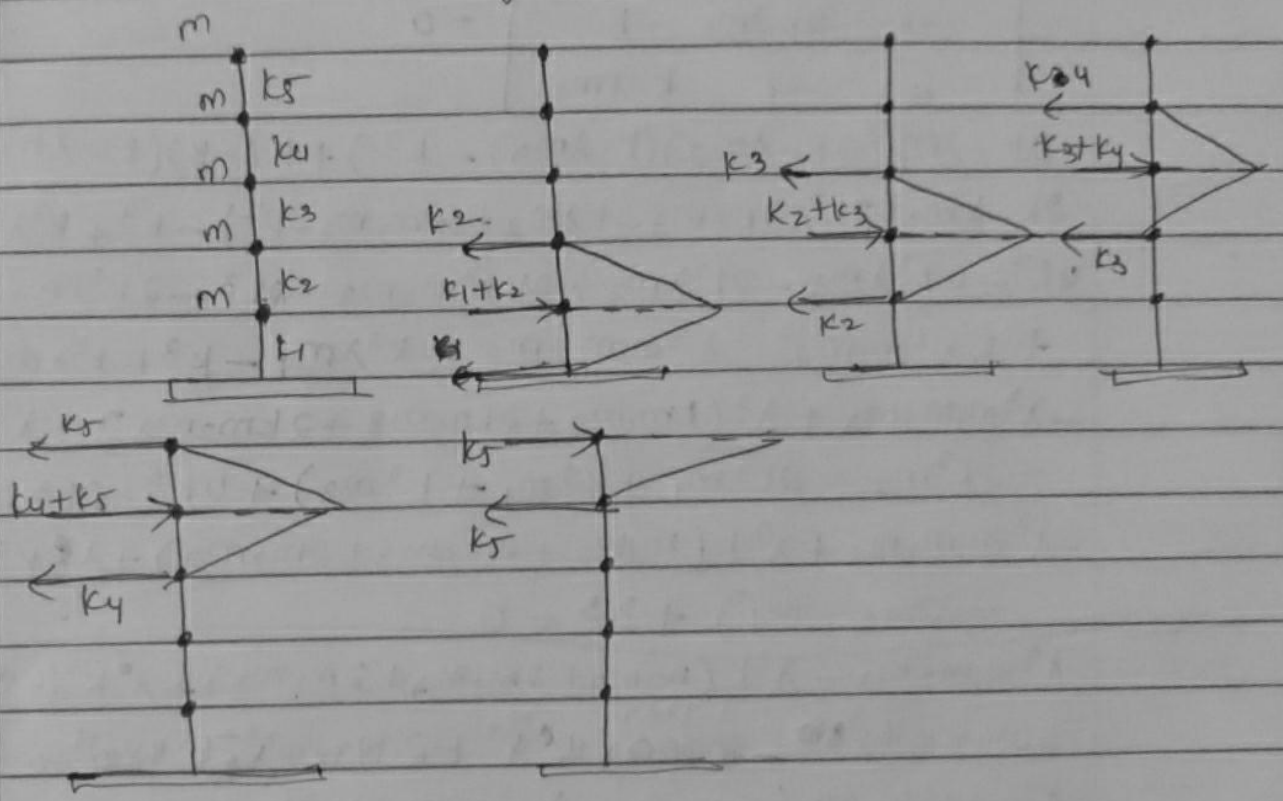
Assignment-05

22/11/22

(1)



$$[M] = \begin{bmatrix} m & 0 & 0 & 0 & 0 \\ 0 & m & 0 & 0 & 0 \\ 0 & 0 & m & 0 & 0 \\ 0 & 0 & 0 & m & 0 \\ 0 & 0 & 0 & 0 & m \end{bmatrix} \quad [k] = \begin{bmatrix} k_1+k_2 & -k_2 & 0 & 0 & 0 \\ -k_2 & k_2+k_3 & -k_3 & 0 & 0 \\ 0 & -k_3 & k_3+k_4 & -k_4 & 0 \\ 0 & 0 & -k_4 & k_4+k_5 & -k_5 \\ 0 & 0 & 0 & -k_5 & k_5 \end{bmatrix}$$

(2) $m_1 = 298648\ kg$, $m_2 = 250000 kg$, $m_3 = 190830\ kg$

$k_1 = k_2 = k_3 = 400 \times 10^6\ N/m$

$$[M] = \begin{bmatrix} 298648 & 0 & 0 \\ 0 & 250000 & 0 \\ 0 & 0 & 190830 \end{bmatrix} \quad [k] = \begin{bmatrix} 800\times10^6 & -400\times10^6 & 0 \\ -400\times10^6 & 800\times10^6 & -400\times10^6 \\ 0 & -400\times10^6 & 400\times10^6 \end{bmatrix}$$

$$|[K] - \omega^2[M]| = 0$$

Let, $\omega^2 = \lambda$

$$|[K] - \lambda[M]| = 0$$

$$\left| \begin{bmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{bmatrix} - \begin{bmatrix} 2k & -k & 0 \\ -k & 2k & -k \\ 0 & -k & k \end{bmatrix} \right| = 0$$

$$\begin{vmatrix} 2k-\lambda m_1 & -k & 0 \\ -k & 2k-\lambda m_2 & -k \\ 0 & -k & k-\lambda m_3 \end{vmatrix} = 0$$

$$2k-\lambda m_1\{(2k-\lambda m_2)(k-\lambda m_3) - k^2\} + k(-k)(k-\lambda m_3) = 0$$

$$2k-\lambda m_1(2k^2 - 2k\lambda m_3 - k\lambda m_2 + \lambda^2 m_2 m_3 - k^2) - k^3 + k^2\lambda m_3 = 0$$

$$4k^3 - 4k^2\lambda m_3 - 2k^2\lambda m_2 + 2k\lambda^2 m_2 m_3 - 2k^3 - 2k^2\lambda m_1 + 2k\lambda^2 m_1 m_3 + k\lambda^2 m_1 m_2 - \lambda^3 m_1 m_2 m_3 + k^2\lambda m_1 - k^3 + k^2\lambda m_3 = 0$$

$$-\lambda^3 m_1 m_2 m_3 + \lambda^2(km_1 m_2 + 2km_1 m_3 + 2km_2 m_3) + \lambda(-4k^2 m_3 - 2k^2 m_2 - 2k^2 m_1 + k^2 m_1 + k^2 m_3) + 4k^3 - 2k^3 - k^3 = 0$$

$$-\lambda^3 m_1 m_2 m_3 + \lambda^2 k(m_1 m_2 + 2m_1 m_3 + 2m_2 m_3) + \lambda k^2(-3m_3 - 2m_2 - m_1) + k^3 = 0$$

By solving we get;

$\lambda_1 = 282.616 \Rightarrow \omega_1 = 16.8112$

$\lambda_2 = 1967.818 \Rightarrow \omega_2 = 44.3601$

$\lambda_3 = 4123.065 \Rightarrow \omega_3 = 64.2111$

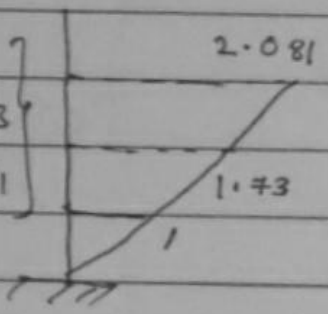
for $\lambda_1 = 282.616$, $\phi_{11} = 1$

$$\begin{bmatrix} 2k - 282.616m_1 & -k & 0 \\ -k & 2k - 282.616m_2 & -k \\ 0 & -k & k - 282.616m_3 \end{bmatrix} \begin{Bmatrix} 1 \\ \phi_{21} \\ \phi_{31} \end{Bmatrix} = 0$$

$$\phi_{21} = \frac{2k - 282.616m_1}{k} = 1.73$$

$$\phi_{31} = \frac{k\phi_{21}}{k - 282.616m_3} = 2.081$$

$$[\phi_1] = \begin{Bmatrix} 1 \\ 1.73 \\ 2.081 \end{Bmatrix}$$

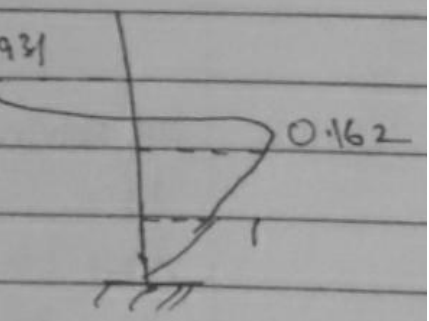


for $\lambda_2 = 1967.818$, $\phi_{12} = 1$

$\phi_{22} = 0.162$

$\phi_{32} = -0.931$

$$[\phi_2] = \begin{Bmatrix} 1 \\ 0.162 \\ -0.931 \end{Bmatrix}$$

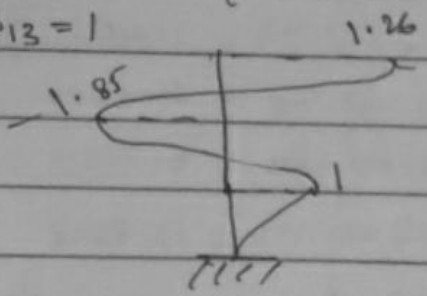


for $\lambda_3 = 4123.065$, $\phi_{13} = 1$

$\phi_{23} = -1.85$

$\phi_{33} = 1.26$

$$[\phi_3] = \begin{Bmatrix} 1 \\ -1.85 \\ 1.26 \end{Bmatrix}$$



Mode participation factor :-

$$P_1 = \frac{[\phi_1]^T[M][1]}{[\phi_1]^T[M][\phi_1]} = 0.60$$

$$P_2 = \frac{[\phi_2]^T[M][1]}{[\phi_2]^T[M][\phi_2]} = 0.34$$

$$P_{\phi_3} = \frac{[\phi_3]^T[M][1]}{[\phi_3]^T[M][\phi_3]} = 0.052$$

$$\Sigma P_i = 0.99 \approx 1$$

(3) Given, $M_1 = M_2 = 2000$ kg & $M_3 = M_4 = 3000$ kg

$K_1 = 350000$ N/m & $K_2 = K_3 = K_4 = 30000$ N/m

$$[M] = \begin{bmatrix} 2000 & 0 & 0 & 0 \\ 0 & 2000 & 0 & 0 \\ 0 & 0 & 3000 & 0 \\ 0 & 0 & 0 & 3000 \end{bmatrix}$$

$$[K] = \begin{bmatrix} K_1+K_2 & -K_2 & 0 & 0 \\ -K_2 & K_2+K_3 & -K_3 & 0 \\ 0 & -K_3 & K_3+K_4 & -K_4 \\ 0 & 0 & -K_4 & K_4 \end{bmatrix} = \begin{bmatrix} 380000 & -30000 & 0 & 0 \\ -30000 & 60000 & -30000 & 0 \\ 0 & -30000 & 60000 & -30000 \\ 0 & 0 & -30000 & 30000 \end{bmatrix}$$

$$|K - \lambda M| = 0$$

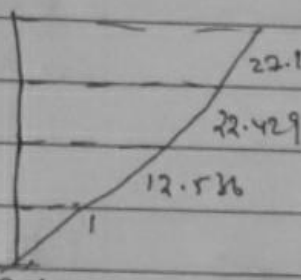
by solving we get;

$\lambda_1 = 1.9652 \Rightarrow \omega_1 = 1.4018$

$\lambda_2 = 18.1214 \Rightarrow \omega_2 = 4.2569$

$\lambda_3 = 38.5118 \Rightarrow \omega_3 = 6.2058$

$\lambda_4 = 191.4017 \Rightarrow \omega_4 = 13.8348$



for $\lambda_1 = 1.9652$, $\phi_{11} = 1$ $\quad \phi_1 = \begin{Bmatrix} 1 \\ \phi_{21} \\ \phi_{31} \\ \phi_{41} \end{Bmatrix} = \begin{Bmatrix} 1 \\ 12.536 \\ 22.429 \\ 27.915 \end{Bmatrix}$

$[K - \lambda_1 M][\phi_1] = 0$

we get,

$\phi_{21} = 12.536$

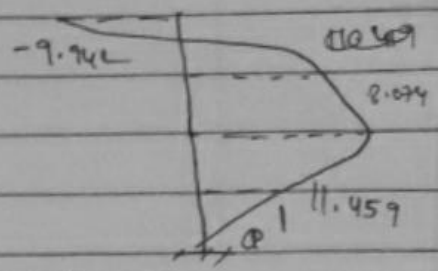
$\phi_{31} = 22.429$

$\phi_{41} = 27.915$

for $\lambda_2 = 18.1214$, $\phi_{12} = 1$

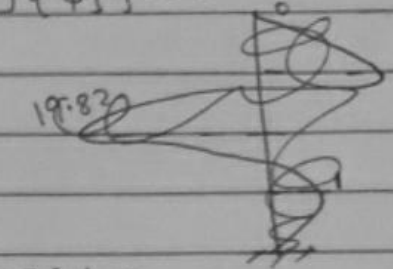
$$[K - \lambda_2 M]\{\phi_2\} = 0$$

$$\phi_2 = \begin{Bmatrix} 1 \\ \phi_{22} \\ \phi_{32} \\ \phi_{42} \end{Bmatrix} \Rightarrow \begin{Bmatrix} 1 \\ 11.459 \\ 8.074 \\ -9.942 \end{Bmatrix}$$



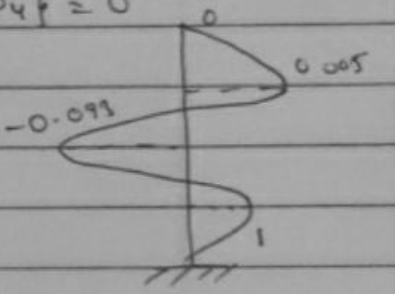
for $\lambda_3 = 38.5118$, $\phi_{13} = 1$, $[K - \lambda_3 M]\{\phi_3\} = 0$

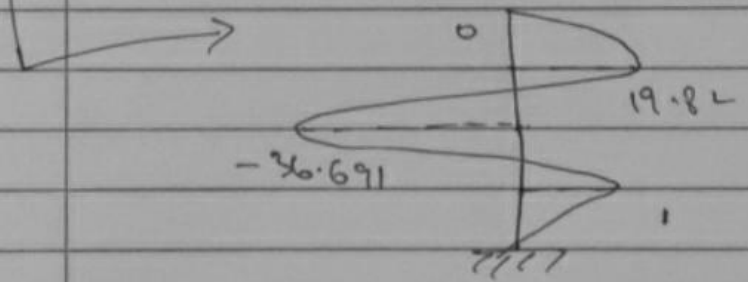
$$\phi_3 = \begin{Bmatrix} 1 \\ \phi_{23} \\ \phi_{33} \\ \phi_{43} \end{Bmatrix} = \begin{Bmatrix} 1 \\ -36.691 \\ 19.82 \\ 0 \end{Bmatrix}$$



for $\lambda_4 = 191.4017$, $\phi_{14} = 1$, $[K - \lambda_4 M]\{\phi_4\} = 0$

$$\phi_4 = \begin{Bmatrix} 1 \\ \phi_{24} \\ \phi_{34} \\ \phi_{44} \end{Bmatrix} = \begin{Bmatrix} 1 \\ -0.093 \\ 0.005 \\ 0 \end{Bmatrix}$$





Mode participation factor:-

$$P_i = \frac{[\phi_i]^T [M] [1]}{[\phi_i]^T [M] [\phi_i]}$$

$P_1 = 0.0427$

$P_2 = 0.0255$

$P_3 = -0.003$

$P_4 = 0.9066$