

Problem set #4: Econ590
 Due Date: November 14, 2001
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1. Parental pre-school investment has long-term consequences in a child's schooling achievements and in labor market performance. Explain how. (You should incorporate the material from the Entwisle paper posted earlier on the course site).
2. We have already analyzed the schooling choice problem of an individual when (s)he is sixteen or so, given his/her parental investment, i.e., given β . Now we formulate how parental investment in children's education (both preschool and up to age sixteen) is determined. Assume simply that if parents invest h in a child's schooling and his/her ability is τ , then the child will earn $w_2(h, \tau)$. We further assume that the form of this earning function is given by $w_2(h, \tau) = H_0(\tau) h^\sigma$, $0 < \sigma \leq 1$. (As discussed in the class, the above relationship is after taking into account the effect of parental investment in the child's optimal schooling choice, which depends on this investment, h). Recall the associated utility maximization problem discussed in the class:

$$\max_h u(w_1 - h) + \gamma U(w_2(h, \tau))$$

Assume Cobb-Douglas utility specification $u(c) = \ln c$ and $U(w_2) = \ln w_2$. Recall that γ represents the degree of parental altruism towards children and assume that $0 \leq \gamma \leq 1$. Derive the optimal h . How does the optimal h relate to parental earnings w_1 , the innate ability of the child τ , and the degree of parental altruism γ ? Derive the intergenerational income dynamics equation

$$\ln w_2 = \alpha_0 + \alpha_1 \ln w_1 + \epsilon$$

where, ϵ absorbs the terms involving τ and γ . What are the values for α_0 and α_1 ? What is a good measure of social mobility in the above equation? What factors are detriment to social mobility in the above economy?

3. Harrod-Domar model of economic growth and development was one of the earliest attempt to give a theory of development. They showed "*under certain assumptions*" that the growth in total output, $\frac{\Delta Y}{Y}$ which we denote as g_Y , is given by

$$g_Y = \frac{s}{k}$$

where s is the saving rate, and k is the capital output ratio.

- (a) Derive the above formula.
- (b) Currently India has a savings rate of 20 percent, and the annual growth rate in total output is 4.0 percent per year. India would like to increase its growth rate of total output from 4.0 percent to 6.0 percent per year, what should be the corresponding savings rate? Recommend three important policies that might help India to increase its savings rate.

- (c) According to Harrod-Domar theory of economic development, *the main development concern is to understand how a less developed economy with low savings rate could be made to have a high savings rate, because then the economy will have a sustained high growth and hence development.* Briefly provide the strengths and weaknesses of this view of economic development in the light of economic development processes of the concurrent less developed economies.
4. State precisely the assumptions of Solow growth model with Harrod neutral technological change and derive the following fundamental difference equation of the Solow growth model

$$\tilde{k}_{t+1} = \frac{sf(\tilde{k}_t) + (1 - \delta)\tilde{k}_t}{(1 + n)(1 + b)}$$

\tilde{k}_0 given

where n is the population growth rate, s is the saving rate, δ is the depreciation rate of capital, b is the rate of Harrod neutral technological change, \tilde{k} denotes the capital labor ratio in efficiency unit, and $f(x) \equiv F(x, 1)$, where $F(K, L)$ denotes the base period production function with the base period unit of capital K and labor L .

Suppose it has been found that all countries have the same rate of technological change $b = .02$, depreciation rate $\delta = .25$ and share the same Cobb-Douglas technology leading to $f(\tilde{k}) = 3(\tilde{k})^{.35}$. Do the following exercises for this economy.

- (a) Suppose at time $t = 0$ the capital-labor ratio in efficiency unit of both the U.S. and Japan were equal to 1.5. and all other parameter values are as in in table 1.
 (i) Compute the capital labor ratio in efficiency unit, per capita income, and rental rates for the next 5 periods. (ii) What are the average of the yearly linear growth rates of per capita income of the U.S. and Japan during the 5 years period that you computed? If they differ, what are the reasons?
- (b) What will be Japan's long-run growth rate in per capita income? What will be the long-run growth rate of U.S., India and Korea?
- (c) Suppose all countries have $b = 0$. Using the basic indicators in table 1, calculate the steady-state (i.e., long-run) level of per capita income for each country in table 1. If these countries differ in the long-run level of per capita income, explain why they differ?
5. Repeat problem 4 when the production function is given by $F(K, L) = 3K + 4L$.

This concludes our study of the problems of less developed countries, and how to use economic model to analyze them and find policies that may help them attain better living

Table 1: Basic indicators of a few selected countries

Basic Indicators	India	China	Thailand	Mexico	Korea	Singapore	U.S.	Japan
P.C.income (\$1994)	320	530	2,410	4,180	8,260	22,500	25,880	34,630
PPP Estimate of PC income	1,280	2,510	6,970	7,040	10,330	21,900	25,880	21,140
pc income gr.rate 1985-94	2.9	7.8	8.6	0.9	7.8	6.1	1.3	3.2
Pop. gr. rate1980-90	2.1	1.5	1.8	2.0	1.2	1.7	0.9	0.6
gr. rate of export 1980-90	5.9	11.5	14.0	6.6	12.0	10.0	5.2	4.8
gr. rate of export 1990-94	13.6	16.0	14.6	4.0	10.6	12.3	6.7	4.0
Savings to GDP ratio 1988	21.0		26.0	23.0	38.0	41.0	13.0	33.0

standards. I hope you keep up with your commitment to think about the problems of the poor, no matter whether they are next door in a developed country or far away in the tropics of Africa or South Asia, they are, in fact, everywhere, five out of every six people you encounter in the whole world.