Development Economics [1]

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[1]To all my students at the University of California- San Diego, University of Hawaii-Manoa, Yale University, and Cal State Fullerton, interaction with whom through teaching formed this book. Your comments are most welcome.

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Overview

Chapter 1

What is Development?

1.1 Introduction

Concepts:
A. Measures of living standards
B. Comparison of living standards of a few selected countries
C. Meaning of Development and why need a separate economics field.

The focus of the course will be the comparison of living standards of economies. The broad topics are: the measurement of living standards of a nation; the rate of growth (i.e., improvement) of living standards; the sources of or determinants of such growth, and the policies that can improve living standards of a nation faster.

1.1.1 Measures of Living Standards

Adam Smith suggested to take GNP, i.e., aggregate income of an economy as a measure of living standards. This has the obvious defect that it does not take into account how many people are sharing this aggregate income. For instance, the GNP of India and Sweden are comparable but India has a population almost eighty times larger than that in Sweden. As we all know that living standards in Sweden is much better than in India. A better measure is perhaps per capita income. While this measure takes care of the above problem, it suffers from other defects to qualify for a good measure of living standards. The one main one is that it is insensitive to income distribution. Consider for instance two nations each having per capita income \$100 and a population of 100 persons. In one country, everybody has equal income, and in the other economy, 5 persons have \$1000 each and 95 persons have \$52.6 each. Which economy should be considered to have better standard of living? Obviously the first one. However, notice that per capita income does not discriminate this.

There are many other aspects such as life expectancy, literacy rate, freedom of speech or democracy, and political and economic stability that should also be included into a good measure of living standards. While per capita income is generally positively related to some of these variables such as life expectancy and literacy rate and thus to some extent represent these aspects, other aspects are not represented by it. There are some suggestions to adjust the per capita income to take into account some of these aspects, however, unadjusted per capita income is most widely used and we do the same.

Generally per capita income is measured in the local currency of the economy. How would you compare the per capita income Rs. 5400 in India with $ 15000 in the U.S.? We need a common unit. The general practice is to express per capita income of an economy in US dollars dividing it by the exchange rate (local currency/US$). This measure suffers from the *purchasing power parity (PPP)* problem of the nature that with $1000 you can buy much more and thus have a better standard of living in say Thailand than in the U.S. Exchange rate, for instance, does not take into account the relative prices of the non-tradables such as construction, hair-cuts etc. Again there are ways to adjust for the purchasing power parity, see in table 1.1 the PPP adjusted per capita income of various countries, and how they differ for countries. But general practice is to use the unadjusted one, and we follow the same practice. To compare the rate of improvement of living standards of various nations, it is appropriate to calculate the growth rate of per capita income in local currency but at constant prices (why constant prices?).

As discussed in the class, it is also important to take into account the extent of non-marketed activities such as home-cooking, child cares provided by parents in exchange of old-age care, which are not reflected in per capita income in the less market oriented economies. A PPP income will take care of some of these problems.

Per capita income does not also take into account the working enviornments in various countries, which are important aspects of living standards. For instance, consider two countries with same per capita income, but in one country there is high incidence of child labor and other bad labor practices, whereas in the other country, only the adults work with proper working enviornments. Obviously the first country has lower standard of living than the second one, but per capita income does not reflect this difference.

In spite of all these deficiencies that per capita income has, economists take it to be an important measure of living standards. In the first part of this course, we will focus on this measure of living standards and examine how various policies improve this measure of living standard faster.

Table 1.1: Basic indicators of a few selected countries

Basic Indicators	India	China	Thailand	Mexico	Korea	Singapore	U.S.	Japan
P.C.income ($1994)	320	530	2,410	4,180	8,260	22,500	25,880	34,630
PPP Estimate of PC income	1,280	2,510	6,970	7,040	10,330	21,900	25,880	21,140
pc income gr.rate 1985-94	2.9	7.8	8.6	0.9	7.8	6.1	1.3	3.2
Pop. gr. rate1980-90	2.1	1.5	1.8	2.0	1.2	1.7	0.9	0.6
gr. rate of export 1980-90	5.9	11.5	14.0	6.6	12.0	10.0	5.2	4.8
gr. rate of export 1990-94	13.6	16.0	14.6	4.0	10.6	12.3	6.7	4.0
Savings to GDP ratio 1988	21.0		26.0	23.0	38.0	41.0	13.0	33.0

Table 1.2: Average annual growth of the US economy over a long period of time

Period	1889-1983	1953-1973	1973-1983
gr. rate. of per capita income	1.8	2.1	0.9

1.1.2 Comparison of living standards of a few selected countries

Table 1.1reports capita income in 1994 for a few selected countries. We find that there is significant difference in the standard of living of these countries. For instance,

In the third row of table 1.1 we report the growth rate of per capita income for each country. We again find a lot of variation.

Now compare the rate of growth of per capita income in the U.S. in different periods in the following table 1.2.

Here again we find a significant variation in the growth rate. The important questions that we will deal with in this course are:

- What is the significance of a small difference in the growth rate of per capita income?
- How long would it take for less developed countries to catch up with the current US standard of living? After how many years these less developed countries will surpass the living standard of the U.S. economy if the current growth rates are maintained?
- Could some of these less developed countries grow at this high rates for ever such that they become the real threat to the U.S. economy? (Convergence hypothesis).
- Why did some countries like Japan, Korea, and Hong Kong grew so fast while other countries like India, and U.S. grew very slow? Or in other words what are the sources of growth in these countries?

- What government policies can achieve higher growth rate in per capita income?

Remark 1 *Growth theory is concerned with the long run growth of potential or full employment output, or the supply side of the economy say in the next ten or twenty years. So the policies we are interested in are those that affect this long-run growth rate of potential output. Macroeconomics is concerned with the stabilization of short-run fluctuations in actual output or the demand side of the economy.*

We have discussed in the class a common structure of the less developed economies on various socio-economic dimensions and how they differ from the structure of developed economies. Development theory is meant not only to understand the process of growth in per capita income, which is the most important indicator of development, and it is meant also to study how the socio-economic-political structure of an underdeveloped economy could be transformed into socio-economic structures that are commonly observed in developed countries. Or in other words, development economics, also examines the mechanics of structural change, and its scope is broader than growth theory. There are other distinctions between economic growth theory and economic development theory, which we will not get into in this course.

1.1.3 Schematic representation of our economy

In the figure 1.1 we have a schematic representation of a closed economy.

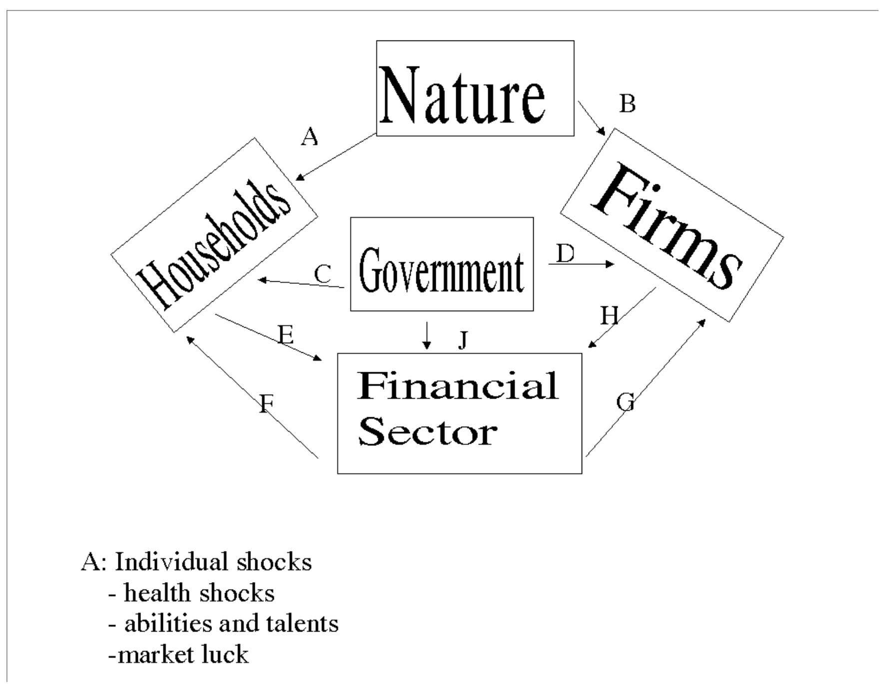

Figure 1.1: Flows of resources and inter-connectedness of different sectors

Chapter 2

Accounting for Observed Growth in Incomes

2.1 Representation of technology by production function

Concepts: *Production function, iso-quant, marginal product of capital, marginal product of labor, marginal productivity theory, returns to scale, Harrod neutral technological change, Solow neutral technological change, and Hicks neutral technological change, business cycles.*

From National Income Accounting we know that gross domestic income (GDY), i.e., the total income of all individuals in an economy, is the same as the gross domestic product (GDP), i.e., the total output that is produced in an economy using all its available inputs, capital, labor raw materials, technological knowledge etc. Our primary interest is to understand what are the different sources of growth in per capita income. We do this by representing the gross domestic product and the aggregate inputs of an economy in a suitable form. Production function is such an useful representation of the relationship. In this chapter we develop a few concepts regarding production function which will be useful in understanding the sources of growth in per capita income.

2.1.1 Production function

Both at the aggregate economy level and at the firm level, output is produced with inputs such as labor and capital. *Production function* is a compact way of expressing the relationship between output and inputs. For instance, the production function $Y = 2K + 3L$ tells us that K units of capital and L units of labor produces $Y = 2K + 3L$ units of output. More specifically, it tells us that when 2 units of

K	**L**	**Y**
2	5	19
2	6	22
3	6	24

Marginal product of labor $\frac{\triangle Y}{\triangle L} = 3$ at the input combination

$$(K, L) = (2, 5)$$

Table 2.1: Input-Output relationship

capital, and 5 units of labor are used we get 19 units of output, and when 3 units of capital and 5 units of labor are used we have 21 units of output and so on. Most technology could be represented by a suitable production function.

Let us take some concrete numerical example. Suppose K measures the number of hours of a 500mhz Pentium-III processor, and L measures the number of student hours (you can think of students of this class). Various combinations of student hours and computer times can produce some output say some computer programs that can be sold in the market. Let us denote by Y the computer program output. The following table summarizes how much output Y is produced by various combinations of input levels K and L.

Marginal product of labor is simply the amount of increment in output that can be obtained by increasing the labor use by one unit, holding the other input levels at constant levels. The above production relation could be simply represented by a linear production function given above.

A few important examples of production functions are

$$Y = c + aK + bL, \quad a, b, c \text{ are constants: linear} \tag{2.1}$$

$$Y = K^{\alpha} L^{\beta}, \alpha, \beta > 0 \qquad \text{Cobb-Douglas} \tag{2.2}$$

$$Y = (bK^{\rho} + cL^{\rho})^{1/\rho}, b, c, \rho > 0 \qquad : \text{CES} \tag{2.3}$$

$$Y = \min\left\{\frac{K}{a}, \frac{L}{b}\right\}, a, b > 0 \qquad : \text{Leontieff} \tag{2.4}$$

Returns to scale

Suppose we double up all the inputs, we may get exactly twice more output, or we may get less than or we may get more than twice output, depending on the nature of the technology. In the notation of production function, suppose we use $2K$ and $2L$ units of inputs, then the amount of output we will get is $Y^* = F(2K, 2L)$, whereas K and L units of inputs produce only $Y = F(K, L).Y^*$ could be $=, >$, or $<$ 2Y. I.e., $F(2K, 2L)$ could be $=, >$, or $< 2F(K, L)$ depending on the nature of the production function.

In general, for any $\lambda > 0$, if we have

$F(\lambda K, \lambda L) = \lambda F(K, L)$: F is called *constant returns to scale* (CRS)

$F(\lambda K, \lambda L) > \lambda F(K, L)$: F is called *increasing returns to scale*

$F(\lambda K, \lambda L) < \lambda F(K, L)$: F is called *decreasing returns to scale*

Example: $Y = (K^2 + L^2)^{1/2}$. We want to check whether this production function is CRS. Notice that for $\lambda > 0$,

$$F(\lambda K, \lambda L) = [(\lambda K)^2 + (\lambda L)^2]^{1/2} = [\lambda^2 (K^2 + L^2)]^{1/2} = \lambda.(K^2 + L^2)^{1/2} = \lambda F(K, L)$$

Hence F is constant returns to scale.

Marginal products and marginal productivity theory of Income distribution

Let the production function be given by $Y = F(K, L)$. If capital is increased by one unit, how much will be the increment in output? The increment in output is given by the the marginal product of capital:

$$F_K \equiv \frac{\partial F(K, L)}{\partial K} \approx \frac{\triangle Y}{\triangle K}$$

Similarly if one unit of labor is increased the increment in output is given by the marginal product of labor,

$$F_L \equiv \frac{\partial F(K, L)}{\partial L} \approx \frac{\triangle Y}{\triangle L}$$

In a competitive system the capitalist pays each unit of a factor of production its marginal product. The story goes as follows: Suppose the factor of production is labor. When the input markets are competitive, the capitalist may say to the worker that one more unit of the worker's labor produces F_L extra units of output, so that is the amount that one unit of the worker's labor hour deserves. Similarly one unit of capital gets its marginal product F_K units of output as its compensation. According to this principle, which is known as *marginal productivity theory of distribution*, in a competitive markets, the wage rate is $w = F_L$, and rental rate, $r = F_K$.

So if the capitalist employs K units of capital and L units of labor, and pays them according to their marginal products, how much output does he produce, and how much of the output he pays to the factors?

Total output produced: $Y = F(K, L)$: same as GDP

His total factor payments: $KR + L.W = K.F_K + L.F_L$: same as GDY

It is a mathematical result that if a production function $F(K, L)$ is constant returns to scale, then we have

$$\begin{aligned} F(K,L) &= K.F_K + L.F_L, i.e. \\ \text{GDP} &= \text{GDY} \end{aligned}$$

Thus, if the production process of the economy is represented by a constant returns to scale production function and if the factors are paid according to the marginal productivity theory then the capitalist will be left with no profit after paying all the factors. Perfect competition comes into the picture in the following way: Suppose the capitalist pays a factor little less than its marginal product, then he will have positive profit. But observing this positive profit, another capitalist will open a new firm demand more labor and capital, given that supply of labor and the stock of capital is fixed in the short-run, higher demand will increase the market wage rate and rental rate. Eventually the wage rate and rental rate will be as high as to wipe out any positive profit the capitalists were making. When the profit is driven to zero because of competition, no capitalist will have any incentive to open a new firm. The economy will be in equilibrium. The above theory fails when there is increasing returns to scale in production.

Properties of CRS production function

- **Result1**:$F(K, L)$ = K.F$_K$ + L.F$_L$
- **Result2**:$F(K, L) = Lf(k)$, where $f(k) \equiv F(k, 1)$, and $k = K/L$, the capital labor ratio.
- **Result3:**

  $$w = f(k) - kf'(k)$$

  $$r = f'(k)$$

  where w and r by definition are the marginal products F_L and F_K (which involve the function F(.); the above result then tells us how to find F_K and F_L from the little f function.

Isoquant:

An *isoquant* is the set of input combinations that produce a given level of output. More formally, an isoquant corresponding to the output level y^*, is

Example 1

$$IQ_{y^*} = \{(K, L) \mid F(K, L) = y^*\}$$

For the Leontieff production function $Y = \min\left\{\frac{K}{2}, \frac{L}{3}\right\}$ *an isoquant corresponding to the output level* $Y = 2$ *is shown in figure 2.1.*

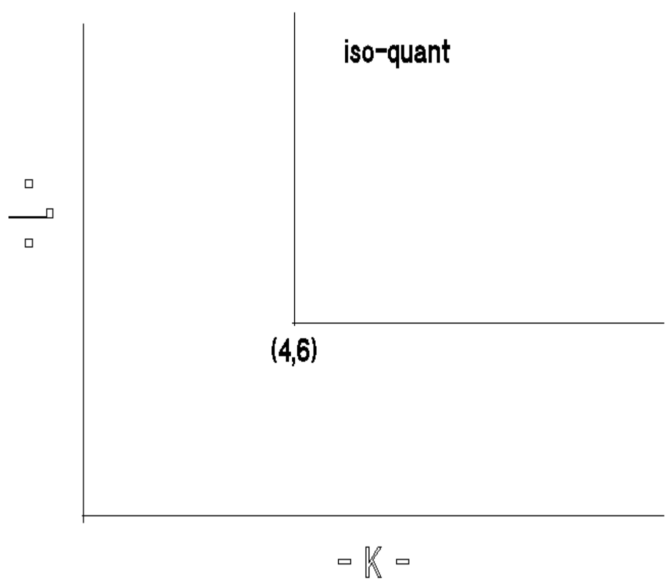

Figure 2.1: An isoquant of the Leontieff production function corresponding to output level $Y = 2$

The distinctive feature of the above function is that inputs are not substitutable in the production process. A typical isoquant corresponding to the Cobb-Douglas production function $Y = K^{\alpha}L^{\beta}$, is shown in figure 2.2.

Notice that in the above production process if we use less of one input we can still produce the given level of output by substituting more of the other input. However, to produce any positive level of output both inputs need to be used in each of the above two production processes. Corresponding to the higher levels of output, the isoquants will shift parallel to the right.

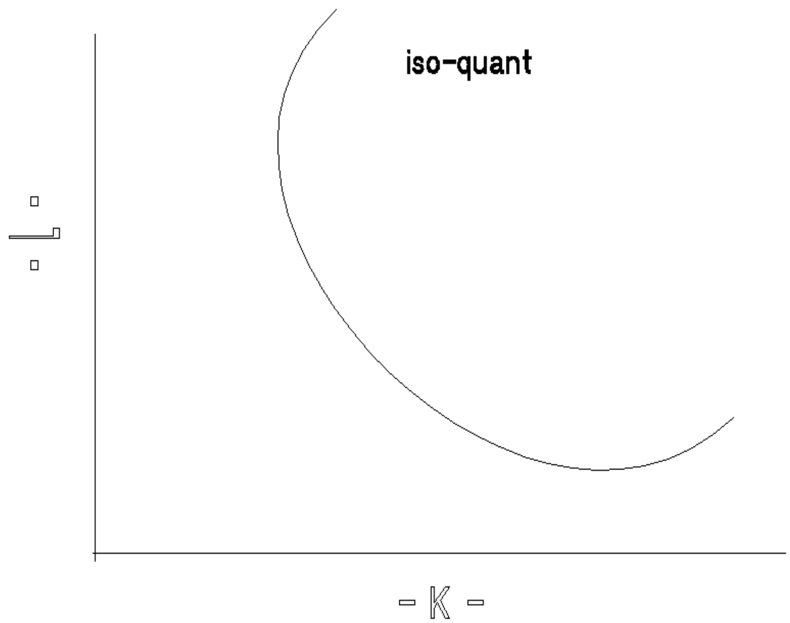

Figure 2.2: an isoquant of a Cobb-Douglas production function corresponding to a given output level

2.1.2 Production over time and different types of technological change

So far we have characterized production processes at a point in time. Now we characterize production over time. Suppose 5 men and $10 worth of capital produced 100 units of an aggregate good 10 years ago. How much output would the above levels of capital and labor be able to produce to-day? Common sense tells us that they will produce more than 100 units of output because both capital and labor are more efficient to-day than 10 years ago. This is tantamount to saying that to produce the same amount of output to-day we need less amount of both capital and labor than 10 years ago. Or in other words, there will be an inward shift in the isoquants of the subsequent periods. Let us denote the production function of period t by $F(K, L, t)$, all we have said so far is that corresponding to any level of output y^*, the isoquant IQ_{y*} at $t = t_1$ will be to the left of IQ_{y*} at $t = t_0$ for any $t_1 > t_0$. This is a form of technological change.

Technical progress implies inward shift of isoquant curves. That is with the same amount of capital and labor more output could be produced if there is a technical change. If this inward shift happens due to mere passage of time, then the technological change is called *disembodied*. Otherwise the technological change is *embodied but exogenous.* Furthermore, if the shift is due to new research and development or investment in human capital, or learning by doing, each of which is determined by individulas' choice so as to maximize their utility, then these technological changes endogenously determined in the model.. We will primarily be interested in disembodied technological change both exogenous and endogenous. Let us define different types of disembodied technological change.

Harrod neutral or labor augmenting technological change:

Consider two time periods, $t = 0$(base period) and $t = 1$(a future period). In both periods there are two factors of production: capital and labor. Suppose the production process in the base period $t = 0$ is represented by the production function $Y = F(K, L)$. Recall the meaning of this function: if L hours of labor work with K units of capital then it will produce $F(K, L)$ units of output. As time passes, workers get on the job training, and also they learn by repeatedly doing certain production activities how to use capital more efficiently. Assume that what he learns is documented and passed over to others, i.e., his knowledge becomes social knowledge. In later periods the workers learn even more such techniques. Or in other words they learn more and more over time to use capital efficiently.

Thus, in period $t = 1$, the same amount of labor hours and capital will produce more output than before. We want to use the production function of the base period to express the output level at $t = 1$. Notice that each unit of labor hour in period $t = 1$ is equivalent to $(1 + b_1)$ units of labor hour of the base period, where $b_1 > 0$ denotes the improvement in efficiency of labor in period $t = 1$ due to on the job training, or learning by doing or education. Since we assume that efficiency of capital remains unchanged, what we have is then K units of capital and L units of labor hour in period one will be equivalent to K units of capital and $(1 + b_1)L$ units of labor in the base period. Since our production function F relates the base period inputs to base period output. So using that production function we can now say that K units of capital and L units of labor in period $t = 1$ will produce $Y_1 = F(K, (1 + b_1)L)$ units of base period output. Repeating the same argument for $t = 2$, we obtain that K units of capital and L hours of labor in period $t = 3$ will produce $Y_2 = F(K, (1 + b_1)(1 + b_2)L)$ units of base period output. Where $b_2 > 0$ denotes the efficiency improvement of labor between period $t = 1$ and $t = 2$. Main point to notice that we have used the same base period production function to express such change in the production technology in future periods. This type of technological change is known as *Harrod neutral* or *labor augmenting* technological change. This is summarized in table 2.2.

The inward shift of isoquants of Leontieff production function due to Harrod

t	in period t units		in base period units		
	K	**L**	**K**	**L**	**Output level**
0	K_0	L_0	K_0	$\beta_0 K_0$	$Y_0 = F(K_0, \beta_0 L_0)$
1	K_1	L_1	K_1	$\beta_1 K_1$	$Y_1 = F(K_1, \beta_1 L_1)$
2	K_2	L_2	K_2	$\beta_2 K_2$	$Y_2 = F(K_2, \beta_2 L_2)$
...	...	...	...	...	...

where, $\beta_0 = 1$,

$\beta_1 = (1 + b_1)$, $\beta_2 = (1 + b_2)(1 + b_1) = (1 + b_2)\beta_1$, ...

Table 2.2: Harrod neutral or labor augmenting technological change

neutral technological change can be analytically shown as follows:

For the Leontieff production function: $Y = \min\left\{\frac{K}{2}, \frac{L}{3}\right\}$, with $b_1 = .5$, we have the relationship $Y_1 = \min\left\{\frac{K}{2}, \frac{1.5L}{3}\right\}$, i.e. $Y_1 = \min\left\{\frac{K}{2}, \frac{L}{2}\right\}$. When we draw the isoquants of these two production functions corresponding to the output level $y^* = 2$ on the same graph, the inward shift in the isoquant will be as in figure 2.3.

The nature of shift for other production functions will depend on the type of production function. Do the following homework problem.

Exercise 1 *For the linear production function draw the shifts in isoquants due to Harrod neutral technological change.*

Remark 2 *As we will see later, this type of technological change will be very important in explaining observed growth pattern of an economy over a long period of time. Bear in mind that efficiency improvements b_1, b_2, etc. of labor in various periods relative to the base period are exogenously given, i.e., taken as parameters. In growth theory we call $\beta_t L_t$, the labor in period t expressed in terms of the base period labor units as* labor in efficiency unit.

Notice that if there is Harrod neutral technological change over time, and the base period production function is constant returns to scale, then why have

$$\begin{aligned} Y_t &= F(K_t, \beta_t L_t) \\ &= \beta_t L_t F\left(\frac{K_t}{\beta_t L_t}, 1\right) \\ &= \beta_t L_t F(\tilde{k}_t, 1) \\ &= \beta_t L_t f(\tilde{k}_t) \end{aligned}$$

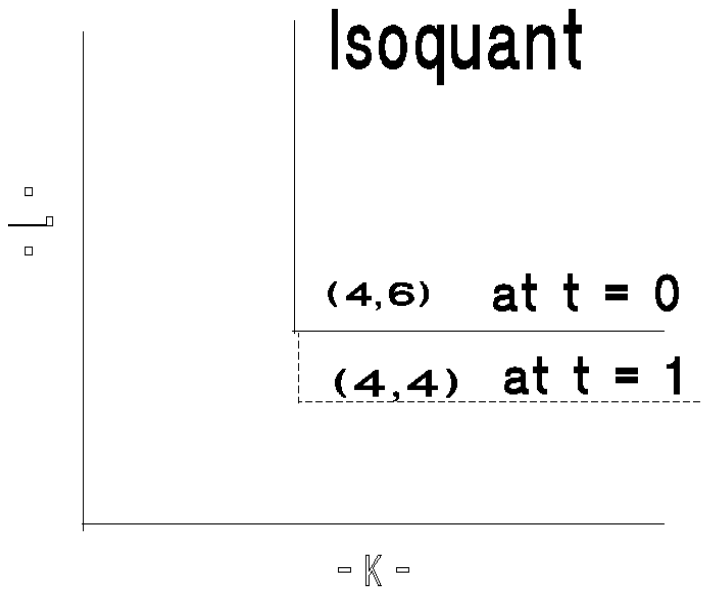

Figure 2.3: Inward shift in an isoquant of Leontieff production function due to Harrod neutral or labor augmenting technological change corresponding to output level $Y = 2$

where $\tilde{k}_t = K_t/\beta_t L_t$ is known as the *capital-labor ratio in efficiency unit*, and $\beta_t = (1+b_1)(1+b_2)...(1+b_t)$; when $b_t = b$ for all $t \geq 1$, we have $\beta_t = (1+b)^t$, for $t \geq 1$.

Remark: In the above, whatever we have said for labor could be symmetrically done for the input capital, and the corresponding type of technological change in the case of capital input is known as *capital augmenting or Solow neutral technological change.*

Hicks neutral or factor neutral technological change

When there is same rate of efficiency gains in labor and capital over time, the production process over time could be represented by

$$Y_t = F(\nu_t K_t, \nu_t L_t), \text{ with } \nu_0 = 1, \nu_1 = (1+v_1), \nu_2 = (1+v_1)(1+v_2)...$$

where v_1 is the common efficiency gains of both capital and labor between period $t = 0$ and $t = 1$, and similarly v_2 is the common efficiency gains between periods

$t = 1$ and $t = 2$, and so on. The inward shift of the isoquants due to Hicks neutral technological change can be derived for the Leontieff production function as follows:

For the Leontieff production function, $Y = \min\left\{\frac{K}{2}, \frac{L}{3}\right\}$, with $v_1 = 1$, we have the relationship $Y_1 = \min\left\{\frac{2K}{2}, \frac{2L}{3}\right\}$, i.e. $Y_1 = \min\left\{\frac{K}{1}, \frac{2L}{3}\right\}$. When we draw the isoquants of these two production functions corresponding to the output level $y^* = 2$ on the same graph, there will be a parallel inward shift of isoquant in period $t = 1$ as shown in figure 2.4.(In fact, it is easy to show that the inward shift of isoquant due to Hicks neutral technological change is always parallel).

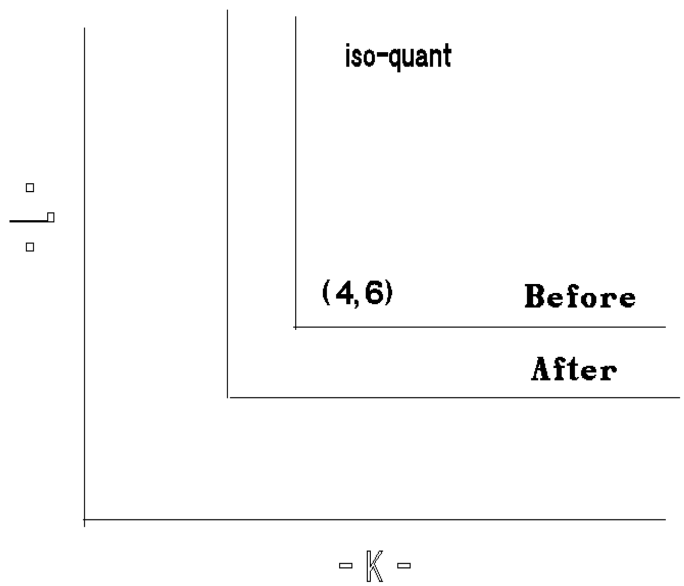

Figure 2.4: Inward shift in an isoquant of Leontieff production function due to Hicks neutral or factor neutral technological change corresponding to output level $Y = 2$

Exercise 2 *For the linear production function draw the shifts in isoquants due to Hicks neutral technological change.*

Example 2 *Notice that if there is Hicks neutral technological change over time, and the base period production function is constant returns to scale, then whey have*

$$
\begin{aligned}
Y_t &= F(v_t K_t, v_t L_t) \\
&= v_t L_t F\left(\frac{K_t}{L_t}, 1\right) \\
&= v_t L_t F(k_t, 1)
\end{aligned}
$$

where k_t is the capital labor ratio in actual units.

Remark 3 *The above derivations for Harrod neutral technological change will be utilized when we discuss Solow growth model. Hicks neutral technological change in continuous time will be utilized when we derive the growth accounting formula to decomposing the growth in output into its sources, namely growth in capital, growth in labor, growth in human capital and growth in productivity or technology which we will assume to be Hicks neutral.*

2.1.3 Potential vs. actual output

Potential or full employment output at any time is the level of output that could be produced if all inputs that are availble in the economy are used in the production process. Actual output is the level of output that will be actually produced to meet the demand (Y = C + G + I, where C = consumption demand, G = government consumption demand, and I = investment demand). Suppose at different time periods, $t = 0, 1, 2, ...,$ K_t is the unit of available capital stock, and L_t is the unit of available labor hours, and the rate of technological change say of the Harrod neutral type is b. Then we can compute potential output using a production function. The details of how exactly this is done is left as a reading exercise and we would not pursue in this course.

When log of potential output and actual output for the U.S. over a 100 years period 1889-1989 is plotted against time we find the graph in figure 2.5. Notice an important features of this graph.

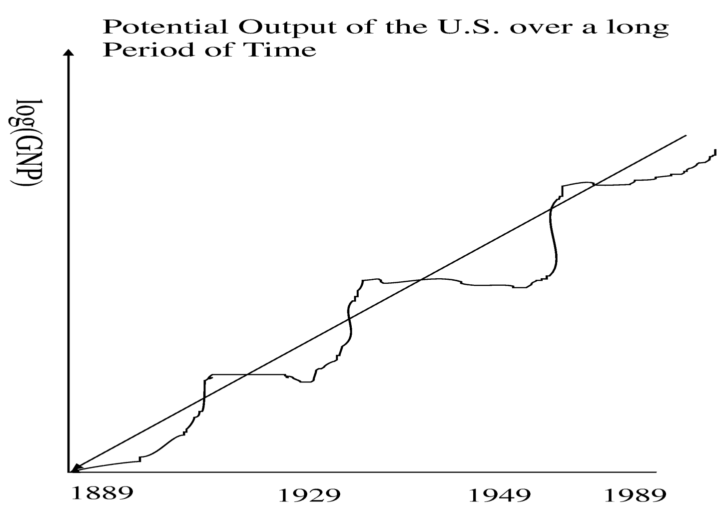

Figure 2.5: Potential ouput of the US during 1889-1989

The actual output fluctuates around the potential output, and thus over a long period of time both variables have the same trend. The purpose of the growth theory is to study this long-term behavior of the potential output and to devise policies that will affect this trend. The fluctuations in the gap between potential and actual output is known as business cycle. We are not concerned with this short-run fluctuations or in business cycles. The policies that deal with the problem of curbing this short-run fluctuations are known as the macro stabilization policies and our main focus is not this. Later we will give some theoretical models that can also generate cycles in output in the short-run around a trend.

2.1.4 Real Business cycle Theory

To be written

2.2 Algebra of Growth Accounting

Concepts: linear or discrete growth rate, continuous or exponential growth rate, catching up time, doubling up time.

————- Suppose you have saved \$100 in a bank which promises a simple interest rate of g per annum, i.e., you will be given interest only at the end of each year which is then reinvested. Both capital and interest of the first period will earn interest again at the end of the second year and so on. Let V(t) denotes the amount of money you will have in the bank at time t. Then your money will be growing discretely as follows:

$$\begin{aligned} V(t) &= 100 && \text{if } 0 \leq t < 1 \\ &= 100(1+g) && \text{if } 1 \leq t < 2 \\ &= 100(1+g)^2 && \text{if } 2 \leq t < 3 \\ &\dots \end{aligned}$$

We will denote the above series as V_0 , V_1 , V_2 , and thus $V_t = 100(1+g)^t$, for $t \geq 0$ is the amount of money you will have at the end of period t. The main point to notice is that your money is growing at discrete points of time. The discrete growth of any other variable such as per capita income or population growth could be understood in the same way. The above annual interest rate in the growth context is called *linear or discrete growth rate.*

To understand continuous growth or compounding, let us give an economic interpretation of the definition of the exponential, e. Suppose you have \$100 in a bank which promises an annual rate of interest r but compounded m times a year. Then at the end of the year you will have

$$V_m = \begin{cases} = 100(1+r) & \text{if } m = 1 \\ = 100(1+r/2)(1+r/2) = (1+r/2)^2 & \text{if } m = 1 \\ \dots \\ = 100(1+r/k)^k & \text{if } m = k \end{cases}$$

Thus if it is compounded continuously, i.e., if $m \longrightarrow \infty$, we have

$$V_\infty = \lim_{m \longrightarrow \infty} 100(1 + r/m)^m = 100e^r$$

With continuous compounding your money at time t will be $V(t) = 100e^{rt}$. In general if you start with V_0 amount of money with continuous compounding at the annual rate r, your money at time t will be given by $V(t) = V_0 e^{rt}$. When a variable say per capita income or population is growing over time according to the above formula, we say that it is growing exponentially at the annual rate r or that the *exponential growth rate or continuous grwoth rate* of the variable is$r$.

For further illustration, suppose that you have saved $100 in a bank which promises r = 100 % annual interest rate with continuous compounding. If At the end of the year you will have e = 2.718 dollars. Whereas with discrete or simple compounding you would have only (1+1) = 2 dollars. You can quickly calculate the simple growth rate, also known as annual yields that corresponds to the above continuous growth rate. Suppose g is the simple growth rate that will give the same amount of money at the end of each year as with the continuous growth rate r, then it is obvious that

$$e^r = (1 + g) \tag{2.5}$$

Thus given one type of growth rate we can always find the other type of growth rate using the above formula.

There are many advantages of using continuous growth rates. For instance, it provides a simpler way to calculate the growth rates of ratio of two variables etc., but in some other situations a discrete growth rates are useful. The continuous growth rate is related to continuous time and discrete growth rate is related to discrete time when modeling of growth process is under consideration.

Definition 1 *A variable*$V(t)$ *is said to be* growing exponentially or continuous grwoth rate *at the annual rate* r *if and only if it could be expressed as* $V(t) = V_0 e^{rt}$, $t \geq 0$, *or equivalently if and only if* $dlnV(t)/dt = r$.

This logarithmic derivative definition is sometimes useful in order to compute the exponential growth rate

Exercise 3 *Suppose a country's per capita income now is $100, and its growing exponentially at the rate r = .05. Find the doubling up time T when the per capita income of this country will be doubled:*

Let T be the doubling up time then it satisfies $100e^{.05T} = 200$ implies T=$\ln(2)/.05 = 13.86$

(a) Do the same calculations for $r = .06, .01, .09$ and understand the significance of small differences in the growth rates.

(b) Do the above exercise when r is instead the discrete growth rate.

Growth formula for functions of variables

Suppose Y(t) = F(K(t),L(t)) be a differentiable function of two variables and we know that K(t) and L(t) are growing exponentially at the rates r_K and r_L respectively. At what exponential rate Y(t) is then growing? Suppose it is growing exponentially at the rate r_Y . Then by applying the definition we can derive that

$$\begin{aligned} r_Y &= d\ln Y(t)/dt = d\ln F(K(t), L(t)/dt \\ &= \eta_{FK} \cdot r_K + \eta_{FL} \cdot r_L \end{aligned}$$

where $\eta_{FK} = dlnF(K,L)/dlnK$, and $\eta_{FL} = dlnF(K,L)/dlnL$. Suppose $F(K,L)$ denotes the production function and K and L denote capital and labor, then η_{FK} is the *output elasticity of capital* which says that if capital is increased by one percent then output will increase by h percent. Similarly, η_{FL} is the *output elasticity of labor*. Notice that the above formula expresses growth in output in terms of growth in input and output elasticities.

Suppose we have Hicks neutral technological change and the production function is given by

$$Y(t) = A(t)F(K(t), L(t)),$$

where F is constant returns to scale function. Proceeding exactly in the above manner, we derive the growth accounting formula

$$r_Y = r_A + \eta_{FK}.r_K + \eta_{FL}.r$$

where $\eta_{FK} = dlnF(K,L)/dlnK$, and $\eta_{FL} = dlnF(K,L)/dlnL$.

The above formula is the single most important growth accounting equation we will use it in the next section. In this section let us work out some examples.

Exercise 4 (1) *Suppose $Y(t) = [K(t)]^{0.4}\,[L(t)]^{0.6}$. Suppose exponential growth rates for K and L are respectively r_K and r_L . Find the exponential growth rate of Y(t) directly using the logarithmic derivative and also using the above formula and convince yourself that they agree.*

Exercise 5 (2) *Use logarithmic derivative to find the exponential growth rate for Y(t) = K(t)/L(t) in terms* r_K *and* r_L *.*

Computation of average growth rates Suppose we have observed data on a variable such as population or per capita income at the end of each year for a few years. We can calculate both discrete growth rates and continuous growth rates between any two consecutive periods. Suppose we want to get an idea about at what average exponential rate the variable has grown during the period. More specifically, we have the following table

We can take one measure of annual average linear growth rate, g, during the period 0-T to be the simple average of the linear growth rates between two consecutive periods. Similarly the average exponential growth rate, r, during the period 0-T as the average of exponential growth rates between two consecutive periods. More appropriate method is the regression technique which gives the average exponential growth rate, and then using formula (2.5) one gets the corresponding average linear growth rate. We fit the following regression equation:

$$\ln Y_t = \alpha + \beta t + \epsilon$$

where natural logarithm of the variable$Y$ is regressed on t and a constant. The coefficient of t gives us the average exponential growth rate during the period under consideration.

2.3 Sources of Growth

Concepts: *Total factor productivity growth, Denison Index, Solow Index.*

Assume that there are two aggregate factors of production, capital and labor. If the supplies of factors of production increase over time, the output will also grow as a result. Moreover, as time passes, both factors become more and more efficient, and this will also lead to higher output. **The questions we address are**: *How much of growth in output over a long period of time is accounted for by the growth in inputs, i.e., growth in capital and labor, and how much is due to the improvement in productivity. How could we empirically measure these for an economy*?

We first derive a growth accounting formula.

2.3.1 Growth accounting formula for functions of several variables

Suppose $Y(t) = A(t)F(K(t), L(t)$ be a differentiable function of two variables and we know that $A(t), K(t)$ and $L(t)$ are growing exponentially at the rates $r_{A,}$ r_K and r_L respectively. Suppose we denote the exponential grwoth rate of $Y(t)$ by r_Y. How is r_Y related to the grwoth rates $r_{A,}$ r_K and r_L? It could be shown easily (see Appendix), that the relationship is given by

$$r_Y = r_A + \eta_{FK}.r_K + \eta_{FL}.r_L \tag{2.6}$$

where $\eta_{FK} = \partial ln\mathrm{F}(K, L)/\partial ln\mathrm{K}$, and $\eta_{FL} = \partial ln\mathrm{F}(K, L)/\partial ln\mathrm{L}$. Suppose $F(K, L)$ denotes the production function and K and L denote capital and labor, then η_{FK} is the *output elasticity of capital* which says that if capital is increased by one percent then output will increase by η_{FK} percent. Similarly, η_{FL} is the *output elasticity of labor*. Notice that the above formula expresses growth in output in terms of growth in input and output elasticities.

The above formula is the single most important growth accounting equation we will use it in the next section. In this section let us work out some examples.

Exercise 6 ***[1]*** *Suppose $Y(t) = A_0 e^{r_A t} K(t)^{.4} L(t)^{.6}$. Suppose exponential growth rates for K and L are respectively r_K and r_L. Find the exponential growth rate of $Y(t)$ directly using the logarithmic derivative and also using the above formula and convince yourself that they agree.*

[2] Use logarithmic derivative to find the exponential growth rate for $Y(t) = K(t)/L(t)$ in terms r_K and r_L.

The above formula in equation (2.6) is known as growth accounting formula. The second and third terms in the right hand side of the above are respectively the part of the growth in output contributed by growth in the factors of production, capital and labor, and the first term is the residual or the contribution of the *total factor productivity growth.*

Suppose from an empirical estimation of a constant returns to scale aggregate production function, we got the estimates $\sigma_{FK} = .7$ and $\sigma_{FL} = .3$, and using the regression techniques suppose we have got the exponential growth rates of output, capital, and labor to be as, $r_Y = .096, r_K = .05$, and $r_L = .07$, then from (2.6) we have an estimate of total factor productivity growth $r_A = .03$. That is the contribution of the growth in total factor productivity to the growth in aggregate output is 3 percent per annum.

While the above paragraph is an illustration of growth accounting using (2.6), the steps for empirical work proceeds also in the same way. The main problem in empirical investigation of the sources of growth is to get an estimate of σ_{FK} and σ_{FL}. There are two main approaches to the estimation of these parameters originally followed by the pioneering works of Solow and Denison. Solow assumed a Cobb-Douglas production function such as

$$Y_t \qquad = e^{r_A t}.K_t^{\alpha}.L_t^{1-\alpha}, 0 < \alpha < 1.$$

which after taking logarithms on both sides give

$$y_t = r_A t + \alpha k_t + (1-\alpha) l_t$$

where y, k, l denote the logarithm of Y, K, L respectively. Using the aggregate output level capital and labor, we can get an estimate of α, and then we get an estimate of total factor productivity growth, known as *Solow Index* or Solow residual as

$$r_A = r_Y - \alpha.r_K - (1-\alpha).r_L$$

Denison does not, however, assume any particular functional form for the aggregate output, but he assumes that the production function is constant returns to scale, and that the factors of production are paid their marginal product. If capital and labor are the only inputs used then GNP = total wages and salaries + total rentals. Under the above assumptions we have

σ_{FL} = wages and salaries/GNP, i.e., wage share of the national income

σ_{FK} = rental income/GNP, i.e., rental share of the national income.

Now taking the mean of the above ratios over a period as an estimate of these parameters,σ_{FL}, σ_{FK}, he calculates an index of total factor productivity growth known as *Denison Index* as

$$r_A = r_Y - \sigma_{FK}.r_K - \sigma_{FL}.r_L$$

The following table provides us with these estimates for the U.S., Korea, and Japan

Sources of growth	**Japan:1953-1971**	**Korea:1963-1982**	**U.S:1948-1973**
Output growth	8.81	8.13	3.79
Contribution of			
gr. in labor	1.85	3.31	1.42
gr. in capital	2.10	1.58	0.71
gr. in TFP	4.86	3.24	1.66

Source: Kim and Park [1985].

Table 2.3: Growth accounting for Korea, Japan and the U.S.

From table 2.3 we can get an idea about the relative importance of different factors that contribute to growth in output of various economies. The most important factors for growth of output in Japan is the productivity growth, in Korea and the U.S. are growth in labor and productivity. Notice that for the U.S. the contribution of these factors are much lower than Korea, and the contribution of TFPg for Japan is so much higher than any other country. If U.S. could have the total factor productivity growth rate of Japan, its growth in output would rise to 6.99 instead of mere 3.79. If it could also have the growth rate of labor as high as in Korea and growth in capital as high as in Japan, then U.S. could enjoy a annual growth rate of 10.27 percent for the total income. **The natural question** is: *Why the U.S. is not doing so?* **Equivalent question is**: *What are the policies that will make capital, labor and TFP grow faster, and are there limits to such growth?* We turn to these issues now.

2.3.2 Growth in Labor

The components of aggregate labor in an economy are the number of workers, their education or skill level, and hours of work. Thus a higher growth rate in population, higher expenditure on education or on the job training, and better incentives to increase the hours of works for each person will contribute to growth in aggregate output.

Population growth

Suppose at any time only a fraction α of the population is of the working age. The value of α will, of course, depend on the age structure of the economy. For instance, consider two economies of equal population. The one with relatively more children or more retired people relative to working adults will have lower α. Higher is the population growth, higher will be proportion of children to adults. There are many other factors such as life expectancy that will affect the relationship between population growth and labor supply. Without getting into such complications, for simplicity let us assume that α is constant. Let us denote by $L(t)$ the labor in period t, and $P(t)$ the population in period t. Then we have $L(t) = \alpha P(t)$. This implies $r_L = r_P$. Thus higher is the growth in population higher is the growth in labor and from (3.2) it follows that higher will be growth in output.

This brings us to the policy question: what determines population growth? We will come back to it later when we discuss growth models with endogenous population.

Question: Is it desirable to have higher growth in population to induce higher growth in aggregate output as far as growth in living standards is concerned?

Hours of Work

Hours of work $= f($ wage rate net of taxes $)$

But we will have income effect and substitution effect as a result of an increase in wage rate, w.

Income effect: As w rises, less number of hours of work the worker can obtain the same income as before. This is the income effect which has negative effect of wage increase on the hours of work.

Substitution effect: Every hour spent in leisure is more expensive now in terms of real income foregone. This implies that the worker will substitute work for leisure. Thus the substitution effect of wage increase is positive.

The net effect of wage increase is undetermined and empirically controversial. We will expect it to be positive at low levels of wage rate as in developing countries. But at high wage rates, only sound empirical work can tell what will be the net effect.

Education:

We have already discussed about the role of education or on the job training on output growth while discussing about Harrod neutral or labor augmenting technological change. There we have seen that education and on the job training increases the efficiency level of labor force when compared with the base period labor force. Suppose the efficiency level of the labor force is growing at the rate of *b*. That is

1 unit of educated labor $\equiv (1 + b)$ units of uneducated worker.

How do we get an estimate of *b*. Compare the earnings of educated and uneducated worker. If we assume that they are paid according to their contribution to production, then their earnings will be related by the same relationship, namely

$$W_S = (1 + b)W_U \Leftrightarrow b = W_S/W_U - 1$$

where W_S and W_U are the wage rates of educated and uneducated workers. The same criterion could be used over time to get an estimate of *b*.

Table 2.4 shows the contribution of education and hours of work to growth in GNP corresponding to the table 2.3.

Sources of growth	**Japan:1953-1971**	**Korea:1963-1982**	**U.S:1948-1973**
Output growth	8.81	8.13	3.79
Contribution of growth in labor	1.85	3.31	1.42
• employment	1.14	2.18	1.22
• hours	0.21	0.43	-0.24
• education	0.34	0.39	0.41

Source: Kim and Park [1985].

Table 2.4: Contribution of employment, hours of work and education to growth in output of Korea, Japan and the U.S.

2.3.3 Growth in capital

Growth in capital comes from investment. You know from national income accounting that

$$I = S + (T - G) + (X - M)$$

where S = domestic household savings, T = tax revenues, G = government expenditure, thus $T - G$ = government budget deficit or government savings, X = export, M = imports, and thus $X - M$ = foreign savings.

Determinants of each components are as follows:

S :
- ○ interest rate
- ○ per capita current income or permanent income
- ○ imperfection in the capital markets
- ○ tax rates on wage income and capital gains
- ○ **demographic characteristics of the population**

Our main interest in this course is in the effects of demographic characteristics on aggregate household savings which we do in the next subsection.

$T - G$: reduction of government spending will increase government saving or increase in tax revenues

$X - M$: higher export relative to imports. Policies such as devaluation of local currency, increasing international competitiveness.

We do not get into the details of these policies here. Later we will come back to industrial policies that can affect the international competitiveness, and thus export.

2.3.4 Total Factor Productivity Growth (TFPg)

The main source of total factor productivity growth is advances in technological knowledge which results from $R\&D$ investment, increasing returns to scale of production, and improvement in management. The chapter by Denison talks about the details, so read this article which is attached. Later we will also come back to the determinants of TFPg.

Chapter 3

Basic Models of Growth and Development

Apart from the stylized facts and development indicators that we subscribed during our tour around the globe, especially from the less developed countries, let us briefly describe the six stylized facts that Kaldor found in 1948. Kaldor's stylized facts led to systematic development of grwoth models.

3.1 Kaldor's six stylized empirical facts

Kaldor in 1948 found six stylized facts about a few macro variables for developed countries. To describe them, let us have the following notations:
$Y(t)$ = total output or GDP at time t
$L(t)$ = labor force which is same as population at time t
$K(t)$ = stock of capital at t
$r(t)$ = rate of profit = total profit at $t/K(t)$
$v(t)$ = $K(t)/Y(t)$, capital output ratio
$S(t)$ = saving rate $= [Y(t) - C(t)]/Y(t)$, where $C(t)$ total consumption at t
$y(t)$ = per capita income $= Y(t)/L(t)$
$k(t)$ = capital- labor ratio $= K(t)/L(t)$

The six stylized facts that Kaldor found are as follows:

I. $y(t)$ grows exponentially more or less at a constant rate over a long period of time possibly with short-run fluctuations, known as business fluctuations.

II. Both $K(t)$ and $L(t)$ are growing exponentially over a long period of time, and the growth rate of $K(t)$ exceeds that of $L(t)$, and hence $k(t)$ is also growing over a long period of time.

III. $K(t)$ and $Y(t)$ are both growing over the long-period of time at the same rate so that $v(t)$ is more or less constant over time.

IV. $r(t)$ is more or less constant over time.

V. Different countries have different growth rates for $y(t)$.

VI. Economies with higher share of profit to income tend to have higher saving rate.

3.1.1 Need for an economic growth model

We have seen that based on the historical data, the growth accounting that we studied in the previous chapter can tell us the sources of variations in the observed grwoth rates. What is missing in that framework as far as we are concerned with the grwoth and development process and devising policies that can affect them? The main deficiency is that it cannot tell us how those historical data were generated and cannot tell us where the economy is heading to. The main purpose of studying growth models is to understand the interaction among various economic forces leading to the economic growth process of an economy so that we can devise policies that can affect the long-run growth rates. In this chapter we want to describe simple theoretical growth models to describe how various economic forces interact with each other to generate economic growth process of a nation. An important criterion for a theoretical model to be useful is that these models should be able to predict some of the empirical stylized facts listed above. There are several models available in the literature. Probably the first serious attempt at modeling economic growth process was by Harrod and Domar who used Leotieff production function. As you know Leontieff production function does not allow for factor substitutions in production, which is an unrealistic feature of Harrod-Domar model. We will first develop a simplified version of the Harrod-Domar growth model. We will examine the kind of predicitons this model produce and use cross country data to see if the predicitons are supported by observed expereinces of the countries. We will then develop Lewis model of grwoth and development and then finally the Solow grwoth model.

3.2 Harrod-Domar model

Their main assumptions are as follows:

A.1 The production function is Leontieff: $Y_t = \min \{vK_t, bL_t\}$

To simplify our exposition we make a stronger assumption that labor is abundant so that labor is not binding in production. Then the assumption A.1 implies that $Y_t = vK_t$, where v is the output-capital ratio, generally found to be of the magnitude $1/3$.

A.2 Let us denote by S_t the total savings in peirod t.They assumed a Keynesian savings function. $S_t = sY_t$, where $0 < s < 1$.

A.3 Assume closed economy, so that $I_t = S_t$, where $I_t = \Delta K_t = K_{t+1} - K_t$.

Notice that from assumption A.1, $Y_t = vK_t$ implies $\Delta Y_t = v\Delta K_t$. From assumption A.3 we have $I_t = S_t$, i.e., $\Delta K_t = S_t$.

$$\begin{array}{ccccl} I_t & = & S_t & & \text{by assumptionA.3} \\ || & & || & \longleftarrow & \text{by assumption A.2} \\ \Delta K_t & = & sY_t & & \\ || & & || & & \\ \Delta Y_t/v & = & sY_t & & \text{.... (*)} \end{array}$$

from (*) we have $\frac{\Delta Y_t}{Y_t} = v.s$. So long as s and v is constant over time, we have constant grwoth rate. The only way a country can have higher grwoth rate in income is by having higher savings rate, or by making The growth rate in output is increasing in savings rate s and output capital ratio v. In this model the variations in the growth rate of total income among countries is due variations in their savings rate. For instance, suppose $v = 1/3$, then if a less developed country has savings rate 6%, then its growth rate is $g_Y = \frac{6\%}{3} = 2\%$. If the country wants to increase its grwoth rate to 9%, it should increase its savings rate to 27%. According to this model, the development problem boils down to the question of how an underdeveloped economy with low savings rate of 6% can increase it to say 27%. It might seems a easy thing to achieve, but in a poorer country with most people living in subsitence it is impossible to introduce any incentive scheme to increase the economy's savings rate. The policy makers in the fifties and sixties recommended that the developing countries should be given foreign aids which increase their investment rate. But it has not worked to achieve this goal. Furthermore, the current savings rates of many developing countries are as high as 27% or even more but their grwoth rates of total income is much lower than what is predicted by the Harrod-Domar model.

We can even put the model to empirical testing directly. We regress the growth rate of output

$$g_Y = \alpha + \beta.s + \epsilon$$

we look at the significance and magnitude of the estimate for β and R^2. of the above regression model. R^2 measures the goodness of fit, and it lies between 0 and 1. If the Harrod-Domar model is a good model of grwoth and development, then the R^2 should be high, and the estimate of β will be significant (i.e., t-statistics will be 1.96 or higher say to be significant at 5%). What do you find?

3.3 Lewis dual economy growth model

We follow closely Todaro's exposition of the Lewis model.

"One of the best-known early theoretical models of development that focused on the structural transformation of a primarily subsistence economy was that formulated by Nobel laureate W. Arthur Lewis in the mid-1950s and later modi-fied, formalized, and extended by John Fei and Gustav Ranis. The Lewis two-sector model became the general theory of the development process in surplus-labor Third World nations during most of the 1960s and early 1970s. It still has many adherents today, especially among American development economists. In the Lewis model, the underdeveloped economy consists of two sectors: (1) a traditional, overpopulated rural subsistence sector characterized by zero marginal labor productivity-a situation that permits Lewis to classify this as surplus labor in the sense that it can be withdrawn from the agricultural sector without any loss of output- and (2) a high-productivity modem urban industrial sector into which labor from the subsistence sector is gradually transferred. The primary focus of the model is on both the process of labor transfer and the growth of output and employment in the modem sector. Both labor transfer and modern-sector employment growth are brought about by output expansion in that sector. The speed with which this expansion occurs is determined by the rate of industrial investment and capital accumulation in the modern sector. Such investment is made possible by the excess of modem-sector profits over wages on the assumption that capitalists reinvest all their profits. Finally, the level of wages in the urban industrial sector is assumed to be constant and determined as a given premium over a fixed average subsistence level of wages in the traditional agricultural sector. (Lewis assumed that urban wages would have to be at least 30% higher than average rural income to induce workers to migrate from their home areas.) At the constant urban wage, the supply curve of rural labor to the modem sector is considered to be perfectly elastic.

We can illustrate the Lewis model of modern-sector growth in a two-sector economy by using Figure 3.1. Consider first the traditional agricultural sector portrayed in the two right-side diagrams of Figure 3.1b. The upper diagram shows how subsistence food production varies with increases in labor inputs. It is a typical agricultural production function where the total output or product

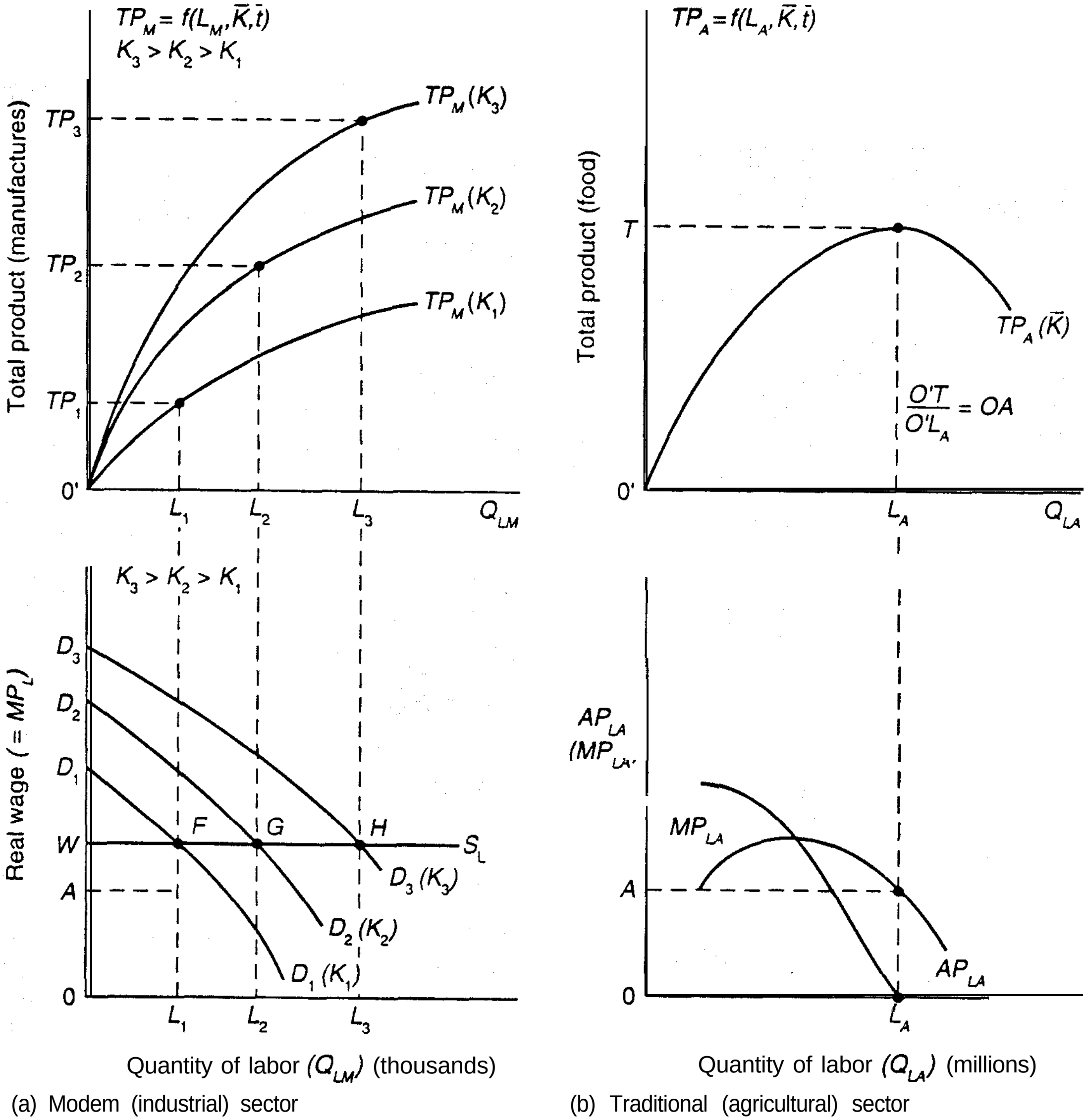

FIGURE 3.1 The Lewis Model of Modem-Sector Growth in a Two-Sector Surplus-Labor Economy

(TP_A) of food is determined by changes in the amount of the only variable input, labor (L_A), given a fixed quantity of capital, $\overline{K}$, and unchanging traditional technology, $\bar{t}$. In the lower right diagram, we have the **average** and marginal product of labor curves, AP_{LA} and MP_{LA}, which are derived from the total product curve shown immediately above. The quantity of agricultural labor (Q_{LA}) available is the same on both horizontal axes and is expressed in millions of workers, as Lewis is describing an underdeveloped economy where 80% to 90% of the population lives and works in rural areas.

Lewis makes two assumptions about the traditional sector. First, there is surplus labor in the sense that MP_{LA} is zero, and second, all rural workers share *equally* in the output so that the rural real wage is determined by the average and not the marginal product of labor (as will be the case in the modern sector). Assume that there are OL_A (= $O'L_A$) agricultural workers producing 0'7' food, which is shared equally as OA food per person (this is the average product, which is equal to $O'T/O'L_A$). The marginal product of these OL_A workers is zero, as shown in the bottom diagram of Figure 3.1b; hence the surplus-labor assumption.

The upper-left diagram of Figure 3.1a portrays the total product (production function) curves for the modern, industrial sector. Once again, output of, say, manufactured goods (TP_M) is a function of a variable labor input, L_M, for a given capital stock ($\overline{K}$) and technology ($\bar{t}$). On the horizontal axes, the quantity of labor employed to produce an output of, say, O'TP,, with capital stock K_1, is expressed in thousands of urban workers, $O'L_1$ (= OL_1). In the Lewis model, the modem-sector capital stock is allowed to increase from K_1 to K_2 to K_3 as a result of the reinvestment of profits by capitalist industrialists. This will cause the total product curves in Figure 3.1a to shift upward from $TP_M(K_1)$ to $TP_M(K_2)$ to $TP_M(K_3)$. The process that will generate these capitalist profits for reinvestment and growth is illustrated in the lower-left diagram of Figure 3.1a. Here we have modem-sector marginal labor product curves derived from the TP_M curves of the upper diagram. Under the assumption of perfectly competitive labor markets in the modem sector, these marginal product curves are in fact the actual demand curves for labor. Here is how the system works.

Segment OA in the lower diagrams of Figures 3.1a and 3.1b represents the average level of real subsistence income in the traditional rural sector. Segment OW in Figure 3.1a is therefore the real wage in the modem capitalist sector. At this wage, the supply of rural labor is assumed to be unlimited or perfectly elastic, as shown by the horizontal labor supply curve WS_L. In other words, Lewis assumes that at urban wage OW above rural average income OA, modern-sector employers can hire as many surplus rural workers as they want without fear of rising wages. (Note again that the quantity of labor in the rural sector, Figure 3.1b, is expressed in millions whereas in the modern urban sector, Figure 3.1a, units of labor are expressed in thousands.) Given a fixed supply of capital K_1, in the initial stage of modern-sector growth, the demand curve for labor is determined by labor's declining marginal product and is shown by the negatively sloped curve $D_1(K_1)$ in the lower-left diagram. Because profit-maximizing modem-sector employers are assumed to hire laborers to the point where their marginal physical product is equal to the real wage (i.e., the point F of intersection between the labor demand and supply curves), total modern-sector employment will be equal to OL_1. Total modern-sector output (O'TP,) would be given by the area bounded by points OD_1FL_1. The share of this total output paid to workers in the form of wages would be equal, therefore, to the area of the rectangle $OWFL_1$. The balance of the output shown by the area WD_1F would be the total profits that accrue to the capitalists. Because Lewis assumes that all of these profits are reinvested, the total capital stock in the modern sector will rise from K_1 to K_2. This larger capital stock causes the total product curve of the modern sector to rise to $TP_M(K_2)$, which in turn induces a rise in the marginal product demand curve for labor. This outward shift in the labor demand curve is shown by line $D_2(K_2)$ in the bottom half of Figure 3.1a. A new equilibrium modern-sector employment level will be established at point G with OL_2 workers now employed. Total output rises to $O'TP_2$ or OD_2GL_2 while total wages and profits increase to $OWGL_2$ and WD_2G, respectively. Once again, these larger (WD_2G) profits are reinvested, increasing the total capital stock to K_3, shifting the total product and labor demand curves to $TP_M(K_3)$ and to $D_3(K_3)$, respectively, and raising the level of modern-sector employment to OL_3.

This process of modem-sector self-sustaining growth and employment expansion is assumed to continue until all surplus rural labor is absorbed in the new industrial sector. Thereafter, additional workers can be withdrawn from the agricultural sector only at a higher cost of lost food production because the declining labor-to-land ratio means that the marginal product of rural labor is no longer zero. Thus the labor supply curve becomes positively sloped as modem-sector wages and employment continue to grow. The structural transformation of the economy will have taken place, with the balance of economic activity shifting from traditional rural agriculture to modern urban industry.

Criticisms of the Lewis Model

Although the Lewis two-sector development model is both simple and roughly in conformity with the historical experience of economic growth in the West, three of its key assumptions do not fit the institutional and economic realities of most contemporary Third World countries.

First, the model implicitly assumes that the rate of labor transfer and employment creation in the modern sector is proportional to the rate of modern-sector capital accumulation. The faster the rate of capital accumulation, the higher the growth rate of the modem sector and the faster the rate of new job creation. But what if capitalist profits are reinvested in more sophisticated laborsaving capital equipment rather than just duplicating the existing capital as is implicitly assumed in the Lewis model? (We are, of course, here accepting the debatable assumption that capitalist profits are in fact reinvested in the local economy and not sent abroad as a form of "capital flight" to be added to the deposits of Western banks!) Figure 3.2 reproduces the lower, modem-sector diagram of Figure 3.1a, only this time the labor demand curves do not shift uniformly outward but in fact cross. Demand curve $D_2(K_2)$ has a greater negative slope than $D_1(K_1)$ to reflect the fact that additions to the capital stock embody laborsav-

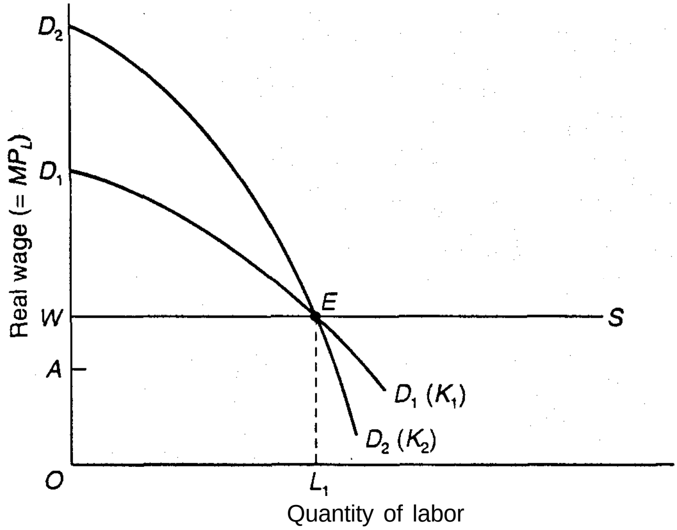

FIGURE 3.2 The Lewis Model Modified by Laborsaving Capital Accumulation: Employment Implications

ing technical progress—that is, K_2 technology requires less labor per unit of output than K_1 technology does.

We see that even though total output has grown substantially (i.e., OD_2EL_1 is significantly greater than OD_1EL_1), total wages ($OWEL_1$) and employment (OL_1) remain unchanged. All of the extra output accrues to capitalists in the form of excess profits. Figure 3.2, therefore, provides an illustration of what some might call "antidevelopmental" economic growth--all the extra income and output growth are distributed to the few owners of capital while income and employment levels for the masses of workers remain largely unchanged. Although total GNP would rise, there would be little or no improvement in aggregate social welfare measured, say, in terms of more widely distributed gains in income and employment.

The second questionable assumption of the Lewis model is the notion that surplus labor exists in rural areas while there is full employment in the urban areas. As we will discover in Chapters 7 and 8, most contemporary research indicates that the reverse is more likely true in many Third World countries-there is substantial unemployment in urban areas but little general surplus labor in rural locations. True, there are both seasonal and geographic exceptions to this rule (e.g., parts of the Asian subcontinent and isolated regions of Latin America where land ownership is very unequal) but by and large, development economists today seem to agree that the assumption of urban surplus labor is empirically more valid than Lewis's assumption of rural surplus labor.

The third unreal assumption is the notion of a competitive modem-sector labor market that guarantees the continued existence of constant real urban wages up to the point where the supply of rural surplus labor is exhausted. It will be demonstrated in Chapter 8 that prior to the 1980s, a striking feature of urban labor markets and wage determination in almost all developing countries was the tendency for these wages to rise substantially over time, both in absolute terms and relative to average rural incomes, even in the presence of rising levels of open modern-sector unemployment and low or zero marginal productivity in agriculture. Institutional factors such as union bargaining power, civil service wage scales, and multinational corporations hiring practices tend to negate whatever competitive forces might exist in Third World modern-sector labor markets. We conclude, therefore, that when one takes into account the laborsaving bias of most modem technological transfer, the existence of substantial capital flight, the widespread nonexistence of rural surplus labor, the growing preva-lence of urban surplus labor, and the tendency for modern-sector wages to rise rapidly even where substantial open unemployment exists, the Lewis two-sector model-though extremely valuable as an early conceptual portrayal of the devel-opment process of sectoral interaction and structural change-requires consider-able modification in assumptions and analysis to fit the reality of contemporary Third World nations." (Todaro [1995])

3.4 Solow Growth Model

Assume that population is growing exogenously as: $L_t = L_0(1+n)^t, t = 0, 1, \ldots$. Assume that economy exhibits an exogenously given constant rate of Harrod neutral technological change in the aggregate production process over time. That is assume that L_t workers are equivalent to $\tilde{L}_t \equiv (1+b)^t L_t$ number of base period workers and there is no improvement in the efficiency of capital. Then as we have seen before, in the period tK_t and L_t amount of capital and labor is going to produce $Y_t = F(K_t, \tilde{L}_t)$ total output, where F is a base period production function and is assumed to be constant returns to scale. Solow further assumes Keynesian savings function: $S_t = sY_t, 0 < s < 1$, where S_t is the aggregate savings. If we further assume that our economy is closed, then in equilibrium $S_t = I_t$, where I_t is aggregate investment. He further assumes that investment takes one period to gestate, and δ is the rate of depreciation per period, then we have

$$\begin{aligned}
K_{t+1} &= (1-\delta)K_t + I_t \\
&= (1-\delta)K_t + S_t, \text{(since } I_t = S_t \text{ in a closed economy)} \\
&= (1-\delta)K_t + sY_t, \text{ (because savings function is Keynesian)} \\
&= (1-\delta)K_t + sF(K_t, \tilde{L}_t) \\
&= (1-\delta)K_t + \tilde{L}_t sF\left(\frac{K_t}{\tilde{L}_t}, 1\right), \text{ (since F(,.,) is CRS)}
\end{aligned}$$

Now dividing both sides by $\tilde{L}_t$, and denoting by $\tilde{k}_t = K_t/\tilde{L}_t$, i.e. the capital-labor ratio in efficiency unit, we have the following

$$\frac{K_{t+1}}{\tilde{L}_{t+1}}.\frac{\tilde{L}_{t+1}}{\tilde{L}_t} = (1-\delta)\frac{K_t}{\tilde{L}_t} + sF\left(\frac{K_t}{\tilde{L}_t}, 1\right)$$

which implies

$$\tilde{k}_{t+1}.(1+b)(1+n) = (1-\delta)\tilde{k}_t + sf(\tilde{k}_t)$$

which implies,

$$\tilde{k}_{t+1} = \tfrac{(1-\delta)\tilde{k}_t + sf(\tilde{k}_t)}{(1+b)(1+n)} \equiv g(\tilde{k}_t) \text{ say}$$

$$t = 0, 1, 2, ... \tag{3.1}$$

$$\text{with } L_0, K_0 \text{ and hence } \tilde{k}_0 = \tfrac{K_0}{L_0} \text{ given}$$

Equation (3.1) is known as the **fundamental difference equation of Solow growth model.** The above difference equation contains all the information about the Solow economy. More specifically notice that once you know $\tilde{k}_0$, then using this in the above equation with $t = 0$ we can get $\tilde{k}_1$. Once we know $\tilde{k}_1$, again using this in the above equation with $t = 1$, we get $\tilde{k}_2$. In fact if we know $\tilde{k}_0$, then we know all future values of $\tilde{k}_t$. But once we know all $\tilde{k}_t$ values as the following formula shows, we know wage rate, interest rate, per capita income total income capital stock and labor in all periods:

$$r_t = f'(\tilde{k}_t) \tag{3.2}$$

$$w_t = (1+b)^t \left[f(\tilde{k}_t) - \tilde{k}_t f'(\tilde{k}_t)\right] \tag{3.3}$$

Since $L_t = (1+n)^t L_0$ $K_t = \tilde{k}_t.\tilde{L}_t = \tilde{k}_t.(1+b)^t L_t$, we can calculate per capita income in each period by

$$y_t = Y_t/L_t = F(K_t, \tilde{L}_t)/L_t = \left(\tilde{L}_t/L_t\right) f(\tilde{k}_t) = (1+b)^t f(\tilde{k}_t) \tag{3.4}$$

Question: Where does this economy lead to over a long period time? What are the long-run effects of an increase in savings rate (s), population growth rate (n), and the rate of technological change (b)? What are their effects in the short-run? The following phase diagram tells the story:

Long-run Implications:

In the long-run the $\tilde{k}_t$ reaches the steady-state level $\tilde{k}^*$, and the steady-state is globally stable. Thus looking back in the formula in the box, we see that in the long-run per capita income and wage rate is growing at the same rate as the rate of technological change, and interest rate is constant. Thus if there is no technological change, i.e., $b = 0$, there is no long-run growth in the per capita income and this counter to one of the six stylized facts listed above.

Now if n is decreased or s is increased, the g-curve in the phase diagram is going to shift upward, this will increase the value of $\tilde{k}^*$. Now looking at the formulas in the box again we can see that such policies have no effect on the long-run growth rate which depends only on the rate of technological change. However, as is obvious, if b increases, i.e., if there is higher rate of Harrod neutral technological change then higher is the rate of growth of per capita income, and wage rate.

Notice that if $b = 0$, then the long-run effect of a decrease in the population growth rate or an increase in saving rate will be to increase the level of capital labor ratio and hence level of per capita income; however they will not affect the long-run growth rates.

3.4.1 Transition dynamics or out of steady-state dynamics

Let us consider the above Solow model without technological change, In this case, $b = 0$, and $\tilde{k}_t$ is same as k_t. In the phase diagram (see figure 1, above) we see that starting from any initial capital-labor ratio k_0, the economy converges to the steady-state capital labor ratio k^* in the long-run. The steady-state is globally stable.

Notice that for $k_0 < k^*$, if the economy starts at k_0, over time the economy's capital labor ratio increases, and hence the per capita income, $y_t = f(k_t)$ also increases (see figure 3.1), but marginal product of capital, $r_t = f'(k_t)$, decreases over time, the growth rate of per capita income g_t between period t-1 and t decreases over time, see the numerical example below for further illustration of these points.

To study the effect of policy parameters such as savings rate, s, and population growth rate, n, on the long-run and short-run growth rates, notice that these variables do not affect long-run growth rates, but they affect the long-run level of capital-labor ratio k^* and the short-run growth rate. To illustrate the the effect on short-run growth rate, suppose countries A and B both start at the same k_0, but suppose country A has a higher savings rate, then its $g(k_t)$ curve will shift upward, and we can see now that if we take average annual growth for 5 years or 10 years, country A with a higher savings rate will have a higher average growth rate of in per capita income over a period of time. Similarly, if country A has a lower population growth, then its $g(k_t)$ curve will shift upward, and over a period of time the country A will have higher average growth rate than country B. Let us illustrate this further with a numerical example.

Example 3 *Suppose $n = .02$, $b = 0$, $\delta = .2$, the production function is Cobb-Douglas type, i.e., $F(K, L) = 6K^{\sigma}L^{1-\sigma}$, $0 < \sigma < 1$. For this production function we have $f(k) = 6k^{\sigma}$. Let us take $\sigma = .4$. Suppose both countries begin with $k_0 = 1.5$, and suppose country A has savings rate, $s = .20$ and for country B, savings rate $s = .10$, compute the per capita income of these countries for next 5 years and compute the average annual discrete growth rate as the average of year to year discrete growth rates. Which country has a higher average growth rate of per capita income? What are the long-run growth rates of these countries? How are the long-run growth rates of these countries affected by population growth rate and savings rate?*

To answer these questions, recall, the dynamical system or the difference equation associated with Solow model without technological change is given by

$$k_{t+1} = \frac{sf(k_t) + (1-\delta)k_t}{(1+n)} = g(k_t)$$

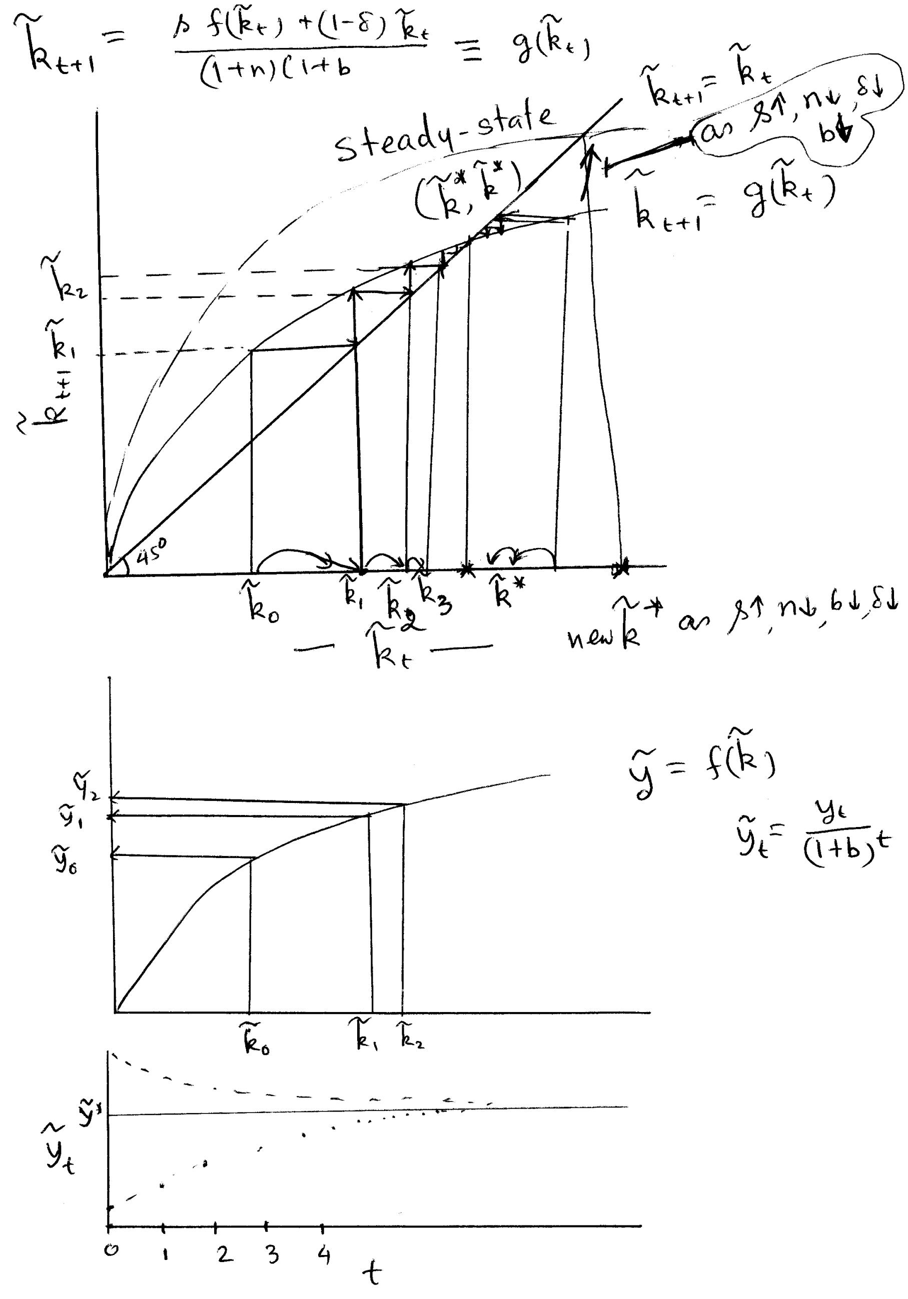

Figure 3.1: Phase diagram and the dynamics of per capita income in Solow growth model

the above for country A becomes:

$$k_{t+1} = \frac{1.2k_t^{.4} + .8k_t}{1.02}, k_0 = 1.5$$

the above for country B becomes:

$$k_{t+1} = \frac{.6k_t^{.4} + .8k_t}{1.02}, k_0 = 1.5$$

Table 3.1 gives for each country the next 5 year's capital labor ratio k_t, and per capita income y_t.

Period	$s = .2$:Country A			$s = .1$:Country B		
	k_t	y_t	g_t	k_t	y_t	g_t
0	1.50	07.05	–	1.50	7.05	–
1	2.56	08.74	0.24	1.87	7.70	0.09
2	3.72	10.15	0.16	2.22	8.26	0.07
3	4.91	11.34	0.12	2.55	8.73	0.06
4	6.07	12.34	0.09	2.85	9.13	0.05
5	7.18	13.20	0.07	3.13	9.48	0.04

Average growth rate of per capita income is
for country A $= 0.136$ and
for country B $= 0.06$.

Table 3.1: Simulated values of the Solow growth model

You can similarly do the exercise when both countries have common savings rate, say $s = .20$, but one country has higher population growth rate than the other. Furthermore, you can do the same analysis when there is Harrod neutral technological change, i.e., $b > 0$. The steps for that will be you begin with $\tilde{k}_0$, and use the dynamical system $\tilde{k}_{t+1} = g(\tilde{k}_t)$, to generate the future values of $\tilde{k}_t$; then use the formula $k_t = (1+b)^t\tilde{k}_t$, and $y_t = (1+b)^t f(\tilde{k}_t)$ and $r_t = f'(\tilde{k}_t)$ to generate the future values of of capital-labor ratio, k_t, per capita income, y_t, and rental rate r_t.

Convergence hypothesis: Notice that solow model predicts a kind of convergence of growth rates of all the countries. In fact, we find that while the countries with higher savings rate or lower population growth rate might have higher growth rates in the short-run, in the long-run the growth rate is the same as the rate of technological change, b. If all countries share the same technology, then in the long-run all countries will converge to the same growth rate, namely to the growth rate of Harrod neutral technological change.

Poverty Trap: In the above standard Solow model we have assumed a concave production function. However, suppose the production function is not concave, and it rather shows first increasing and then decreasing returns to scale. This will lead to the following phase diagram. It explains the poverty trap phenomenon.

Cycles and Chaotic dynamics of per capita income:

If the graph of the g-function looks as in figure 3.2, then as we can see that the capital-labor ratio and hence the per capita income will cycle around the steady state value. For cycles and chaos, the g-function should satisfy some conditions, which is beyond the scope of this course.

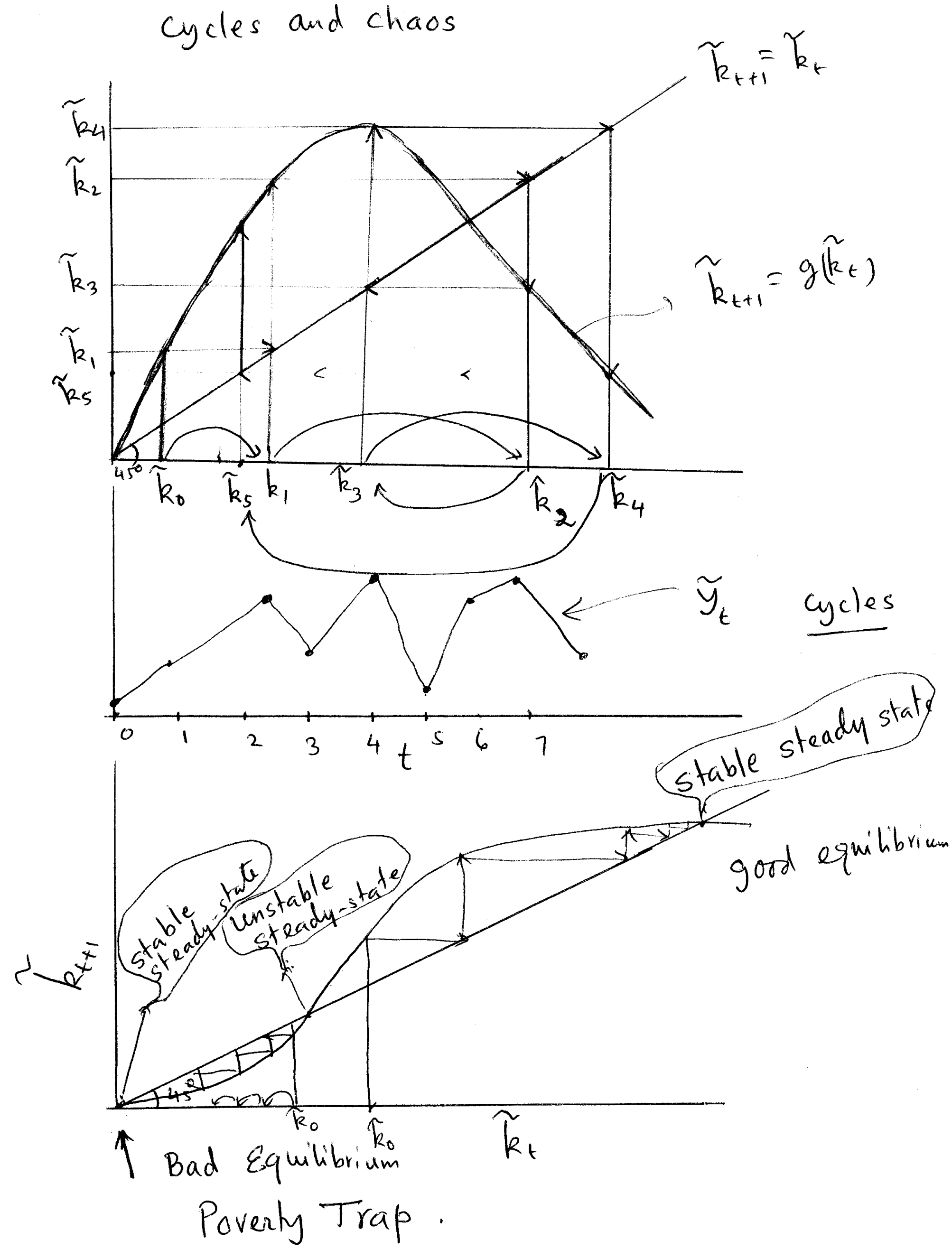

Figure 3.2: Phase diagram and the dynamics of per capita income in Solow growth model