary Burtless (ed), *Does Money Matter: The Link Between Schools, Achievement, and Adult Success”*, Washington DC, Brookings Institution.

Heckman, James J. [1995], ”Cracked Bell”, *Reason*, March:49-56.

Herrnstein, R.J. and C. Murray [1994], *”The Bell Curve: Intelligence and Class Structure in American Life*,”, New York:Free Press.

Jaffe, A. [1986], Technological opportunity and spillovers of R&D: Evidence from firm’s patents, profits and market value,”, *American Economic Review*, vol.76(5):984-1001.

Jovanovich, Boyan [1979], ”Job Matching and the Theory of Turnover”, *J. Polit. Econ.*, vol. 87(5), part 1:972-990.

Jovanovich, Boyan and Robert Moffit [1990], ”An Estimate of a Sectoral Model of Labor Mobility”, *J. Polit. Econ.*, 98(4):827-852.

Kreps, D. [1990], *A Course on Microeconomic Theory*, Princeton University Press, New Jersey, Princeton.

Levine J. and D. Suzuki [1993], ”The Secret of Life: Redesigning the living world”, WGBH, Boston:Mass.

Loury, G. [1981], ”Intergenerational Transfers and the distribution of earnings”, Econometrica.

Lucas, R. [1988], ”On the mechanics of economic development”, J. Monetary Econ.

Mincer, Jacob [1958], ”Investment in human capital and personal income distribution,” *J. Polit. Econ.*, 281-302.

Raut, L.K. [1991], ”R&D spillover and productivity growth: Evidence from Indian private firms,” *Journal of Development Economics*, Vol. 48:1-23.

Raut, L.K. [1995], ”Capital Accumulation, Income Inequality, and Endogenous Fertility in an Overlapping Generations General Equilibrium Model,” *Journal of Development Economics*, Vol. 34.

Rosen, Sherwin [1977], ”Human capital: Relations between earnings and education,” in (ed) M. Intrilligator, *”Frontiers of Quantitative Economics*,” Amsterdam, North-Holland, Vol. 3B.

Roy, A.D. [1951], ”Some thoughts on the distribution of earnings”, *Oxford Economic Papers*,vol. 3:135-146.

Sicherman N. and O. Galor [1990], ”A Theory of Career Mobility”, *J. Polit. Econ.*, 98(1):169-192.

Spence, M. [1974], ”Market Signaling: Information transfer in Hiring and Related Screening Processes”, Harvard University Press, Cambridge:Mass.

Stiglitz, Joseph E. [1975], ”The theory screening, education, and the distribution of income,” *American Economic Review*, vol.65:283-300.

Stokey, Nancy L. and Robert E. Lucas, Jr. [1989], *Recursive Methods in Economic Dynamics*,” Harvard University Press, Cambridge:Massachusetts.

Willis, Robert J. [1986], ”Wage Determinants: A survey and reinterpretation of human capital earnings functions”, in (eds) O. Ashenfelter and R. Layard, *Handbook of Labor Economics: Vol. I”*, Elsevier Science Publishers BV.

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